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On Combining SDMA and B-IFDMA: Multi-User Detection and Channel Estimation

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***Abstract*—In this paper, we combine two different multiple access, namely, Space Division Multiple Access (SDMA) and Block-Interleaved Division Multiple Access (B-IFDMA), for uplink transmission, and we name the combined multiple access SD/B-IFDMA. Since SDMA suffers from multiple access interference (MAI) and B-IFDMA from intersymbol interference (ISI), a multi-user detector that can counteract both MAI and ISI is needed for SD/B-IFDMA. We propose three low complexity linear multi-user detectors for SD/B-IFDMA, namely, Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Non-Iterative Constrained Least Squares (NICLS). The NICLS is a heuristic multi-user detector that tries to improve the performance of ZF without the necessity of having to estimate the noise variance. Additionally, we also address the channel estimation procedure for SD/B-IFDMA by applying orthogonal time multiplexing training and using Chu sequence as the training sequence. Two estimators are considered for channel estimation for SD/B-IFDMA, namely, frequency domain Least Squares (LS) and time domain low complexity Maximum Likelihood (lcML). From bit error performance, it is shown that the MMSE multi-user detector performs best followed by NICLS and ZF. For performance assessment of the channel estimators, ZF multi-user detection is used. It is shown that the lcML outperforms LS with the penalty of having higher computational complexity.**

***Index Terms*—hybrid multiple access; multi-carrier and multi-user MIMO; multi-user detection; channel estimation.**

I. Introduction

The requirement for having high data rate transmissions in IMT Advanced (International Mobile Telecommunications Advanced) has led to various in-depth researches to find suitable modulation and multiple access schemes. In recent years, several promising modulation and multiple access schemes have been assessed. One of them is Interleaved Frequency Division Multiple Access (IFDMA), which is considered as a promising candidate for uplink transmission [1].

As a special form of Code Division Multiple Access (CDMA) and Orthogonal Frequency Division Multiple Access (OFDMA), IFDMA owns the properties of both CDMA and OFDMA. Two of its properties are low envelope fluctuations and high frequency diversity [2]. The low envelope fluctuations provides a higher power efficiency, which saves the battery power of the user equipment [1], [2].

An increase in the system capacity can be obtained by adapting to the individual small-scale fading of the time-frequency resources. However, this frequency adaptive transmission becomes less reliable when operating below a certain Signal to Interference and Noise Ratio (SINR) threshold and when the users move above a certain velocity [3]. In such cases, it is preferred to use non-frequency adaptive transmission and to employ an air interface providing high frequency diversity [4]. As IFDMA provides high frequency diversity, it is suitable for non-frequency adaptive uplink transmission [4].

Nevertheless, IFDMA has the disadvantages of being sensitive to frequency offsets and of needing high training sequence overhead for channel estimation [2], [3]. One way to reduce these disadvantages is by interleaving blocks of adjacent subcarriers instead of single subcarrier, as proposed in [3], [5], and being introduced as Block-IFDMA (B-IFDMA). In B-IFDMA, each user has its own allocated frequencies and they are orthogonal to the allocated frequencies of other users. The allocated frequencies are equidistantly frequency-separated blocks where each block consists of a few subcarriers. A Discrete Fourier Transform (DFT) precoding step is performed on each Orthogonal Frequency Division Multiplexing (OFDM) symbol before transmission. Despite of the penalty of having a slightly higher peak-to-average power ratio (PAPR), B-IFDMA is more robust against frequency offset and reduces the overhead for channel estimation while maintaining the frequency diversity gain [3], [5]. Thus, B-IFDMA is considered as a candidate for non-frequency adaptive multiple access scheme in uplink since it fulfils, or provides good trade-off between, the requirements for non-frequency adaptive transmission [5].

As bandwidth is an expensive and limited resource, communication schemes which offer high bandwidth efficiency are preferred. It has been widely known from literatures, e.g. [6], that the use of multiple antennas at the Base Station (BS) increases the spectral efficiency of the system. In an uplink scenario, having multiple antennas, the BS may perform Space Division Multiple Access (SDMA) to accommodate more users by separating the users spatially. SDMA increases the bandwidth efficiency of the system by allowing different users to share the same time-frequency resources in space. Therefore, combining SDMA and B-IFDMA leads a way to improve the spectral efficiency of the system.

In this paper, we combine SDMA with B-IFDMA for an uplink scenario and we name it hybrid Space Division/Block-Interleaved Frequency Division Multiple Access (SD/B-IFDMA). Since SDMA introduces multiple access interference (MAI) at the receiver, because it is a non-orthogonal multiple access, and B-IFDMA, on the other hand, suffers from intersymbol interference (ISI) due to the time-dispersion of the channel, multi-user detectors which can simultaneously counteract the MAI and the ISI are needed for SD/B-IFDMA. Moreover, from practical point of view, the multi-user detectors need to have low computational complexity. Therefore, in this paper, we derive the system model of SD/B-IFDMA and propose low complexity linear multi-user detectors for SD/B-IFDMA. These make SD/B-IFDMA very promising for practical implementation.

We design three linear multi-user detectors for SD/B-IFDMA. The first two detectors are based on the Zero Forcing (ZF) and the Minimum Mean Square Error (MMSE) criteria. The other detector is a Non-Iterative Constrained Least Squares (NICLS) detector, which can be seen as a hybrid ZF and Constrained Least Squares (CLS) detector. In terms of computational complexity, CLS has the same processing complexity as ZF and MMSE with additional complexity for iteration process [7]. The CLS needs several iterations for finding the optimum Lagrange multiplier λ which is used for the detection process. Having $\lambda = 0$, the CLS is indeed a ZF [7]. Our aim for NICLS is to avoid the iteration of CLS while having a better performance compared to ZF. Since NICLS avoids the iteration process, it has the same computational complexity as ZF and MMSE. Moreover, the aim of NICLS is to improve the performance of ZF while avoiding the estimation of the noise variance, which is needed for MMSE. Thus, compared to MMSE, NICLS has a lower estimation effort.

The performance of communication systems depends on the availability of the Channel State Information (CSI). In order to obtain the CSI, the BS needs to perform channel estimation. In this paper, we propose a procedure for channel estimation at the BS for SD/B-IFDMA using a training sequence. We choose Chu sequence [8] as the training sequence due to its autocorrelation property. In this work, we adjust two channel estimators, namely, Least Squares (LS) [9] and low complexity Maximum Likelihood (lcML) [10], in order to use them for channel estimation for SD/B-IFDMA. The lcML is based on a parametric model with the channel length as the only parameter [10]. While the channel estimation for LS is performed in frequency domain, the channel estimation for lcML is performed in time domain.

