Hierarchical Beamforming for Downlink Fog RAN

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Abstract—A scenario consisting of several cells is considered where each cell contains a multiple-antenna enhanced remote radio head (eRRH) and several single-antenna mobile stations (MSs). All eRRHs can access the cloud with limited capacity fronthaul links. The beamforming problem aiming at maximizing the sum rate with per eRRH power constraint is considered. In this paper, a hierarchical one-shot top-down approach that decomposes the beamforming problem between the cloud and eRRHs is proposed. The basic idea of our approach is that each eRRH leaves out part of its transmit signal space for minimizing the inter-cell interference to other cells while exploiting the rest of the signal space termed transmission subspace for data transmission. This idea is elaborated in a hierarchical order. First, the cloud, using only the knowledge of link qualities among eRRHs and MSs averaged over all antennas, finds the size of the transmission subspace at every eRRH. Using this information, each eRRH finds the basis of its transmission subspace and correspondingly optimizes its beamforming vectors employing only local channel knowledge. The results show that feeding back only link qualities to the cloud, a significant performance gain can be achieved compared to uncoordinated eRRH based approaches. Furthermore, our proposed approach achieves sum rates close to the global maximum at all signal to noise ratios.

Index Terms—fog RAN, enhanced RRH, hierarchical beamforming, sum rate maximization, limited fronthaul capacity.

I. Introduction

In conventional cellular networks, baseband processing is efficiently performed at the level of base stations (BSs) with an acceptable amount of signaling among the BSs. With the significant increase in user density and traffic demands, it becomes challenging to coordinate many BSs within the limited channel coherence time duration. Thanks to the considerable advances in cloud computing, baseband functions can be flexibly virtualized and delegated to the cloud for central processing, which is known as cloud radio access networks (RAN) [1], [2]. The main challenge cloud RAN technology faces is that the fronthaul links are capacity-limited, and because the baseband processing strongly relies on the precision of the available time varying channel knowledge, the amount of signaling and/or data that can reach the cloud for processing becomes limited. One solution to this issue is by jointly optimizing fronthaul data compression together with beamforming vectors at the cloud [3], [4]. The second solution is by feeding back channel state information (CSI) with different accuracy to the cloud, i.e., instantaneous CSI of the significant links and statistical CSI for the remaining links [5]. Recently, Fog RAN technology has been proposed which offers baseband processing capabilities at both the cloud as well as the eRRHs [6]. This way, the requirement of CSI availability at the cloud is relaxed and the baseband

processing can be performed at both the cloud and at the eRRHs. In [7], the baseband processing is jointly optimized at the cloud and at the eRRHs. In contrast to this, our approach is based on performing baseband processing in a hierarchical way. In particular, the cloud with limited abstract version of the channel knowledge sets constraints for every eRRH. Considering these constraints, every eRRH with only local knowledge can optimize its baseband functions.

To elaborate the concept of hierarchical baseband processing in Fog RAN, this paper focuses on hierarchical beamforming design in multiuser downlink cellular networks. Before explaining the idea of hierarchical beamforming, the structure of the beamforming problem for sum rate maximization will be discussed. In general, beamforming optimization problems aiming at maximizing sum rate with power constraints is non-convex. Accordingly, global optimization methods, such as the branch and bound algorithm, are used to find the globally optimum beamforming vectors [8]. Several suboptimum central algorithms have been proposed for solving the sum rate maximization beamforming problem [9]–[12]. The authors of [9] show that only rank 1 transmit covariance matrices are relevant to achieve a Pareto optimal point in the rate region. Furthermore, the sum rate maximization beamforming problem is formulated as either a weighted mean square error minimization problem, a multiconvex problem or a sequence of second order cone problems in [10], [11] and [12], respectively. Although these formulations lead to tractable iterative algorithms, they require all the instantaneous CSI to be available at a central entity.

Using standard decomposition methods from optimization theory, distributed beamforming algorithms based on primal and dual decomposition are proposed in [13] and [14], respectively. Moreover, the distributed beamforming algorithms with limited eRRH cooperation are proposed in [15] and [16]. The main challenge these distributed algorithms face is that they require frequent signaling updates of the optimized variables among the different eRRHs. The key difficulty which does not make the decomposition straightforward is the inter-cell interference terms in the SINR expression. In other words, each eRRH needs to know the exact design of the beamforming vectors at other eRRHs as well as the CSI of inter-cell interference links to its MSs so that it can optimize its beamforming vectors. One way to overcome this problem is by adding inter-cell interference temperature constraints at every eRRH. This way, the problem becomes naturally decomposed. If the inter-cell interference temperature constraints are a priori known, the problem can be
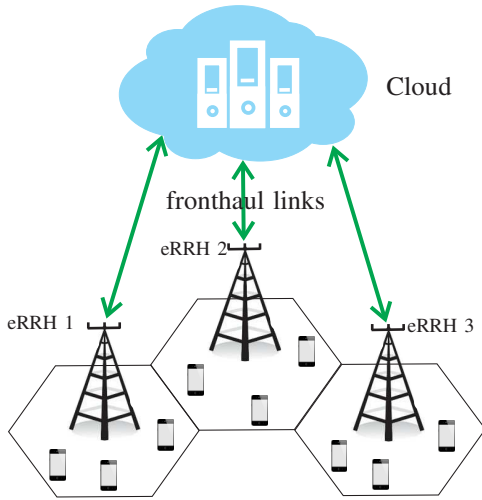
Fig. 1: An Illustration of basic fog RAN architecture.

