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## **On the Performance of Relay Stations with Multiple Antennas in the Two-Way Relay Channel**

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# On the Performance of Relay Stations with Multiple Antennas in the Two-Way Relay Channel

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## Abstract

This paper considers the two-hop relaying case where two nodes  $S1$  and  $S2$  in a wireless network can communicate with each other via an intermediate relay station (RS). Assuming that a RS can only receive and transmit on orthogonal channel resources, the required resources in amplify-and-forward relaying are doubled compared to a direct communication between two nodes. Recently, two-way relaying has been introduced as a promising protocol in order to compensate for this drawback. In this paper, the two-way relaying protocol is extended to the case of nodes and RSs with multiple antennas where both nodes  $S1$  and  $S2$  have the same number of antennas and the number of antennas at the RS is at least twice as much. This multiple input multiple output (MIMO) two-way relaying protocol only requires channel state information (CSI) at the RS which reduces the CSI signalling overhead in the two-way relay network compared to previous work. Comparing with other relaying protocols, the performance gain in terms of network sum rate of the proposed MIMO two-way relaying protocol is verified. It is shown that the network sum rate may be further increased for the case of different channel qualities on the two links to the RS.

## 1 Introduction

Recently, relaying and in particular cooperative relaying gain much attention in the wireless communications research community [1]. This paper considers the amplify-and-forward (AF) two-hop relaying case where two nodes  $S1$  and  $S2$  can communicate with each other via an intermediate relay station (RS) assuming that a direct communication between the two nodes is not possible, e.g., due to shadowing or limited transmit power. The RS receives a signal from node  $S1$  on a first hop, amplifies this signal and retransmits it to node  $S2$  on a second hop. AF relaying is a low complexity relaying strategy. Nevertheless, the results of this work can be easily extended to decode-and-forward (DF) relaying [2] which requires further effort at the RS.

Assuming a one-way relaying protocol from node  $S1$  to node  $S2$ , where a RS can only receive and transmit on orthogonal channel resources, the required resources in AF relaying are doubled compared to a direct communication between  $S1$  and  $S2$ , i.e., although two-hop relaying aims at an increase in spectral efficiency there exists also a conceptual degradation of the spectral efficiency by the factor 2. There exist several protocols which aim at compensating for this conceptual drawback of two-hop relaying. Common to all these protocols is that it is not possible to improve the spectral efficiency of a single two-hop connection between source and destination, but the overall spectral efficiency of different two-hop connections. In [3], the authors consider several two-hop connections with multiple RSs. These RSs are divided into two groups. While the first group of RSs receives signals, the second group of RSs transmits signals on the same channel resource and vice versa. This protocol significantly increases the spectral efficiency of the network.

For this paper, the two-way relaying protocol introduced in [4] which is restricted to the case of nodes and RSs with single antennas is of particular interest. The principle of this protocol is based on the framework of network coding [5] in which data packets from different sources in a multi-node computer network are jointly encoded at intermediate network nodes, thus saving network resources. For two-way relaying, the nodes  $S1$  and  $S2$  transmit simultaneously on a first channel resource to a RS which receives a superposition of both signals. On the second channel resource, the RS retransmits this superposition. Due to the broadcast nature of the wireless channel, both nodes receive this superposition and may detect the desired signal from the other node by subtracting their own known signal. In [6], it is shown that the spectral efficiency of the two-way relaying protocol is significantly increased compared to one-way relaying protocols. However, in order to design an adequate receive filter both nodes of the two-way relay network require channel state information (CSI) about their own link to the RS as well as CSI about the link from the other node to the RS. Exchanging this CSI requires a high signalling effort. Compared to this signalling effort, it is relatively easy to obtain CSI at the RS, e.g., by estimating the channel at the RS in a time division duplex system. In [7], RSs with multiple antennas which apply CSI in order to separate multiple users' signals by linear beamforming filters in the one-way relay channel are considered.

In this paper, the idea of applying CSI at the RS is applied to the two-way relaying protocol of [4]. The two-way relaying protocol is extended to nodes and RSs with multiple antennas leading to a multiple input multiple output (MIMO) two-way relaying protocol. For the proposed MIMO two-way relaying protocol, both nodes  $S1$  and  $S2$  have the same number of antennas and the number of antennas at the RS is at least twice as much. Since the RS in the two-way relay channel is a transmitter as well as a receiver, spatial transmit and receive processing may be combined at the RS. The spatial filter matrix at the RS is termed transceive filter matrix in the following since it combines transmit and receive processing. Its design may be divided into three steps. Firstly, the receive filter matrix separates the signals from  $S1$  and  $S2$ . Secondly, the RS mapping matrix is introduced which ensures that each node is provided with its desired signal after retransmission from the RS. Thirdly, the transmit filter matrix is applied, which substitutes receive processing and in particular subtracting the own signal at  $S1$  and  $S2$ .

