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SEAMLESS TRANSITION OF POWER ALLOCATION IN MULTI-USER XDSL SYSTEMS

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ABSTRACT

In non-static scenarios where Dynamic Spectrum Management (DSM) Level 2 is employed, an optimal power allocation can become invalid e.g. due to a joining user and thus has to be recomputed. To not unnecessarily interrupt service, the spectra should be updated successively as individual users start a new session. If not done carefully, this results in sub-optimal intermediate joint power allocations where the actual signal-to-noise ratio (SNR) margin of users decreases drastically. This paper investigates a novel approach to gradually update transmit spectra of modems to a new optimal power allocation while assuring that the actual SNR margin of all users does not fall below a specified value during the transition phase. Also, a low-complexity solution for the involved hard optimization problem is described. Simulations carried out demonstrate the interdependency between the minimum margin value and convergence time of the proposed scheme.

1. INTRODUCTION

It is well-known that far-end crosstalk (FEXT) between copper wires in a binder is the dominant impairment in current Digital Subscriber Line (DSL) systems [6], severely limiting achievable data rates. *DSM Level 2* promises to mitigate the capacity loss due to crosstalk by centrally coordinating the modems transmit power allocation, effectively introducing *politeness* between users. In the recent past, the Spectrum Management problem has been studied extensively in literature, and although the problem has been proved to be NP-hard [5], a number of low-complexity algorithms have been proposed, e.g. [1, 7], which are able to efficiently compute the majority of user rate tuples achieved by *Optimal Spectrum Balancing* [2]. A key issue with most of the proposed solutions is that they assume a scenario with static channel conditions and a fixed set of active users and as such, do not cope with the case that some optimal joint power allocation computed by the Spectrum Management Center (SMC) according to the current channel conditions may become invalid at some point in the future when the channel or DSM system parameters change, e.g.:

1. *A user is joining or leaving the system.* This is likely to happen in an unbundled environment where customers change service providers which all operate their proprietary DSM system.
2. *A user is changing service.* If a user upgrades e.g. from ADSL2 to a VDSL2 service, then necessarily will the transmit spectrum change which in turn will alter the crosstalk profile on other users' lines in the binder.

In occurrence of any of the two events, the SMC has to determine a new joint allocation that is optimal for the new situation. A major problem that arises here is that transmit spectra of modems already in showtime cannot be reconfigured without interrupting service which is why such an intervention is usually avoided by the provider. Instead, updating a transmit power profile should be delayed until the modem enters initialization phase of the following session where the configured power allocation is then kept until the session is terminated. Generally, however, by only updating spectra of part of the users in the system as individual modems reinitialize, one ends up with an *intermediate* joint power allocation that is a mixture of old and new optimal spectra. These intermediate allocations generally are not guaranteed to maintain transmission with the desired target bit error rate (BER) until the system has fully updated to the new allocation. At the least, they are likely to result in a severe drop of the SNR margin intended for protection against fluctuation of out-of-domain crosstalk and noise, thus threatening line stability.

Therefore, a novel approach is studied in this paper which allows to gradually update power allocations in a DSM system while assuring that at each point during the transition phase, the actual SNR margin of each user does not fall below a given minimum value, thus improving reliability of transmission.

The remainder of this work is organized as follows: Section 2 defines the system model for a multi-user DSL channel. Section 3 discusses the problem of updating multi-user power allocations and a solution is proposed in Section 4. In Section 5, a technique for low-complexity implementation of the studied approach is presented. Finally, Section 6 gives some results from numerical simulations.

