# Multicast Interference Alignment in a Multi-Group Multi-Way Relaying Network

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***Abstract*—In this paper, a multi-group multi-way relaying network is considered. Multiple half-duplex nodes form a group and each node wants to share its message with all other nodes in its group via an intermediate half-duplex relay. The nodes of the whole network are equally distributed over the groups. In this paper, we propose a multicast interference alignment algorithm for a multi-group multi-way relay network, in which the minimum required number of antennas at the relay is independent of the number of nodes per group. This is an important property, because physical antenna resources are limited in general. In order to achieve this, we consider a transmission scheme with several multiple access phases and several multicast phases. In each of the multicast phases we create a MIMO interference multicast channel, by separating the antennas of the relay into clusters. Each of these clusters serves a specific group of nodes and transmits in such a way that the signals transmitted from different clusters are aligned at the non-intended multicast groups. It is shown that the proposed multicast algorithm outperforms a reference algorithm from the literature for a broad range of Signal-to-Noise Ratio (SNR) values, while still requiring less antennas at the relay.**

## I. Introduction

In recent years, applications like video conferences, area monitoring, health care monitoring, text based chats and multi-player gaming are becoming more popular in our daily life. In such applications, multiple nodes form a group. Within a group, each node wants to share its message with all other nodes in its group. In its general form, such networks consist of multiple groups containing an arbitrary number of nodes each. The assumption that all nodes inside a group are connected via direct links is not realistic due to physical propagation phenomena, e.g., path loss or shadowing. Hence, we consider a network topology in which the whole communication takes place via an intermediate relay. This leads to a multi-way relay network as introduced in [1]. A full-duplex communication, where full-duplex nodes communicate with each other through an intermediate full-duplex relay is considered in [1]. However, half-duplex devices are more realistic than full-duplex devices [2]. Hence, in this paper we focus on half-duplex nodes and a half-duplex amplify-and-forward relay.

In a multi-group multi-way relaying network, the nodes transmit their data in several multiple access (MAC) phases to the relay. The relay broadcasts processed versions of this received data to all nodes in the different groups in several broadcast (BC) phases. An upper bound for the degrees of freedom (DoF) of a multi-group multi-way relay network was derived in [3], where the DoF characterize the interference-free signal dimensions, or the prelog factor of the capacity. The derived conditions in [3] show that the number of required antennas at the relay increases with the number of nodes per group. Interference alignment (IA) algorithms can maximize the achievable DoF in multi-user networks [4]. The key idea of IA is to design the transmit filters in such a way that all interference signals are aligned in the smallest number of time/frequency/space dimensions at the receiver [5]. This maximizes the number of independent data streams which can be transmitted. A special case of a multi-way relay network is a two-way relay network, investigated in [6] and the references therein. Such a two-way relaying network consists of several groups containing only two nodes each.

An algorithm performing IA in a multi-way relaying network which maximizes the DoF was proposed in [7]. In [7], all nodes are equally distributed over multiple groups transmitting simultaneously to the relay in one MAC phase. Then, the relay broadcasts multiple linearly processed versions of the received signal in the MAC phase to the nodes in several BC phases. The minimum required number of antennas at the relay derived in [7] scales linearly with the number of nodes per group and the number of groups. For the mentioned multi-group applications, the algorithms proposed in [3] and [7] are not really practical due to the increase of the required number of antennas at the relay with the number of nodes per group.

In this paper, we propose a more scalable solution to deal with an increased number of nodes per group. A broadcast to all nodes inside a group in each BC phase is not necessary, because each node knows its own transmitted data stream, i.e., this node would receive only self-interference. In order to mitigate this self-interference in this paper we replace the BC phases by multicast (MC) phases. IA in a multiple input multiple output (MIMO) interference multicast network was investigated in [8]. The authors of [8] proposed a framework minimizing the required CSI feedback dimensions subject to IA feasibility constraints. Furthermore, they derived the feasibility conditions for an IA multicast network. The feasibility conditions are topology specific [8]–[10], hence the feasibility conditions of a multicast network derived in [8] are different from that of a MIMO interference network derived in [10] or from a two-way relaying network derived in [11]. Since all nodes inside a group want to receive the data from all other nodes in their group after a certain number of MC phases, the amount of alignment constraints is less than for other topologies.