In this paper, the performance of the MIMO two-way relaying protocol is verified by means of the network sum rate. It is shown that the sum rate may be further increased for the case of different channel qualities on the two links to the RS.

The paper is organized as follows: In Section 2, the system model of MIMO two-way relaying is introduced. Section 3 gives a discussion about obtaining CSI at the RS. In Section 4, the sum rate of MIMO two-way relaying is given and the optimum sum rate is derived. The performance of MIMO two-way relaying is analyzed by means of simulations in Section 5. Section 6 concludes this work.

## 2 System Model

In the following, the communication between two nodes  $S1$  and  $S2$  is considered which cannot exchange information directly, e.g., due to shadowing conditions, but via an intermediate RS.  $S1$  and  $S2$  are equipped with  $M^{(1)}$  and  $M^{(2)}$  antennas, respectively. For the introduced MIMO two-way relaying protocol

$$M^{(1)} = M^{(2)} = M \quad (1)$$

is required while it is assumed that the RS is equipped with

$$M_{\text{RS}} \geq M^{(1)} + M^{(2)} = 2M \quad (2)$$

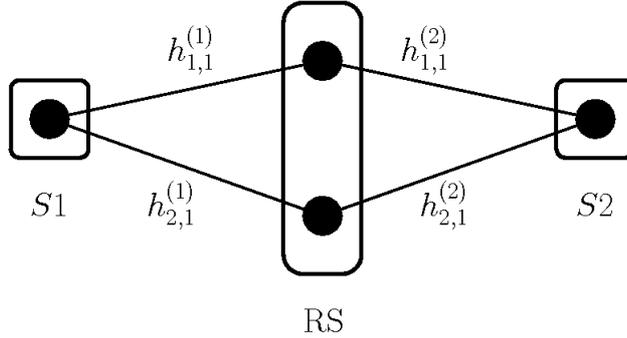


Figure 1: Relay network for  $M^{(1)} = 1$  antenna at  $S1$ ,  $M^{(2)} = 1$  antenna at  $S2$ , and  $M_{\text{RS}} = 2$  antennas at the RS

antennas.

Data vector  $\mathbf{x}^{(1)} = [x_1^{(1)}, \dots, x_M^{(1)}]^T$  of data symbols  $x_l^{(1)}$ ,  $l = 1, \dots, M$ , shall be transmitted from  $S1$  to  $S2$ , and data vector  $\mathbf{x}^{(2)} = [x_1^{(2)}, \dots, x_M^{(2)}]^T$  of data symbols  $x_l^{(2)}$ ,  $l = 1, \dots, M$ , shall be transmitted from  $S2$  to  $S1$ , where  $[\cdot]^T$  denotes the transpose. For simplicity, but without loss of generality, the wireless channel is assumed to be flat fading. Hence, the channel between  $Sk$ ,  $k = 1, 2$ , and the RS may be described by the channel matrix

$$\mathbf{H}^{(k)} = \begin{bmatrix} h_{1,1}^{(k)} & \dots & h_{1,M}^{(k)} \\ \vdots & \ddots & \vdots \\ h_{M_{\text{RS}},1}^{(k)} & \dots & h_{M_{\text{RS}},M}^{(k)} \end{bmatrix}, \quad (3)$$

where  $h_{m,n}^{(k)}$ ,  $m = 1, \dots, M_{\text{RS}}$  and  $n = 1, \dots, M$ , are complex fading coefficients. In Fig. 1, the described relay network is depicted for the case of  $M = 1$  and  $M_{\text{RS}} = 2$ . In MIMO two-way relaying, the data vectors  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$  are exchanged during two orthogonal time slots. During the first time slot,  $S1$  and  $S2$  transmit simultaneously to the RS. The overall data vector  $\mathbf{x} = [\mathbf{x}^{(1)T}, \mathbf{x}^{(2)T}]^T$  is defined with covariance matrix  $\mathbf{R}_{\mathbf{x}} = \mathbf{E} \{ \mathbf{x} \mathbf{x}^H \}$  where  $\mathbf{E} \{ \cdot \}$  and  $[\cdot]^H$  denote the expectation and the conjugate transpose, respectively. Since spatial filtering shall only be applied at the RS, only a scalar transmit filter  $\mathbf{Q} = q\mathbf{I}$  at  $S1$  and  $S2$  is required in order to fulfill the transmit energy constraint where  $\mathbf{I}$  is an identity matrix. Assuming that  $E_{\text{T}}$  is the maximum sum transmit energy of nodes  $S1$  and  $S2$ , the transmit energy constraint is given by

$$\mathbf{E} \{ \| q\mathbf{x} \|_2^2 \} \leq E_{\text{T}}, \quad (4)$$

where  $\| \cdot \|_2^2$  is the Euclidian norm of a vector. Defining the overall channel matrix  $\mathbf{H} = [\mathbf{H}^{(1)}, \mathbf{H}^{(2)}]$ , the receive vector  $\mathbf{y}_{\text{RS}}$  at the RS is given by

$$\mathbf{y}_{\text{RS}} = q\mathbf{H}\mathbf{x} + \mathbf{n}_{\text{RS}}, \quad (5)$$

where  $\mathbf{n}_{\text{RS}}$  is an additive white Gaussian noise vector.

