Dynamic Subcarrier, Bit and Power Allocation in OFDMA-Based Relay Networks

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Abstract— **In this paper, the downlink of a cell is considered in an OFDMA-based relay network. In the cell, a base station (BS) and fixed relay stations (RSs) are deployed. Multiple subscriber stations (SS) are located in the cell. A SS is served either by a direct or a two hop connection. In a direct connection, a link between BS and SS exists. In a two hop connection, a link exists between BS and RS and between RS and SS. Using instantaneous channel state information, a method for resource allocation is presented which dynamically allocates subcarriers, bits and power to the existing links. An optimization problem is formulated aiming at the minimization of the power required for the transmissions of the BS and the RSs. The optimization problem is subject to a requested data rate on each link. The constraint is considered that a RS cannot transmit and receive simultaneously in order to avoid strong intercarrier interference. The evaluation of the presented resource allocation method shows that the required power is near to the theoretical minimum value and is lower than the power required by static resource allocation methods in a defined scenario of a relay network.**

I. INTRODUCTION

The use of relay stations (RSs) and the use of orthogonal frequency division multiple access (OFDMA) are expected in future wireless cellular networks. The use of RSs enables a coverage extension, capacity enhancement or cost reduction compared to nowadays wireless cellular networks [1], [2]. Knowing the channel state information at the transmitter of an OFDMA system, dynamic subcarrier, bit and power allocation enables a power efficient transmission [3]. In a multiuser scenario without considering RSs, the optimization problem of minimizing the transmit power of a BS in the downlink subject to the constraint that each subscriber station (SS) achieves a requested data rate is defined in [3], [4].

In this paper, the downlink of a cell is considered in an OFDMA-based relay network. In the cell, a base station (BS) and a fixed number of RSs are deployed. Multiple subscriber stations (SSs) are located in the cell. A SS is served either by a direct or by a two hop connection from the BS. A direct connection consists of a link between the BS and a SS. A two hop connection is formed by a link between the BS and a RS and between the RS and a SS. In this paper, the resource allocation problem is formulated how to allocate subcarriers, bits and power such that the transmit power of the BS and of the RSs are minimzed. The resource allocation problem is subject to a requested data rate of each link. The resource allocation problem is additionally subject to the constraint that a RS cannot transmit on a subcarrier and simultaneously receive on another subcarrier in order to avoid strong intercarrier interference. Compared to a scenario without considering RSs, the additional question arises how the resources can be shared between the BS and the RSs.

The complexity of the resource allocation problem increases exponentially with the number of subcarriers, the possible number of bits per subcarrier and the number of links in a cell. Since an optimal solution of the resource allocation problem is too complex, a method is presented in which the solution of the resource allocation problem is split into two steps. Firstly, the transmissions of the BS and of the RSs are separated in time domain. A novel algorithm dividing the resources in time domain is presented. In the second step, subcarriers, bits and power are allcoated for the BS and for the RSs. The division in time domain enables an independent subcarrier, bit and power allocation for the BS and for the RSs.

The paper is structured as follows: In Section II, the formulation of the optimization problem describing the resource allocation problem is presented. The proposed method solving the introduced resource allocation problem is presented in Section III. Section IV gives a performance evaluation of the presented method. In Section V, conclusions are drawn.

II. FORMULATION OF THE RESOURCE ALLOCATION PROBLEM

The cell of the considered relay network is modelled as follows. A BS and a number of RSs and of SSs are in the cell. A direct or a two hop connection is established from the BS to each SS, e.g., by applying the assignment algorithm given in [2]. An index $k = 1, 2, ..., K$ is used to represent all the K links of the cell. The links from the BS to a RS or to a SS are grouped in the set $\mathbf{K}_1$. The links between all RSs and the SSs are in the set $\mathbf{K}_2$. An OFDMA system is considered with N subcarriers and a subcarrier index $n = 1, 2, ..., N$ is defined. The subcarriers, bits and power are allocated to the links by the BS. The BS has perfect knowledge about the noise power and the instantaneous channel gain of all subcarriers of

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the links. On each subcarrier n of a link k the same noise power σ_k^2 is assumed. The BS knows about a requested data rate R_k for each link.