solved distributedly [17]. Unfortunately, finding the right inter-cell interference temperatures is a non-convex problem and requires complete CSI knowledge [18]. The authors of [19] propose a distributed beamforming algorithm where the inter-cell interference temperature constraints from all eRRHs are jointly updated in a central pool. As a summary, decomposing the downlink beamforming problem for sum rate maximization is non-trivial and requires frequent signaling updates among the optimizing entities.

In the present paper, we propose to exploit the hierarchical architecture of fog RAN and perform part of the beamforming vector design at the cloud level while the remaining part is done at the respective eRRH. To the best of authors' knowledge, hierarchical decomposition of the downlink beamforming problem is not investigated in the literature. The paper is aiming at proposing a hierarchical decomposition approach with a low CSI signaling through the fronthaul links and formulating the proper optimization problems at the cloud and eRRHs. Our approach is a top-down approach such that the cloud will first find a set of constraints for every eRRH using only an abstract version of the channel knowledge. Then, each eRRH designs its beamforming vectors using only local CSI considering the constraints coming from the cloud. It is assumed that eRRHs cannot coordinate directly with each other and the signaling between eRRHs and the cloud is done only once.

The rest of this paper is organized as follows. The next section introduces the system model and signaling between eRRHs and the cloud. In Section III, the problem of downlink beamforming is stated. Our proposed hierarchical approach is explained in Section IV. In Section V, the performance of the proposed approach is discussed. Finally, the conclusions are drawn in Section VI.

II. SYSTEM MODEL AND SIGNALING

A downlink cellular scenario consisting of K cells, illustrated in Fig. 1, is considered. Each cell contains a eRRH with T antennas and M MSs with single antenna each. All eRRHs

have access to the cloud with capacity limited fronthaul links. Throughout the paper, we denote a MS m at the k-th cell with the tuple (k, m). Let Υ, $\mathcal{M}_k$ and $\mathcal{I}_k$ be the set of all MSs in the system, the set of all MSs in the k-th cell and the set of all MSs not in cell k, but with dominant inter-cell interference links to the k-th eRRH, respectively. It is assumed that every MS (k, m) is intended to receive a single data symbol $d_{(k,m)} \in \mathbb{C}$ from the corresponding eRRH k with a unit average data symbol energy $\mathrm{E}\left\{\left|d_{(k,m)}\right|^2\right\} = 1, \forall (k, m) \in \Upsilon$. Accordingly, each eRRH transmits simultaneously M data symbols with $M \leq T$. In this work, we assume that the data is already available at the respective eRRH while the beamforming design is done at eRRHs as well as the cloud.

The channels between the eRRHs and MSs are assumed to be constant throughout the transmission. Let $\mathbf{h}_{(k,m);l} \in \mathbb{C}^{1 \times T}$ be the channel vector between the l-th eRRH and MS (k, m). Moreover, every eRRH k needs to design M beamforming vectors, i.e., beamforming vector $\mathbf{v}_{(k,m)} \in \mathbb{C}^{T \times 1}$ corresponds the m-th data symbol at the k-th eRRH intended for MS (k, m). Then, the received signal at MS (k, m) reads

$$y_{(k,m)} = \mathbf{h}_{(k,m);k}\mathbf{v}_{(k,m)}d_{(k,m)} + \sum_{n \neq m} \mathbf{h}_{(k,m);k}\mathbf{v}_{(k,n)}d_{(k,n)}$$
$$+ \sum_{l \neq k} \sum_{n=1}^{M} \mathbf{h}_{(k,m);l}\mathbf{v}_{(l,n)}d_{(l,n)} + z_{(k,m)}, \quad (1)$$

where $z_{(k,m)}$ denotes the received noise at MS (k, m) which is modeled as independently identically distributed (i.i.d.) Gaussian with zero mean and variance $\sigma_{(k,m)}^2$. The first term of (1) represents the received useful data while the second and third terms of (1) describe the received intra-cell interference and inter-cell interference, respectively. Based on this, the SINR at MS (k, m) is calculated as

$$\gamma_{(k,m)} =$$
$$\frac{\left|\mathbf{h}_{(k,m);k}\mathbf{v}_{(k,m)}\right|^2}{\sigma_{(k,m)}^2 + \sum_{n \neq m} \left|\mathbf{h}_{(k,m);k}\mathbf{v}_{(k,n)}\right|^2 + \sum_{l \neq k} \sum_{n=1}^{M} \left|\mathbf{h}_{(k,m);l}\mathbf{v}_{(l,n)}\right|^2}.$$
$$(2)$$

Furthermore, it is assumed that each eRRH has a total power constraint P.