In the following, a linear transceiver filter  $\mathbf{G}$  at the RS shall be designed. This linear transceiver filter  $\mathbf{G}$  is a combination of a linear receive filter  $\mathbf{G}_{\text{R}}$  and a linear transmit filter  $\mathbf{G}_{\text{T}}$  where both filters can be determined independently.

The receive vector  $\mathbf{y}_{\text{RS}}$  is multiplied with the linear receive filter  $\mathbf{G}_{\text{R}}$  resulting in the RS estimation vector

$$\hat{\mathbf{x}}_{\text{RS}} = \left[ \hat{\mathbf{x}}_{\text{RS}}^{(1)T}, \hat{\mathbf{x}}_{\text{RS}}^{(2)T} \right]^T = \mathbf{G}_{\text{R}} \mathbf{y}_{\text{RS}} = q \mathbf{G}_{\text{R}} \mathbf{H} \mathbf{x} + \mathbf{G}_{\text{R}} \mathbf{n}_{\text{RS}} \quad (6)$$

with the estimate  $\hat{\mathbf{x}}_{\text{RS}}^{(1)}$  for  $\mathbf{x}^{(1)}$  and the estimate  $\hat{\mathbf{x}}_{\text{RS}}^{(2)}$  for  $\mathbf{x}^{(2)}$ , respectively. In the second time slot,  $S1$  should receive an estimate of data vector  $\mathbf{x}^{(2)}$  and  $S2$  should receive an estimate of data vector  $\mathbf{x}^{(1)}$ . Therefore, before applying the transmit filter  $\mathbf{G}_{\text{T}}$ , the RS estimation vector  $\hat{\mathbf{x}}_{\text{RS}}$  is multiplied with the RS mapping matrix

$$\mathbf{G}_{\text{II}} = \begin{bmatrix} \emptyset_M & \mathbf{I}_M \\ \mathbf{I}_M & \emptyset_M \end{bmatrix} \quad (7)$$

where  $\emptyset_M$  is a null matrix with  $M$  rows and  $M$  columns. Note that the RS mapping matrix  $\mathbf{G}_{\text{II}}$  is an essential part of the introduced MIMO two-way relaying protocol since  $\mathbf{G}_{\text{II}}$  ensures that the RS transmits the estimate  $\hat{\mathbf{x}}_{\text{RS}}^{(2)}$  in the direction of  $S1$  and the estimate  $\hat{\mathbf{x}}_{\text{RS}}^{(1)}$  in the direction of  $S2$ . Finally, the transmit filter  $\mathbf{G}_{\text{T}}$  can be applied yielding the RS transmit vector

$$\mathbf{x}_{\text{RS}} = \mathbf{G} \mathbf{y}_{\text{RS}} = q \mathbf{G} \mathbf{H} \mathbf{x} + \mathbf{G} \mathbf{n}_{\text{RS}} \quad (8)$$

of the second time slot with the overall transceive filter

$$\mathbf{G} = \mathbf{G}_{\text{T}} \mathbf{G}_{\text{II}} \mathbf{G}_{\text{R}}. \quad (9)$$

In a linear system, the transceive filter  $\mathbf{G}$  of (9) is a multiplication of a receive filter  $\mathbf{G}_{\text{R}}$ , the RS mapping matrix  $\mathbf{G}_{\text{II}}$ , and a transmit filter  $\mathbf{G}_{\text{T}}$  where  $\mathbf{G}_{\text{R}}$  and  $\mathbf{G}_{\text{T}}$  can be designed independently, i.e., the joint design of  $\mathbf{G}_{\text{R}}$  and  $\mathbf{G}_{\text{T}}$  leads to the same overall transceive filter as the independent design of  $\mathbf{G}_{\text{R}}$  and  $\mathbf{G}_{\text{T}}$ . The RS transmit vector has to fulfill the transmit energy constraint at the RS

$$\mathbb{E} \{ \|\mathbf{x}_{\text{RS}}\|_2^2 \} \leq E_{\text{RS}}, \quad (10)$$

where  $E_{\text{RS}}$  is the maximum transmit energy at the RS. Note that the channel matrix from the RS to nodes  $S1$  and  $S2$  is the transpose  $\mathbf{H}^T$  of channel matrix  $\mathbf{H}$  assuming that the channel is constant during two consecutive time slots. In the following, the estimate for data vector  $\mathbf{x}^{(2)}$  at  $S1$  is termed  $\hat{\mathbf{x}}^{(1)}$  and the estimate for data vector  $\mathbf{x}^{(1)}$  at  $S2$  is termed  $\hat{\mathbf{x}}^{(2)}$ . For each receiving node  $S1$  and  $S2$ , a scalar receive filter is assumed which results in an overall receive filter matrix

