On the Performance of Distributed Space-Time Block Codes in Cooperative Relay Networks

Timo Unger, *Graduate Student Member, IEEE,* and Anja Klein, *Member, IEEE*

Abstract—In this paper, space-time block codes are applied in a distributed fashion in a scenario with multiple cooperating relay stations (RSs) having multiple antennas. By applying the Chernoff bound to the theoretical bit error rate (BER) in Rayleigh fading channels, it turns out that the BER performance has a higher sensitivity to spatial correlation in Multiple Input Multiple Output channels than to different receive powers at the receiver from different cooperating RSs. If the number of overall available antennas exceeds the number of required antennas for the considered STBC, based on the theoretical analysis a criterion for the selection of the antennas which should cooperate in order to achieve the best BER performance is given.

Index Terms—Distributed space-time block coding, cooperative relaying, antenna selection.

I. I

Cooperative relaying is a promising extension to the recent research field of relay networks where several relay stations (RSs) transmit jointly to the same receive node (RN) [1]. Assuming no channel state information (CSI) at the transmitting RSs, cooperative relaying can be used to achieve transmit diversity gain without large effort at the RSs [2]. The impact of the diversity gain was shown by Mietzner and Hoeher [3] for the case of two cooperating RSs each with one antenna by applying the two-antenna Alamouti code [4] in a distributed fashion. If there are more than two cooperating RSs or two RSs each with more than one antenna, arbitrary space-time block codes (STBCs) for the according overall number of antennas can be applied in a distributed fashion [5][6].

In the following, it is assumed that an STBC which is designed for a fixed number of antennas is given. This STBC is applied in a scenario with multiple RSs having multiple antennas each, where the number of overall available antennas exceeds the number of required antennas for the considered STBC. In this case, it is still an open problem which antennas should cooperate in order to achieve the best bit error rate (BER) performance. There are two variables which mainly influence the BER performance of the STBC, namely channel gain and spatial correlation. If different RSs cooperate, a diversity gain can only be expected if the different channel gains from the RSs to the RN are in the same order of magnitude, otherwise the contribution of the RSs with low channel gains is only marginal. If the transmit antennas of the same RS are within a range of a few wavelengths, it is very likely that the channel fading coefficients are spatially correlated. In this case, diversity cannot be fully exploited. However, for spatially separated cooperating RSs, correlated channel coefficients are less likely and a higher diversity gain can be expected. In this paper, it is shown that by applying the

Chernoff bound to the BER, the influence of both variables may be described theoretically for the case of orthogonal space-time block codes (OSTBCs) and quasi OSTBCs (Q-OSTBCs). With this theoretical framework, it is possible to define an antenna selection criterion in a scenario with multiple RSs having multiple antennas each.

The paper is organized as follows: The system model of a cooperative relay network applying distributed STBCs is given in Section II. In Section III, the theoretical BER performance of distributed (Q-)OSTBCs is given. The resulting antenna selection criterion is explained for the case of four antennas in Section IV. Section V gives some simulation results.

II. C R S M

In this section, the downlink system model for a cooperative relay network applying distributed STBCs is introduced. Throughout the whole paper, it is assumed that a direct transmission from the source node (SN) to the RN is not possible, i.e., the RN receives no symbols from the SN. It is assumed that $M_{RS}^{(k)}$ transmit antennas are used at the k-th RS, $k = 1, \ldots, K$, and M_{RN} receive antennas at the RN. Distributed STBCs are applied to groups of N symbols in T symbol intervals with the total number of used antennas $M_{RS} = \sum_{k=1}^{K} M_{RS}^{(k)}$ at K RSs. At each RS, the received vector from the SN is processed according to the considered relaying strategy (e.g., amplify-and-forward or decode-and-forward [1]) resulting in the symbol vector $\mathbf{x}_{RS}^{(k)}$ of elements $x_{RS}^{(k)}(n)$, $n = 0, 1, \ldots, N - 1$. It is assumed that the K RSs are synchronized and from each antenna at each RS a complex linear combination of the symbols $x_{RS}^{(k)}(n)$ and their conjugate complexes $x_{RS}^{(k)*}(n)$ is transmitted in T time intervals according to the applied distributed STBC. Let $\mathbf{X}_{RS}^{(k)}$ be the coded transmit matrix of dimension $T \times M_{RS}^{(k)}$ at the k-th RS consisting of complex linear combinations of symbols $x_{RS}^{(k)}(n)$ and $x_{RS}^{(k)*}(n)$. All coded transmit matrices $\mathbf{X}_{RS}^{(k)}$ are combined in the overall code matrix

$$\mathbf{X}_{RS} = \left[\mathbf{X}_{RS}^{(1)}, \ldots, \mathbf{X}_{RS}^{(K)}\right]. \tag{1}$$