2. SYSTEM MODEL FOR STATIC SCENARIO

We start out by defining the channel model for a static DSL system with a fixed set $\mathcal{N}$ of users sharing the same binder, thus causing mutual FEXT on each other's line. By employing Discrete Multitone (DMT) transmission with K orthogonal tones $k = 1, \ldots, K$, the interference channel is effectively divided into K independent subchannels k. Augmenting each DMT symbol with a sufficiently long cyclic extension avoids intersymbol interference (ISI) and allows the direct channel of user $n \in \mathcal{N}$ on tone k to be fully described by a single complex coefficient $h_k^{n,n}$. Similarly, the crosstalk channel from disturber m to victim line n on tone k is given by the complex scalar $h_k^{n,m}$ $(m \neq n)$. Let s_k^n denote the power spectral density (PSD) of the transmit signal of user n and σ_k^2 the combined PSD of alien FEXT and receiver background noise on tone k. Then, using Shannon gap approximation [6], the number of bits $b_k^n(\gamma)$ per symbol user n can load on tone k with a given SNR margin $\gamma \geq 1$ is

$$b_k^n(\gamma) = \log_2 \left(1 + \frac{1}{\gamma \Gamma} \frac{g_k^{n,n} s_k^n}{\sum_{m \neq n} g_k^{n,m} s_k^m + \sigma_k^2} \right) \qquad (1)$$

where $\Gamma > 1$ denotes the *gap to capacity* [6] which is a function of the target BER and $g_k^{n,m} = |h_k^{n,m}|^2$ are the crosstalk and direct channel gain coefficients. Furthermore, the total utilized power P^n and data rate R^n of user n are given by

$$P^n = \Delta f \sum_k s_k^n \quad \text{and} \quad R^n(\gamma) = f_s \sum_k b_k^n(\gamma), \qquad (2)$$

respectively, where f_s is the symbol rate of the DMT system.

3. UPDATE OF MULTI-USER POWER ALLOCATION IN NON-STATIC SCENARIOS

In a DSM system, regardless whether operating in *rate-adaptive*, *margin-adaptive* or *fixed-margin* mode, the optimal joint power allocation is determined using a spectrum balancing algorithm which typically accounts for at least three per-user constraints in the optimization process: the total power constraint

$$P^n \leq P_{\max}^n \qquad \forall n \qquad (3)$$

where $P_{\max}^n$ is the *maximum aggregate transmit power* specified in the respective xDSL standard, a spectral mask constraint

$$0 \leq s_k^n \leq s_{k,\text{mask}}^n \qquad \forall n, k \qquad (4)$$

where $s_{k,\text{mask}}^n$ is the PSD mask determined by the band profile used and a rate constraint

$$R^n(\gamma) \geq R_{\text{target}}^n \qquad \forall n \qquad (5)$$

where R_{target}^n is the *target data rate* of user n chosen according to the *Service Level Agreement* and γ takes some value $\gamma_{\text{target}} > 1$ which is the *target SNR margin* chosen by the provider.

Now, consider a non-static scenario in which an optimal allocation $\boldsymbol{s}_{\text{old}} = \{s_{k,\text{old}}^n | n \in \mathcal{N}; k = 1, \ldots, K\}$ computed by the SMC becomes invalid at some time instance $t = t_0$ when any of the mentioned events such as a user joining or leaving a DSM system occurs. In this case, a new allocation $\boldsymbol{s}_{\text{new}} = \{s_{k,\text{new}}^n | n \in \mathcal{N}; k = 1, \ldots, K\}$ has to be determined that is optimal for time $t \geq t_0$ but, as has been pointed out, cannot be applied instantly to those users already in show-time without interruption of service.

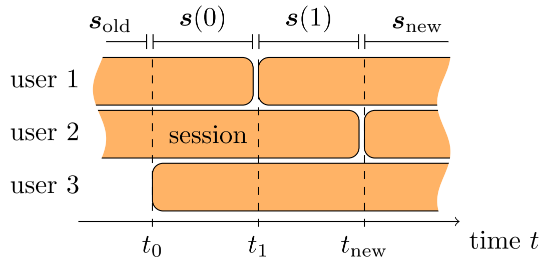


Figure 1: Transition of multi-user power allocation $\boldsymbol{s}$ after user 3 has joined the DSM system at time $t = t_0$.

