Exploiting Additional Transmit Antennas for More Degrees of Freedom in 3-User MIMO Interference Channel with Delayed CSIT

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***Abstract*—In this paper, we consider the three-user multiple-input multiple-output interference channel, where the transmitters and the receivers have M and N antennas, respectively, and there is the delayed channel state information at the transmitters (CSIT). For this scenario, we propose a new transmission scheme, which achieves a number of degrees of freedom greater than that known in the literature for the region of antenna configurations of $1 < M/N < 2$. The proposed transmission scheme has a three-phase transmission structure, where in phases 1 and 2 a novel approach of using the delayed CSIT is employed.**

I. Introduction

The number of degrees of freedom (DoF) of the three-user multiple-input multiple-output (MIMO) interference channel (IC) where each transmitter has M antennas and each receiver has N antennas was evaluated in [1], where the achievability was based on a combination of beamforming and interference alignment. The result of [1] assumes perfect and instantaneous channel state information at the transmitters (CSIT), an assumption which does not always hold in practice. Under identically and independently distributed (i.i.d.) fading, the number of DoF of [1] is not achievable with the delayed CSIT, hence it is interesting whether the delayed CSIT can be used to achieve a number of DoF greater than that in absence of CSIT in this network.

In the context of the K-user multiple-input single-output (MISO) broadcast channel (BC), this question has been answered affirmatively by Maddah-Ali and Tse in [2] by showing that the number of DoF of this network with the delayed CSIT is greater than that in absence of CSIT. The number of DoF was achieved by a multi-phase transmission strategy, where in each phase the interference overheard in the previous phases was reconstructed at the transmitters using the delayed CSIT and retransmitted. [2] proposed an iterative procedure to construct the signals which were simultaneously useful for a larger subset of receivers in each phase, where the number of the phases equals the number of the receivers in the network.

An extension of the approach of [2] to the 3-user single-input single-output (SISO) IC as well as for the SISO X-channel (XC) is challenging since the transmitters do not share the information symbols and hence, the interference due to multiple simultaneously active interferers cannot be reconstructed using the delayed CSIT. This problem has been circumvented in [3] and [4] by applying a transmission technique where the transmitters transmit the information symbols along with some redundancy, such that each transmitter occupies only part of the receive signal space of each receiver. In such case, the receivers can cancel the signal of one of the interferers from the received signal, where the remaining interference originates from only a single transmitter and can be reconstructed using the delayed CSIT. [3] and [4] showed that 9/8 DoF and 36/31 DoF are achievable in the 3-user SISO IC, respectively, where [5] and [6] extended the schemes of [3] and [4] to the 3-user MIMO IC for $M/N \leq 1$, respectively, and [7] studied the case of $M/N > 1$.

For the MIMO XC, [8] and [9] showed that instead of applying redundancy transmission, the transmitters can be forced to occupy the receive signal space of the unintended receivers only partially by exploiting the delayed CSIT. [8] and [9] proposed to split the transmission into two parts, where in part 2 the interference overheard at the unintended receivers in part 1 was retransmitted. As compared to redundancy transmission, such approach reduces the size of the signal space occupied by the interference at the unintended receivers, which increases the number of the interference terms obtained at the receivers after the interference cancellation and increases the achievable number of DoF.

In this paper, we apply the approach of [8] to design a novel transmission scheme for the 3-user MIMO IC, which achieves a number of DoF greater than that known in literature for the region of antenna configurations of $1 < M/N < 2$. The proposed transmission scheme has a three-phase transmission structure, where the transmissions in phases 1 and 2 are split into three and two parts, respectively, where in each part the interference overheard at the unintended receivers in the previous parts is retransmitted. Recently, [10] proposed a transmission scheme for the K-user MIMO IC, which similarly to the scheme proposed in this paper uses the multi-part transmission principle of [8]. As compared to [10] for $K = 3$, the scheme proposed in this paper has a more effective transmission in phase 2 and in phase 1 for the antenna configurations of $5/3 < M/N < 2$, which results in greater achieved number of DoF.

