Multicast transmission performance improvement through adaptive antenna arrays

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***Abstract*— This paper evaluates and compares different adaptive antenna techniques applied in the context of multicast services. Traditional unicast algorithms, such as the matched filter, zero-forcing, Tomlinson-Harashima precoding, and switched fixed beams are formulated for the multicast case. Algorithms which try to improve the performance of the worst user within the multicast group are also analyzed and a new algorithm called USMF is proposed. It is seen that the techniques which treat all the users alike do not perform well when compared to those which focus on the worst user. The spatial correlation of the channel is shown to have a significant impact on the results. The presence of line-of-sight is verified to be beneficial to the performance of the proposed algorithm and the switched fixed beams. Other aspects, such as the transmitter/receiver design and the impact of the multicast group size, are investigated as well.**

I. Introduction

In the context of next-generation wireless systems, it is expected that services targeted at mass content distribution become widely popular, especially considering the 3GPP standardization activities for their implementation within GSM/EDGE and WCDMA networks [1]. Examples of such services are audio/video streaming, mobile TV, messaging, news clips, localized services, download, among others. Their common characteristic is that the same information has to be transmitted to a group (multicast) or to all users (broadcast) within a certain coverage area.

The implementation of such services in mobile cellular networks raises some issues concerning their feasibility and efficiency within the different layers of the transmission chain [1]. From a higher layer perspective, there are for example questions regarding how best to distribute the data generated by the content providers among the different elements of the core network. However, the scope of this article lies on the radio link between the mobile and base stations, for which one of the main issues concerns the optimization of the use of the radio resources for multicast services.

Radio resource management (RRM) techniques, such as channel allocation, power control, link adaptation, adaptive antennas, among others, need to be adjusted in order to cope with the introduction of multicast services. Multicasting assumes that there exists a group of users expecting the same information, therefore the more users that can be allocated to the same radio resource the more spectrally efficient the system will be. Such sharing of resources leads, however, to a higher complexity of the RRM algorithms, because instead of dealing with a single user per resource, all users of the multicast group have to be considered. Each RRM technique needs, therefore, to determine which resource (which channel, which transmit power, which modulation/coding scheme, which weight vector) is more adequate to the group as a whole.

Among these RRM techniques, this paper focuses on the application of adaptive antenna arrays to multicast services. Different transmit processing algorithms are formulated and evaluated in a multicast context, taking into account those originally proposed for unicast services, as well as some specifically designed for multicast groups. Additionally, lower-complexity adaptive techniques, such as switched fixed beams, are also considered within a multicast context.

This paper is organized as follows. In section II, the different adaptive antenna techniques are formulated, and a simple sub-optimum algorithm for multicast (USMF - User Selective Matched Filter) is proposed. The evaluation of the algorithms is presented in section III, considering their relative performance as well as the impact of the multicast group size and the spatial correlation of the channel. Finally, section IV draws some conclusions and indicates perspectives for further studies.

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II. Adaptive beamforming for multicast

A multi-user multi-carrier system is considered, which assumes flat-fading per sub-carrier and negligible inter-symbol interference (ISI), so that the data symbols can be treated individually. The base station has an antenna array composed of M elements and serves a group of N single-antenna users. For the unicast case, considering a vector $\mathbf{d}_{N\times 1}$ with N data symbols (each addressed to a different user), which are modulated by a matrix $\mathbf{M}_{M\times N}$, transmitted over the radio channel $\mathbf{H}_{N\times M}$, subject to additive white Gaussian noise $\mathbf{n}_{N\times 1}$, and demodulated by a matrix $\mathbf{D}_{N\times N}$, the N downlink estimates $\hat{\mathbf{d}}_{\mathrm{N}\times 1}$ of the N transmitted symbols $\mathbf{d}$ may be written as

$$\hat{\mathbf{d}} = \mathbf{DHMd} + \mathbf{Dn}. \quad (1)$$

The multicast scenario can then be seen as a particular case of a MIMO multiuser system [2], for which all users expect the same symbols, i.e., $\mathbf{d} = s\mathbf{1}$, where $\mathbf{1}_{N\times 1}$ is a vector of ones and s is the data symbol. Equation (1) may then be rewritten as

$$\hat{\mathbf{d}} = \mathbf{DHw}s + \mathbf{Dn}, \quad (2)$$

where $\mathbf{w}_{M\times 1} = \mathbf{M1}$ is the resulting weight vector, which is the sum of the weight vectors of the individual users contained within $\mathbf{M}$.