Concerning the channel knowledge available at eRRHs and the cloud, it is assumed that at the beginning of every coherence time period, a training signaling among the eRRHs and MSs takes place to estimate the channel exploiting the reciprocal property of the downlink-uplink channels assuming a time division duplex (TDD) system. After the training signaling period, every eRRH k knows the instantaneous local CSI perfectly, i.e., $\mathbf{h}_{(k,m);k}, \forall m \in \mathcal{M}_k$, and the instantaneous CSI of the dominant inter-cell interference links to MSs in other cells, i.e., $\mathbf{h}_{(l,n);k}, \forall (l, n) \in \mathcal{I}_k$. In this work, the feedback through the fronthaul links will not be modeled but rather the limited signaling required to be available at the cloud will be assumed and it will be shown how the cloud can exploit this limited signaling. Due to the limited capacity

of the fronthaul links, eRRHs feed back only the link qualities $g_{(k,m);l} = \left\| \mathbf{h}_{(k,m);l} \right\|^2$, $\forall (k,m), l$ to the cloud. This means, the cloud is aware of the average channel gain over antennas between every eRRH and MS, but it does not know more details such as phase information or direction of the channel vectors. It is interesting to point out here that this way, the amount of signaling fed back to the cloud is reduced by a factor of $\frac{1}{2T}$ as compared to feeding back all CSI since every channel vector has T complex elements.

III. PROBLEM STATEMENT

The goal of this work is to design the beamforming vectors at all eRRHs aiming at maximizing the sum rate with a per eRRH power constraint. Mathematically, the optimization problem can be stated as

$$\left(\mathbf{v}_{(k,m)}^{\text{ub}} \right)_{(k,m) \in \Upsilon} =$$

$$\underset{\left\{ \mathbf{v}_{(k,m)} \right\}_{(k,m) \in \Upsilon}}{\operatorname{argmax}} \left\{ \sum_{k=1}^{K} \sum_{m=1}^{M} \log_2 \left(1 + \gamma_{(k,m)} \right) \right\} \quad (3)$$

subject to

$$\sum_{m=1}^{M} \left\| \mathbf{v}_{(k,m)} \right\|^2 \leq P, \quad \forall k. \quad (4)$$

This problem is non-convex. However, many algorithms have been proposed for finding optimal and suboptimal solutions [8]–[12]. Unfortunately, the problem of (3)–(4) cannot be decomposed among eRRHs because of the inter-cell interference terms, see (2). In other words, all eRRHs need to design their beamforming vectors jointly to find the optimum amount of inter-cell interference they should allow so that the maximum sum rate in the whole system is achieved.

IV. HIERARCHICAL BEAMFORMING

A. Key Idea

In this section, it will be shown that the hierarchical architecture of the considered scenario can be exploited to design the beamforming vectors using only the available signaling information in the cloud and eRRHs as explained in Section II. Basically, the problem of (3)–(4) is sub-optimally decomposed among eRRHs where the inter-cell interference term in the objective is eliminated and substituted by a set of zero forcing constraints. In other words, every eRRH k splits its T-dimensional signal space into two subspaces: First, a t_k-dimensional subspace which is employed for data transmission which is termed transmission subspace. The remaining $(T-t_k)$ dimensions will be unoccupied for the sake of inter-cell interference minimization at other cells and this subspace will be termed unoccupied subspace. In the next section, the best sizes of the transmission subspaces at all eRRHs will be derived. In Section IV-C, the zero forcing constraints which define the unoccupied subspace of every eRRH are analyzed. Finally, the decomposed beamforming problem with zero forcing constraints at every eRRH is stated in Section IV-D.