The organisation of the paper is as follows. Section II describes the system model. In Section III, the proposed

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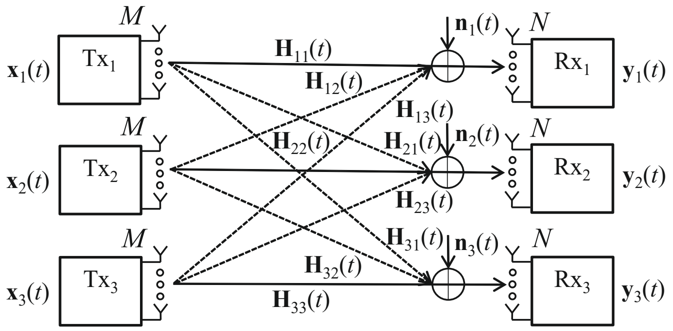

Fig. 1. Three-user MIMO IC

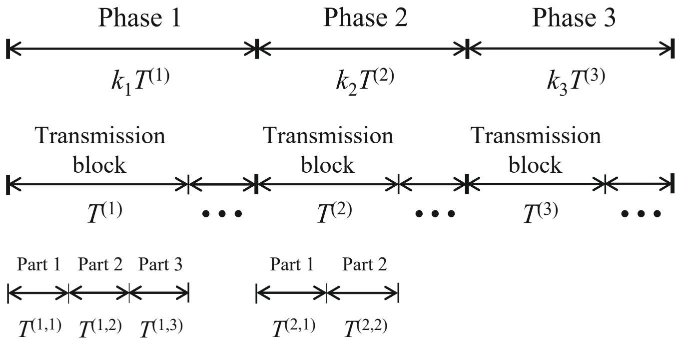

Fig. 2. Structure of transmission scheme

transmission scheme is described and the achieved number of DoF is given in Section IV.

II. System Model

We consider the three-user MIMO IC depicted in Fig. 1, where the transmitters and the receivers have M and N antennas, respectively. During the transmission period of T_Σ time slots, each of the transmitters Tx_i intends to communicate a vector of information symbols $\mathbf{u}'_i \in \mathbb{C}^{b\times 1}$ to the receiver Rx_i, $i \in \{1,2,3\}$, where b denotes the number of the information symbols transmitted by each transmitter.

Let $\mathbf{x}_i(t)$ denote the signal which is transmitted by Tx_i in time slot t, $1 \le t \le T_\Sigma$. The signal received by Rx_j in time slot t, $j \in \{1,2,3\}$, is evaluated as

$$\mathbf{y}_j(t) = \sum_{i=1}^{3} \mathbf{H}_{ji}(t)\,\mathbf{x}_i(t) + \mathbf{n}_j(t), \tag{1}$$

where $\mathbf{H}_{ji}(t) \in \mathbb{C}^{N\times M}$ is the channel matrix between Tx_i and Rx_j and $\mathbf{n}_j(t) \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$. The signal transmitted by Tx_i is subject to the average power constraint of $\frac{1}{T_\Sigma}\sum_{t=1}^{T_\Sigma} \mathrm{E}\left[\mathbf{x}_i^{\mathrm{H}}(t)\,\mathbf{x}_i(t)\right] \le P$, where P is the maximum transmit power.

The entries of the channel matrices $\mathbf{H}_{ji}(t), \forall i,j \in \{1,2,3\}$ are drawn from a continuous distribution and are i.i.d. for different transmitter and receiver pairs as well as for different antennas and time slots. We suppose that in time slot t, $1 \le t \le T_\Sigma$, each of the receivers has the global channel knowledge for the current and the previous time slots, which corresponds to the knowledge about the sets of channel matrices of $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t}$, $\forall i,j \in \{1,2,3\}$. The transmitters have the identical channel knowledge delayed by a single time slot, which corresponds to the knowledge about the sets of channel matrices of $\{\mathbf{H}_{ji}(\tau)\}_{\tau=1}^{t-1}$, $\forall i,j \in \{1,2,3\}$.

We say that the number of DoF $d = 3b/T_\Sigma$ is achievable in the 3-user MIMO IC if each of the information symbol vectors $\mathbf{u}'_i$ transmitted by Tx_i to Rx_i, $i \in \{1,2,3\}$, is decodable with probability one.

III. Proposed Transmission Scheme

In this section, the proposed transmission scheme is described. The first subsection introduces the three-phase structure of the transmission scheme and in the following subsections, each phase of the scheme is given in details.