Figure 1 shows an example scenario where user $n^* = 3$ who has not used a DSL service before joins the DSM system and becomes active for the first time at $t = t_0$. $\boldsymbol{s}_{\text{new}}$ is determined such that both constraints (3) and (5) are satisfied for all users $n \in \mathcal{N}$ where $\mathcal{N} = \{1, 2, 3\}$ while with the old allocation $\boldsymbol{s}_{\text{old}}$, (3) and (5) are satisfied only for users $n \in \{1, 2\}$ and user n^* is not yet active for $t < t_0$, i.e. $s_{k,\text{old}}^{n^*} = 0 \; \forall k$. Of course, the existence of a feasible allocation $\boldsymbol{s}_{\text{new}}$ requires an anticipatory planning of the operator when assigning service levels and hence target data rates to individual customers. Assuming all users $n \neq n^*$ are already in show-time at $t = t_0$, then only the transmit PSD $s_k^{n^*}$ of the newly joined user n^* can be updated at this point.

We define the time instances $i = 0, 1, 2, \ldots$ in discrete time corresponding to the instances $t = t_i$ $(t_i < t_{i+1})$ in continuous where any of the users initiates a new session and therefore is about to reconfigure its transmit PSD. Furthermore, let $\boldsymbol{s}(i) = \{s_k^n(i) | n \in \mathcal{N}; k = 1, \ldots, K\}$ denote the power allocation used by the system in the time interval $\theta_i = t_i \leq t < t_{i+1}$. Consequently, if a user n does not retrain at instance i, then $s_k^n(i) = s_k^n(i-1) \forall k$.

A straight-forward, albeit naive approach would be to initialize user n^*'s PSD $s_k^{n^*}(0)$ at instance $i = 0$ to the new optimal allocation $s_{k,\text{new}}^{n^*}$ while users $n \neq n^*$ continue transmission with old optimal spectra, i.e.

$$s_k^n(0) = \begin{cases} s_{k,\text{new}}^n & n = 3 \\ s_{k,\text{old}}^n & n \in \{1, 2\} \end{cases} \qquad \forall k = 1, \ldots, K. \qquad (6)$$

At the following instance $i = 1$, assume that user 1 is about to reinitialize transmit PSD $s_k^2(1)$ which would be set to $s_{k,\text{new}}^2$ while user 2 keeps his spectrum since he is still in show-time, i.e.

$$s_k^n(1) = \begin{cases} s_{k,\text{new}}^n & n \in \{1, 3\} \\ s_k^n(0) & n = 2 \end{cases} \qquad \forall k = 1, \ldots, K. \qquad (7)$$

Once each of the users has reinitialized at least one time, the DSM system has been fully updated to $\boldsymbol{s}_{\text{new}}$ and therefore regained an optimal power allocation. In this consideration, we ignored the possibility that during the transition phase, another event that would invalidate $\boldsymbol{s}_{\text{new}}$ could occur. However, in this case, we would simply start over by determining a new optimal $\boldsymbol{s}_{\text{new}}$ and setting $\boldsymbol{s}_{\mathrm{old}}$ to the allocation that was used by the system at that point. Thus, without loss of generality, this case is not further considered explicitly in the remainder of this work.

During each interval θ_i, the *actual SNR margin* $\gamma^n(i)$ of user n resulting from a given multi-user power allocation $\boldsymbol{s}(i)$ is the solution of

$$R^n\big(\gamma^n(i)\big)|_{\boldsymbol{s}(i)} - R^n_{\mathrm{target}} = 0 \tag{8}$$

However, as has already been mentioned, it cannot be guaranteed that any of the allocations $\boldsymbol{s}(i)$, which are a mixture of old and new optimal power spectra, are feasible, i.e. yield a solution $\gamma^n(i) \geq 1$ to eq. (8). This makes the described naive approach impractical as it is likely to compromise stability of service.

4. PROPOSAL FOR NEW UPDATING SCHEME

In this section, we describe a novel algorithm which enables seamless transition from power allocation $\boldsymbol{s}_{\mathrm{old}}$ to $\boldsymbol{s}_{\mathrm{new}}$ in a DSM system. By this, we mean to determine the intermediate spectra $\boldsymbol{s}(i)$ in such a manner that at all times the actual SNR margin $\gamma^n(i)$ is guaranteed to not fall below a specified minimum margin value $\bar{\gamma}$.