The radio channel is assumed to be flat fading, i.e., all following considerations are applicable to multi-carrier systems. Hence, the k-th RS-to-RN channel may be described by matrix $\mathbf{H}^{(k)}$ with $M_{RS}^{(k)} \times M_{RN}$ complex channel coefficients. Let $\mathrm{E}\{\cdot\}$, $\mathrm{tr}\{\cdot\}$, $[\cdot]^T$, and $[\cdot]^H$ denote the expectation, the sum of the main diagonal elements of a matrix, the transpose, and the conjugate transpose, respectively. The average normalized channel gain of $\mathbf{H}^{(k)}$ is defined by $\mathrm{E}\left\{\mathrm{tr}\left\{\mathbf{H}^{(k)}\mathbf{H}^{(k)^H}\right\}\right\} = \alpha^{(k)}M_{RN}M_{RS}^{(k)}$, where $\alpha^{(k)}$ models different channel gains from the k-th RS to the

RN under the constraint

$$\sum_{k=1}^{K} \alpha^{(k)} M_{RS}^{(k)} = M_{RS}. \qquad (2)$$

With Eq. (2), the overall channel matrix

$$\mathbf{H} = \left[\mathbf{H}^{(1)^T}, \ldots, \mathbf{H}^{(K)^T} \right]^T \qquad (3)$$

has a normalized average channel gain of $\mathrm{E}\left\{ \mathrm{tr}\left\{ \mathbf{H}\mathbf{H}^H \right\} \right\} = M_{RN} M_{RS}$, i.e., the overall average channel gain stays constant while different RS-to-RN channels contribute different fractions of this channel gain. It is assumed that the overall transmit energy E_s per transmit symbol at the RSs is shared equally among the M_{RS} transmit antennas of all RSs. The overall receive matrix $\mathbf{Y}_{RN}$ at the RN of dimension $T \times M_{RN}$ is a superposition of all single receive matrices from K RSs after T time intervals and results in

$$\mathbf{Y}_{RN} = \sqrt{\frac{E_s}{M_{RS}}} \mathbf{X}_{RS} \mathbf{H} + \mathbf{N}_{RN}, \qquad (4)$$

where the elements of the noise matrix $\mathbf{N}_{RN}$ are zero mean complex Gaussian random variables with constant power density spectrum $N_0/2$.

III. T   BER P

In the following, the theoretical BER analysis for orthogonal diversity branches is applied [7]. Since Q-OSTBCs may be orthogonalized by the RN at the cost of some complexity [8], this analysis is valid for OSTBCs as well as for Q-OSTBCs. Note that there exists no OSTBC for more than two transmit antennas which achieves a transmission rate of one [9]. Since Q-OSTBCs are rate one codes they are preferable if complexity is no determining factor at the RN. Omitting power control in case of several cooperating RSs, different symbols of the distributed (Q-)OSTBC are received with different average powers for different average channel gains $\alpha^{(k)}$. Furthermore, it is assumed that channel coefficients assigned to different RSs are spatially uncorrelated. Channel coefficients assigned to the k-th RS are assumed to be correlated and the channel matrix $\mathbf{H}^{(k)}$ is modeled by

$$\mathrm{vec}\left\{ \mathbf{H}^{(k)} \right\} = \mathbf{S}^{(k)^{1/2}} \mathrm{vec}\left\{ \mathbf{H}_{\mathbf{W}}^{(k)} \right\} \qquad (5)$$

where $\mathrm{vec}\{\cdot\}$ stacks $\{\cdot\}$ into a column vector columnwise, $\mathbf{H}_{\mathbf{W}}^{(k)}$ is spatially white and $\mathbf{S}^{(k)}$ is a $M_{RS}^{(k)} \times M_{RS}^{(k)}$ covariance matrix defined as

$$\mathbf{S}^{(k)} = \begin{bmatrix} 1 & \rho^1 & \ldots & \rho^{M_{RS}^{(k)}-1} \\ \rho^1 & 1 & \ldots & \rho^{M_{RS}^{(k)}-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{M_{RS}^{(k)}-1} & \rho^{M_{RS}^{(k)}-2} & \ldots & 1 \end{bmatrix} \qquad (6)$$

with correlation coefficient $0 \leq \rho < 1$ and $\rho = 0$ defining uncorrelated channel coefficients [7].