Even though this paper focuses on the investigation of adaptive algorithms to be implemented on the transmitter side of a multicast system, some comments need to be made regarding the structure of the receivers. Since the number of users within a multicast group may well exceed the number of transmit antennas, it is not possible in such cases to guarantee that the received symbols will be in-phase with the actual symbols through pure transmit processing.

Note that in the case of multicast the demodulation matrix $\mathbf{D}$ is diagonal, since independent single antenna users are considered. Therefore, the system equation can be expressed for each user i as

$$\hat{\mathrm{d}}_i = D_{ii}\mathbf{H}_i\mathbf{w}s + D_{ii}n_i, \quad (3)$$

and the optimization problem for determining each element of $\mathbf{D}$, such that the SNR is maximized, becomes

$$D_{ii,opt} = \underset{D_{ii}}{\text{argmax}} \frac{E\{|D_{ii}\mathbf{H}_i\mathbf{w}s|^2\}}{E\{|D_{ii}n_i|^2\}}, \quad \text{subject to: } \begin{cases} Im\{D_{ii}\mathbf{H}_i\mathbf{w}\} = 0 \\ |D_{ii}|^2 = 1 \end{cases}, \quad (4)$$

where D_{ii} denotes the i^{th} element of the main diagonal of $\mathbf{D}$, $\mathbf{H}_{i\,1\times M}$ corresponds to the channel of user i, i.e., the i^{th} row of matrix $\mathbf{H}$, $|\cdot|$ is the absolute value of a scalar, and $Im\{\cdot\}$ is the imaginary part of the argument. Note that it is assumed that the users have knowledge of their equivalent channel $\mathbf{H}_i\mathbf{w}$.

The solution of the problem for each user and the corresponding matrix form are expressed as

$$D_{ii,opt} = |\mathbf{H}_i\mathbf{w}|^{-1}(\mathbf{H}_i\mathbf{w})^*,$$
$$\mathbf{D}_{opt} = diag(|\mathbf{H}_1\mathbf{w}|, \ldots, |\mathbf{H}_N\mathbf{w}|)^{-1}\, diag(\mathbf{Hw})^H, \quad (5)$$

where $diag(\cdot)$ is a diagonal matrix with the arguments on the diagonal and $(\cdot)^H$ denotes the conjugate transpose of a matrix.

In the following subsections, the herein investigated transmit processing algorithms are formulated. Among the techniques already known from unicast optimization are the matched filter (MF), zero-forcing (ZF), and Tomlinson-Harashima precoding (THP) [3-6]. The multicast-specific algorithms, which aim at the maximization of the lowest signal-to-noise ratio (SNR) among the multicast connections [7-9], are presented next, followed by the switched fixed beams [10].

A. Matched filter

The matched filter optimization for a single user scenario consists of finding the weight vector which maximizes the SNR perceived at the receiver. In the case of multicast, it can be expressed as the maximization of the sum/average SNR perceived by the users within the multicast group [3]. The optimization problem may thus be written as

$$\mathbf{w}_{opt} = \underset{\mathbf{w}}{\text{argmax}} \frac{E\{||\mathbf{Hw}s||^2\}}{E\{||\mathbf{n}||^2\}} \quad \text{subject to: } ||\mathbf{w}s||^2 \le E_{tr}, \quad (6)$$

where $\mathbf{w}_{opt}$ is the optimal weight vector, E_{tr} is the available transmit energy, $E\{\cdot\}$ is the expectation operator, and $||\cdot||$ is the Euclidean norm of a vector. This optimization leads to an eigenvalue problem, with solution

$$\mathbf{w}_{opt} = \beta \cdot (\text{principal eigenvector of } \mathbf{H}^H\mathbf{H}),$$
$$\beta = \sqrt{E_{tr}/\sigma_s^2}, \quad (7)$$

where σ_s^2 is the average symbol energy [3].