In this paper, we propose a multicast algorithm for a multi-group multi-way relay network, in which the minimum required number of antennas at the relay is independent of the number of nodes per group. In order to achieve this, we consider a transmission scheme with multiple MAC phases and multiple MC phases. The idea behind this algorithm is, that in each of the MC phases, we create a MIMO interference multicast channel by dividing the number of antennas at the relay into as many clusters as groups in the network. Each of these clusters serves a specific group of nodes and transmits its signals such that they are aligned at the non-intended multicast group. The great advantage of the proposed algorithm is that one can easily increase the number of nodes per group without changing physical components, i.e., without changing the number of antennas.

The present paper is organized as follows: Section II introduces the system model of the considered multi-group multi-way relaying network with multicast IA. In Section III, the proposed IA multicast algorithm is presented. In Section IV, the performance of the proposed algorithm is investigated. Section V concludes this paper.

***Notation:*** In the following, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $\mathbb{C}$ represents the set of complex numbers. $(.)^{\mathrm{H}}$, $(.)^{-1}$ denote the complex conjugate transpose and the inverse of the element inside the brackets, respectively. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix. The Frobenious norm of $\mathbf{A}$ is denoted by $\|\mathbf{A}\|_{\mathrm{F}} = \sqrt{\mathrm{tr}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)}$. The trace of a matrix is denoted by $\mathrm{tr}(.)$. $\mathbb{E}[.]$ denotes the expectation of the element inside the brackets. $\nu_{\min,d}(.)$ denotes an operation delivering a matrix containing the eigenvectors corresponding to the $d$ smallest eigenvalues of the matrix within the brackets, as its columns.

## II. System Model

In this paper, we consider a multi-group multi-way relay network [1] consisting of $L \geq 1$ groups, as shown in Figure 1. Each of the $L$ groups contains $K \geq 2$ multi-antenna half-duplex nodes. Let $l \in \mathcal{L} = \{1, ..., L\}$ denote the group index and $k \in \mathcal{K} = \{1, ..., K\}$ the node index, respectively. The $k$-th node in the $l$-th group is denoted by $(l, k)$ and is equipped with $N$ antennas. Each of the $K$ nodes in the $l$-th group wants to share $d \leq N$ data streams with the $K-1$ other nodes in its group. Nodes cannot overhear the data streams transmitted by other nodes, i.e., there are no direct links between the nodes themselves. Hence, the communication between the nodes inside a group takes place via an intermediate amplify-and-forward half-duplex relay equipped with $R$ antennas. All $LK$ nodes in the entire network are connected to this intermediate relay.

To exchange information between the $K$ nodes in each group in a bidirectional manner, we consider a transmission scheme with $M$ multiple access (MAC) phases and $K$ multicast (MC) phases. $K$ MC phases are required, because $Kd$ data streams must be exchanged, omitting self-interference, in each group. In each of the $K$ MC phases, $K-1$ nodes per group receive the data stream previously transmitted from the node which is not served in this MC phase. Hence, the relay needs estimates of all data streams after $M$ MAC phases, in order to multicast a specific data stream only to nodes which want to receive this data stream. For the MAC phase, we assume that at least one node of each group is transmitting $d$ data streams in each MAC phase. Due to this assumption and the condition that the relay has to separate all received data streams, the minimum required number of antennas at the relay is given by $R \geq Ld$.

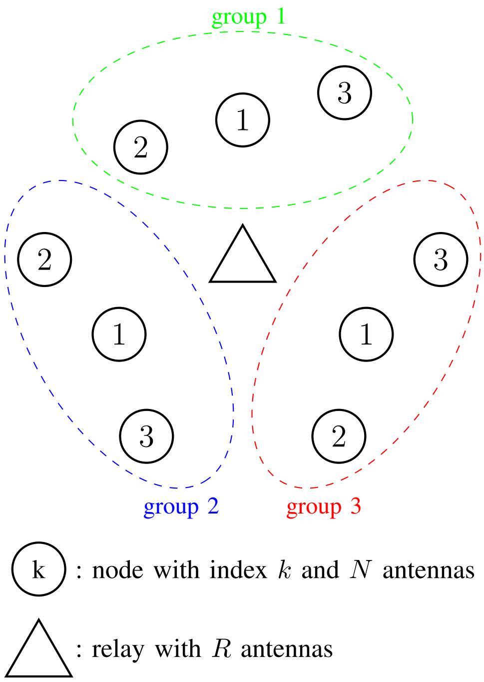


Fig. 1. Multi-group multi-way relay network with $L = 3$ groups and $K = 3$ nodes per group