The key idea of our proposed scheme is to shape spectra $\boldsymbol{s}(i)$ at each instance i as similar as possible to the target allocation $\boldsymbol{s}_{\mathrm{new}}$ while accounting for per-user power and target rate constraints. To characterize similarity between $\boldsymbol{s}(i)$ and $\boldsymbol{s}_{\mathrm{new}}$, we define the distance function

$$\Delta\big(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}}\big) = \sum_n \sum_k \left(\frac{s^n_k(i)}{s^n_{k,\mathrm{new}}} - 1 \right)^2 \tag{9}$$

which becomes 0 for $\boldsymbol{s}(i) = \boldsymbol{s}_{\mathrm{new}}$. In order to avoid division by zero, $s^n_{k,\mathrm{new}}$ should be lower-bounded to some sufficiently small positive value $s_{\min}$. In our simulations, a value of -130 dBm/Hz has shown to be reasonable for DSL applications and is easily handled by single-precision floating point arithmetic.

Assuming that at instance i, users $n \in \mathcal{G}_i \subseteq \mathcal{N}$ are about to resynchronize, then, given an *minimum SNR margin* $\bar{\gamma}$, the intermediate power allocation $\boldsymbol{s}(i)$ is obtained by solving the optimization problem

$$\begin{aligned} \min_{s^n_k(i) \forall n \in \mathcal{G}_i, k} \quad & \Delta\big(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}}\big) \\ \text{s.t.} \quad & R^n(\bar{\gamma})|_{\boldsymbol{s}(i)} \geq R^n_{\mathrm{target}} && \forall n \\ & \sum_k s^n_k(i) \leq P^n_{\max} && \forall n \\ & 0 \leq s^n_k(i) \leq s^n_{k,\mathrm{mask}} && \forall n, k \end{aligned} \tag{10}$$

where the spectra for users $n \notin \mathcal{G}_i$ are kept fixed according to

$$s^n_k(i) = \begin{cases} s^n_{k,\mathrm{old}} & i = 0 \\ s^n_k(i-1) & i > 0 \end{cases} \qquad \forall n \notin \mathcal{G}_i; k = 1, \ldots, K. \tag{11}$$

The efficient solution of problem (10) is discussed later on in this work.

We here limit ourselves to an informal convergence analysis of the proposed scheme. Generally, the sequence of optimal solutions $\{\Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}})\}$ is monotonously decreasing, i.e. $\Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}}) \leq \Delta(\boldsymbol{s}(i-1), \boldsymbol{s}_{\mathrm{new}})$. In practical scenarios where all DSL sessions are of finite duration, for every instance i where $\boldsymbol{s} \neq \boldsymbol{s}_{\mathrm{new}}$, there will always be a later instance $j > i$ such that $\Delta(\boldsymbol{s}(j), \boldsymbol{s}_{\mathrm{new}}) < \Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}})$ which implies convergence of the system to $\boldsymbol{s}_{\mathrm{new}}$ in a finite number of time steps.

Existence of a feasible intermediate allocation $\boldsymbol{s}(i)$ is shown by a simple induction: if a feasible solution for $\boldsymbol{s}(i)$ exists, then this solution is also feasible for $\boldsymbol{s}(i+1)$. The remaining issue is to find an initial power allocation $\boldsymbol{s}(0)$ that is feasible. Coming back to the example scenario from the previous section, we saw that generally, there is no guarantee that a service with the defined target rates and target BER can be maintained for all users once the newly joined user n^* becomes active. Thus, at the initial step $i = 0$, we have to check whether a minimum SNR margin $\bar{\gamma}$ with $1 \leq \bar{\gamma} \leq \gamma_{\mathrm{target}}$ exists so that Problem (10) with $\mathcal{G}_0 = \{n^*\}$ is feasible for $\boldsymbol{s}(0)$. If such a $\bar{\gamma}$ is found, then this value can be used to determine all intermediate power allocations. Obviously, by enlarging the set of feasible power allocations, a low intermediate margin $\bar{\gamma}$ generally increases flexibility in shaping the spectra and thus tends to reduce the number of required intermediate steps i before all users can be configured to the target allocation $\boldsymbol{s}_{\mathrm{new}}$. At the same time, a trade-off has to be made between faster convergence and reduced protection against fluctuation of noise.