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B. Zero-forcing

It has been shown in [5] that the transmit zero-forcing filter minimizes the mean square error (MSE) subject to certain constraints. For a multicast scenario, the MSE relates to the difference between the symbols estimated at the receivers $\hat{\mathbf{d}}$ and the actual data symbol s. The zero-forcing optimization can be written as

$$\begin{aligned} &\mathbf{w}_{opt} = \underset{\mathbf{w}}{\text{argmin}}\, E\{||\hat{\mathbf{d}} - s\mathbf{1}||^2\} \\ &\text{subject to:} \begin{cases} ||\mathbf{w}s||^2 \leq E_{tr} \\ \hat{\mathbf{d}}|_{\mathbf{n}=\mathbf{0}} = s\mathbf{1} \end{cases}, \end{aligned} \tag{8}$$

where the second constraint corresponds to the zero-forcing constraint, which means that in the absence of noise there should be no difference between the estimated and the actual data symbols. In the case of multicast, it leads to $\mathbf{Hw} = \mathbf{1}$. The multicast zero-forcing solution is

$$\begin{aligned} &\mathbf{w}_{opt} = \beta \cdot \mathbf{H}^H(\mathbf{HH}^H)^{-1}\mathbf{1}, \\ &\beta = \sqrt{\frac{E_{tr}}{\sigma_s^2 \cdot tr((\mathbf{HH}^H)^{-1}\mathbf{1}\mathbf{1}^T)}}, \end{aligned} \tag{9}$$

where $tr(\cdot)$ denotes the trace and $(\cdot)^T$ the transpose of a matrix or vector.

Due to the channel inversion in (9), this algorithm has the limitation that the number of users cannot exceed the number of transmit antennas. Another aspect worth mentioning is that the zero-forcing constraint forces the received signals to be in-phase with those transmitted, thus not requiring, according to (5), the previously mentioned receiver structure. When substituting the weight vector given by (9) in (5), it results that $\mathbf{D}_{opt} = \mathbf{I}$, where $\mathbf{I}_{N\times N}$ denotes the identity matrix.

C. Tomlinson-Harashima precoding

This precoding algorithm introduces a feedback filter $\mathbf{F}_{N\times N}$ at the transmitter and a modulo operator at both transmitter and receivers [5, 6]. The system equation, before the modulo operation at the receiver, then becomes

$$\begin{aligned} &\hat{\mathbf{d}} = \mathbf{HMv} + \mathbf{n}, \\ &\mathbf{v} = (\mathbf{I} - \mathbf{F})^{-1}(\mathbf{d} + \mathbf{a}), \end{aligned} \tag{10}$$

where $\mathbf{v}_{N\times 1}$ is the precoded data vector and $\mathbf{a}$ is an auxiliary signal that models the modulo operation within the feedback loop at the transmitter.

The optimization problem for a zero-forcing THP filter is similar to that in (8), with the additional constraint that $\mathbf{F}$ has to be spatially causal, i.e., it is a lower triangular matrix with zero main diagonal [5]. The solution is

$$\begin{aligned} &\mathbf{M}_{opt} = \beta \cdot \mathbf{H}^H(\mathbf{L}^H)^{-1}\mathbf{L}_d^{-1}, \\ &\mathbf{F} = \mathbf{I} - \mathbf{L}\,\mathbf{L}_d^{-1}, \\ &\beta = \sqrt{\frac{E_{tr}}{tr(\mathbf{R}_v \mathbf{L}_d{}^{-2})}}, \end{aligned} \tag{11}$$

where $\mathbf{L}_{N\times N}$ comes from the Cholesky factorization of the channel ($\mathbf{HH}^H = \mathbf{LL}^H$), $\mathbf{L}_{d\,N\times N}$ is a diagonal matrix containing the elements of the main diagonal of $\mathbf{L}$, and $\mathbf{R}_{v\,N\times N}$ is the covariance matrix of the precoded data vector $\mathbf{v}$.

In the case of multicast, even though the same symbol is transmitted to all users, the precoded data vector will contain different elements, due to the different channel profiles perceived by each user. Therefore, the THP procedure presented here is the same for both unicast and multicast.

Similarly to zero-forcing, the THP algorithm is subject to the same limitation regarding the number of users, and no further receive processing (besides the modulo operation) is required, i.e., $\mathbf{D}_{opt} = \mathbf{I}$.