B. Finding Transmission Subspace Sizes

As mentioned in Section II, the cloud knows only link qualities $g_{(k,m);l}$, $\forall (k,m), l$. Using this information, it will optimize the size of the transmission subspace of every eRRH. In the following, we will explain how the cloud can find the optimum size of the transmission subspaces for maximizing the system sum rate given only link qualities. Based on the system model, the received power at MS (k,m) resulting by transmitting the n-th data symbol from eRRH l is calculated as

$$p_{(k,m);l} = \left| \mathbf{h}_{(k,m);l} \mathbf{v}_{(l,n)} \right|^2. \quad (5)$$

Based on the selection of k, l, m and n, this received power can be considered as useful power for $l = k$, $n = m$, intra-cell interference power for $l = k$, $n \neq m$ or inter-cell interference power for $l \neq k$, $n \neq m$. For the l-th eRRH signal space, let $\mathbf{A}_l \in \mathbb{C}^{T \times (T-t_l)}$ be a matrix whose column space spans the unoccupied subspace. Then, the projection matrix Π_l into the t_l-dimensional transmission subspace is calculated as

$$\Pi_l = \mathbf{I}_T - \mathbf{A}_l \left(\mathbf{A}_l^{\text{H}} \mathbf{A}_l \right)^{-1} \mathbf{A}_l^{\text{H}}, \quad (6)$$

where $\mathbf{I}_T$ is an identity matrix of size $T \times T$. Furthermore, the received power calculated in (5) can be written as

$$p_{(k,m);l}^{\text{proj}} = \left| \mathbf{h}_{(k,m);l} \Pi_l \mathbf{v}_{(l,n)} \right|^2 = \left| \operatorname{tr} \left(\Pi_l \mathbf{v}_{(l,n)} \mathbf{h}_{(k,m);l} \right) \right|^2. \quad (7)$$

Using Cauchy-Schwarz inequality and the fact that the projection matrix is idempotent, i.e., $\Pi_l = \Pi_l^2$, the received power of (7) is upper bounded as

$$p_{(k,m);l}^{\text{proj}} \leq \operatorname{tr} (\Pi_l) \operatorname{tr} \left(\mathbf{v}_{(l,n)} \mathbf{h}_{(k,m);l} \mathbf{h}_{(k,m);l}^{\text{H}} \mathbf{v}_{(l,n)}^{\text{H}} \right)$$

$$\leq \operatorname{tr} (\Pi_l) \operatorname{tr} \left(\mathbf{v}_{(l,n)} \mathbf{v}_{(l,n)}^{\text{H}} \right) g_{(k,m);l}, \quad (8)$$

where the term $\mathbf{h}_{(k,m);l} \mathbf{h}_{(k,m);l}^{\text{H}}$ is scalar and equals $g_{(k,m);l}$. As the transmit power is upper bounded by $\operatorname{tr} \left(\mathbf{v}_{(l,n)} \mathbf{v}_{(l,n)}^{\text{H}} \right) \leq P$, the received power at MS (k,m) is upper bounded by

$$p_{(k,m);l}^{\text{proj}} \leq \operatorname{tr} (\Pi_l) P g_{(k,m);l}, \quad (9)$$

with equality when the l-th eRRH matches its beamforming vector to the projected channel vector, i.e., $\mathbf{v}_{(l,n)} = \Pi_l \mathbf{h}_{(k,m);l}^{\text{H}}$ and transmits with maximum power P. Based on idempotent matrix properties, the size of the subspace that this matrix projects into is

$$t_l = \operatorname{rank} (\Pi_l) = \operatorname{tr} (\Pi_l). \quad (10)$$

Therefore, (9) can be rewritten as

$$p_{(k,m);l}^{\text{proj}} \leq t_l P g_{(k,m);l}, \quad (11)$$

where (11) in case of equality describes the highest power that can be received if all the t_l-dimensions are employed for serving/interfering only MS (k,m). Because a eRRH serves multiple MSs, its transmission subspace needs to be split further into at most M subspaces. Let $\alpha_{(k,m);l}$ be the size of the signal subspace reserved for the channel link between the l-th eRRH and MS (k,m). Accordingly, the dimensions of

subspaces reserved for each MS in the l-th cell are constrained as

$$\sum_{m=1}^{M} \alpha_{(l,m);l} = t_l. \tag{12}$$

Because the links are either useful or interference links, two cases can be distinguished:

- Useful link $l = k$: As a single data symbol is intended for every MS, a one-dimensional subspace is sufficient to serve MS (k,m). However, larger dimensions can be utilized for increasing the transmission diversity at this MS. Accordingly, the possible values of the size $\alpha_{(k,m);k}$ of signal subspace reserved for the useful link between eRRH k and MS (k,m) is $\alpha_{(k,m);k} \in \{0, 1, \ldots, T\}$. Accordingly, the received useful power can be approximated as

$$p_{(k,m);k}^{\text{proj}} \approx \alpha_{(k,m);k} P g_{(k,m);k}. \tag{13}$$

- Interference link $l \neq k$: Because MSs are equipped with a single antenna each, each inter-cell interference link can span only a single-dimensional subspace of the T-dimensional signal space at the l-th eRRH. Hence, a single-dimensional subspace is needed to zero force an inter-cell interference link. However, the inter-cell interference from a eRRH can be ignored or mitigated if it is weak as compared to the useful signal strength. Therefore, the possible values of the size $\alpha_{(k,m);l}$ of the subspace reserved for the interference link between eRRH l and MS (k,m) is $0 \leq \alpha_{(k,m);l} \leq 1$, $\forall l \neq k$. This means that the received inter-cell interference power can be written as

$$p_{(k,m);l}^{\text{proj}} \approx \left(1 - \alpha_{(k,m);l}\right) P g_{(k,m);l}, \quad \forall l \neq k. \tag{14}$$