$$\mathbf{P} = p \tilde{\mathbf{P}} = p \begin{bmatrix} \beta \mathbf{I}_M & \emptyset_M \\ \emptyset_M & (1 - \beta) \mathbf{I}_M \end{bmatrix} \quad (11)$$

with the filter coefficient  $p$  and the parameter  $\beta$  with  $0 \leq \beta \leq 1$  modelling the weight of the receive vectors at nodes  $S1$  and  $S2$ , respectively. For  $\beta = 0.5$ , the receive vectors at both nodes are equally weighted, for  $\beta = 1$  only  $S1$  receives its designated vector and for  $\beta = 0$  only  $S2$  receives its designated vector. The overall estimated data vector  $\hat{\mathbf{x}} = \left[ \hat{\mathbf{x}}^{(1)T}, \hat{\mathbf{x}}^{(2)T} \right]^T$  is given by

$$\hat{\mathbf{x}} = \mathbf{P} (q \mathbf{H}^T \mathbf{G} \mathbf{H} \mathbf{x} + \mathbf{H}^T \mathbf{G} \mathbf{n}_{\text{RS}} + \mathbf{n}_{\text{R}}) \quad (12)$$

where it is assumed that  $\mathbf{n}_{\text{R}} = \left[ \mathbf{n}_{\text{R}}^{(1)T}, \mathbf{n}_{\text{R}}^{(2)T} \right]^T$  is an additive white Gaussian noise vector. For purposes of further investigations, Eq. (12) may be rewritten as

$$\begin{aligned} \hat{\mathbf{x}} &= \mathbf{A}_{\text{TW}} \mathbf{x} + [\mathbf{D} \ \mathbf{P}] \mathbf{n} \\ &= \begin{bmatrix} \mathbf{A}_{\text{TW}}^{(1)} \\ \mathbf{A}_{\text{TW}}^{(2)} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{B}_{\text{TW}}^{(1)} \\ \mathbf{B}_{\text{TW}}^{(2)} \end{bmatrix} \mathbf{n} \end{aligned} \quad (13)$$

with

$$\mathbf{A}_{\text{TW}} = q\mathbf{P}\mathbf{H}^T\mathbf{G}\mathbf{H} \quad (14)$$

$$\mathbf{D} = \mathbf{P}\mathbf{H}^T\mathbf{G} \quad (15)$$

$$\mathbf{n} = [\mathbf{n}_{\text{RS}}^T, \mathbf{n}_{\text{R}}^T]^T \quad (16)$$

leading to the separated estimates at nodes  $S1$  and  $S2$

$$\hat{\mathbf{x}}^{(k)} = \mathbf{A}_{\text{TW}}^{(k)}\mathbf{x} + \mathbf{B}_{\text{TW}}^{(k)}\mathbf{n} \quad \text{for } k = 1, 2 \quad (17)$$

with  $\mathbf{A}_{\text{TW}}^{(k)}$  of dimension  $M \times 2M$  and  $\mathbf{B}_{\text{TW}}^{(k)}$  of dimension  $M \times 4M$ .

### 3 Obtaining CSI at the RS

Since the reduced effort in obtaining CSI is a main advantage of the proposed MIMO two-way relaying protocol this aspect is discussed in the following.

In the two-way relaying protocol of [4], both nodes  $S1$  and  $S2$  require CSI about their own link to the RS as well as CSI about the link of the other node to the RS. Exchanging this CSI requires a feedback channel. In [7], it is proposed that transmit and receive processing can be restricted to the RS for one-way relaying, i.e., CSI is only required at the RS. A similar approach may be applied in MIMO two-way relaying. However, there is one significant difference between the effort for obtaining CSI in [7] and the proposed MIMO two-way relaying protocol. In [7], CSI of the channels from nodes  $S1$  and  $S2$  to the RS can be achieved by pilot signalling. But CSI of the channels from the RS to the nodes can only be obtained by feedback from the nodes to the RS since up- and downlink are separated on orthogonal channel resources. In MIMO two-way relaying, CSI of the channels from the nodes to the RS as well as from the RS to nodes  $S1$  and  $S2$  is obtained by only one pilot signal since up- and downlink are processed simultaneously. Since feedback channels are costly MIMO two-way relaying protocol is very promising in terms of CSI signalling effort.

### 4 Sum Rate of MIMO Two-Way Relaying

In the following, the sum rate of a system is defined as the sum of the mutual information for all transmissions using the same channel resources. In [8], it is shown that for a MIMO system with

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{n} \quad (18)$$

the mutual information is given by

$$C_{\text{MIMO}} = \log_2 \left( \det \left[ \mathbf{I} + \frac{\mathbf{A}\mathbf{R}_{\mathbf{x}}\mathbf{A}^H}{\mathbf{B}\mathbf{R}_{\mathbf{n}}\mathbf{B}^H} \right] \right) \quad (19)$$

where  $\mathbf{A}$  and  $\mathbf{B}$  depend on the underlying MIMO system and  $\mathbf{R}_{\mathbf{x}}$  and  $\mathbf{R}_{\mathbf{n}}$  are the transmit vector and receive noise vector covariance matrices, respectively.