The contribution of this paper can be summarised as follows: 1. We combine SDMA and B-IFDMA, and derive the system model of the hybrid SD/B-IFDMA. 2. We design three low complexity multi-user detectors for SD/B-IFDMA, namely, ZF, MMSE and NICLS. 3. We propose channel estimation procedure with two different estimators, namely, LS and lcML, for SD/B-IFDMA.

This paper is organised as follows. Section II introduces the system model of SD/B-IFDMA. The linear multi-user detectors for SD/B-IFDMA are explained in Section III. Section IV explains the channel estimation for SD/B-IFDMA. The performance analysis is given in Section V and Section VI provides the conclusions.

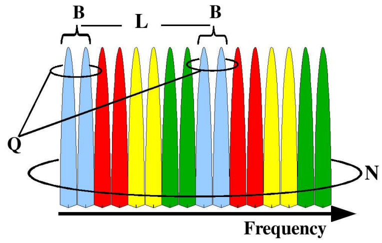

Fig. 1. Pictorial example of block-interleaved subcarrier allocation: $U = 4, N = 16, Q = 4, B = 2, L = 2$

II. System Model

In this section, the system model considered in this work is described. In the following, normal letters indicate scalar quantities, boldface letters indicate vectors, and boldface capitals indicate matrices. $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote the transpose and the conjugate transpose of a vector or a matrix, while $(\cdot)^{\dagger}$ and $(\cdot)^{-1}$ denote the pseudo-inverse and the inverse of a matrix, respectively.

A. B-IFDMA Subcarrier Allocation

In B-IFDMA systems, the available bandwidth is divided into several orthogonal sets of block-interleaved subcarriers. Each of the subcarrier sets is scheduled to a particular user. Assuming there are U orthogonal subcarrier sets, each subcarrier set $u, u = 1, \cdots, U$, is allocated to different user. The distribution of the subcarrier sets is defined by the mapping matrix $\mathbf{M}^{(u)}$.

Given B as the number of subcarriers per block, L as the number of blocks, $Q = B \cdot L$ as the number of block-interleaved subcarriers within one set, and $N = U \cdot Q$ as the total number of available subcarriers, the elements of the $N \times Q$ mapping matrix $\mathbf{M}^{(u)}$, $M_{n,q}^{(u)}$ in the n-th row, $n = 1, \cdots, N$, and q-th column, $q = (l-1) \cdot B + b$; $l = 1, \cdots, L$; $b = 1, \cdots, B$, can be written as [5], [11]

$$M_{n,q}^{(u)} = \begin{cases} 1, & \text{if } n = (l-1)\frac{N}{L} + b + (u-1)B, \text{ and} \\ 0, & \text{else.} \end{cases} \quad (1)$$

Figure 1 shows a pictorial example of block-interleaved subcarrier allocation for $U = 4, N = 16, Q = 4, B = 2$ and $L = 2$ where different colors show the U different block-interleaved subcarrier sets resulting from different $\mathbf{M}^{(u)}$.

B. SD/B-IFDMA System Model

An SD/B-IFDMA system allows several users who are separated in space to access the same block-interleaved subcarrier set at the same time. If there are K spatially compatible users (i.e., users who can efficiently share the same time-frequency resources through SDMA [12]) scheduled in each of the block-interleaved subcarrier sets with index u, then the total number of users is given by $U \cdot K$. This in turn increases the spectral efficiency of the system.

In the following, it is assumed that all users are already grouped into U SDMA groups using a certain SDMA grouping method, such as in [12]. Each group occupies a different

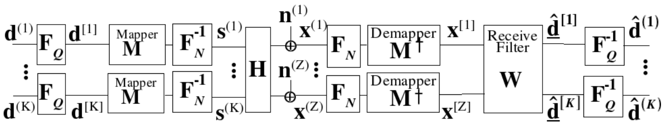

Fig. 2. Block diagram of SD/B-IFDMA for uplink transmission

block-interleaved subcarrier set. Without loss of generality, since B-IFDMA is an orthogonal multiple access, for the rest of the paper only one SDMA group is considered. In this SDMA group, there exist K spatially compatible users who are scheduled to use the same block-interleaved subcarrier set.

It is assumed that the BS is equipped with Z antennas. The K users' mobile terminals are equipped with one antenna. Within the specific group u, the K users are using the same mapping matrix $\mathbf{M}^{(u)}$. For the sake of simplicity, the specific group's variable u is dropped in the following. The variable $k, k = 1, \cdots, K$, denote a specific user in the SDMA group who is spatially separated to the others and may not be fully spatially orthogonal to them. The variable $z, z = 1, \cdots, Z$, denote a specific antenna at the BS. Figure 2 shows the block diagram of SD/B-IFDMA for uplink transmission.

Let

$$\mathbf{d}^{(k)} = (d_1^{(k)}, \cdots, d_Q^{(k)})^{\mathrm{T}} \tag{2}$$

denote a block of Q data symbols $d_q^{(k)}$, $q = 1, \cdots, Q$, at a symbol rate of $1/T_s$ of user k. The data symbols $d_q^{(k)}$ are taken from the alphabet of a bit mapping scheme such as Quadrature Phase Shift Keying (QPSK) or Quadrature Amplitude Modulation (QAM), which, in the case of coded transmission, is applied after coding and interleaving. Assuming perfect synchronisation at the BS, all of the K users' data symbols can be stacked into a $KQ \times 1$ vector

$$\mathbf{d} = (\mathbf{d}^{(1)\mathrm{T}}, \cdots, \mathbf{d}^{(K)\mathrm{T}})^{\mathrm{T}}. \tag{3}$$

Let $\mathbf{F}_Q$ denote a $Q \times Q$ FFT matrix, $\mathbf{I}_K$ a $K \times K$ identity matrix and '$\otimes$' the Kronecker product of two matrices. All K users use the same $\mathbf{F}_Q$, $\mathbf{M}$ and $\mathbf{F}_N^{-1}$, thus,

$$\mathbf{s} = (\mathbf{I}_K \otimes \mathbf{F}_N^{-1})(\mathbf{I}_K \otimes \mathbf{M})(\mathbf{I}_K \otimes \mathbf{F}_Q)\,\mathbf{d}. \tag{4}$$