Note that intra-cell interference is not considered at the cloud level, because the cloud has only link qualities and not channel vectors. Thus, the cloud reserves different subspaces for different MSs in the cell such that the intra-cell interference is avoided. Nevertheless, the cloud provides at the end the total size t_k of the transmission subspace to the k-th eRRH rather than the sizes $\alpha_{(k,m);k}$ of reserved subspaces of the individual useful links. Therefore, the k-th eRRH can later on optimize its beamforming vectors using local CSI which may require allowing some intra-cell interference such that its sum rate is maximized, i.e., at low signal to noise ratios (SNRs).

Based on this analysis, the subspace sizes reserved for each link in the system are optimized as

$$\left(\alpha_{(k,m);l}^{\text{opt}}\right)_{\forall (k,m),l} = \underset{\left\{\alpha_{(k,m);l}\right\}_{\forall (k,m),l}}{\operatorname{argmax}}$$

$$\left\{ \sum_{k=1}^{K} \sum_{m=1}^{M} \log_2 \left(1 + \frac{\alpha_{(k,m);k} P g_{(k,m);k}}{\sigma_{(k,m)}^2 + \sum_{l \neq k} \left(1 - \alpha_{(k,m);l}\right) P g_{(k,m);l}} \right) \right\} \tag{15}$$

subject to

$$\sum_{(k,m)\in\Upsilon} \alpha_{(k,m);l} = T, \quad \forall l, \tag{16}$$

$$\alpha_{(k,m);k} \in \{0, 1, \ldots, T\}, \quad \forall k, m, \tag{17}$$

and

$$0 \leq \alpha_{(k,m);l} \leq 1, \quad \forall k, l, m, \quad l \neq k. \tag{18}$$

The optimization problem of (15)–(18) is a mixed integer non-linear program which can be solved using standard global optimization methods [20]–[22]. From the solution of (15)–(18), the optimum size of the transmission subspace at the k-th eRRH is calculated as

$$t_k = \sum_{m=1}^{M} \alpha_{(k,m);k}^{\text{opt}}. \tag{19}$$

Finally, the sizes of transmission subspaces t_k, $\forall k$ are forwarded to the corresponding eRRHs.

C. Finding the Basis of the Unoccupied Subspaces

In this section, every eRRH l, using the size t_l of its transmission subspace provided by the cloud, will find the basis of the subspace of size $T - t_l$ within the T-dimensional signal space, which will not be occupied for data transmission. Recalling matrix $\mathbf{A}_l$ whose column space spans the unoccupied subspace of eRRH l introduced in Section IV-B and considering a single inter-cell interference channel $\mathbf{h}_{(k,m);l}$ between eRRH l and MS (k,m) with $l \neq k$, the amount of the inter-cell interference power which results by projecting $\mathbf{h}_{(k,m);l}$ onto the unoccupied subspace with basis described by $\mathbf{A}_l$ is $\mathbf{h}_{(k,m);l} \mathbf{A}_l \mathbf{A}_l^{\text{H}} \mathbf{h}_{(k,m);l}^{\text{H}}$. As the cloud asks every eRRH l to leave out a $(T - t_l)$-dimensional subspace unoccupied from data transmission, the best a eRRH l can do is to select a subspace where it produces the highest inter-cell interference to the system. Considering all the dominant inter-cell interference links, matrix $\mathbf{A}_l$ can be optimized for maximizing the sum inter-cell interference as

$$\left(\mathbf{A}_l^{\text{opt}}\right) = \underset{\mathbf{A}_l}{\operatorname{argmax}} \left\{ \sum_{(k,m)\in\mathcal{I}_l} \mathbf{h}_{(k,m);l} \mathbf{A}_l \mathbf{A}_l^{\text{H}} \mathbf{h}_{(k,m);l}^{\text{H}} \right\}. \tag{20}$$