Assuming maximum likelihood detection at the RN with $M_{RN} = 1$ receive antenna and applying the Chernoff bound

for channel coefficients with Rayleigh distributed amplitudes, the average BER may be upper bounded by

$$\mathrm{BER} \leq \bar{N}_e \left[\det\left\{ \mathbf{I}_{M_{RS}} + \frac{E_s d_{min}^2}{4 M_{RS} N_0} \mathbf{W} \right\} \right]^{-1} \qquad (7)$$

[7], where $\det\{\cdot\}$ denotes the determinant, $\mathbf{I}_{M_{RS}}$ is an $M_{RS} \times M_{RS}$ identity matrix, $\mathbf{W} = \mathrm{E}\left\{ \mathrm{vec}\{\mathbf{H}\} \mathrm{vec}\{\mathbf{H}\}^H \right\}$ is the covariance matrix of the overall channel and $\bar{N}_e$ and d_{min} are the number of nearest neighbors and minimum Euclidean distance in the constellation diagram, respectively. In the high SNR region, the BER bound of (7) may be approximated by

$$\mathrm{BER} \leq \bar{N}_e \left(\frac{E_s d_{min}^2}{4 M_{RS} N_0} \right)^{-M_{RS}} [\det \mathbf{W}]^{-1} \qquad (8)$$

where $\mathbf{W}$ is assumed to have full rank. For further investigations, $[\det \mathbf{W}]^{-1}$ is defined as the BER degradation factor

$$\beta_{deg} = [\det \mathbf{W}]^{-1} \qquad (9)$$

which is a function of ρ and $\alpha^{(k)}$, $k = 1, \ldots, K$. In the non-distributed case with spatially uncorrelated channel coefficients, the BER degradation factor results in $\beta_{deg} = 1$ which gives the well-known BER performance of (Q-)OSTBCs with diversity order M_{RS} [7].

IV. A   S   C

In the following, the BER degradation factor β_{deg} is used as an antenna selection criterion at the example of a four-antenna (Q-)OSTBC. For that purpose, a scenario is assumed where one can choose between two different arrangements of the overall four transmit antennas:

(i) $K = 1$, $M_{RS}^{(1)} = 4$

(ii) $K = 2$, $M_{RS}^{(1)} = M_{RS}^{(2)} = 2$

Assuming (8), a comparison of the BER degradation factors directly corresponds to a comparison of the BER performances. With the underlying channel model, the BER degradation factors of cases (i) and (ii) may be calculated as

$$\beta_{deg}^{(i)} = (1 - 6\rho + 15\rho^2 - 20\rho^3 + 15\rho^4 - 6\rho^5 + \rho^6)^{-1} \qquad (10)$$

$$\beta_{deg}^{(ii)} = \left(\left(4\alpha^{(1)^2} - 4\alpha^{(1)^3} + \alpha^{(1)^4} \right) \left(1 - 4\rho + 6\rho^2 - 4\rho^3 + \rho^4 \right) \right)^{-1} \qquad (11)$$

From (10) and (11), it can be seen that the BER degradation factor $\beta_{deg}^{(i)}$ depends only on the correlation factor ρ, while the BER degradation factor $\beta_{deg}^{(ii)}$ depends on ρ and on the normalized channel gain factor $\alpha^{(1)}$, with $\alpha^{(1)} + \alpha^{(2)} = 2$ from Eq. (2). In Fig. 1, $\beta_{deg}^{(i)}$ and $\beta_{deg}^{(ii)}$ are depicted depending on the correlation factor ρ where $\alpha^{(1)}$ is a parameter for case (ii). In both cases, β_{deg} increases monotonically with increasing ρ and for case (ii) it also increases with increasing $\alpha^{(1)}$.