For AF relaying, where the RS can only receive and transmit on orthogonal channel resources, the pre-log factor  $1/2$  is introduced in order to indicate the increase in required channel resources leading to the mutual information

$$C_{\text{AF}} = \frac{1}{2} \log_2 \left( \det \left[ \mathbf{I} + \frac{\mathbf{A}_{\text{AF}}\mathbf{R}_{\mathbf{x}}\mathbf{A}_{\text{AF}}^H}{\mathbf{B}_{\text{AF}}\mathbf{R}_{\mathbf{n}}\mathbf{B}_{\text{AF}}^H} \right] \right). \quad (20)$$

For this conventional AF relaying protocol, the sum rate equals the mutual information since communication only takes place in one direction, either from  $S1$  to  $S2$  or from  $S2$  to  $S1$ .

For the introduced MIMO two-way relaying protocol, at each receive node the mutual information is given by

$$C_{\text{TW}}^{(k)} = \frac{1}{2} \log_2 \left( \det \left[ \mathbf{I}_M + \frac{\mathbf{A}_{\text{TW}}^{(k)} \mathbf{R}_x \mathbf{A}_{\text{TW}}^{(k)H}}{\mathbf{B}_{\text{TW}}^{(k)} \mathbf{R}_n \mathbf{B}_{\text{TW}}^{(k)H}} \right] \right) \text{ for } k = 1, 2. \quad (21)$$

with  $\mathbf{A}_{\text{TW}}^{(k)}$  and  $\mathbf{B}_{\text{TW}}^{(k)}$  from Eq. (17). Since communication takes place in two directions by using the same channel resources, the sum rate of the MIMO two-way relaying protocol results in

$$C_{\text{TW}} = C_{\text{TW}}^{(1)} + C_{\text{TW}}^{(2)}. \quad (22)$$

Both mutual informations  $C_{\text{TW}}^{(1)}$  and  $C_{\text{TW}}^{(2)}$  depend on the quality of both channels  $\mathbf{H}^{(1)}$  between  $S1$  and the RS and  $\mathbf{H}^{(2)}$  between  $S2$  and the RS, i.e., even if one channel is much better than the other channel, both directions of communication are degraded in the same way by the bad channel. Because of this fact, both mutual informations may be assumed in the same range for all previous considerations. However, assigning equal power to both received data vectors at the RS before retransmission may lead to a sub-optimum sum rate. Different power assignments to the two data vectors are modeled by the factor  $\beta$  of Eq. (11). If one data vector is received over a good channel while the other data vector is received over a bad channel, it may increase the sum rate if the data vector which is received over the good channel gets assigned a higher power before retransmission. Otherwise power is wasted for the data vector which is already badly received and which unavoidably leads to a low mutual information. Hence, the sum rate of Eq. (22) can be maximized by optimizing the parameter  $\beta$ . The underlying optimization problem is formulated as

$$\beta_{\text{opt}} = \arg \max_{\beta} \left\{ C_{\text{TW}}^{(1)} + C_{\text{TW}}^{(2)} \right\} \quad (23a)$$

$$\text{subject to: } 0 \leq \beta \leq 1. \quad (23b)$$

There exists no closed form solution to this optimization problem. However, it may be solved by computer optimization.

In the following, the previous optimization problem is simplified leading to a closed form solution for  $\beta$ . Let us assume, that the average signal-to-noise ratio (SNR) on the channel from  $S1$  to the RS is given by  $\rho^{(1)}$  and the SNR on the channel from  $S2$  to the RS is given by  $\rho^{(2)}$ . In this case, the overall SNR for AF relaying at receiving node  $S1$  results in

$$\rho_{\text{ov}}^{(1)} = \frac{\beta \rho^{(1)} \rho^{(2)}}{\rho^{(1)} + \beta \rho^{(2)} + 1} \quad (24)$$

while the overall SNR at receiving node  $S2$  results in

$$\rho_{\text{ov}}^{(2)} = \frac{(1 - \beta) \rho^{(1)} \rho^{(2)}}{(1 - \beta) \rho^{(1)} + \rho^{(2)} + 1}. \quad (25)$$

Approximating the mutual information of Eq. (22) by the single input single output (SISO) mutual information

$$\tilde{C}_{\text{TW}}^{(k)} = \frac{1}{2} \log_2 \left( 1 + \rho_{\text{ov}}^{(k)} \right) \text{ for } k = 1, 2 \quad (26)$$

the sum rate may be approximated in the high SNR region by

$$\tilde{C}_{\text{TW}} = \frac{1}{2} \log_2 \left( \rho_{\text{ov}}^{(1)} \right) + \frac{1}{2} \log_2 \left( \rho_{\text{ov}}^{(2)} \right). \quad (27)$$