The $KN \times 1$ vector $\mathbf{s}$ is the stacked version of the B-IFDMA modulated signals of the K users which is given by

$$\mathbf{s} = (\mathbf{s}^{(1)\mathrm{T}}, \cdots, \mathbf{s}^{(K)\mathrm{T}})^{\mathrm{T}}, \tag{5}$$

where

$$\mathbf{s}^{(k)} = \mathbf{F}_N^{-1}\,\mathbf{M}\,\mathbf{F}_Q\,\mathbf{d}^{(k)}. \tag{6}$$

It is well known that the insertion at the transmitter and the removal at the receiver of the Cyclic Prefix (CP) make the $N \times N$ channel matrix $\tilde{\mathbf{H}}^{(z,k)}$ of the user k's antenna to the BS receive antenna z circular [13]. Stacking all the channel matrices $\tilde{\mathbf{H}}^{(z,k)}$ results in a $ZN \times KN$ matrix

$$\tilde{\mathbf{H}} = \begin{pmatrix} \tilde{\mathbf{H}}^{(1,1)} & \dots & \tilde{\mathbf{H}}^{(1,K)} \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{H}}^{(Z,1)} & \dots & \tilde{\mathbf{H}}^{(Z,K)} \end{pmatrix}. \tag{7}$$

The received signal vector being distorted by Additive White Gaussian Noise (AWGN) can be written as

$$\mathbf{x} = \tilde{\mathbf{H}}\,\mathbf{s} + \mathbf{n}, \tag{8}$$

where $\mathbf{n}$ is a $ZN \times 1$ vector of complex Gaussian noise and $\mathbf{x}$ is a $ZN \times 1$ stacked vector of all the received signal vectors from all Z antennas, which is given by

$$\mathbf{x} = (\mathbf{x}^{(1)\mathrm{T}}, \cdots, \mathbf{x}^{(Z)\mathrm{T}})^{\mathrm{T}}, \tag{9}$$

with $\mathbf{x}^{(z)} = (x_1^{(z)}, \cdots, x_N^{(z)})^{\mathrm{T}}$.

The $KQ \times 1$ vector $\hat{\mathbf{d}}$ from the output of the detection process at the BS can be written as

$$\hat{\mathbf{d}} = (\mathbf{I}_K \otimes \mathbf{F}_Q^{-1})\,\mathbf{W}\,(\mathbf{I}_Z \otimes \mathbf{M}^{\dagger})(\mathbf{I}_Z \otimes \mathbf{F}_N)\,\mathbf{x}, \tag{10}$$

where $\mathbf{W}$ is a $KQ \times ZQ$ receive filter matrix, $\mathbf{I}_Z$ is a $Z \times Z$ identity matrix and $\mathbf{M}^{\dagger}$ is a $Q \times N$ demapper matrix which is the pseudo-inverse of $\mathbf{M}$.

Inserting (4) and (8) into (10) results in

$$\begin{aligned} \hat{\mathbf{d}} =& (\mathbf{I}_K \otimes \mathbf{F}_Q^{-1})\mathbf{W}(\mathbf{I}_Z \otimes \mathbf{M}^{\dagger})(\mathbf{I}_Z \otimes \mathbf{F}_N)\dots \\ & (\tilde{\mathbf{H}}(\mathbf{I}_K \otimes \mathbf{F}_N^{-1})(\mathbf{I}_K \otimes \mathbf{M})(\mathbf{I}_K \otimes \mathbf{F}_Q)\,\mathbf{d} + \mathbf{n}). \end{aligned} \tag{11}$$

At this point, a $ZQ \times KQ$ system matrix $\mathbf{A}$ of an SD/B-IFDMA system is introduced as

$$\mathbf{A} = (\mathbf{I}_Z \otimes \mathbf{M}^{\dagger})(\mathbf{I}_Z \otimes \mathbf{F}_N)\tilde{\mathbf{H}}(\mathbf{I}_K \otimes \mathbf{F}_N^{-1})(\mathbf{I}_K \otimes \mathbf{M}) \tag{12}$$

and a $Q \times Q$ subsystem matrix $\bar{\mathbf{A}}^{(z,k)}$ of antenna z and user k as

$$\bar{\mathbf{A}}^{(z,k)} = \mathbf{M}^{\dagger}\boldsymbol{\Delta}^{(z,k)}\mathbf{M}, \tag{13}$$

where

$$\boldsymbol{\Delta}^{(z,k)} = \mathbf{F}_N\tilde{\mathbf{H}}^{(z,k)}\mathbf{F}_N^{-1} \tag{14}$$

is an $N \times N$ diagonal matrix and contains the eigenvalues of $\tilde{\mathbf{H}}^{(z,k)}$ in its main diagonal. Equation (14) can be computed using the Block-Fourier Algorithm [14] as

$$\mathrm{diag}[\boldsymbol{\Delta}^{(z,k)}] = \sqrt{N}\mathbf{F}_N\tilde{\mathbf{H}}^{(z,k|1)}, \tag{15}$$

where $\tilde{\mathbf{H}}^{(z,k|1)}$ is the first column of the circular channel matrix $\tilde{\mathbf{H}}^{(z,k)}$ and $\mathrm{diag}[\cdot]$ denote the main diagonal elements of a matrix. The subsystem matrix $\bar{\mathbf{A}}^{(z,k)}$ is obtained from

$$\mathrm{diag}[\bar{\mathbf{A}}^{(z,k)}] = \mathbf{M}^{\dagger}\sqrt{N}\mathbf{F}_N\tilde{\mathbf{H}}^{(z,k|1)}. \tag{16}$$