A. Structure of Transmission Scheme

The proposed transmission scheme is comprised of three phases. In phase 1, the original information symbols are transmitted, where from the interference terms overheard at the unintended receivers in phase 1, terms simultaneously useful for subsets of two receivers are generated. These terms are transmitted in phase 2, where from the interference terms overheard at the remaining unintended receivers in phase 2, terms useful for subsets of two receivers and known at the remaining unintended receivers are generated. The transmission of these terms is performed in phase 3.

As shown in Fig. 2, phase l, $l \in \{1,2,3\}$, is comprised of k_l transmission periods of $T^{(l)}$ time slots, referred to as transmission blocks throughout the paper, where the terms transmitted in a single transmission block can be decoded independently from the terms transmitted in other transmission blocks. In a transmission block of phase l, a subset of the transmitters is scheduled for the transmission, where a scheduled transmitter Tx_i transmits $b_i^{(l)}$ terms, with $b_\Sigma^{(l)}$ denoting the number of the terms transmitted by all transmitters. The transmitters are shuffled between the transmission blocks to ensure an equal number of terms is transmitted by each transmitter in each phase. After the transmission of a transmission block of phase l, $l \in \{1,2\}$, $q^{(l)}$ terms to be transmitted in phase $l+1$ are generated. The numbers of the blocks are chosen to ensure the number of the terms generated after phase l is equal to the number of the terms transmitted in phase $l+1$, $l \in \{1,2\}$, i.e.

$$k_l q^{(l)} = k_{l+1} b_\Sigma^{(l+1)}, l \in \{1,2\} \tag{2}$$

holds. Due to the identical structure of the transmission blocks, only the first transmission block of each phase is described.

The transmission blocks of phases 1 and 2 are divided into transmission periods, referred to as parts throughout the paper, where each transmission block of phases 1 and 2 is split into three and two parts, respectively. In each part, a subset of the scheduled transmitters retransmits the interference overheard at the unintended receivers in the previous parts of the transmission block. The duration of part k of phase 1, $k \in \{1,2,3\}$, is denoted as $T^{(1,k)}$ and the duration of part k of phase 2, $k \in \{1,2\}$, is denoted as $T^{(2,k)}$, $T^{(l)} = \sum_k T^{(l,k)}$, $l \in \{1,2\}$.

B. Transmission in Phase 1

In phase 1, all transmitters are scheduled to transmit simultaneously. In the first transmission block of phase 1, Tx_1, Tx_2 and Tx_3 are scheduled to transmit information symbol vectors $\mathbf{u}_1 \in \mathbb{C}^{b_1^{(1)} \times 1}$, $\mathbf{u}_2 \in \mathbb{C}^{b_2^{(1)} \times 1}$ and $\mathbf{u}_3 \in \mathbb{C}^{b_3^{(1)} \times 1}$, respectively, where $b_1^{(1)} = b_2^{(1)} = M\left(T^{(1,1)} + T^{(1,2)}\right)$ and $b_3^{(1)} \geq NT^{(1,1)}$.

Part 1: In part 1 of the transmission block, Tx_1, Tx_2 and Tx_3 transmit information symbol vectors $\mathbf{u}_1^{(1)} \in \mathbb{C}^{MT^{(1,1)} \times 1}$, $\mathbf{u}_2^{(1)} \in \mathbb{C}^{MT^{(1,1)} \times 1}$ and $\mathbf{u}_3$, respectively, where $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_2^{(1)}$ denote the vectors comprising the first $MT^{(1,1)}$ elements of $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively. Tx_1 and Tx_2 apply a precoding, where a new information symbol is transmitted from each antenna in each time slot and Tx_3 employs random precoding.

Let $\mathbf{x}_i^{(l,k)} \in \mathbb{C}^{MT^{(l,k)} \times 1}$ denote the vertical concatenation of the signal vectors transmitted by Tx_i, in part k of the transmission block of phase l. The signals transmitted by Tx_1, Tx_2 and Tx_3 are calculated as $\mathbf{x}_1^{(1,1)} = \mathbf{u}_1^{(1)}$, $\mathbf{x}_2^{(1,1)} = \mathbf{u}_2^{(1)}$ and $\mathbf{x}_3^{(1,1)} = \mathbf{C}_3^{(1)} \mathbf{u}_3$, where $\mathbf{C}_3^{(1)} \in \mathbb{C}^{MT^{(1,1)} \times b_3^{(1)}}$ is the concatenation of the random precoding matrices used by Tx_3.