A frame based transmission is applied. A frame consists of S slots where a slot has the duration of an OFDM symbol. A frame based time division multiplexing is applied to separate reception and transmission of a RS. A frame is divided into two subframes. The first subframe consists of S_1 slots with index 1 to S_1. In the first subframe, the BS transmits to the RSs and to those SSs which use direct connections. The BS uses all N subcarriers. During the second subframe of length

$$S_2 = S - S_1, \tag{1}$$

the RSs transmit from slot S_1+1 until slot S. The variable m denotes the subframe index, i.e., $m \in \{1;2\}$. The coherence time of the channel is assumed to be larger than the duration of a frame.

A subcarrier, bit and power allocation method is applied to a complete frame. Each subcarrier can be allocated only to one link in a subframe. A subcarrier may be loaded with no information or with a modulation symbol carrying a number of bits depending on the chosen constellation size of the modulation scheme. It is assumed that QPSK, 16-QAM or 64-QAM can be used. The number of bits loaded on a subcarrier during a slot is c. Note that c bits can represent coded as well as uncoded information. The possible values of c are given as the elements of a set called $\mathbf{D} = \{0,2,4,6\}$. The bits c must be transmitted according to a maximally tolerated bit error probability on a subcarrier. The function $f_k(c)$ describes the required receive power on a subcarrier for the reception of c bits per symbol according to a noise power and a tolerated bit error probability on link k. Derived from the formula of the bit error probability P_e of QPSK and QAM depending on the signal to noise ratio [5], the function $f_k(c)$ is given by

$$f_k(c) = \frac{(2^c-1)\sigma_k^2}{3}\left(Q^{-1}\left(\frac{P_e}{4}\right)\right)^2 \tag{2}$$

where $Q^{-1}(\bullet)$ denotes the inverse complementary error function. The function $f_k(c)$ is monotonically increasing with $f_k(0) = 0$. The transmit power required on a subcarrier is given by

$$P_{k,n} = \frac{f_k(c)}{\alpha_{k,n}^2} \tag{3}$$

where $\alpha_{k,n}^2$ represents the instantaneous channel gain of link k and subcarrier n.

An indicator variable $u_{k,n,c}^{(m)}$ is introduced which describes if subcarrier n is allocated to link k and if subcarrier n is loaded with c bits. The indicator variable $u_{k,n,c}^{(m)}$is defined as

$$u_{k,n,c}^{(m)} = \begin{cases} 1 & \text{if } c \text{ bits are mapped} \\ & \text{on subcarrier } n \text{ allocated to} \\ & \text{link } k \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The transmit power used in subframe m is given by

$$P_m = \sum_{k\in\mathbf{K}_m}\sum_{n=1}^{N}\sum_{c\in\mathbf{D}} \frac{f_k(c)}{\alpha_{k,n}^2} u_{k,n,c}^{(m)}. \tag{5}$$

where P_1 is the transmit power of the BS and P_2 is the transmit power of the RSs. Since bits of a link are only transmitted in one of the two subframes, an instantaneous data rate

$$R_k^{(m)} = \frac{S}{S_m} R_k \tag{6}$$

of a link is defined where $R_k^{(m)}$ gives the data rate which is achieved in the subframe m in which the link k is served. The instantaneous data rate of a link k during the frame in which this link shall not be served is zero. Minimizing the transmit power of the BS and the power of the RSs is written as

$$P_{min} = \min_{u_{k,n,c}^{(m)}} \max\{P_1, P_2\} \tag{7a}$$

subject to:

$$\sum_{n=1}^{N}\sum_{c\in\mathbf{D}} c u_{k,n,c}^{(m)} \ \geq R_k^{(m)};\ \forall k \in \mathbf{K}_m;\ \ m \in \{1;2\} \tag{7b}$$

$$\sum_{k\in\mathbf{K}_m}\sum_{c\in\mathbf{D}} u_{k,n,c}^{(m)} = 1\ ;\ \forall n;\ \ m \in \{1;2\}. \tag{7c}$$