Stacking all of the diagonal subsystem matrices $\bar{\mathbf{A}}^{(z,k)}$, the system matrix $\mathbf{A}$ becomes

$$\mathbf{A} = \begin{pmatrix} \bar{\mathbf{A}}^{(1,1)} & \dots & \bar{\mathbf{A}}^{(1,K)} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{A}}^{(Z,1)} & \dots & \bar{\mathbf{A}}^{(Z,K)} \end{pmatrix}, \tag{17}$$

which is a $ZQ \times KQ$ matrix that consits of $Z \times K$ blocks of $Q \times Q$ diagonal matrices. The system matrix $\mathbf{A}$ can be further manipulated through column and row permutation into a block diagonal (BD) matrix

$$\breve{\mathbf{A}} = \begin{pmatrix} \underline{\mathbf{A}}[1] & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \underline{\mathbf{A}}[2] & & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \underline{\mathbf{A}}[Q] \end{pmatrix}, \tag{18}$$

where

$$\underline{\mathbf{A}}[q] = \begin{pmatrix} \bar{\mathbf{A}}_{q,q}^{(1,1)} & \dots & \bar{\mathbf{A}}_{q,q}^{(1,K)} \\ \vdots & \ddots & \vdots \\ \bar{\mathbf{A}}_{q,q}^{(Z,1)} & \dots & \bar{\mathbf{A}}_{q,q}^{(Z,K)} \end{pmatrix} \tag{19}$$

is a $Z \times K$ matrix with $\bar{\mathbf{A}}_{q,q}^{(z,k)}$ defined as the element of the main diagonal of $\bar{\mathbf{A}}^{(z,k)}$ at the q-th row and q-th column.

Since the system matrix $\breve{\mathbf{A}}$ is a BD matrix, the equalisation and detection process can be performed per-subcarrier. As a result, the computational complexity is lower than for time-domain processing [15]. The per-subcarrier equation can be written as

$$\underbrace{\begin{pmatrix} \mathbf{x}^{[1]}[q] \\ \vdots \\ \mathbf{x}^{[Z]}[q] \end{pmatrix}}_{\underline{\mathbf{x}}[q]} = \underline{\mathbf{A}}[q] \cdot \underbrace{\begin{pmatrix} \mathbf{d}^{[1]}[q] \\ \vdots \\ \mathbf{d}^{[K]}[q] \end{pmatrix}}_{\underline{\mathbf{d}}[q]} + \underbrace{\begin{pmatrix} \mathbf{n}^{[1]}[q] \\ \vdots \\ \mathbf{n}^{[Z]}[q] \end{pmatrix}}_{\underline{\mathbf{n}}[q]} \tag{20}$$

where $\mathbf{x}^{[z]}, \mathbf{d}^{[k]}$ and $\mathbf{n}^{[z]}$ are $Q \times 1$ vectors of the frequency domain signals of the received signal at BS's antenna z after the demapping process, the transmitted signal of user k before the mapping process, and the noise signal at BS's antenna z after the demapping process, respectively, and $[q]$ denote the frequency domain signal at subcarrier q.

III. Linear Multi User Detectors

In this section, the linear multi-user detectors for SD/B-IFDMA are described. As the detection process is performed per-subcarrier, the subcarrier index $[q]$ is omitted in the following for notational simplicity, unless being stated otherwise.

In this work, it is assumed that the noise is temporally white with a covariance matrix of

$$\mathbf{R}_{\underline{n}} = \mathbf{E}\{\underline{\mathbf{n}}\,\underline{\mathbf{n}}^{\mathrm{H}}\} = \sigma^2 \mathbf{I}_Z. \tag{21}$$

The data covariance matrix is given by

$$\mathbf{R}_{\underline{d}} = \mathbf{E}\{\underline{\mathbf{d}}\,\underline{\mathbf{d}}^{\mathrm{H}}\} = \sigma_d^2 \mathbf{I}_K. \tag{22}$$

A. ZF and MMSE Joint Detection

For both linear ZF and MMSE detectors, the per-subcarrier $K \times 1$ vector $\hat{\underline{\mathbf{d}}}$ as the output of the receive filtering using $K \times Z$ per-subcarrier receive filter $\underline{\mathbf{W}}$ is given by

$$\hat{\underline{\mathbf{d}}} = \underline{\mathbf{W}} \cdot \underline{\mathbf{x}}. \tag{23}$$

ZF linear solution minimises the quadratic form of [16]

$$(\underline{\mathbf{x}} - \underline{\mathbf{A}}\,\hat{\underline{\mathbf{d}}})^{\mathrm{H}} \mathbf{R}_{\underline{n}}^{-1} (\underline{\mathbf{x}} - \underline{\mathbf{A}}\,\hat{\underline{\mathbf{d}}}) \tag{24}$$

and the receive filter $\underline{\mathbf{W}}$ solving (24) is given by [16]

$$\underline{\mathbf{W}} = (\underline{\mathbf{A}}^{\mathrm{H}} \underline{\mathbf{A}})^{-1} \underline{\mathbf{A}}^{\mathrm{H}}. \tag{25}$$

The MMSE linear solution minimises [16]

$$\mathbf{E}\{(\hat{\underline{\mathbf{d}}} - \underline{\mathbf{d}})^{\mathrm{H}} (\hat{\underline{\mathbf{d}}} - \underline{\mathbf{d}})\} \tag{26}$$

and the solution of (26) is given by [16]

$$\underline{\mathbf{W}} = (\underline{\mathbf{A}}^{\mathrm{H}} \underline{\mathbf{A}} + \frac{\sigma}{\sigma_d} \mathbf{I}_K)^{-1} \underline{\mathbf{A}}^{\mathrm{H}}. \tag{27}$$

B. Non-Iterative Constrained Least Squares (NICLS)

The Constrained Least Squares (CLS) detector was proposed as an SDMA detector for both single carrier [15] and OFDM [7]. The CLS tries to solve a least squares (LS) problem under the constraint that $\underline{\mathbf{d}}^{\mathrm{H}} \underline{\mathbf{d}} = K E_s$, assuming that all K users' signals have symbol energy E_s [7]. In the following, the CLS solution for SD/B-IFDMA is explained and followed by the proposed NICLS.