This problem finds a $(T - t_l)$-dimensional subspace in which the highest amount of inter-cell interference from eRRH l resulted if the interference channel vectors $\mathbf{h}_{(k,m);l}$, $\forall (k,m) \in \mathcal{I}_l$ projected onto this subspace. Let $\mathbf{H}_l$ be a $(|\mathcal{I}_l| \times T)$-dimensional matrix with rows being the inter-cell interference channel vectors $\mathbf{h}_{(k,m);l}$, $\forall (k,m) \in \mathcal{I}_l$ to the l-th eRRH. Then, the objective function in (20) can be rewritten as

$$\sum_{(k,m)\in\mathcal{I}_l} \mathbf{h}_{(k,m);l} \mathbf{A}_l \mathbf{A}_l^{\text{H}} \mathbf{h}_{(k,m);l}^{\text{H}} = \operatorname{tr}\left(\mathbf{H}_l \mathbf{A}_l \mathbf{A}_l^{\text{H}} \mathbf{H}_l^{\text{H}}\right)$$

$$= \|\mathbf{H}_l \mathbf{A}_l\|_{\text{F}}^2, \tag{21}$$

where $\|.\|_{\text{F}}^2$ is the Frobenius norm of a matrix. Accordingly, the optimization problem of (20) can be rewritten as

$$\left(\mathbf{A}_l^{\text{opt}}\right) = \underset{\mathbf{A}_l}{\operatorname{argmax}} \left\{ \|\mathbf{H}_l \mathbf{A}_l\|_{\text{F}}^2 \right\}. \tag{22}$$

The columns of the optimum matrix $\mathbf{A}_l^{\text{opt}}$ are the eigenvectors of $\mathbf{H}_l$ corresponding to the highest $(T - t_l)$ eigenvalues of
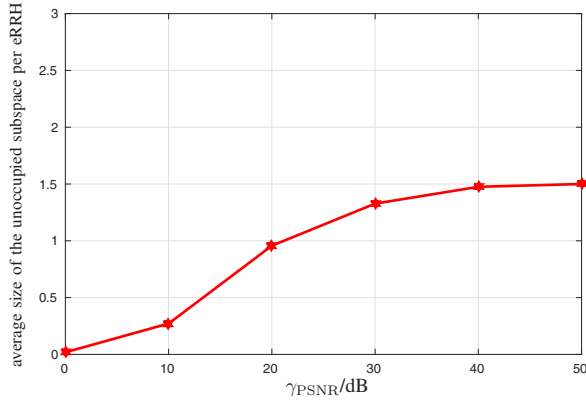
Fig. 2: Average size of the unoccupied subspaces per eRRH.

$\mathbf{H}_l$. In this way, eRRH l can design its beamforming vectors using local CSI considering that the optimum vectors have to be orthogonal to the unoccupied subspace spanned by the columns of $\mathbf{A}_l^{\text{opt}}$.

D. Local Beamforming with Zero Forcing constraints

In this section, the problem of optimizing the beamforming vectors at every eRRH l is formulated. As described in the previous sections, the beamforming vector optimization problem stated in Section III is decomposed over eRRHs. Each eRRH will not consider the inter-cell interference terms at the objective function, i.e., sum rate function, but instead, it will leave out a $(T-t_l)$-dimensional subspace where the l-th eRRH can produce the highest inter-cell interference to other cells. Mathematically, the optimization problem can be stated as

$$\left(\mathbf{v}_{(k,m)}\right)_{m\in\mathcal{M}_k} = \operatorname*{argmax}_{\left\{\mathbf{v}_{(k,m)}\right\}_{m\in\mathcal{M}_k}}$$

$$\left\{ \sum_{m=1}^{M} \log_2 \left(1 + \frac{\left|\mathbf{h}_{(k,m);k}\mathbf{v}_{(k,m)}\right|^2}{\sigma_{(k,m)}^2 + \sum_{n\neq m} \left|\mathbf{h}_{(k,m);k}\mathbf{v}_{(k,n)}\right|^2} \right) \right\} \quad (23)$$

subject to

$$\sum_{m=1}^{M} \left\|\mathbf{v}_{(k,m)}\right\|^2 \leq P. \quad (24)$$

and

$$\left(\mathbf{A}_k^{\text{opt}}\right)^{\text{H}} \mathbf{v}_{(k,m)} = \mathbf{0}, \quad \forall m. \quad (25)$$

This problem has the same structure as the problem of (3)–(4) except the additional affine set of zero-forcing constraints of (25). It is also non-convex. However, many off-the-shelf beamforming algorithms can be adopted to solve the optimization problem of (23)–(25).

It is worth to mention that in this paper, we are not after proposing new algorithms for solving the optimization problems of (15)–(18) and (23)–(25). Rather, formulating and characterizing these problems are our target as the proposed formulations are similar to well known optimization problems and existing solvers can be used.

V. NUMERICAL RESULTS

In this section, the performance of the proposed hierarchical beamforming approach is investigated as a function of the pseudo SNR γ_{PSNR} which is defined as the ratio of the total transmit power over the noise variance $\gamma_{\text{PSNR}} = \frac{KP}{\sigma^2}$ with $\sigma^2 = \sigma_{(k,m)}^2$, $\forall(k,m) \in \Upsilon$. The sum rate, in bits per channel use, is chosen as a performance metric. A $(K = 2)$-cell downlink scenario is investigated. Each cell has a central eRRH with $T = 3$ antennas and $M = 2$ MSs randomly positioned with a uniform distribution. Note that the number of antennas is carefully selected such that the optimum solution is not trivial, e.g., simple transmit zero forcing is optimum at high SNRs if the number of antennas is large enough. A dual slope pathloss model with a break-point at cell border is considered where the attenuation exponent of the first and second segments are set to 2 and 4, respectively. I.i.d. complex Gaussian channels with average channel gain being normalized to one are assumed. For all the simulations, the optimization problems are solved using Baron global optimization solver [22], [23].