If, however, no feasible $\bar{\gamma} \geq 1$ exists, then the only choice that remains is to augment $\mathcal{G}_0$ by one or more additional users whose spectra are to be reshaped at $t = t_0$. In this case, a forced resynchronization of these users cannot be avoided.

5. LOW-COMPLEXITY SOLUTION

Clearly, the objective $\Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}})$ to be minimized is convex in $s^n_k(i)$ and separable in the tones k while the target rate constraint $R^n(\bar{\gamma})|_{\boldsymbol{s}(i)} \geq R^n_{\mathrm{target}}$ leads to a non-convex set of feasible solutions, making it difficult to find a solution that is guaranteed to be globally optimal. Numerous algorithms for solving spectrum management problems with similar structure as Problem (10) have been proposed in the literature which differ both in complexity and accuracy. A standard approach is to decompose the Lagrangian

$$\begin{aligned} \Lambda = \Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\mathrm{new}}) + \sum_n \omega^n \Big(R^n_{\mathrm{target}} - R^n(i) \Big) \\ + \sum_n \lambda^n \Big(\sum_k s^n_k(i) - P^n_{\max} \Big) \end{aligned} \tag{12}$$

into per-tone Lagrangians Λ_k according to

$$\Lambda = \sum_k \Lambda_k + \underbrace{\sum_n \omega^n R^n_{\mathrm{target}} - \sum_n \lambda^n P^n_{\max}}_{\text{const. in } \boldsymbol{s}(i)} \tag{13}$$

with

$$\Lambda_k = \sum_n \left(\frac{s_k^n(i)}{s_{k,\text{new}}^n} - 1 \right)^2 + \sum_n \lambda^n s_k^n(i)$$
$$- f_s \sum_n \omega^n \log_2 \left(1 + \frac{1}{\bar{\gamma}\Gamma} \frac{g_k^{n,n} s_k^n(i)}{\sum_{m \neq n} g_k^{n,m} s_k^m(i) + \sigma_k^2} \right). \tag{14}$$

This allows solving the dual

$$\max_{\omega^n, \lambda^n \forall n \in \mathcal{G}_i} \quad \min_{s_k^n(i) \forall n \in \mathcal{G}_i, k} \Lambda \tag{15}$$
$$\text{s.t. } \omega^n, \lambda^n \geq 0 \qquad \forall n$$
$$0 \leq s_k^n(i) \leq s_{k,\text{mask}}^n \quad \forall n, k$$

of Problem (10) by solving K independent sub-problems

$$\min_{s_k^n(i) \forall n \in \mathcal{G}_i} \Lambda_k \tag{16}$$
$$\text{s.t. } 0 \leq s_k^n(i) \leq s_{k,\text{mask}}^n \qquad \forall n$$

per Lagrange multiplier search step, thus rendering the overall algorithm complexity linear in K.

As Λ_k is non-convex, minimization however still requires an exhaustive search with exponential complexity in the number of users N. For the rate-adaptive spectrum management problem, the authors of [7] propose an efficient algorithm based on convex relaxation by noting that the Lagrangian can be rewritten as a difference of convex (d.c.) functions. Rewriting Λ_k as

$$\Lambda_k = \sum_n \left(\frac{s_k^n(i)}{s_{k,\text{new}}^n} - 1 \right)^2 + \sum_n \lambda^n s_k^n(i)$$
$$\underbrace{-f_s \sum_n \omega^n \log_2 \left(\sum_{m \neq n} g_k^{n,m} s_k^m(i) + \sigma_k^2 + \frac{g_k^{n,n} s_k^n(i)}{\bar{\gamma}\Gamma} \right)}_{A}$$
$$+ \underbrace{f_s \sum_n \omega^n \log_2 \left(\sum_{m \neq n} g_k^{n,m} s_k^m(i) + \sigma_k^2 \right)}_{B} \tag{17}$$

where part A is convex and part B is concave, one finds that Problem (10) as well exposes a d.c. structure and can thus be solved efficiently using the techniques described in [7]. The key idea here is to approximate the solution for the per-tone sub-problem (16) by iteratively solving a sequence of relaxed convex minimization problems, where the solution of one iteration is used as an approximation point for finding a convex relaxation of Λ_k in the next iteration. Adaption of the low-complexity algorithm to Problem (10) is straight-forward and therefore omitted here.