For this approximation, the optimization problem of (23) is solved by

$$\beta_{\text{app}} = \frac{\rho^{(1)} + 1 - \sqrt{(\rho^{(1)} + 1)(\rho^{(2)} + 1)}}{\rho^{(1)} - \rho^{(2)}}. \quad (28)$$

Note that the sum rate which is calculated by Eq. (27) is different from the exact sum rate in Eq. (22). However, in order to determine the optimum parameter  $\beta$  this approximation provides reasonable results with low effort, which is confirmed by the following simulations.

## 5 Simulation Results

In this section, the sum rates of different relaying protocols are compared to each other by means of simulations. For all protocols, it is assumed that nodes  $S1$  and  $S2$  are each equipped with  $M = 1$  antenna and the RS is equipped with  $M_{\text{RS}} = 2$  antennas according to the requirement in Eq. (2) for the MIMO two-way relaying protocol. Firstly, the one-way relaying protocol is considered where  $S1$  and  $S2$  have to use the available channel resources successively for their transmission to  $S2$  and  $S1$ , respectively. Secondly, the two-way relaying protocol of [4] is considered with the modification of  $M_{\text{RS}} = 2$  antennas at the RS in order to guarantee a fair comparison. In this case, the filter matrix at the RS is a simple diagonal matrix with a constant amplification factor on the main diagonal which fulfills the transmit power constraint at the RS. Thirdly, the introduced MIMO two-way relaying protocol with a zero forcing (ZF) transceive filter at the RS is considered. The ZF transceive filter is derived according to the receive and transmit filter design given in [9]. Note that other linear transceive filters could be applied, e.g., a matched filter or a minimum mean square error filter, but they are omitted since they provide no other results concerning the network sum rate.

The average transmit energy  $E_{\text{RS}}$  at the RS is equal to the average transmit energy at nodes  $S1$  and  $S2$ , i.e.,  $E_{\text{RS}} = E_{\text{T}}$  while  $S1$  and  $S2$  share  $E_{\text{T}}$  equally. The channel coefficients are spatially white and their amplitude is Rayleigh distributed.

Fig. 2 gives the sum rate for the considered relaying protocols. The sum rate is depicted depending on the  $\text{SNR}^{(2)}$  of the channel from  $S2$  to the RS with  $\text{SNR}^{(1)}$  of the channel from  $S1$  to the RS as a parameter. The sum rate strongly depends on  $\text{SNR}^{(1)}$ . For low  $\text{SNR}^{(1)} = 10\text{dB}$ , the sum rate goes into saturation for increasing  $\text{SNR}^{(2)}$  while it monotonically increases for high  $\text{SNR}^{(1)} = 30\text{dB}$  in the considered region of  $\text{SNR}^{(2)}$ . Obviously, the sum rate of both two-way relaying protocols outperforms the mutual information of the one-way relaying protocol. Especially in the high SNR region, there exists an increase by the factor of 2. The proposed MIMO two-way relaying with the ZF transceive filter outperforms the sum rate of the two-way relaying protocol of [4]. Since no transmit or receive beamforming is applied in [4], no antenna array gain can be exploited. However, for the ZF transceive filter the antenna array gain can be exploited at the RS which leads to an increase of the sum rate. Note that the MIMO two-way relaying protocol may not achieve the mutual information of a system with two transmit and two receive antennas since the transmission and reception at  $S1$  and  $S2$  cannot be encoded and decoded jointly, respectively.

For MIMO two-way relaying in case of different channel qualities on the two links, the sum rate may be even increased if different powers are assigned to the two data vectors  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$  at the RS. Fig. 3 gives the sum rate for the ZF transceive filter for varying  $\beta$  with fixed  $\text{SNR}^{(1)} = 10\text{dB}$