D. Max-min algorithms

The quality perceived by the users within a multicast group may vary significantly, depending on their radio channel conditions. Fairness among the users could therefore be introduced by the following optimization procedure, which tries to maximize the minimum SNR within the group:

$$\begin{aligned} &\mathbf{w}_{opt} = \underset{\mathbf{w}}{\text{argmax}}\, \min\{SNR_i\} \\ &\text{with } SNR_i = |\mathbf{H}_i\mathbf{w}s|^2/\sigma_n^2\,, \quad i = 1, \ldots, N \\ &\text{subject to: } ||\mathbf{w}s||^2 \leq E_{tr}, \end{aligned} \tag{12}$$

where σ_n^2 is the noise power.

This optimization problem is a quadratically constrained quadratic programming problem and does not have a closed-form solution. In [7] the problem is solved using sequential quadratic programming, while other articles [8, 9] have presented different approaches for simplifying the problem and finding more computationally efficient solutions. In [8], the problem is relaxed by removing one of the constraints of an equivalent optimization problem, which can then be solved efficiently by semidefinite programming methods.

E. User selective matched filter

In this paper we propose a heuristic algorithm, called User Selective Matched Filter (USMF), which does not claim to provide the optimum for (12), but which tries to improve the performance of the matched filter in a multicast scenario.

If it were assumed that there is a point-to-point connection for each user i, the ideal solution in the sense of maximizing the SNR would be to employ a transmit matched filter, i.e., $\mathbf{w} = \mathbf{H}_i^H$. The idea of USMF is to stack these individual weight vectors within a matrix ($\mathbf{H}^H$), but disregarding the weight vectors within it that do not positively contribute to the goal of maximizing the lowest SNR. The algorithm may be written as

$$\mathbf{w}_{USMF} = \beta \cdot \mathbf{H}^H \mathbf{P1}, \\ \beta = \sqrt{\frac{E_{tr}}{\sigma_s^2 \cdot tr(\mathbf{P}^T \mathbf{H}\mathbf{H}^H \mathbf{P11}^T)}}, \quad (13)$$

where $\mathbf{P}_{N \times N}$ is a non-zero diagonal matrix, with elements $\mathrm{p}_{ii} \in \{0, 1\}$, for $i = 1, \ldots, N$. Since there are N users, and the diagonal elements of $\mathbf{P}$ are restricted to binary values, there exists a total of $2^N - 1$ possibilities.

For small group sizes, all possible $\mathbf{P}$ matrices can be evaluated, from which the one providing the highest minimum SNR can be chosen. However, complexity grows exponentially with an increasing number of users. An alternative for making it computationally efficient would be to evaluate only a limited number of possibilities, which could for example be selected through randomization.

F. Switched fixed beams

Besides the fully-adaptive algorithms presented in the previous sections, another option for deploying antenna arrays in cellular networks is the use of switched fixed beams. They represent a low-cost solution which can be implemented, among other methods, through a Butler matrix [10]. The set of weight vectors is selected so that beams spanning the whole cell area are made available.

In the case of unicast users, the beam providing the highest SNR, which can be identified through feedback on the uplink, is selected. For multicast, however, all users within the group need to be taken into account. The solution herein considered is to activate all those beams which are currently being requested by the users. The result is then a normalized linear combination of the selected weight vectors.

III. Performance Evaluation

The simulation scenario considered for the performance evaluation of multicast consists of a single cell equipped with a four-element uniform linear antenna array, single antenna mobile terminals, and QPSK modulation. The implemented channel model regards both line-of-sight (LOS) and non-line-of-sight (NLOS) components [11], and can be written as

$$\mathbf{H} = \sqrt{K/(1+K)}\,\overline{\mathbf{H}} + \sqrt{1/(1+K)}\,\mathbf{H}_w, \quad (14)$$

where K is the Ricean factor which determines the ratio of deterministic-to-scattered power, $\mathbf{H}_{wN \times M}$ is composed of zero mean circularly symmetric complex Gaussian random variables with unit variance, and $\overline{\mathbf{H}}$ models the LOS component, which has each row given by

$$\overline{\mathbf{H}}_i = [1, e^{j2\pi d cos(\theta)}, \ldots, e^{j2\pi d(M-1)cos(\theta)}], \quad (15)$$

where d is the antenna spacing in wavelengths and θ is the direction of the user, which is assumed to be uniformly distributed within $[0,\ 2\pi/3]$ (base station at the corner of the sectorized cell). Note that the effects of path-loss and log-normal fading are not taken into account.