Let us denote the diagonal concatenation of the channel matrices between Tx_i and Rx_j in part k of the transmission block of phase l as $\mathbf{H}_{ji}^{(l,k)} \in \mathbb{C}^{NT^{(l,k)} \times MT^{(l,k)}}$. Let $\mathbf{y}_j^{(l,k)} \in \mathbb{C}^{NT^{(l,k)} \times 1}$ denote the vertical concatenation of the signal vectors received by Rx_j in part k of the transmission block of phase l. $\mathbf{y}_j^{(1,1)}$ is evaluated as

$$\mathbf{y}_j^{(1,1)} = \mathbf{H}_{j1}^{(1,1)} \mathbf{u}_1^{(1)} + \mathbf{H}_{j2}^{(1,1)} \mathbf{u}_2^{(1)} + \mathbf{H}_{j3}^{(1,1)} \mathbf{C}_3^{(1)} \mathbf{u}_3 + \mathbf{n}_j^{(1,1)}, \quad (3)$$

where $\mathbf{n}_j^{(1,1)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(1,1)}}\right)$.

Part 2: In part 2 of the transmission block, Tx_1 and Tx_2 transmit information symbol vectors $\mathbf{u}_1^{(2)} \in \mathbb{C}^{MT^{(1,2)} \times 1}$ and $\mathbf{u}_2^{(2)} \in \mathbb{C}^{MT^{(1,2)} \times 1}$ which correspond to the last $MT^{(1,2)}$ elements of $\mathbf{u}_1$ and $\mathbf{u}_2$, respectively, where the signals transmitted by Tx_1 and Tx_2 are evaluated as $\mathbf{x}_1^{(1,2)} = \mathbf{u}_1^{(2)}$ and $\mathbf{x}_2^{(1,2)} = \mathbf{u}_2^{(2)}$. Tx_3 retransmits linear combinations of $\mathbf{u}_3$ which are linearly dependent on the linear combinations of $\mathbf{u}_3$ overheard by Rx_1 and Rx_2 in part 1 of the transmission block. Such transmission ensures that the sizes of the spaces spanned by the signals of Tx_3 at Rx_1 and Rx_2 do not increase during the transmission in part 2 of the transmission block.

In part 1 of the transmission block, Rx_1 and Rx_2 received the interference from Tx_3 denoted as $\mathbf{H}_{13}^{(1,1)} \mathbf{C}_3^{(1)} \mathbf{u}_3$ and $\mathbf{H}_{23}^{(1,1)} \mathbf{C}_3^{(1)} \mathbf{u}_3$, respectively. The entries of $\mathbf{H}_{13}^{(1,1)}$, $\mathbf{H}_{23}^{(1,1)}$ and $\mathbf{C}_3^{(1)}$ are distributed independently, hence the spaces spanned by the rows of $\mathbf{H}_{13}^{(1,1)} \mathbf{C}_3^{(1)}$ and $\mathbf{H}_{23}^{(1,1)} \mathbf{C}_3^{(1)}$ have an intersection space of a size $\delta_3 = 2NT^{(1,1)} - b_3^{(1)}$ almost surely. It means there exist full rank projection matrices $\mathbf{V}_{13} \in \mathbb{C}^{\delta_3 \times NT^{(1,1)}}$ and $\mathbf{V}_{23} \in \mathbb{C}^{\delta_3 \times NT^{(1,1)}}$ for which

$$\mathbf{V}_{13} \mathbf{H}_{13}^{(1,1)} \mathbf{C}_3^{(1)} = \mathbf{V}_{23} \mathbf{H}_{23}^{(1,1)} \mathbf{C}_3^{(1)} = \mathbf{V}_{1,2;3} \quad (4)$$

holds, where the rows of $\mathbf{V}_{1,2;3} \in \mathbb{C}^{\delta_3 \times b_3^{(1)}}$ contain the coefficients of the linear combinations of $\mathbf{u}_3$, which are linearly dependent on the linear combinations of $\mathbf{u}_3$ received by Rx_1 and Rx_2 in part 1 of the transmission block. The signal transmitted by Tx_3 is evaluated as $\mathbf{x}_3^{(1,2)} = \mathbf{C}_3^{(2)} \mathbf{V}_{1,2;3} \mathbf{u}_3$, where $\mathbf{C}_3^{(2)} \in \mathbb{C}^{MT^{(1,2)} \times \delta_3}$ denotes the concatenation of the random precoding matrices.