The optimization problem (7) is chosen as a min-max optimization to enable that the power of the BS and the sum of the power of all RSs is minimized without favoring one of them. The constraint (7b) ensures that each link achieves its requested data rate. The constraint (7c) represents that a subcarrier is allocated to only one link in a subframe. Problem (7) can be solved by an exhaustive search algorithm. Since the complexity of such an exhaustive search algorithm increases exponentially with the number of variables, such a solution is not applicable in practice. An applicable resource allocation method is proposed in the next section.

III. DYNAMIC RESOURCE ALLOCATION METHOD

To enable an applicable solution of problem (7), the problem is split into the following subproblems: Firstly, the subframe sizes S_1 and S_2 are determined. The solution of this subproblem is given by a novel algorithm which adapts the subframe size to the channel state and the requested data rate on the links. Secondly, a dynamic subcarrier, bit and power allocation is applied. For this, an algorithm defined for a network without RSs is adapted to a relay network.

A. Subframe Size

The subframe sizes S_1 and S_2 must be determined without knowing the allocation of the subcarriers to the links, the bits transmitted on a subcarrier or the power used on a subcarrier. Thus, the required power of the BS and of the RSs is estimated instead of precisely determinted. For all possible sizes of the subframes, the maximum of the transmit power of the BS and of the RSs is estimated. The subframe size is chosen which leads to the smallest maximum.

The estimate of the required transmit power is based on a representative number of bits per subcarrier and on a representative channel gain of a link. The representative channel gain of a link is calculated by

$$\overline{\alpha}_k^2 = \frac{1}{N}\sum_{n=1}^{N} \alpha_{k,n}^2. \tag{8}$$

In (8), the arithmetic mean value is chosen in which each channel has an equal weight, because no knowledge of the subcarrier allocation is given. The representative number of bits per subcarrier is $\overline{c}_m$ with $\overline{c}_m \in \mathbb{R}^+$. In order to find S_1, each subcarrier is assumed to carry $\overline{c}_m$ bits. The number B_m of bits which must be transmitted in a subframe is equal to the number of slots in a subframe times the sum of the requested data rates in a subframe given by

$$B_m = S_m \sum_{k\in\mathbf{K}_m} R_k^{(m)}. \tag{9}$$

For all possible sizes of the first subframe and the corresponding sizes of the second subframe, the representative number of bits is calculated by

$$\overline{c}_m = \frac{B_m}{NS_m}. \tag{10}$$

The smallest possible size of the first subframe is given if the number B_1 of bits is offered by loading all subcarriers with the highest number of bits defined in $\mathbf{D}$. The size of the first subframe is lower bounded by

$$S_1 \geq \lceil \frac{B_1}{N\max\{\mathbf{D}\}} \rceil \tag{11}$$

where $\lceil \bullet \rceil$ denotes the rounding to the next greater integer value and $\max\{\mathbf{D}\}$ is the greatest element of the set $\mathbf{D}$. The size S_2 is given by (1). The size S_1 is upper bounded by assuming that the highest number of bits is loaded on all subcarriers in the second subframe, i.e.,

$$S_1 \leq S - \lceil \frac{B_2}{N\max\{\mathbf{D}\}} \rceil. \tag{12}$$

The estimate of the transmit power of the BS and the RSs, respectively, is given by

$$\overline{P}_m = \sum_{k\in\mathbf{K}_m} \frac{R_k}{\sum\limits_{l\in\mathbf{K}_m} R_l} \frac{N f_k(\overline{c}_m)}{\overline{\alpha}_k^2}. \tag{13}$$