In each MAC-phase $m = 1, ..., M$, $K_{\mathrm{MAC}} = \frac{R}{Ld}$ nodes per group can be active and transmit data to the relay. Consequently, the relay receives $K_{\mathrm{MAC}}L$ data streams in each MAC-phase. This leads to a total number of $M = \frac{KLd}{R}$ required MAC phases. In a special case, where $R = Ld$, only one node per group transmits in each of the MAC phases to the relay. In this special case, $M = K$ holds. Let $\mathcal{K}_{l,m}$ denote the set of nodes selected for the transmission in group $l$ in phase $m$. In each MC phase $b = 1, ..., K$, we create a MIMO interference multicast channel, by separating the $R$ antennas of the relay into $L$ clusters, each one serving a specific group of nodes and aligning the interference generated to the other groups. The relay multicasts a linearly processed version of the signal received in the MAC phases to $K-1$ nodes per group which want to receive this signal, in each MC phase, i.e., no node receives self-interference.

Let $\mathbf{H}_{l,k}^{\mathrm{m}} \in \mathbb{C}^{R \times N}$ and $\mathbf{H}_{l,k}^{\mathrm{b}} \in \mathbb{C}^{N \times R}$ denote the MIMO channel matrix between node $(l, k)$ and the relay during the MAC phases and between the relay and node $(l,k)$ in the MC phases, respectively. The channels are assumed to be constant over the $M$ MAC phases and over the $K$ MC phases. Furthermore, global channel state information (CSI) is assumed to be available at the nodes and the relay.

Let $\mathbf{d}_{l,k} \in \mathbb{C}^{d\times 1}$ and $\mathbf{V}_{l,k} \in \mathbb{C}^{N\times d}$ denote the data vector originating of node $(l,k)$ and its precoding matrix, respectively. It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_{l,k}\mathbf{d}_{l,k}^{\mathrm{H}}] = \mathbf{I}_d$, $\forall k \in \mathcal{K}$ and $\forall l \in \mathcal{L}$ holds. Each of the nodes has a maximum transmit power denoted by $P_{\text{n,max}}$. To satisfy the maximum transmit power constraint, the precoders are normalized, i.e., $\|\mathbf{V}_{l,k}\|_F^2 \leq P_{\text{n,max}}$. Further, $\mathbf{n}_{\mathrm{r},m} = \mathcal{CN}(0,\sigma_{\mathrm{r},m}^2) \in \mathbb{C}^{R\times 1}$ denotes the noise at the relay in phase $m$ and $\mathbf{n}_{l,k,b} = \mathcal{CN}(0,\sigma_{l,k}^2) \in \mathbb{C}^{d\times N}$ denotes the noise at node $(l,k)$ in phase $b$. The components of the noise vectors $\mathbf{n}_{\mathrm{r},m}$ and $\mathbf{n}_{l,k,b}$ are i.i.d. complex Gaussian random variables.

The signal received at the relay in phase $m$ is given by

$$\mathbf{r}_m = \mathbf{G}_m \left( \sum_{l=1}^{L} \sum_{k\in\mathcal{K}_{l,m}} \mathbf{H}_{l,k}^{\mathrm{m}} \mathbf{V}_{l,k}\mathbf{d}_{l,k} + \mathbf{n}_{\mathrm{r},m} \right) \in \mathbb{C}^{R\times 1}, \quad (1)$$

where $\mathbf{G}_m \in \mathbb{C}^{R\times R}$ denotes the receive processing matrix of the relay.