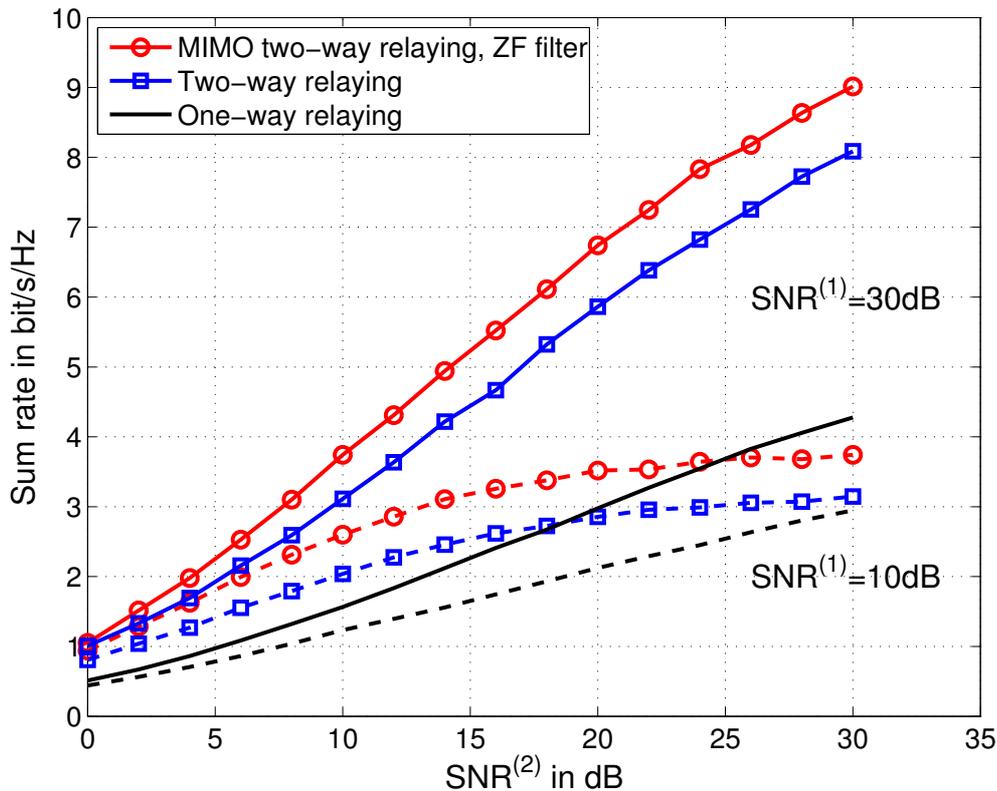


Figure 2: Sum rate of different relaying protocols depending on  $\text{SNR}^{(2)}$  with  $\text{SNR}^{(1)}$  as a parameter;  $\beta = 0.5$  for MIMO two-way relaying; dashed lines:  $\text{SNR}^{(1)} = 10\text{dB}$ , solid lines:  $\text{SNR}^{(1)} = 30\text{dB}$

and  $\text{SNR}^{(2)}$  as a parameter. For  $\beta = 0.5$ , both data vectors get assigned equal power at the RS which leads to the maximum sum rate if both links have the same average channel quality, i.e.,  $\text{SNR}^{(1)} = \text{SNR}^{(2)} = 10\text{dB}$ . If the channels have different SNR, the sum rate may be increased by introducing even more power to the data vector which has already a higher reliability after the first hop. For each channel realization, the optimum value  $\beta_{opt}$  may be calculated by computer optimization according to Eq. (23). Fig. 3 gives the mean value  $\bar{\beta}_{opt}$  for the considered average values of  $\text{SNR}^{(1)}$  and  $\text{SNR}^{(2)}$  from the simulations. Furthermore, the standard deviation of the optimum value  $\beta_{opt}$  is depicted in the figure. Obviously, there is a strong variation of  $\beta_{opt}$  which strongly depends on the current channel state. Hence,  $\beta_{opt}$  has to be adapted dynamically.

In Fig. 4, depending on  $\text{SNR}^{(2)}$  for fixed  $\text{SNR}^{(1)} = 5\text{dB}$  the average sum rate from the exact computer optimization is compared to the value achieved by the approximation of Eq.(28), and the simplest approach with fixed  $\beta = 0.5$ . For similar channel qualities on both links  $\text{SNR}^{(1)} \approx \text{SNR}^{(2)}$ , all approaches provide the same sum rate. However, for increasing difference of the channel qualities on both links the sum rates diverge. The exact solution provides the largest sum rate and the approximation comes close to this sum rate while  $\beta = 0.5$  clearly provides the worst performance.

## 6 Conclusion

In this paper, the MIMO two-way relaying protocol has been introduced which compensates for the degradation of the spectral efficiency by the factor 2 in conventional AF two-hop relay networks. Compared to previous work on the two-way relay channel, MIMO two-way relaying only requires CSI at the RS which significantly reduces the CSI signalling overhead. Furthermore, the introduced