An arithmetic mean value is chosen in which each power is weighted by its normalized requested data rate because it is assumed that the higher the data rate of a link the more subcarriers are allocated to that link. Out of all possible combinations of S_1 and S_2, the combination $(S_1^*; S_2^*)$ is chosen which fulfills

$$(S_1^*; S_2^*) = \arg\min_{S_1;S_2} \max\{\overline{P_1}; \overline{P_2}\}. \tag{14}$$

B. Subcarrier, Bit and Power Allocation

Using (14), the power of the BS and of the RSs can be minimized separately by finding an optimal subcarrier, bit and power allocation per subframe, i.e. the optimization problem (7) is split into two optimization problems. The problem how to allocate subcarriers, bits and power such that the transmit power is minimized subject to a requested data rate on each link is formulated and solved for a scenario without RSs in [4]. In the following, a proposal is made how to solve this problem in a scenario of a relay network.

Keeping the constraints (7b) and (7c), the power used in a subframe is minimized given by

$$P_{min}^{(m)} = \min_{u_{k,n,c}^{(m)}} \sum_{k\in\mathbf{K}_m} \sum_{n=1}^{N} \sum_{c\in\mathbf{D}} \frac{f_k(c)}{\alpha_{k,n}^2} u_{k,n,c}^{(m)} \tag{15a}$$

subject to:

$$\sum_{n=1}^{N} \sum_{c\in\mathbf{D}} c u_{k,n,c}^{(m)} \geq R_k^{(m)};\ \forall k \in \mathbf{K}_m \tag{15b}$$

$$\sum_{k\in\mathbf{K}_m} \sum_{c\in\mathbf{D}} u_{k,n,c}^{(m)} = 1\ ;\ \forall n. \tag{15c}$$

In contrast to problem (7), the subframe size is fixed. Subcarriers, bits and power are allocated independently in both subframes. During the first subframe, the BS is the only transmitter and multiple receivers which are RSs or SSs exist. Thus, problem (15) is equivalent for $m = 1$ to the one defined and solved in [4]. During the second subframe, the RSs transmit simultaneously separated in frequency domain. Although multiple RSs are allowed to transmit simultaneously, the cost function is the same as in the first subframe since the power required on all subcarriers is minimized. Thus, problem (15) can be also solved for $m = 2$ by the algorithm defined in [4].

Problem (15) is solved according to [4] as follows. First, an algorithm for the allocation of subcarriers and then an algorithm for bit and power loading is applied. The subcarrier allocation is based on the assumption that the same number $\overline{c}_k$ of bits is loaded on the subcarriers allocated to a link k. The number $\overline{c}_k$ of bits is assumed to be a real value, i.e., $\overline{c}_m \in \mathbb{R}^+$. Under these assumptions, the problem of (15) can be rewritten as

$$P_{SC,min}^{(m)} = \min_{u_{k,n}^{(m)}} \sum_{k\in\mathbf{K}_m} \sum_{n=1}^{N} \frac{f_k(\overline{c}_k)}{\alpha_{k,n}^2} u_{k,n}^{(m)} \tag{16a}$$

subject to:

$$\sum_{n=1}^{N} u_{k,n}^{(m)} = \frac{R_k^{(m)}}{\overline{c}_k};\ \forall k \in \mathbf{K}_m \tag{16b}$$

$$\sum_{k\in\mathbf{K}_m} u_{k,n}^{(m)} = 1\ ;\ \forall n. \tag{16c}$$

which is given in [4]. Note that $u_{k,n}^{(m)}$ does not depend any longer on c as in the problem of (15) and that the summation over the possible values of c is dropped because a constant number $\overline{c}_k$ of bits is assumed for each

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TABLE I
PARAMETERS

Parameter	Value
Side length of hexagons	400 m
Bandwidth	5 MHz
Power of white Gaussian noise	-99 dBm
Number of subcarriers N	128
Path loss from BS to RS in dB where d is the distance in meters	$38.5 + 23.5\log_{10}(d)$
Path loss from BS to SS and from RS to SS in dB	$38.4 + 35\log_{10}(d)$
Standard deviation log-normal fading between BS and RS	3.4 dB
Standard deviation log-normal fading between BS and SS and between RS and SS	8 dB
Antenna gain between BS and RS	17 dBi
Requested sum rate in cell	192 bits/slot
Maximally tolerated bit error probability per connection $P_{e,c}$	10^{-2}
Frame duration	40 slots

SS. The problem of (16) gives the subcarrier allocation problem which must be solved for each subframe. The subcarrier allocation problem is an integer program. How to find $\overline{c}_k$ and a suboptimal algorithm of solving (16) is given in [4].