After all MAC phases are completed, the relay has separate estimates of all data vectors. The estimated data vector at the relay, related to a given node, can be expressed as

$$\mathbf{r}_{l,k} = \mathbf{G}_{l,k} \left( \sum_{l'=1}^{L} \sum_{k'\in\mathcal{K}_{l',\check{m}}} \mathbf{H}_{l',k'}^{\mathrm{m}} \mathbf{V}_{l',k'}\mathbf{d}_{l',k'} + \mathbf{n}_{\mathrm{r},\check{m}} \right) \in \mathbb{C}^{d\times 1}, \quad (2)$$

where $\mathbf{G}_{l,k} \in \mathbb{C}^{d\times R}$ is a submatrix of $\mathbf{G}_m$ obtained by extracting the $d$ rows related to user $(l,k)$ from $\mathbf{G}_m = \left[\mathbf{G}_{1,k_1'}^{\mathrm{H}} \cdots \mathbf{G}_{L,k_2'}^{\mathrm{H}}\right]^{\mathrm{H}}$, $\forall k_1', k_2' \in \mathcal{K}_{l,m}$, taking into account the phase $\check{m}$ in which the node's signal was transmitted. Equation (2) can be rewritten as follows:

$$\mathbf{r}_{l,k} = \mathbf{G}_{l,k}\mathbf{H}_{l,k}^{\mathrm{m}}\mathbf{V}_{l,k}\mathbf{d}_{l,k} + \mathbf{G}_{l,k} \sum_{\substack{k'\in\mathcal{K}_{l,\check{m}},\\ k'\neq k}} \mathbf{H}_{l,k'}^{\mathrm{m}}\mathbf{V}_{l,k'}\mathbf{d}_{l,k'} + \mathbf{G}_{l,k} \sum_{\substack{l'=1,\\ l'\neq l}}^{L} \sum_{k'\in\mathcal{K}_{l',\check{m}}} \mathbf{H}_{l',k'}^{\mathrm{m}}\mathbf{V}_{l',k'}\mathbf{d}_{l',k'} + \mathbf{G}_{l,k}\mathbf{n}_{\mathrm{r},\check{m}}. \quad (3)$$

Assuming that ZF is applied to determine the receive processing matrix $\mathbf{G}_m$, (3) simplifies to

$$\mathbf{r}_{l,k} = \mathbf{d}_{l,k} + \mathbf{G}_{l,k}\mathbf{n}_{\mathrm{r},\check{m}}. \quad (4)$$

Let us define $\mathbf{H}_{l,k,j}^{\mathrm{b}} \in \mathbb{C}^{N\times R/L}$ as a channel submatrix of $\mathbf{H}_{l,k}^{\mathrm{b}} = \left[\mathbf{H}_{l,k,1}^{\mathrm{b}} \cdots \mathbf{H}_{l,k,L}^{\mathrm{b}}\right]$, representing a channel between the $j$-cluster of antennas at the relay to user $k$ in group $l$, with $j \in \{1,\dots,L\}$. This splitting of the $R$ relay antennas into $L$ clusters creates a MIMO interference multicast channel. In order to perform IA, we define the precoding matrices $\mathbf{G}_{j,b}^{b} \in \mathbb{C}^{R/L\times d}$ for each relay antenna cluster $j$ in phase $b$, such that the interference signals arriving at the non-intended multicast groups are aligned at the nodes in these non-intended multicast groups. The relay precoding matrix $\mathbf{G}_{j,b}^{b}$ is normalized such that the maximum power constraint $P_{\mathrm{r}}$ is fulfilled. Let $\tilde{\mathbf{G}}_{j,b}^{b} \in \mathbb{C}^{R/L\times d}$ denote the unnormalized precoders and $\beta_b$ the normalization factor related to phase $b$. The following total power constraint needs to be satisfied in each phase $b$:

$$\sum_{j=1}^{L} \mathrm{E}\left\{ \left\| \beta_b \tilde{\mathbf{G}}_{j,b}^{b}\mathbf{r}_{j,b} \right\|_{\mathrm{F}}^{2} \right\} \leq P_r\,. \quad (5)$$

Plugging (4) into (5) leads to

$$\beta_b = \sqrt{\frac{P_r}{\sum_{j=1}^{L} \mathrm{tr}\left[\tilde{\mathbf{G}}_{j,b}^{b,\mathrm{H}}\tilde{\mathbf{G}}_{j,b}^{b}\left(\mathbf{I} + \sigma_{\mathrm{r},m}^2\mathbf{G}_{j,b}\mathbf{G}_{j,b}^{\mathrm{H}}\right)\right]}}. \quad (6)$$