6. SIMULATION RESULTS

To numerically evaluate the benefit of the proposed scheme, we consider an upstream scenario depicted in

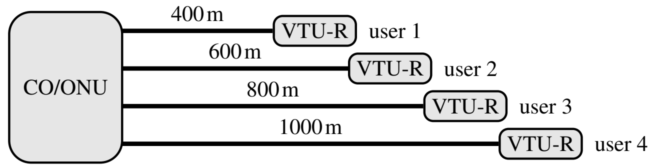


Figure 2: 4-user VDSL2 scenario

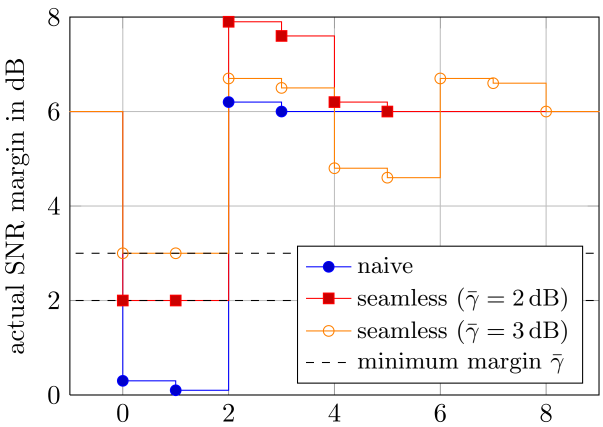
Table 1: Simulation parameters

cable type	TP100 [3]
FEXT model	ETSI 1% worst-case [3] without FSAN sum
background noise level N_0	$-140\,\text{dBm/Hz}$
Alien noise model	ETSI MD_EX [3]
VDSL2 band profile	998-ADE17M2x-B with US0 [4]
symbol rate f_s	4 kHz
tone spacing Δf	4.3125 kHz
SNR gap Γ including coding gain	6.8 dB
Target SNR margin γ_{target}	6 dB

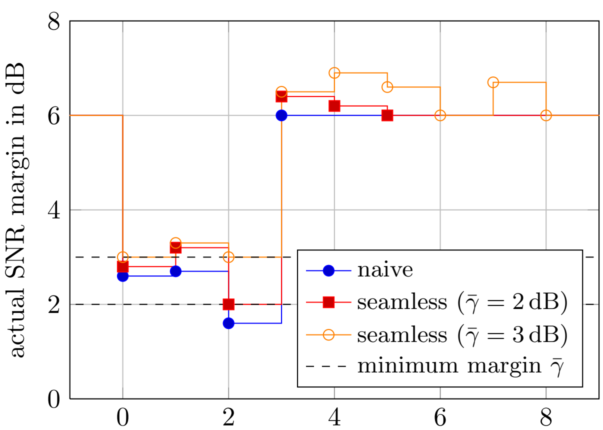
Figure 2 where, for time $t < t_0$, only users 1,3 and 4 use a VDSL2 service and are controlled by an SMC while the VDSL2 service for user 2 is not active yet. The SMC operates in fixed-margin mode, i.e. has computed a joint power allocation $\boldsymbol{s}_{\text{old}}$ for users 1,3 and 4 with minimum sum power $\sum_{n \in \{1,3,4\}} P_n$ while accounting for target rate constraints (5) with $R^{1,\text{target}} = 25\,\text{Mbps}$, $R^{3,\text{target}} = 10\,\text{Mbps}$ and $R^{4,\text{target}} = 10\,\text{Mbps}$ as well as a target SNR margin $\gamma_{\text{target}} = 6\,\text{dB}$.