Bit and power loading is done for each link separately using the set of allocated subcarriers. Bit and power loading is made by a greedy algorithm described in [3]. Roughly speaking, the number of bits per subcarrier and per slot is initially set to zero. The number of bits and the allocated power is incremented on the subcarrier which needs the lowest amount of power to be loaded with further bits.

IV. PERFORMANCE EVALUATION

A. Scenario

The proposed resource allocation method is evaluated in a cell where a BS and two RSs are deployed. The cell consist of three hexagons of equal size. In the center of one hexagon, a BS is placed. The RSs are placed in the centers of the neighboring hexagons. The SSs are uniformly distributed in the scenario and assigned to the BS or RSs according to a best server algorithm explained in detail in [2]. The parameters chosen for the evaluation are given in Table I. The parameters do not conform to a standard, but specify a general OFDMA system capturing basic features of a system according to IEEE 802.16, LTE or WINNER. The channel between BS and RS is modeled by a line of sight scenario called B5a and defined in [7]. The channels between BS and SS and RS and SS are modeled by a non-line of sight scenario called C2 and defined in [7]. An antenna gain between BS and RSs is assumed to achieve an improved channel condition on the first hop of a two-hop connection. An omnidirectional antenna is used for the transmissions between the BS and a SS and between a RS and a SS. The sum of the requested data rates of all SSs called sum rate is always constant to make the results comparable when the number of SSs is changed. To consider SSs with different data rate requests, the following traffic model is applied. The requested data rate of a SS is given by a random portion of the sum rate which can be between 0% and 100%. The transmission between two nodes is only reliable according to a given bit error probability given in (2). A bit error probability $P_{e,c}$ maximally tolerated on a connection is given. For a two hop connection, the maximally tolerated bit error probability is well approximated [6] by

$$P_{e,c} = 1 - (1 - P_e)^2. \tag{17}$$

B. Evaluation Results

To evaluate the proposed resource allocation method of Section III, this method is compared to the following resource allocation methods:

- near optimum,
- fixed subframe size,
- static.

The resource allocation method called near optimum solves the problem (7) by testing all possible combinations of the subframe sizes S_1 and S_2. For all combinations, the problem (15) is solved by a suboptimal algorithm proposed in [4] and is not optimally solved in order to reduce complexity. The near optimum method is a close approximation to the optimal solution of problem (7) because the difference between the optimal solution of problem (15) and its suboptimal solution is less than 0.25 dB in the analysis of [4]. This method is chosen as a lower bound of the performance of the resource allocation method proposed in Section III.

Using the method called fixed subframe size, the frame is divided in two subframes of equal size. The subcarriers, bits and power are allocated according to the algorithm given in Section III-B. A comparison between this method and the resource allocation method proposed in Section III shows the gain of finding a suitable subframe size.

In the method called static, the frame is also divided in two subframes of equal size. The number of subcarriers allocated to a link is proportional to the requested data rate on that link, i.e., if a link requests 40% of the data rate in a subframe, the link is allocated 40% of the subcarriers. The subcarriers are chosen without taking into account channel state information. The same number of bits is allocated to each subcarrier which must be two, four or six bits. The power is allocated to the subcarriers such that the maximally tolerated bit error probability and the requested data rate are achieved. A comparison of this method and the method proposed in Section III shows the gain achieved by a suitable subframe size, subcarrier allocation and bit loading.