Let $\mathbf{U}_{l,k,b}^{\mathrm{H}} \in \mathbb{C}^{d\times N}$ denote the receive zero-forcing filter at node $(l,k)$ nullifying the interference signals. The estimated data vector at node $(l,k)$ in phase $b$ (with $b \neq k$) is given by

$$\hat{\mathbf{d}}_{l,k,b} = \mathbf{U}_{l,k,b}^{\mathrm{H}} \left( \sum_{j=1}^{L} \mathbf{H}_{l,k,j}^{\mathrm{b}}\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{r}_{j,b} + \mathbf{n}_{l,k,b} \right). \quad (7)$$

Taking (4) into account, (7) can be written as

$$\hat{\mathbf{d}}_{l,k,b} = \mathbf{U}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k,l}^{\mathrm{b}}\mathbf{G}_{l,b}^{\mathrm{b}}\mathbf{d}_{l,b} + \mathbf{U}_{l,k,b}^{\mathrm{H}} \sum_{\substack{j=1,\\ j\neq l}}^{L} \mathbf{H}_{l,k,j}^{\mathrm{b}}\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{d}_{j,b} + \mathbf{U}_{l,k,b}^{\mathrm{H}} \left( \sum_{j=1}^{L} \mathbf{H}_{l,k,j}^{\mathrm{b}}\mathbf{G}_{j,b}^{\mathrm{b}}\mathbf{G}_{j,b}\mathbf{n}_{\mathrm{r},\check{b}} + \mathbf{n}_{l,k,b} \right). \quad (8)$$

## III. Proposed algorithm

This section describes the transmit and receive filter design of the nodes as well as the relay precoder design.

### *A. Transmit filter design of the nodes*

Since the relay has enough antennas to spatially separate all received data streams during each MAC phase, the relay is able to cancel the whole interference. Hence, the node's precoding matrix is designed in order to maximize the SNR. Based on the assumed channel knowledge, this can be done by assigning the $d$ strongest singular values of the channel matrix $\mathbf{H}_{l,k}^{\mathrm{m}}$ to the precoding matrix $\mathbf{V}_{l,k}$. The singular-value decomposition (SVD) of $\mathbf{H}_{l,k}^{\mathrm{m}}$ is given by

$$\mathrm{SVD}\left(\mathbf{H}_{l,k}^{\mathrm{m}}\right) = \mathbf{\Gamma}_{l,k}\mathbf{\Sigma}_{l,k}\mathbf{\Lambda}_{l,k}^{\mathrm{H}}, \quad (9)$$

where $\mathbf{\Gamma}_{l,k} \in \mathbb{C}^{R\times R}$ and $\mathbf{\Lambda}_{l,k} \in \mathbb{C}^{N\times N}$ are orthogonal matrices containing the singular vectors of $\mathbf{H}_{l,k}^{\mathrm{m}}$. Matrix $\mathbf{\Sigma}_{l,k} \in \mathbb{C}^{R\times N}$ contains the singular values. The precoding matrix of node $(l,k)$ is given by

$$\mathbf{V}_{l,k} = \sqrt{\frac{P_{\text{n,max}}}{d}}\mathbf{\Lambda}_{l,k,1\dots d}, \quad (10)$$

where $\mathbf{\Lambda}_{l,k,1\dots d}$ contains the $d$ strongest singular vectors, respectively.

### B. Relay receive processing matrix

Let us define the following equivalent channel for each MAC phase $m$:

$$\mathbf{H}_m^{\mathrm{m}} = \left[\mathbf{H}_{1,k1}^{\mathrm{m}}\mathbf{V}_{1,k1} \cdots \mathbf{H}_{1,k2}^{\mathrm{m}}\mathbf{V}_{1,k2} \cdots \mathbf{H}_{L,k1}^{\mathrm{m}}\mathbf{V}_{L,k1} \cdots \mathbf{H}_{L,k2}^{\mathrm{m}}\mathbf{V}_{L,k2}\right] \in \mathbb{C}^{R\times R}, \quad k_1,\, k_2 \in \mathcal{K}_{l,m},\; k_1 \neq k_2. \tag{11}$$

The relay receive processing matrix $\mathbf{G}_m$ can be determined using a Zero-Forcing approach by taking the inverse of the squared equivalent channel matrix given by

$$\mathbf{G}_m = \left(\mathbf{H}_m^{\mathrm{m}}\right)^{-1}. \tag{12}$$

### C. Relay precoding matrix

Let us define $\mathbf{\Phi}_{l,k,b} \in \mathbb{C}^{N\times d}$ as an orthonormal basis for the interference subspace at node $k$ in group $l$ at phase $b$, and let $\nu_{\min,d}(.)$ denote an operation delivering a matrix containing the eigenvectors corresponding to the $d$ smallest eigenvalues of the matrix within the brackets as its columns. The concept of alternating optimization can be employed to determine the relay precoding matrix. The algorithm applied in each MC phase $b$ in order to determine the relay precoding matrix is described in Algorithm 1.