At $t = t_0$ corresponding to instance $i = 0$, the service for user 2 with $R^{2,\text{target}} = 25\,\text{Mbps}$ is activated the first time so that the SMC has to determine a new optimal joint power allocation $\boldsymbol{s}_{\text{new}}$ for the 4-user DSM system again using a target margin $\gamma_{\text{target}} = 6\,\text{dB}$. The modems are assumed to resynchronize successively in cyclic order $2, 3, 1, 4, 2, 3, \ldots$ at instances $i = 0, 1, 2, \ldots$ where the transmit PSDs are updated in order to eventually converge to the target allocation $\boldsymbol{s}_{\text{new}}$. Two schemes for updating transmit spectra were compared in the simulations: The *naive* approach described in Section 3 and the proposed algorithm for *seamless* update of power allocations where the minimum desired SNR margin $\bar{\gamma}$ was configured to either 2 dB or 3 dB. The remaining important simulation parameters are summarized in Table 1.

Figure 3 shows the resulting actual SNR margin values $\gamma^1(i)$ and $\gamma^4(i)$ of users 1 and 4 during the transition phase. Given an intermediate allocation $\boldsymbol{s}(i)$ at instance i, the value for $\gamma^n(i)$ is obtained by solving eq. (8) e.g. via bisection method. Clearly, one finds that for instances $i = 0, 1, 2$, the actual margin of both users decreases drastically when naively updating power spectra. In contrast, using our proposed seamless scheme, $\gamma^n(i)$ is guaranteed to not fall below the specified minimum margin $\bar{\gamma}$. This comes at the expense of a drop in



(a) user 1 (400 m)



(b) user 4 (1000 m)

Figure 3: Actual SNR margin $\gamma^n(i)$ during transition phase

margin for user 1 at instances $i = 4, 5$ compared to the naive scheme.

Finally, the distances $\Delta(\boldsymbol{s}(i), \boldsymbol{s}_{\text{new}})$ as a function of instance i are depicted in Figure 4 for the different schemes. Clearly, all three curves show the expected behavior of monotonously converging towards a value of 0. As the naive method updates all spectra directly to the new optimal allocation, it converges only after 4 time steps, that is, before the seamless update method. However, we have already seen that this comes at the price of a drastically reduced actual SNR margin. Comparing curves of the seamless method for different values of $\bar{\gamma}$, we find the expected result that a higher minimum SNR margin value tends to increase convergence time.

7. CONCLUSION

In this work, we studied the problem of migrating transmit spectra in a non-static multi-user xDSL system. It was found that using a naive approach can strongly threaten line stability during the transition phase. Therefore, a novel power allocation scheme was proposed to gradually update spectra to the desired new optimal joint power allocation while respecting a given minimum SNR margin at any point in time. The optimization associated with the proposed approach was shown to expose a d.c. structure which allows the solution to be found efficiently based on convex relaxation. It has been shown that choosing a large minimum margin in the shaping of the intermediate spectra generally increases convergence time of the scheme. Due to this trade-off, an interesting issue for future work is how to choose the minimum margin value so that the *loss of margin* probability is minimized.

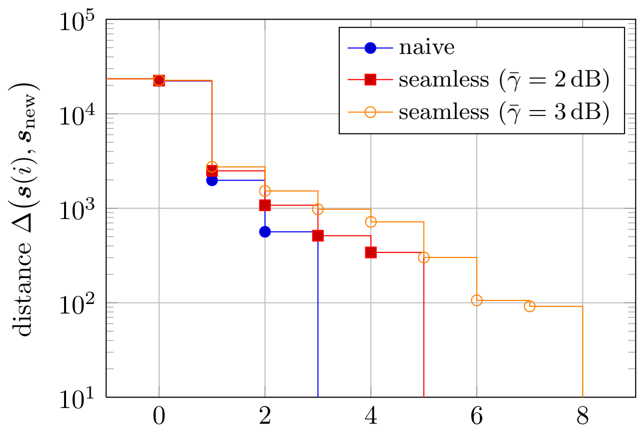


Figure 4: Normalized distance function $\tilde{\Delta}$ as transmit spectra converge towards target power allocation.

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