In Fig. 1, the cumulative distribution functions (CDFs) of the maximum out of the transmit powers of the BS and the sum of the powers of the RSs are given for the introduced

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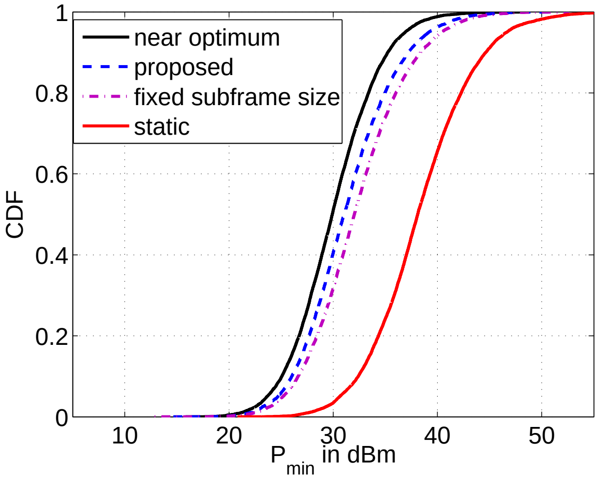

Fig. 1. CDF of the maximum out of the transmit powers of the BS and of all RSs for eight SSs.

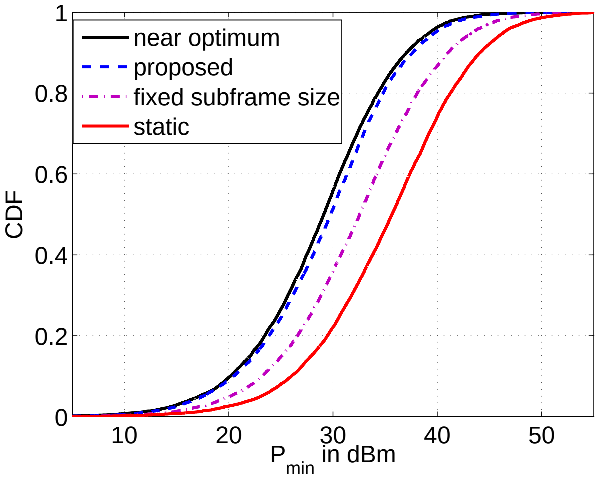

Fig. 2. CDF of the maximum out of the transmit powers of the BS and of all RSs for two SSs.

methods. The connections of eight SSs are considered within a frame. The median of the proposed method is 1.2 dB larger than the median of the near optimum method, 0.9 dB smaller than the median of the fixed size method and 7 dB smaller than the median of the static method.

In Fig. 2, the CDFs of the introduced methods are shown considering the connections of two SSs in a frame. The proposed resource allocation method is close to the near optimum method. The difference of the medians is approximately 0.6 dB. The methods called fixed subframe size and static show an inferior performance. Since the number of SSs is small, it happens quite often that no SS is assigned to any RS so that the resources of the second subframe are unused.

A comparison of Fig. 1 and Fig. 2 shows that the slope increases for all CDFs with an increasing number of SSs resulting in an decreased variance of the maxima of the transmit powers. For a high number of SSs, cases which require an extremely low or large power are infrequent because neither all SSs are close to the BS or RS nor all SSs are at the cell border. Since such extreme cases become infrequent for high number of SSs, the gain of the proposed resource allocation method compared to the fixed subframe size method decreases. Further evaluations of the introduced methods have shown that differences in the performance of the methods are almost independent of the maximally tolerated bit error probability.

V. CONCLUSION

In this paper, a formulation of the problem how to allocate subcarriers, bits and power dynamically in a OFDMA-based relay network is given such that the power of the BS and the RSs in a cell is minimized. An applicable resource allocation method is presented considering a requested data rate and that a RS cannot simultaneously transmit and receive. A frame based time division multiplexing is applied to separate reception and transmission of a RS. Based on an applicable algorithm known from a scenario without relay stations, a dynamic subcarrier, bit and power allocation is presented for a relay network. The presented resource allocation method shows a nearly optimum performance in a scenario representing basic features of a system according to IEEE 802.16, LTE or WINNER.

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