**Algorithm 1** Alternating optimization algorithm applied to interference multicast scenario

1) Randomly initialize precoders $\tilde{\mathbf{G}}_{j,b}^{\mathrm{b}}$ for $j = 1, \ldots, L$.
2) Find the basis of the interference subspace
$$\mathbf{\Phi}_{l,k,b} = \nu_{\min,d}\left(\sum_{\substack{j=1,\\ j\neq l}}^{L} \mathbf{H}_{l,k,j}^{\mathrm{b}}\tilde{\mathbf{G}}_{j,b}^{\mathrm{b}}\tilde{\mathbf{G}}_{j,b}^{\mathrm{b,H}}\mathbf{H}_{l,k,j}^{\mathrm{b,H}}\right)$$
for $l = 1, \ldots, L$ and $k = 1, \ldots, K$, with $k \neq b$.
3) Find the unnormalized precoders
$$\tilde{\mathbf{G}}_{j,b}^{\mathrm{b}} = \nu_{\min,d}\left(\sum_{\substack{l=1,\\ l\neq j}}^{L}\sum_{\substack{k=1,\\ k\neq b}}^{K} \mathbf{H}_{l,k,j}^{\mathrm{b,H}}\mathbf{\Phi}_{l,k,b}\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k,j}^{\mathrm{b}}\right)$$
for $j = 1, \ldots, L$.
4) Repeat steps 2 and 3 until convergence.
5) Calculate $\beta_b$ according to (6) and then normalize precoders as $\mathbf{G}_{j,b}^{\mathrm{b}} = \beta_b\tilde{\mathbf{G}}_{j,b}^{\mathrm{b}}$ for $j = 1, \ldots, L$.

### D. Receive processing matrix

After applying Algorithm 1, the interference signals among the groups are aligned at the receivers. In order to cancel this interference, the nodes design their receive processing ZF-matrix $\mathbf{U}_{l,k,b}^{\mathrm{H}}$ such that the useful signal will be projected to a subspace orthogonal to that of the aligned interference signals. The receive processing matrix in each phase $b$ is given by

$$\mathbf{U}_{l,k,b}^{\mathrm{H}} = \left(\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}\mathbf{H}_{l,k,l}^{\mathrm{b}}\mathbf{G}_{l,b}^{\mathrm{b}}\right)^{-1}\mathbf{\Phi}_{l,k,b}^{\mathrm{H}}, \quad \forall l \in \mathcal{L};\quad \forall k \in \mathcal{K},\; k \neq b. \tag{13}$$

## IV. Degrees of Freedom and Performance analysis

This section presents a discussion on aspects related to the Degrees of Freedom (DoF) and performance of the proposed algorithm.

### A. Degrees of Freedom analysis

In each MC phase, a MIMO interference multicast channel is created by using a subset of relay antennas for each multicast group. The feasibility constraints of such a MIMO interference multicast scenario have been derived in [8]. Considering the parameters defined in Section II, these can be expressed as:

$$\min(R/L,\, N) \geq d \tag{14}$$
$$K(Ld - N) \leq R/L - d\,. \tag{15}$$

For the case in which $N = Ld$ and $R/L \geq d$ hold, the system is always feasible, independent of the number $K$ of nodes per group. Thus, when satisfying these constraints, the dimensioning of the number of relay and node antennas does not depend on $K$.

The DoF of the proposed algorithm can be determined by calculating the ratio between the total number $(LK(K-1)d)$ of delivered streams and the total number $(KLd/R + K)$ of phases.

Table I presents a comparison between the proposed multicast algorithm and the reference algorithm in [7], with regard to their dimensioning parameters and DoF expressions. The reference algorithm requires a single MAC phase, $K-1$ BC phases, and implements IA for interference cancelation, which is achieved through group signal alignment, group channel alignment and transceive zero forcing [7]. Note that the reference algorithm considers that the nodes may have different numbers of antennas, with $N_{lk}$ denoting the number of antennas at node $k$ in group $l$.