The beamforming algorithms are implemented according to their description in section II. For the THP algorithm, the suboptimum stream ordering procedure presented in [5] is assumed, the ”Max-min opt.” algorithm refers to the optimization problem in (12) solved through numerical optimization, and the LOS and NLOS scenarios correspond to $K \to \infty$ and $K = 0$ in (14), respectively.

Figs. 1 and 2 show the average bit error rate (BER) performance of the different algorithms for a multicast group composed of four users in NLOS and LOS scenarios, respectively. The BER is depicted as a function of the E_s/N_0, which represents the ratio of the symbol energy to the spectral noise density.

In Fig. 1, the THP and zero-forcing algorithms present the worst performance, achieving results comparable to those of spatial multiplexing with multiple unicast streams [5], for which THP outperforms zero-forcing for higher SNR values. The reason of their poor performance with regard to the other algorithms is due to the fact that they spend a considerable amount of effort trying to suppress interference among the data streams, which in the case of multicast is not necessary.

Still in Fig. 1 it can be seen that the solution of the max-min problem in (12) presents the lowest bit

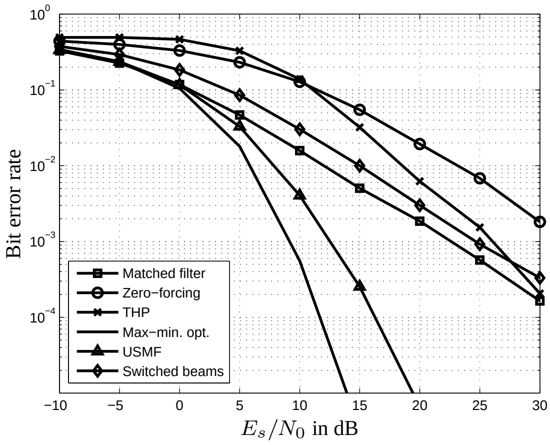

Fig. 1. BER performance of multicast beamforming algorithms with four-element antenna array and NLOS channel.

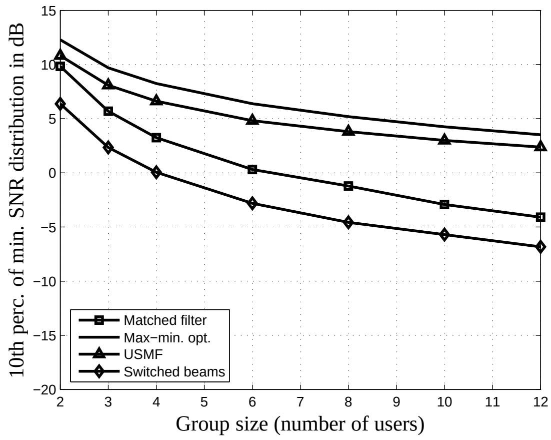

Fig. 3. Impact of the group size on the minimum SNR for the NLOS scenario and an input SNR of 10dB.

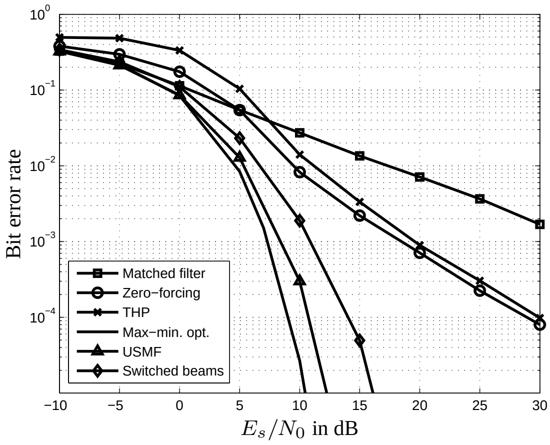

Fig. 2. BER performance of multicast beamforming algorithms with four-element antenna array and LOS channel.

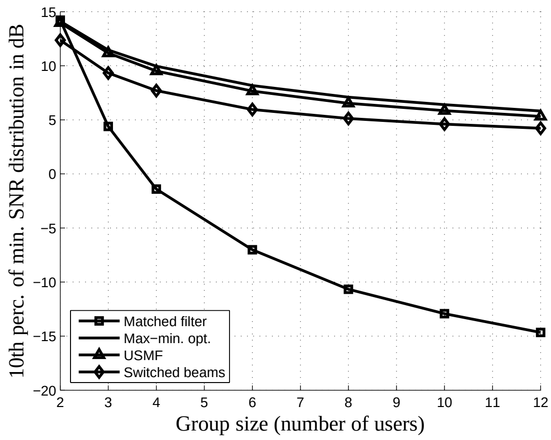

Fig. 4. Impact of the group size on the minimum SNR for the LOS scenario and an input SNR of 10dB.

error rates, being followed by the USMF algorithm, which requires approximately an extra 2.5dB in order to provide a BER of $5 \cdot 10^{-3}$. The matched filter and switched beams achieve an intermediate performance when compared to the others, approaching that of THP for higher SNR.