The CLS problem for SD/B-IFDMA is given by

$$\hat{\underline{\mathbf{d}}} = \arg \min_{\underline{\mathbf{d}}^{\mathrm{H}} \underline{\mathbf{d}} = K E_s} \|\underline{\mathbf{A}} \cdot \underline{\mathbf{d}} - \underline{\mathbf{x}}\|_2^2. \tag{28}$$

Using Lagrange multipliers, the solution is

$$\hat{\underline{\mathbf{d}}} = (\mathbf{R} + \lambda \mathbf{I}_K)^{-1} \underline{\mathbf{A}}^{\mathrm{H}} \underline{\mathbf{x}}, \tag{29}$$

with $\mathbf{R} = \underline{\mathbf{A}}^{\mathrm{H}} \underline{\mathbf{A}}$ [7]. Since CLS constitutes a quadratic LS problem, as in [7], the optimum Lagrange multiplier, λ^{opt}, can be calculated using *compact* Singular Value Decomposition (SVD) of $\underline{\mathbf{A}}$, according to $\underline{\mathbf{A}} = \mathbf{U}_{\mathrm{r}} \mathbf{\Sigma}_{\mathrm{r}} \mathbf{V}_{\mathrm{r}}^{\mathrm{H}}$, where $\mathbf{\Sigma}_r$ is an $r \times r$ diagonal matrix of the nonzero singular values of $\underline{\mathbf{A}}$, $\mathbf{U}_r$ and $\mathbf{V}_r$ are $Z \times r$ and $K \times r$ matrices of the r column vectors of the $Z \times Z$ and $K \times K$ unitary matrices $\mathbf{U}$ and $\mathbf{V}$, with r the rank of $\underline{\mathbf{A}}$. Thus, (28) can be rewritten as

$$\arg \min_{\bar{\mathbf{d}}^{\mathrm{H}} \bar{\mathbf{d}} = K E_s} \|\mathbf{\Sigma}_{\mathrm{r}} \bar{\mathbf{d}} - \bar{\mathbf{x}}\|_2^2 = \arg \min_{\bar{\mathbf{d}}^{\mathrm{H}} \bar{\mathbf{d}} = K E_s} \sum_{i=1}^{r} (\sigma_i \bar{d}_i - \bar{x}_i)^2, \tag{30}$$

with $\bar{\mathbf{d}} = \mathbf{V}_{\mathrm{r}}^{\mathrm{H}} \underline{\mathbf{d}}$, $\bar{\mathbf{x}} = \mathbf{U}_{\mathrm{r}}^{\mathrm{H}} \underline{\mathbf{x}}$ [7]. The solution using Lagrange multiplier is given by

$$\bar{d}_i(\lambda) = \frac{\sigma_i \bar{x}_i}{\sigma_i^2 + \lambda}, \qquad i = 1, \cdots, r, \tag{31}$$

where λ^{opt} needs to be found iteratively to fulfil

$$|\phi(\lambda) - K E_s| \leq T_i, \tag{32}$$

with T_i the threshold in order to reach a desired accuracy and

$$\phi(\lambda) = \|\bar{\mathbf{d}}(\lambda)\|_2^2 = \sum_{i=1}^{r} \frac{\sigma_i^2 |\bar{x}_i|^2}{|\sigma_i^2 + \lambda|^2}. \tag{33}$$

A good starting point is given in [7] as

$$\lambda^{start} = -\sigma_r^2 + \frac{\sigma_r |\bar{x}_r|}{\sqrt{K E_s}}. \tag{34}$$

After iteratively finding λ^{opt}, the $K \times 1$ data detected vector per-subcarrier is obtained from

$$\hat{\underline{\mathbf{d}}} = \mathbf{V}_{\mathrm{r}} \bar{\mathbf{d}}(\lambda^{opt}), \tag{35}$$

where $\bar{\mathbf{d}}(\lambda^{opt}) = (\bar{d}_1(\lambda^{opt}), \cdots, \bar{d}_r(\lambda^{opt}))$.

Although the CLS is a linear detector [7], [15], λ^{opt} needs to be calculated iteratively. Equation (29) is basically the same as the solution of MMSE in (27) and the Generalised MMSE (GMMSE) [17]. The GMMSE itself has a more relaxed constraint than the CLS, which is $\underline{\mathbf{d}}^{\mathrm{H}}\underline{\mathbf{d}} \leq KE_s$ [7], [17]. It also uses an iterative method on finding λ^{opt}. The λ^{opt} for GMMSE is given by [17]

$$\lambda^{opt} = \max(0, \bar{\lambda}). \tag{36}$$

where $\bar{\lambda}$ is the result of the iteration process with a reasonable step size.

In this paper an NICLS detector is proposed. It can be seen as a hybrid ZF and Constrained Least Square (CLS) detector. As explained in the previous paragraphs, the CLS needs several iterations for finding the optimum Lagrange multiplier λ^{opt} which is used for the detection process. If $\lambda^{opt} = 0$, the CLS is indeed a ZF [7]. For NICLS, we propose a suboptimum λ^{sub} to avoid the iteration process on finding the λ^{opt}. Our aim for NICLS is to avoid the iteration of CLS while having a better performance compared to ZF. Therefore, we compare the starting value of λ^{start} with zero and we take the maximum out of them. This is a heuristic way to find a performance improvement for ZF which ensure that at least, in worst cases where λ is always equal to or less than zero, we have a performance exactly like ZF. The λ^{sub} of the NICLS is defined by

$$\lambda^{sub} = \max(0, \lambda^{start}). \tag{37}$$

If $\lambda^{sub} = 0$, then the NICLS becomes a ZF detector and when $\lambda^{sub} = \lambda^{start}$, the performance of NICLS depends on the λ^{start} in (34). The λ^{sub} is used to calculate $\bar{d}_i(\lambda)$ as in (31). The per-subcarrier $K \times 1$ data detected vector of NICLS detector is given by

$$\hat{\underline{\mathbf{d}}} = \mathbf{V}_{\mathrm{r}}\bar{\mathbf{d}}(\lambda^{(sub)}), \tag{38}$$

where $\bar{\mathbf{d}}(\lambda^{sub}) = (\bar{d}_1(\lambda^{sub}), \cdots, \bar{d}_r(\lambda^{sub}))$.

For all three linear multi-user detectors discussed above, after performing the per-subcarrier data detection for all Q subcarriers, all $K \times 1$ vectors $\hat{\underline{\mathbf{d}}}[q]$ can be stacked into one $KQ \times 1$ vector

$$\hat{\underline{\mathbf{d}}}^{det} = (\hat{\underline{\mathbf{d}}}[1]^{\mathrm{T}}, \cdots, \hat{\underline{\mathbf{d}}}[Q]^{\mathrm{T}})^{\mathrm{T}}, \tag{39}$$

where now the subcarrier index $[q]$ is used again.