It can be seen from Table I that the required number of relay antennas of the proposed algorithm is independent of $K$. The dependency on $K$ is shifted to the number of required MAC phases, which might be a favorable property, as we are trading physical antenna resources for additional time phases. The reference algorithm, in contrast, has a single MAC phase, but at the cost of a potentially high number of relay antennas as $K$ increases. In terms of DoF, the multicast approach suffers a penalty when using the smallest number of relay antennas,

TABLE I
System dimensioning and DoF expressions.

| Approach | Ref. [7] | Multicast |
|---|---|---|
| Relay antennas | $R = Ld(K-1)$ | $R \geq Ld$ |
| Node antennas | $\sum_k N_{lk} \geq R + d,\ \forall l$ | $N = Ld$ |
| MAC phases | $1$ | $KLd/R$ |
| BC/MC phases | $K-1$ | $K$ |
| DoF | $L(K-1)d$ | $\dfrac{L(K-1)d}{(Ld/R)+1}$ |

but it has the flexibility to use more relay antennas[1]. As the number of relay antennas increases, the DoF performance of the proposed algorithm approaches that of the reference algorithm.

In order to better illustrate the comparison between the algorithms, the following two scenarios are considered: Scenario $A$ with $K = 3$ nodes per group and Scenario $B$ with $K = 6$ nodes per group, both assuming $L = 3$ groups and $d = 1$ data stream per node. These different values of $K$ allow to verify the impact of the group size on the dimensioning of the system parameteres, such as the number of relay antennas.

The values obtained for the system parameters and DoF are presented in Table II. For Scenario $A$, the multicast algorithm can employ half the number of relay antennas of the reference case, achieving half the number of DoF. Increasing the number of relay antennas is not beneficial in this case, since a larger number of antennas than the reference case is required and still a lower number of DoF is achieved. For Scenario $B$, however, the multicast case has a higher flexibility. It can use, for example, a fifth of the number of antennas of the reference case, achieving half of its DoF, or a larger number of antennas with increased DoF.

Note that the restriction that the number $KLd/R$ of MAC phases should be an integer does not allow, for the considered scenarios, a comparison between the reference and the proposed algorithms with the same number of relay antennas.

### B. Performance analysis

The DoF analysis, however, is only valid for an asymptotically high SNR. In order to assess the performance of the algorithms for a larger range of SNR values, Figures 2 and 3 present sum rate results for Scenarios $A$ and $B$, respectively. The results were obtained through Monte Carlo simulations, taken into account 1,000 realizations for each SNR point and considering the i.i.d. frequency flat Rayleigh MIMO channel model. The alternate optimization Algorithm 1 was implemented considering 10 iterations, which were verified to achieve convergence.

From both figures it can be seen that the multicast algorithm presents a better sum rate performance up to a certain SNR value, in comparison to the reference algorithm. As the number of antennas is increased for each scenario, the slope of the sum rate curves increases as well, pushing the point at which the reference algorithm outperforms the multicast approach to a higher SNR.

In Figure 2, some performance gain can be perceived for the multicast algorithm with $R = 3$, with it being surpassed by the reference algorithm at an SNR of roughly 20 dB. The performance improves for $R = 9$, but at the cost of using more antennas than the reference case. Nevertheless, as we increase the number of users per group from 3 to 6, rather significant gains are achieved by the proposed multicast approach, as shown in Figure 3. In particular, the cases with

[1]It should be an integer multiple of $Ld$, but such that $KLd/R$ (number of MAC phases) is also an integer.

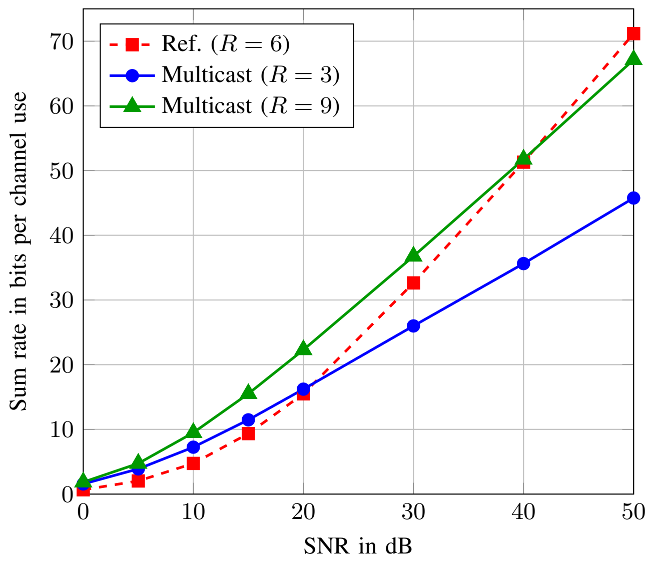


Fig. 2. Sum rate results for Scenario $A$ with $L = 3, d = 1, K = 3$.