The signal received by Rx_j in part 2 of the transmission block reads as

$$\begin{aligned} \mathbf{y}_j^{(1,2)} = {} & \mathbf{H}_{j1}^{(1,2)} \mathbf{u}_1^{(2)} + \mathbf{H}_{j2}^{(1,2)} \mathbf{u}_2^{(2)} + \\ & \mathbf{H}_{j3}^{(1,2)} \mathbf{C}_3^{(2)} \mathbf{V}_{1,2;3} \mathbf{u}_3 + \mathbf{n}_j^{(1,2)}, \end{aligned} \quad (5)$$

where $\mathbf{n}_j^{(1,2)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(1,2)}}\right)$.

Part 3: In part 3 of the transmission block, all transmitters transmit the linear combinations of the information symbols which are linearly dependent on the linear combinations of the information symbols overheard at the unintended receivers in parts 1 and 2 of the transmission block.

Similarly to (4), we define matrices $\mathbf{V}_{2,3;1}^{(1)} \in \mathbb{C}^{\delta_1^{(1)} \times NT^{(1,1)}}$ and $\mathbf{V}_{2,3;1}^{(2)} \in \mathbb{C}^{\delta_1^{(2)} \times NT^{(1,2)}}$, $\delta_1^{(1)} = (2N - M)\, T^{(1,1)}$, $\delta_1^{(2)} = (2N - M)\, T^{(1,2)}$, the rows of which contain the coefficients of the linear combinations of $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_1^{(2)}$ which are linearly dependent on the linear combinations of $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_1^{(2)}$ received by Rx_2 and Rx_3 in parts 1 and 2 of the transmission block, respectively. The signal transmitted by Tx_1 is evaluated as $\mathbf{x}_1^{(1,3)} = \mathbf{C}_1^{(1)} \mathbf{V}_{2,3;1}^{(1)} + \mathbf{C}_1^{(2)} \mathbf{V}_{2,3;1}^{(2)}$, where $\mathbf{C}_1^{(1)} \in \mathbb{C}^{MT^{(1,3)} \times \delta_1^{(1)}}$ and $\mathbf{C}_1^{(2)} \in \mathbb{C}^{MT^{(1,3)} \times \delta_1^{(2)}}$ are the concatenations of the random precoding matrices. Similarly, Tx_2 and Tx_3 transmit $\mathbf{x}_2^{(1,3)} = \mathbf{C}_2^{(1)} \mathbf{V}_{1,3;2}^{(1)} + \mathbf{C}_2^{(2)} \mathbf{V}_{1,3;2}^{(2)}$ and $\mathbf{x}_3^{(1,3)} = \mathbf{C}_3^{(3)} \mathbf{V}_{1,2;3}$, where $\mathbf{V}_{1,3;2}^{(1)} \in \mathbb{C}^{\delta_1^{(1)} \times NT^{(1,1)}}$ and $\mathbf{V}_{1,3;2}^{(2)} \in \mathbb{C}^{\delta_1^{(2)} \times NT^{(1,2)}}$ are defined similarly to (4) and $\mathbf{C}_2^{(1)} \in \mathbb{C}^{MT^{(1,3)} \times \delta_1^{(1)}}$, $\mathbf{C}_2^{(2)} \in \mathbb{C}^{MT^{(1,3)} \times \delta_1^{(2)}}$ and $\mathbf{C}_3^{(3)} \in \mathbb{C}^{MT^{(1,3)} \times \delta_3}$ are the concatenations of the random precoding matrices.