As the final step, the $\hat{\underline{\mathbf{d}}}^{det}$ needs to be permutated into a $KQ \times 1$ vector

$$\hat{\underline{\mathbf{d}}}^{permut} = (\hat{\underline{\mathbf{d}}}^{[1]^{\mathrm{T}}}, \cdots, \hat{\underline{\mathbf{d}}}^{[K]^{\mathrm{T}}})^{\mathrm{T}}, \tag{40}$$

where $\hat{\underline{\mathbf{d}}}^{[k]}$ is a $Q \times 1$ data detected vector in frequency domain of user k for all the Q subcarriers. The $KQ \times 1$ SD/B-IFDMA data detected vector in time domain as in (10) is then given by

$$\hat{\mathbf{d}} = (\mathbf{I}_K \otimes \mathbf{F}_Q^{-1})\, \hat{\underline{\mathbf{d}}}^{permut}, \tag{41}$$

where $\hat{\mathbf{d}} = (\hat{\mathbf{d}}^{(1)^{\mathrm{T}}}, \cdots, \hat{\mathbf{d}}^{(K)^{\mathrm{T}}})^{\mathrm{T}}$.

It has been shown in [7] that ZF, MMSE and CLS detectors have an overall processing complexity of the same order of $\mathcal{O}(KZ)$. However, CLS detector needs to perform additional iterations for finding λ^{opt} [7]. At this point it can be said that, by avoiding the iteration, the NICLS has the same overall processing complexity as ZF and MMSE.

IV. Channel Estimation

In order to perform the multi-user detection process, the BS needs the information of the channel matrix $\tilde{\mathbf{H}}$. In reality, the channel matrix needs to be estimated at the BS. In this section, the channel estimation procedure at the BS for SD/B-IFDMA is addressed. The first subsection explains the orthogonal training multiplexing and the Chu sequence which is used as the training sequence. The following subsections explain the channel estimators under consideration, namely, LS and lcML. A remark on this section is that we only address one channel estimation procedure using training sequence. One of the aim is to provide a performance comparison between LS and lcML. In this work, we assume that the channel (by channel, it means the concatenation of the front end filters and the propagation channel [10]) is time invariant during the channel estimation and the data detection of all nodes.

A. Training Sequence

In SD/B-IFDMA, the channel estimation needs to be performed for all available links between the K users and the BS. In order to have a better estimation, each of the training sequence from each user should be received without interference.

Two orthogonal training schemes, namely, Time Multiplexed Training (TMT) and Frequency Multiplexed Training (TMT), have been used in [18] to estimate the channel of IFDMA for uplink SDMA. It is shown in [18] that as long as the orthogonality of the training sequence is maintained, either using time or frequency multiplexing scheme does not bring impact in the performance in time-varying channel.

Using TMT, each user transmits its training sequence in different and non-overlapping time slots. The time slots for channel estimation are different with the time slot for users' data. Figure 3 shows the time slots allocation for TMT. In FMT, the training sequence from each user is sent at the same time slot with the data, however at different subcarriers. To avoid MAI at channel estimation process, each user sends the training sequence in different and non-overlapping subcarriers. TMT maintains the low PAPR with the expense of higher overhead, while FMT leads to a higher PAPR [19].

The structure of B-IFDMA, which is using block instead of single interleaved subcarrier, results in a slightly higher PAPR penalty than IFDMA. Consequently, the use of FMT for channel estimation purpose for B-IFDMA will increase the PAPR even more [20]. Therefore, in this work, TMT scheme as in Figure 3 is proposed for SD/B-IFDMA.

A CAZAC sequence has a constant amplitude and its DFT is also CAZAC [21]. Thus, training sequence within the class of CAZAC is suitable for channel estimation both in time and frequency domain. Polyphase sequence is a non-binary class of CAZAC whose elements are roots of unity. One of periodic Polyphase sequences which can be generated for any arbitrary

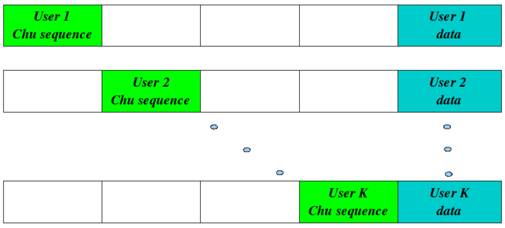

Fig. 3. Time Multiplexing Training

period length is Chu sequence [8]. It is generated using two relatively prime integers. A sequence with an even length T can be generated by [8]

$$\{a_v\}_{v=0,\ldots,T-1} = e^{j\frac{W\pi v^2}{T}} \tag{42}$$

with W an integer relatively prime to T. If length T is odd, then [8]

$$\{a_v\}_{v=0,\ldots,T-1} = e^{j\frac{W\pi v(v+1)}{T}}. \tag{43}$$

B. Channel Estimators

Several estimators for frequency domain channel estimation have been proposed mainly for OFDM transmission, e.g., [9], [10], [22]. An LS estimator [9] offers a straight forward approach with low complexity. Another estimator is based on the SVD of the channel correlation matrix [22]. Using SVD, only p best singular values of the channel correlation matrix are used to estimate the channel. The lower p the lower the computational complexity at the expense of a higher approximation error [22]. A low complexity estimator is proposed in [10] and introduced as low complexity Maximum-Likelihood (lcML). It has lower complexity and has a better performance compared to [22]. Moreover, lcML has been successfully implemented on an OFDM application-specified integrated circuit (ASIC) [10], which shows its practicality. Therefore, in this work only LS and lcML are considered and assessed. Regarding LS estimator, it has been used in channel estimation for IFDMA in [20] and for B-IFDMA in [23], [24].

1) Least Squares: In the following, let $\mathbf{p}^{(k)}$ denote the training sequence sent by user k. For the purpose of explaining the channel estimators, without loss of generality, $\mathbf{p}^{(k)}$ can replace $\mathbf{d}^{(k)}$ in Figure 2 and in the corresponding equations in Section II.

After the CP removal and the multiplication with the user-specific demapping matrix $\mathbf{M}^{(k)\dagger}$, the received signal in frequency domain at antenna element z of the BS is given by

$$\mathfrak{x}^{(z,k)} = \mathbf{M}^{(k)\dagger}\,\mathbf{F}_N\,(\tilde{\mathbf{H}}\,\mathbf{F}_N^H\,\mathbf{M}^{(k)}\,\mathbf{F}_Q\,\mathbf{p}^{(k)} + \mathbf{n}) \tag{44}$$

which can be rewritten as

$$\mathfrak{x}^{(z,k)} = \mathrm{diag}(\mathfrak{P})\,\mathfrak{h}^{(z,k)} + \mathfrak{n}^{(z)} \tag{45}$$

with

$$\mathfrak{h}^{(z,k)} = \mathbf{M}^{(k)\dagger}\,\mathbf{F}_N\,\tilde{\mathbf{H}}\,\mathbf{F}_N^H\,\mathbf{M}^{(k)} \tag{46}$$

the channel transfer function vector for the allocated B-IFDMA subcarriers, $\mathfrak{P} = \mathbf{F}_Q\;\mathbf{p}^{(k)}$ the vector of the FFT of the training sequence and $\mathfrak{n}^{(z)}$ the FFT of the noise vector at the corresponding allocated subcarriers at antenna z.