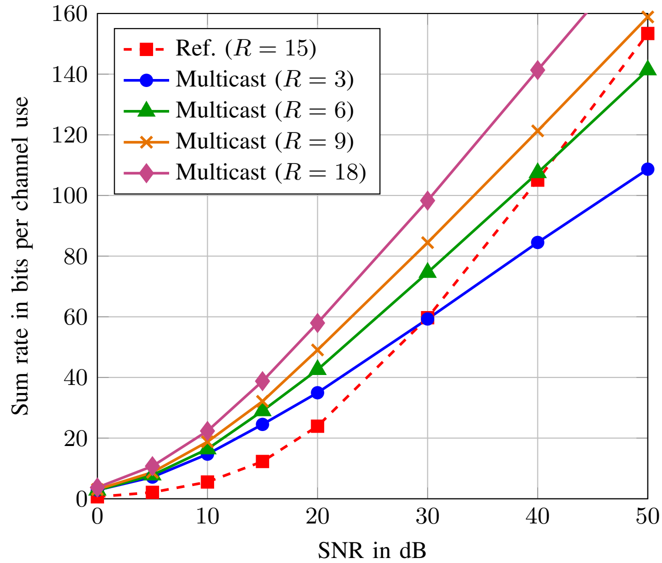


Fig. 3. Sum rate results for Scenario $B$ with $L = 3, d = 1, K = 6$.

$R = 6$ and $R = 9$ antennas present a good trade-off between number of antennas and sum rate performance, outperforming the reference algorithm (with $R = 15$) for a broad range of SNR values and still requiring less antennas. In numerical terms, for $R = 3, 6$, and 9, the proposed algorithm achieves higher sum rate than the reference algorithm up to the SNR values of roughly $30, 40$, and 50 dB, respectively.

## V. Conclusion

In this paper, an algorithm based on multicast interference alignment is proposed for multi-group multi-way relaying networks. In the proposed algorithm, the minimum required number of antennas at the relay is independent of the number of users per group, which is an important property, since physical antenna resources are limited in general. The algorithm is

TABLE II
DoF ANALYSIS FOR BOTH SCENARIOS, WITH $L = 3, d = 1$.

| | Scenario $A$ ($K = 3$) | | | Scenario $B$ ($K = 6$) | | | | |
|---|---|---|---|---|---|---|---|---|
| | Ref. | $R = Ld$ | $R = 3Ld$ | Ref. | $R = Ld$ | $R = 2Ld$ | $R = 3Ld$ | $R = 6Ld$ |
| Relay antennas | 6 | 3 | 9 | 15 | 3 | 6 | 9 | 18 |
| Node antennas | $\{2, 2, 3\}$ | 3 | 3 | $\{2, 2, 3, 3, 3, 3\}$ | 3 | 3 | 3 | 3 |
| MAC phases | 1 | 3 | 1 | 1 | 6 | 3 | 2 | 1 |
| BC/MC phases | 2 | 3 | 3 | 5 | 6 | 6 | 6 | 6 |
| DoF | 6 | 3 | 4.5 | 15 | 7.5 | 10 | 11.25 | 12.86 |

flexible in the sense that it supports different numbers of antennas at the relay for each given system configuration, which allows to achieve different trade-offs between performance and required hardware resources.

When compared to a reference algorithm from the literature, it is shown that the proposed algorithm requires less antennas at the relay. Even though this reduction in the number of relay antennas leads to an increase in the number of MAC phases, the simulation results indicate that the proposed algorithm outperforms the reference algorithm for a broad range of SNR values. It is also shown that the upper limit of this SNR range, up to which the proposed algorithm achieves higher sum rate than the reference algorithm, increases as the number of relay antennas is increased.

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