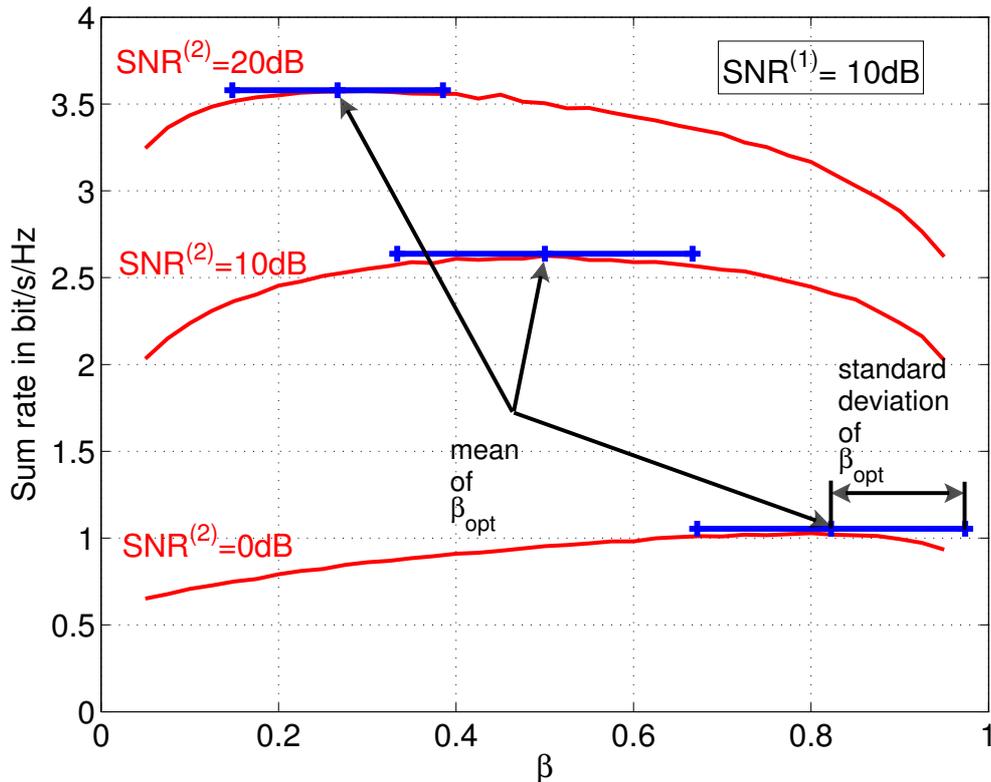


Figure 3: Sum rate depending on  $\beta$  with fixed  $\text{SNR}^{(1)} = 10\text{dB}$  and  $\text{SNR}^{(2)}$  as a parameter

protocol outperforms previous protocols in terms of network sum rate. It is shown that the sum rate may be even increased in case of different channel qualities on the two different links if the power is not uniformly distributed among the communicating nodes. The derived approximation of the power distribution provides reasonable results for practical applications.

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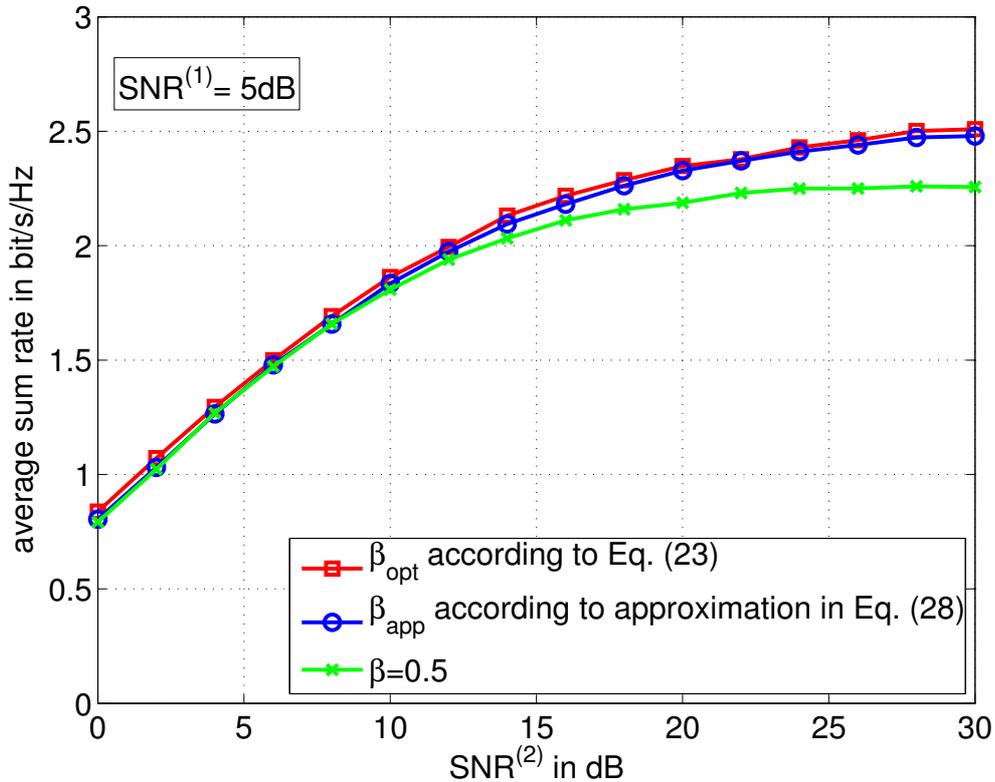


Figure 4: Average sum capacity depending on  $\text{SNR}^{(2)}$  for fixed  $\text{SNR}^{(1)} = 5\text{dB}$ : comparison of results from Eq. (23), Eq. (28), and  $\beta = 0.5$

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