The LS estimator tries to minimize $(\mathfrak{x} - \mathrm{diag}(\mathfrak{P})\,\mathfrak{h})^H(\mathfrak{x} - \mathrm{diag}(\mathfrak{P})\,\mathfrak{h})$ which has the solution of [9]

$$\hat{\mathfrak{h}}_{LS}^{(z,k)} = \mathrm{diag}(\mathfrak{P})^{-1}\mathfrak{x}^{(z,k)}. \tag{47}$$

The LS estimator is equal to ZF estimator [9].

2) Low Complexity Maximum Likelihood FD-CE: The lcML estimator performs the actual estimation in time domain where the channel length is smaller than the CP length [10]. It can be interpreted as a transformation from frequency domain to time domain, where the actual estimation is performed, and another transformation to transform the results of the estimation back to the frequency domain. It needs the knowledge of the channel length in order to perform the estimation in time domain. In the following, the explanation of lcML is given based on [10]. As the system under consideration in [10] is OFDM, then what follows after explains the adjustment to the B-IFDMA case.

Even though in this work we use Chu sequence, without loss of generality, for the purpose of explaining the lcML only, let us assume that the training sequence $\mathbf{p}$ is all one, $\mathbf{p} = [1, 1, ..., 1]^{\mathrm{T}}$. The received signal in frequency domain can be written for all the N OFDM subcarriers as

$$\mathbf{x} = \mathfrak{h} + \mathbf{n} \tag{48}$$

which can be rewritten as

$$\mathbf{x} = \mathbf{F}_N \begin{pmatrix} \mathbf{h} \\ \mathbf{0} \end{pmatrix} + \mathbf{n} \tag{49}$$

with $\mathbf{h}$ the channel in time domain with length h which is smaller than the number of subcarriers N. $\mathbf{F}_N$ can be splitted into "signal subspace" and "noise subspace", resulting in

$$\mathbf{x} = [\mathbf{F}_h\;\mathbf{F}_{ns}] \begin{pmatrix} \mathbf{h} \\ \mathbf{0} \end{pmatrix} + \mathbf{n} \tag{50}$$

with $\mathbf{F}_h$ part of $\mathbf{F}_N$ for the corresponding "signal subspace" $\mathbf{h}$ and $\mathbf{F}_{ns}$ part of $\mathbf{F}_N$ for the "noise-only subspace". The reduced signal space becomes

$$\mathbf{r} = \mathbf{F}_h^{\dagger}\mathbf{x} = \mathbf{h} + \mathbf{F}_h^{\dagger}\mathbf{n} = \mathbf{h} + \mathfrak{v} \tag{51}$$

with $\mathfrak{v}$ a zero-mean Gaussian noise of covariance $C_{\mathfrak{v}\mathfrak{v}} = \mathbf{F}_h^H C_{\mathfrak{n}\mathfrak{n}} \mathbf{F}_h$. It is assumed that $C_{\mathfrak{n}\mathfrak{n}} = \sigma^2 \mathbf{I}_h$. Equation (51) has a log-likelihood function of

$$log\,f(\mathbf{r}) = -log\,(\pi det(C_{\mathfrak{v}\mathfrak{v}})) - (\mathbf{F}_h^{\dagger}\mathbf{x} - \mathbf{h})^H C_{\mathfrak{v}\mathfrak{v}}^{-1}(\mathbf{F}_h^{\dagger}\mathbf{x} - \mathbf{h}). \tag{52}$$

The lcML estimator is obtained through maximising (52) with respect to $\mathbf{h}$, which leads to

$$\hat{\mathfrak{H}} = \mathbf{F}_h\mathbf{F}_h^{\dagger}\mathbf{x} = \mathbf{P}_{\mathbf{F}_h}\mathbf{x} \tag{53}$$

with $\hat{\mathfrak{H}}$ the estimated channel transfer function vector and $\mathbf{P}_{\mathbf{F}_h}$ the orthogonal projection on the column of $\mathbf{F}_h$.

Each user in B-IFDMA uses only Q out of N subcarriers. Moreover, in this work to estimate the channel, a training sequence is used. Considering (45) and (51) for the case of

TABLE I
SD/B-IFDMA SYSTEM SETUP

Modulation	QPSK
Total subcarriers N	1024
Allocated subcarriers Q	128
Subcarriers per block B	2
System's Bandwidth	40 MHz
Length of Guard Interval	3.2e-6 s
B-IFDMA subcarriers allocation	all K users use the allocated subcarriers defined by $M^{(1)}$
BS antenna elements' distance	5λ
MS velocity	50 m/s
Scenario	C2 (Urban Macro) NLOS
Carrier	3.7 GHz

B-IFDMA with training sequence and bring them into (53) results in

$$\hat{\mathfrak{h}}_{lcML}^{(z,k)} = \mathfrak{F}_h \mathfrak{F}_h^{\dagger} \mathrm{diag}(\mathfrak{P})^{-1} \mathfrak{r}^{(z,k)} \tag{54}$$

with $\hat{\mathfrak{h}}_{lcML}^{(z,k)}$ the estimated channel transfer function vector at antenna z for the B-IFDMA allocated subcarriers and $\mathfrak{F}_h$ is the $\mathbf{F}_h$ which rows are already mapped according to the B-IFDMA allocated subcarriers. Referring to (47), (54) can be written as

$$\hat{\mathfrak{h}}_{lcML}^{(z,k)} = \mathfrak{F}_h \mathfrak{F}_h^{\dagger} \hat{\mathfrak{h}}_{LS}^{(z,k)} \tag{55}$$

which shows that for B-IFDMA, lcML estimator can be obtained from the LS estimator with additional low complexity.