In part 3 of the transmission block Rx_j receives

$$\begin{aligned} \mathbf{y}_j^{(1,3)} = {} & \mathbf{H}_{j1}^{(1,3)} \left(\mathbf{C}_1^{(1)} \mathbf{V}_{2,3;1}^{(1)} \mathbf{u}_1^{(1)} + \mathbf{C}_1^{(2)} \mathbf{V}_{2,3;1}^{(2)} \mathbf{u}_1^{(2)} \right) + \\ & \mathbf{H}_{j2}^{(1,3)} \left(\mathbf{C}_2^{(1)} \mathbf{V}_{1,3;2}^{(1)} \mathbf{u}_2^{(1)} + \mathbf{C}_2^{(2)} \mathbf{V}_{1,3;2}^{(2)} \mathbf{u}_2^{(2)} \right) + \\ & \mathbf{H}_{j3}^{(1,3)} \mathbf{C}_3^{(3)} \mathbf{V}_{1,2;3} \mathbf{u}_3 + \mathbf{n}_j^{(1,3)}, \end{aligned} \quad (6)$$

where $\mathbf{n}_j^{(1,3)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(1,3)}}\right)$.

Generation of overheard interference terms: To generate the terms to be transmitted in phase 2 of the transmission scheme, each of the receivers alternatively cancels the signal of one of the interferers from the received signal, where the remaining interference constitutes the signal to be retransmitted. We consider the generation of the signals at Rx_1, where the processing at the other receivers is performed similarly.

Let us denote the concatenation of the signal vectors received by Rx_1 during the transmission block of phase 1 as $\mathbf{y}_j^{(1)} = \left[\mathbf{y}_j^{(1,1)\mathrm{T}} \quad \mathbf{y}_j^{(1,2)\mathrm{T}} \quad \mathbf{y}_j^{(1,3)\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{C}^{NT^{(1)} \times 1}$. We write the signal received by Rx_1 in a form of

$$\mathbf{y}_1^{(1)} = \mathbf{H}_{11}^{(1)} \mathbf{u}_1 + \mathbf{H}_{12}^{(1)} \mathbf{u}_2 + \mathbf{H}_{13}^{(1)} \mathbf{u}_3 + \mathbf{n}_1^{(1)}, \quad (7)$$

where $\mathbf{H}_{11}^{(1)}, \mathbf{H}_{12}^{(1)} \in \mathbb{C}^{NT^{(1)} \times M\left(T^{(1,1)} + T^{(1,2)}\right)}$ and $\mathbf{H}_{13}^{(1)} \in \mathbb{C}^{NT^{(1)} \times b_3^{(1)}}$ are the effective channel matrices and $\mathbf{n}_1^{(1)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(1)}}\right)$.

Let us consider the interference of Tx_2 in (7). $\mathbf{H}_{12}^{(1)}$ can be written as

$$\mathbf{H}_{12}^{(1)} = \begin{bmatrix} \mathbf{I}_{NT^{(1,1)}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{NT^{(1,2)}} \\ \mathbf{H}_{12}^{(1,3)}\mathbf{C}_2^{(1)}\mathbf{V}_{12}^{(1)} & \mathbf{H}_{12}^{(1,3)}\mathbf{C}_2^{(2)}\mathbf{V}_{12}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{H}_{12}^{(1,1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{12}^{(1,2)} \end{bmatrix},$$

which almost surely has a rank of $N\left(T^{(1,1)} + T^{(1,2)}\right)$ with $\left(NT^{(1,3)}\right)$-dimensional left null space. It means there exists a full rank matrix $\mathbf{W}_{12} \in \mathbb{C}^{NT^{(1,3)} \times NT^{(1)}}$, for which $\mathbf{W}_{12}\mathbf{H}_{12}^{(1)} = \mathbf{0}_{NT^{(1,3)} \times M\left(T^{(1,1)}+T^{(1,2)}\right)}$ holds. By multiplying $\mathbf{y}_1^{(1)}$ with $\mathbf{W}_{12}$, Rx_1 cancels the signal of Tx_2 and obtains

$$\mathbf{W}_{12}\mathbf{y}_1^{(1)} = \mathbf{W}_{12}\mathbf{H}_{11}^{(1)}\mathbf{u}_1 + \mathbf{W}_{12}\mathbf{H}_{13}^{(1)}\mathbf{u}_3, \tag{8}$$

where the noise term has been omitted since it does not influence the DoF analysis.