Fig. 2 shows the average size $T - \hat{t}_l$ of the unoccupied subspace out of $T = 3$ dimensions per eRRH as a function of the pseudo SNR. At low pseudo SNR regime, the inter-cell interference is weak and the noise is dominant in this regime. Therefore, every eRRH employs all its signal space for data transmission. As the pseudo SNR increases, the inter-cell interference becomes significant and less dimensions of the signal space of each eRRH will be exploited for data transmission. At high pseudo SNRs, only one half of the dimensions, i.e., $\hat{t}_l = 1.5$ dimensions out of $T = 3$-dimensional signal space, will be used for transmission because the scenario is symmetric and there are $K = 2$ eRRHs.

In the following, the performance of our proposed approach is assessed and compared with three different reference approaches. Firstly, the central beamforming approach which uses all CSI and is simply the solution of the optimization problem of (3)–(4). It can be noted that this approach is impractical but it can serve as an upper bound. Secondly, the local beamforming approach where every eRRH designs its beamforming vectors independently using only local CSI, i.e., ignoring the inter-cell interference links. This approach can be achieved by solving the optimization problem of (23)–(24) without the zero forcing constraints of (25) for every eRRH. Finally, the zero forcing approach where each eRRH transmits orthogonal to the two-dimensional subspace spanned by the channels towards the two MSs in the other cell. It can be pointed out here that the central beamforming approach assumes full cooperation among eRRHs whereas the local beamforming and the zero forcing approaches assume no cooperation at all among eRRHs.

In Fig. 3, the performances of the proposed approach and the reference approaches are compared. It can be seen that the central beamforming achieves the maximum sum rate for all pseudo SNRs as it reaches the global maximum sum rates. Furthermore, the local beamforming approach achieves sum
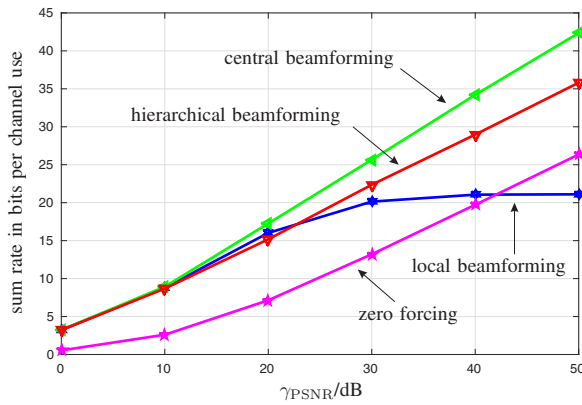
Fig. 3: Achieved sum rates as a function of pseudo SNRs.

rates close to the global optimum only at low pseudo SNRs where the noise is dominant. However, this approach saturates at high pseudo SNRs because the inter-cell interference becomes significant but this approach does not consider the inter-cell interference at all. Concerning the zero forcing approach where each eRRH left out a two-dimensional subspace of its three-dimensional signal space and exploits only a one-dimensional subspace for serving only one of its MSs, it performs poorly at all pseudo SNRs as compared to the global optimum. Finally, our proposed approach termed hierarchical beamforming performs better than the uncoordinated eRRHs based approaches at all pseudo SNRs. Also, our approach achieves sum rates close to the global maximum at all γ_{PSNR}.

VI. CONCLUSION

In this paper, it is shown that the hierarchical architecture of the future cellular systems can be exploited for baseband processing. As a case study, we investigate designing the downlink beamforming problem in hierarchical order, namely at the cloud and eRRHs. The main idea of our approach is that the central beamforming problem is decomposed per eRRH by considering zero forcing constraints instead of the inter-cell interference term. In other words, each eRRH will transmit in a part of its signal space and leave out the other part of its signal space unoccupied such that the inter-cell interference is minimized. The cloud finds the size of the subspaces at every eRRH and forwards this information to the respective eRRH. Afterwards, each eRRH independently finds the basis of the unoccupied subspace and optimizes its beamforming vectors, constrained to be orthogonal to the unoccupied subspace and considering only local CSI. The results show a performance gain of our proposed approach over the uncoordinated eRRH based approaches and a performance close the upper bound.

REFERENCES

[1] A. Checko, H. Christiansen, Y. Yan, L. Scolari, G. Kardaras, M. Berger, and L. Dittmann, "Cloud RAN for mobile networks– a technology overview," *IEEE Communications Surveys and Tutorials*, vol. 17, no. 1, pp. 405–426, First Quarter 2015.