One note regarding lcML for B-IFDMA, it has a similar fashion to post processing method proposed in [23], [24]. Both method can be seen as "noise" reduction process in channel estimation procedure. Nonetheless, they have different interpretation of "noise". In [23], [24], the "noise" is all coefficients of the estimated channel impulse response which are smaller than a given threshold.

V. Performance Analysis

In this section, the performance analysis of the proposed multi-user detectors and channel estimators for SD/B-IFDMA is given. The K users are randomly selected without performing any SDMA grouping method. The random selection provides the worst case scenario, which can be improved by applying SDMA grouping, such as in [12]. All K users access the same block-interleaved subcarrier set and use the same matrix $\mathbf{M}^{(u)}$, with the same index $u = 1$. A channel model according to [25] is used. The parameters of the system setup and the channel are given in Table I. As NICLS multi-user detector is based on CLS detector, which is suitable for constant modulus, we use uncoded QPSK.

Figure 4 shows the bit error rate (BER) performance of all three detectors in a scenario when only one B-IFDMA user, $K = 1$, communicates to the BS with one antenna, $Z = 1$. In such scenario, all three detectors act as an equaliser to counteract only the ISI, since there is no MAI. MMSE detector performs the best followed by NICLS and ZF. The NICLS performs better than ZF, which shows that the heuristic approach leads to a better result. The NICLS gains about 4 dB Eb/No performance for BER 10^{-2} compared to ZF.

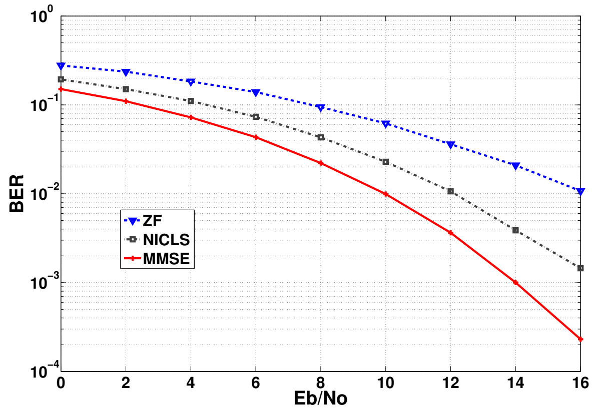

Fig. 4. BER performance of the B-IFDMA's equaliser.

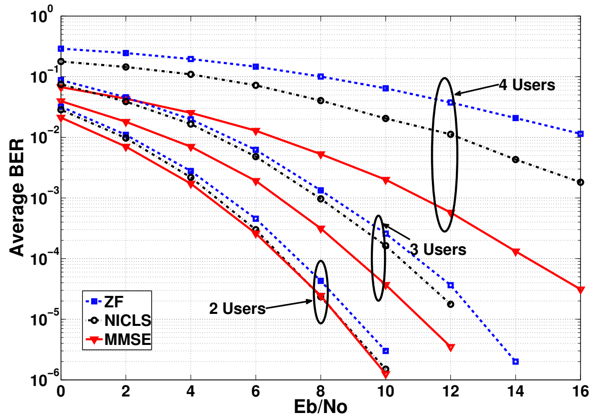

Fig. 5. Average BER performance of multi-user detector for SD/B-IFDMA.

Figure 5 shows the performance of the detectors in a multi-user scenario when the number of users K runs from 2 to 4 and the number of BS's antennas is fixed to $Z = 4$. Comparing both fully loaded cases, $Z = K = 1$ in Fig. 4 and $Z = K = 4$ in Fig. 5, NICLS obtains 4 dB performance gain compared to ZF. It can be seen as well that MMSE detector is able to achieve the spatial gain offered by the multiple antennas at the BS, while both NICLS and ZF detectors are not.

The lower the number of users, the better the performance of the detectors as the MAI is getting smaller. The performance gap between them is also getting smaller as both ZF and NICLS are able to achieve the spatial gain in a non-fully loaded case. Again, both MMSE and NICLS detectors outperform the ZF detector in the multi-user case. NICLS detector is once again better than ZF. Thus, the objective of improving the performance of ZF detector, while avoiding the necessity of having the information of noise variance, can be obtained by using NICLS detector. Even though MMSE performs best, it needs to know the noise variance of the BS, which means that there is a need to perform another estimation procedure to obtain this information.

Figure 6 shows the average BER performance of the channel

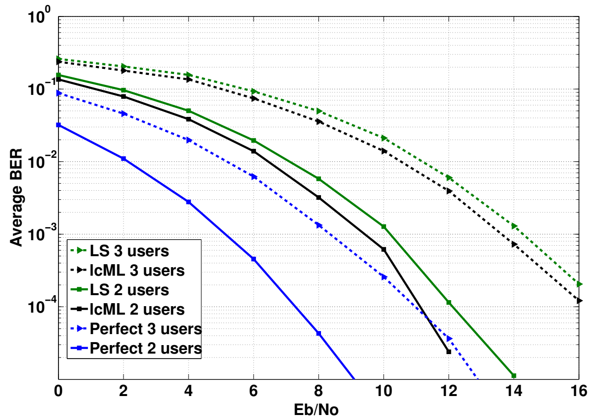

Fig. 6. Average BER performance of SD/B-IFDMA with estimated channel.

estimators for SD/B-IFDMA for the case of $Z = 4$ antennas at the BS and the number of user $K = 2$ and $K = 3$. ZF multi-user detector detector is applied at the BS. As the number of users increases, the performance degradation compared to the perfect channel is getting higher. It can be seen that the lcML performs better than the LS estimator. The lcML gains about 1 dB performance compared to the LS estimator. Nonetheless, lcML requires higher computational complexity and needs the information of the channel length.

VI. Conclusion

In this paper, a hybrid multiple access scheme SD/B-IFDMA is introduced. Three multi-user detectors, namely, ZF, MMSE and NICLS, are proposed and newly investigated as multi-user detectors for SD/B-IFDMA. The NICLS is a heuristic multi-user detector that tries to improve the performance of ZF without the need of the noise variance. Channel estimation procedure with two practical channel estimators, namely, LS and lcML, is proposed and newly investigated for SD/B-IFDMA. From BER, MMSE and NICLS detectors outperform the ZF detector. NICLS outperforms ZF without the necessity of having the estimated noise variance. Hence, NICLS has lower estimation complexity compared to MMSE and it provides a good trade-off between performance and complexity for SD/B-IFDMA. Regarding the channel estimators, lcML outperforms LS with higher computational complexity and the need of knowing the length of the channel tap.

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