For $\beta_{deg}^{(i)} < \beta_{deg}^{(ii)}$, case (i) has a better BER performance than case (ii) and vice versa. Fig. 2 gives the curve for $\beta_{deg}^{(i)} = \beta_{deg}^{(ii)}$ depending on ρ and $\alpha^{(1)}$. For all parameter pairs $\left\{ \rho, \alpha^{(1)} \right\}$ on the right of the dashed curve, case (ii) provides a better BER performance than case (i), and for all parameter pairs $\left\{ \rho, \alpha^{(1)} \right\}$ on the left of the dashed curve, case (i) has a better BER performance than case (ii). Obviously, there are more parameter pairs which correspond to a better BER performance
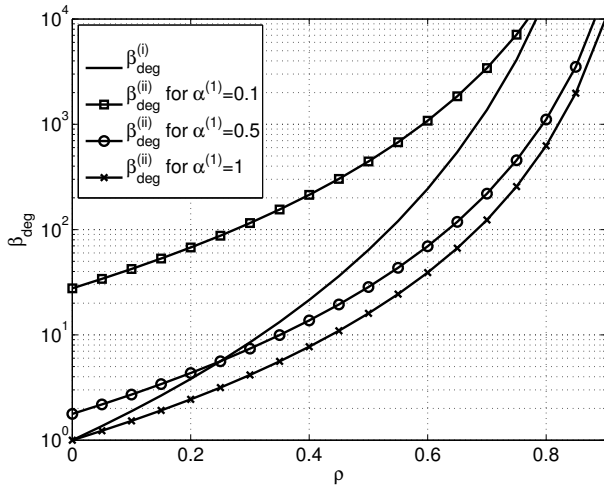
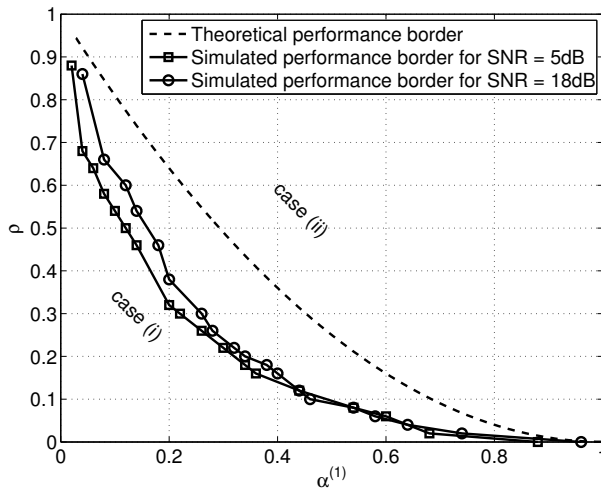Fig. 1. BER degradation factor depending on ρ with $\alpha^{(1)}$ as a parameter

Fig. 2. BER performance borders of cases (i) and (ii): $\beta_{deg}^{(i)} = \beta_{deg}^{(ii)}$

simplicity, error-free SN-to-RS links are assumed as the paper focuses on the cooperation between the RSs on the RS-to-RN links. The RN has perfect CSI and the receive signals from different cooperating RSs are synchronized in time.

The BER of cases (i) and (ii) has been simulated for low (5 dB) and high (18 dB) E_s/N_0 values with different parameter pairs $\{\rho, \alpha^{(1)}\}$. Fig. 2 gives the comparison of the performance borders for low and high E_s/N_0 and the theoretical performance border derived in Section IV for high E_s/N_0. There exists no essential difference between the low and high E_s/N_0 value, although the results for the high E_s/N_0 value are closer to the theory. The increasing difference between theory and simulation for increasing ρ comes from the fact that with increasing ρ covariance matrix $\mathbf{W}$ becomes closer to singular which degrades the approximation in (8). However as expected from Section IV, the BER performance of case (i) has a higher degradation due to correlated channel coefficients than case (ii) due to different channel gains.

VI. C

In this paper, the application of distributed STBCs in cooperative relay networks consisting of multiple RSs with multiple transmit antennas is considered. By applying the Chernoff bound to the BER, the BER degradation factor is derived which shows that the BER performance has a higher sensitivity to spatial correlation in Multiple Input Multiple Output channels than to different receive powers at the receiver from different cooperating RSs. The BER degradation factors can be used as a criterion for the selection of the best antenna arrangement in a scenario where the number of overall available antennas significantly exceeds the number of required antennas for the considered STBC.

for case (ii). Hence, the BER performance is less sensitive to different channel gains in case (ii) than to correlated channel coefficients in case (i). Note that similar considerations can be made for the comparison of other arrangements of the transmit antennas than case (i) and case (ii) by using the respective BER degradation factors.

In a scenario, where the number of overall available antennas exceeds the number of required antennas for the considered (Q-)OSTBC, the BER degradation factors may be used as an antenna selection criterion. For that purpose, all available RSs transmit training sequences to the RN. The RN which anyway requires CSI for decoding the underlying (Q-)OSTBC calculates the BER degradation factor according to (9) for all possible antenna arrangements and determines the antenna arrangement with the lowest BER degradation factor which shall be used for the cooperation. Using a feedback channel, the RN signals its decision to the chosen RSs.

V. S   R

The following simulation results are valid for the four-antenna Q-OSTBC with constellation rotation from [8]. For

R

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