[2] J. Liu, S. Xu, S. Zhou, and Z. Niu, "Redesigning fronthaul for next-generation networks: Beyond baseband samples and point-to-point links," *IEEE Wireless Communications*, vol. 22, no. 5, pp. 90–97, Oct 2015.

[3] S.-H. Park, O. Simeone, O. Sahin, and S. Shamai, "Inter-cluster design of precoding and fronthaul compression for cloud radio access networks," *IEEE Wireless Communications Letters*, vol. 3, no. 4, pp. 369–372, Aug 2014.

[4] J. Kang, O. Simeone, J. Kang, and S. Shamai, "Joint precoding and fronthaul optimization for C-RANs in ergodic fading channels," in *Proc. IEEE International Conference on Communication Workshop*, London, June 2015, pp. 2683–2688.

[5] Y. Shi, J. Zhang, and K. Bin Lataief, "CSI overhead reduction with stochastic beamforming for cloud radio access networks," in *Proc. IEEE International Conference on Communications*, Sydney, June 2014, pp. 5154–5159.

[6] M. Peng, S. Yan, K. Zhang, and C. Wang, "Fog-computing-based radio access networks: Issues and challenges," *IEEE Network*, vol. 30, no. 4, pp. 46–53, July 2016.

[7] S. Park, O. Simeone, and S. Shamai, "Joint optimization of cloud and edge processing for fog radio access networks," *IEEE Transactions on Wireless Communications*, vol. 15, no. 11, pp. 7621–7632, Nov 2016.

[8] S. Joshi, P. Weeraddana, M. Codreanu, and M. Latva-aho, "Weighted sum-rate maximization for MISO downlink cellular networks via branch and bound," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 2090–2095, April 2012.

[9] R. Mochaourab and E. Jorswieck, "Optimal beamforming in interference networks with perfect local channel information," *IEEE Transactions on Signal Processing*, vol. 59, no. 3, pp. 1128–1141, Mar 2011.

[10] Q. Shi, M. Razaviyayn, Z. Luo, and C. He, "An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broacast channel," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4331–4340, Sep 2011.

[11] H. Al-Shatri, X. Li, R. S. Genesan, A. Klein, and T. Weber, "Maximizing the sum rate in cellular networks using multiconvex optimization," *IEEE Transactions on Wireless Communications*, vol. 15, no. 5, pp. 3199–3211, May 2016.

[12] L. Tran, M. F. Hanif, A. Tölli, and M. Juntti, "Fast converging algorithm for weighted sum rate maximization in multicell MISO downlink," *IEEE Signal Processing Letters*, vol. 19, no. 12, pp. 872–875, Dec 2012.

[13] H. Pennanen, A. Tölli, and M. Latva-aho, "Decentralized coordinated downlink beamforming via primal decomposition," *IEEE Signal Processing Letters*, vol. 18, no. 11, pp. 647–650, Nov 2011.

[14] A. Tölli, H. Pennanen, and P. Komulainen, "Decentralized minimum power multi-cell beamforming with limited backhaul signaling," *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 570–580, Feb 2011.

[15] J. Qiu, R. Zhang, Z. Luo, and S. Cui, "Optimal distributed beamforming for MISO interference channels," *IEEE Transactions on Signal Processing*, vol. 59, no. 11, pp. 5638–5643, Nov 2011.

[16] Y. Huang, G. Zheng, M. Bengtsson, K. Wong, L. Yang, and B. Ottersten, "Distributed multicell beamforming with limited intercell coordination," *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 728–738, Feb 2011.

[17] H. Huh, H. Papadopoulos, and G. Caire, "Multiuser MISO transmitter optimization for intercell interference mitigation," *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4272–4285, Aug 2010.

[18] A. Dotzler, W. Utschik, and G. Dietl, "Efficient zero-forcing based interference coordination for MISO networks," in *Proc. IEEE 73rd Vehicular Technology Conference*, Budapest, May 2011, pp. 1–5.

[19] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 10, pp. 5450–5458, Oct 2010.

[20] A. Philipp, "Mixed-integer nonlinear programming with application to wireless communication systems," PhD Thesis, Technische Universität Darmstadt, Nov 2015.

[21] R. Burkard, *Combinatorial and Global Optimization*, P. Pardalos, A. Migdalas, and R. E. Burkard, Eds. World Scientific Publishing Company, April 2002.

[22] M. Tawarmalani and N. V. Sahinidis, "A polyhedral branch-and-cut approach to global optimization," *Mathematical Programming*, vol. 103, no. 2, pp. 225–249, May 2005.

[23] N. V. Sahinidis, *BARON 14.3.1: Global Optimization of Mixed-Integer Nonlinear Programs,* User's Manual, 2014.