When we compare the results obtained for a rich scattering scenario in Fig. 1 to those obtained for a purely line-of-sight situation in Fig. 2, it becomes clear that the channel profile has a considerable impact on the performance of the algorithms. The USMF gets much closer to the optimal solution and the switched beams have their performance greatly improved in the presence of LOS. The increased spatial correlation of this scenario has a positive effect on USMF, which can be explained due to the fact that it increases the probability that the rows of $\mathbf{H}^H$ be correlated, resulting in more zero entries within $\mathbf{P}$ in (13), which brings it closer to the single-user beamforming case. For the switched beams the reason is similar, with an increased probability that less beams be requested by the users, and therefore allowing more energy to be concentrated in certain directions.

In Fig. 2, the THP and zero-forcing algorithms have similar performance, with zero-forcing presenting slightly lower bit error rates. The matched filter, however, goes through a considerable degradation. Even though it maximizes the average SNR, the quality of the users within the group may vary significantly, which in a LOS scenario has quite a negative impact on the bit error rates.

The impact that the group size has on the performance of the algorithms can be seen in Figs. 3 and 4, for the NLOS and LOS scenarios, respectively. Note that the THP and zero-forcing algorithms are not displayed, since for a number of users larger than the number of transmit antennas they do not apply.

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An input E_s/N_0 of 10dB is assumed and the results are presented in terms of the 10^{th} percentile of the cumulative distribution of the minimum SNR within the multicast group. This indicates that there is a 90% probability that the SNR perceived by the worst user within the group is higher than the given value.

For all algorithms it can be seen that the more users there are within a group, the lower the SNR that can be guaranteed. Up to a certain number of users (roughly 4-5 for most algorithms) the descent is steeper, but then it tends to slowly saturate for larger group sizes.

The relative behavior among the algorithms, for both NLOS and LOS scenarios, is very similar to that verified through the BER evaluation. The performance of the USMF and switched beams improves for the LOS channel, getting closer to the optimum, while the matched filter is severely degraded as the number of users increases in a channel with line-of-sight.

The poor performance of the matched filter for the LOS channel, in terms of the worst-user SNR, is due to the fact that the objective of the algorithm is to maximize the average and not the minimum SNR. For this channel, the eigendecomposition of $\mathbf{H}^H\mathbf{H}$ results in a large ratio of the largest to smallest singular values (ill-conditioned matrix), which means that more energy is concentrated on the principal eigenmode. This has a positive effect on the average, but leads to a more uneven energy distribution within the group, i.e., some users achieve very high SNR at the expense of others with very low quality.

IV. Conclusions

The application of multiple antenna arrays and adaptive beamforming to multicast presents some peculiarities when compared to the unicast case, since the users of a multicast group share the same radio resources and yet are subject to different radio channel conditions.

Different beamforming algorithms which can be applied to this problem have been presented and evaluated, including the traditional unicast algorithms (matched filter, zero-forcing, and THP), the one which tries to provide fairness among the users (max-min algorithm), and the switched fixed beams. Additionally, an algorithm called user-selective matched filter (USMF) has been proposed for multicast.

It has been shown that the zero-forcing and THP algorithms are not appropriate for the multicast scenario, due to their unnecessary interference suppression characteristics and their limitation regarding the group size.

The USMF algorithm provides a reasonable approximation with regard to the optimal solution of the max-min problem, especially for scenarios with a stronger LOS. The switched beams also perform better in LOS than in NLOS scenarios, representing an efficient low-cost solution for such cases. The matched filter, which maximizes the average SNR, has exactly the opposite behavior, presenting a much worse performance in the presence of line of sight.

The joint evaluation of unicast and multicast users, and how they can be scheduled and spatially multiplexed, each with their own requirements, are interesting topics for further studies.

Acknowledgments

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