The sum in (8) is comprised of the term useful for Rx_1 of $\mathbf{W}_{12}\mathbf{H}_{11}^{(1)}\mathbf{u}_1$ and the remaining interference term of Tx_3 of $\mathbf{W}_{12}\mathbf{H}_{13}^{(1)}\mathbf{u}_3$, which is useful for both Rx_1 and Rx_3 as follows:

- it can be subtracted from $\mathbf{W}_{12}\mathbf{y}_1^{(1)}$ to yield $\mathbf{W}_{12}\mathbf{H}_{11}^{(1)}\mathbf{u}_1$;
- it is a term useful for Rx_3.

Further, we will use the notation of order-2 symbols, where an order-2 symbol is a term which is desired by two receivers simultaneously. By $\mathbf{u}_{l|i,j} \in \mathbb{C}^{q\times 1}$ we denote a vector of $q \in \mathbb{N}$ order-2 symbols, which is desired by Rx_i and Rx_j and is available at Tx_l, $1 \le i, j, l \le 3, i \ne j, l \in \{i, j\}$. From (8), the vector of order-2 symbols $\mathbf{u}_{3|1,3} = \mathbf{W}_{12}\mathbf{H}_{13}^{(1)}\mathbf{u}_3 \in \mathbb{C}^{NT^{(1,3)}\times 1}$ is generated at Tx_3.

Now, let us consider the interference of Tx_3 in (7). $\mathbf{H}_{13}^{(1)}$ can be written as

$$\mathbf{H}_{13}^{(1)} = \begin{bmatrix} \mathbf{I}_{NT^{(1,1)}} \\ \mathbf{H}_{13}^{(1,2)}\mathbf{C}_3^{(2)}\mathbf{V}_{13} \\ \mathbf{H}_{13}^{(1,3)}\mathbf{C}_3^{(3)}\mathbf{V}_{13} \end{bmatrix} \mathbf{H}_{13}^{(1,1)}\mathbf{C}_3^{(1)}, \tag{9}$$

which almost surely has a rank of $NT^{(1,1)}$ with $\left(N\left(T^{(1,2)} + T^{(1,3)}\right)\right)$-dimensional left null space. It means there exists a full rank matrix $\mathbf{W}_{13} \in \mathbb{C}^{N\left(T^{(1,2)}+T^{(1,3)}\right)\times NT^{(1)}}$, for which $\mathbf{W}_{13}\mathbf{H}_{13}^{(1)} = \mathbf{0}_{N\left(T^{(1,2)}+T^{(1,3)}\right)\times b_3^{(1)}}$ holds. By multiplying $\mathbf{y}_1^{(1)}$ with $\mathbf{W}_{13}$, Rx_1 cancels the signal of Tx_3, where from the remaining interference term of Tx_2, the vector of order-2 symbols $\mathbf{u}_{2|1,2} = \mathbf{W}_{13}\mathbf{H}_{12}^{(1)}\mathbf{u}_2^{(1)} \in \mathbb{C}^{N\left(T^{(1,2)}+T^{(1,3)}\right)\times 1}$ is generated at Tx_2.

By applying the processing similar to that of Rx_1, Rx_2 obtains vectors of order-2 symbols $\mathbf{u}_{3|2,3} \in \mathbb{C}^{NT^{(1,3)}\times 1}$ and $\mathbf{u}_{1|1,2} \in \mathbb{C}^{N\left(T^{(1,2)}+T^{(1,3)}\right)\times 1}$ and Rx_3 obtains vectors of order-2 symbols $\mathbf{u}_{2|2,3} \in \mathbb{C}^{NT^{(1,3)}\times 1}$ and $\mathbf{u}_{1|1,3} \in \mathbb{C}^{NT^{(1,3)}\times 1}$. This results in overall $q^{(1)} = 2N\left(T^{(1,2)} + 3T^{(1,3)}\right)$ order-2 symbols generated after the transmission of the transmission block of phase 1.

Choice of $T^{(1,1)}$, $T^{(1,2)}$, $T^{(1)}$ and $b_3^{(1)}$: The parameters of the transmission block are designed to maximize the normalized number of the transmitted information symbols $\frac{b_\Sigma^{(1)}}{T^{(1)}}$, while ensuring the transmitted information symbols can be decoded given all order-2 symbols are provided to the receivers which desire them.

Order-2 symbol vectors $\mathbf{u}_{3|1,3}$, $\mathbf{u}_{3|2,3}$, $\mathbf{u}_{2|2,3}$ and $\mathbf{u}_{1|1,3}$ provide $4NT^{(1,3)}$ linear combinations of $\mathbf{u}_3$ to Rx_3. To ensure the decodability of $\mathbf{u}_3$, we require the number of the linear combinations to be equal to the number of the unknowns, i.e.

$$b_3^{(1)} = 4NT^{(1,3)}. \tag{10}$$

For Rx_1, order-2 symbol vectors $\mathbf{u}_{3|1,3}$, $\mathbf{u}_{2|1,2}$, $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{1|1,3}$ provide $2N\left(T^{(1,2)} + 2T^{(1,3)}\right)$ linear combinations of $\mathbf{u}_1$. Similarly, to ensure the decodability of $\mathbf{u}_1$, we require $M\left(T^{(1,1)} + T^{(1,2)}\right) = 2N\left(T^{(1,2)} + 2T^{(1,3)}\right)$, which can be rewritten in terms of $\frac{T^{(1,1)}}{T^{(1)}}$ and $\frac{T^{(1,2)}}{T^{(1)}}$ as

$$\frac{4N+M}{4N}\frac{T^{(1,1)}}{T^{(1)}} + \frac{2N+M}{4N}\frac{T^{(1,2)}}{T^{(1)}} = 1. \tag{11}$$

Due to symmetry, the identical requirement holds for the decodability of $\mathbf{u}_2$ by Rx_2. The following theorem introduces an additional constraint on $\frac{T^{(1,1)}}{T^{(1)}}$ and $\frac{T^{(1,2)}}{T^{(1)}}$, which is necessary for the decodability.

Theorem 1*:* $\mathbf{u}_1^{(1)}$ and $\mathbf{u}_2^{(1)}$ are decodable only if

$$\frac{9N-2M}{4N}\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}} \ge 1. \tag{12}$$

Proof: The proof shows that the linear combinations of $\mathbf{u}_1^{(1)}$ obtained by Rx_1 from order-2 symbol vectors $\mathbf{u}_{3|1,3}$, $\mathbf{u}_{2|1,2}$, $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{1|1,3}$ are linearly dependent when (12) does not hold. A similar statement holds for $\mathbf{u}_2^{(1)}$ due to symmetry. First, we construct a matrix, the rows of which are comprised of the coefficients of the linear combinations of $\mathbf{u}_1^{(1)}$ obtained by Rx_1. By using rank properties of sums and products of matrices, we obtain an upper bound on the rank of this matrix, where the obtained upper bound is less than the maximum rank when (12) does not hold. Due to space limitation, the details of the proof are omitted. ■

To choose $T^{(1,1)}$, $T^{(1,2)}$ and $T^{(1)}$, we first express $\frac{b_\Sigma^{(1)}}{T^{(1)}}$ as

$$\frac{b_\Sigma^{(1)}}{T^{(1)}} = 4N - 2\left(2N - M\right)\left(\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}}\right), \tag{13}$$

which is inversely proportional to $\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}}$. To maximize $\frac{b_\Sigma^{(1)}}{T^{(1)}}$ while ensuring decodability, we would like to minimize $\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}}$ while ensuring that (11) and (12) hold. Next, we consider the following regions of antenna configurations.

Region 1.1: $1 < M/N \le 5/3$, only (11) is active. To minimize $\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}}$ while ensuring (11), we choose

$$T^{(1,1)} = 4N,\ T^{(1,2)} = 0,\ T^{(1)} = 4N + M. \tag{14}$$

Region 1.2: $5/3 < M/N < 2$, both (11) and (12) are active. To minimize $\frac{T^{(1,1)}}{T^{(1)}} + \frac{T^{(1,2)}}{T^{(1)}}$ while ensuring (11) and (12), we choose

$$T^{(1,1)} = 4N\left(2N - M\right),\ T^{(1,2)} = 4N\left(3N - 5M\right),$$
$$T^{(1)} = 2M^2 - MN - 2N^2. \tag{15}$$

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C. Transmission in Phase 2

In phase 2, the order-2 symbols generated in phase 1 are transmitted. The transmitters are scheduled to transmit in pairs, where the scheduled transmitters Tx_i and Tx_j transmit the order-2 symbols simultaneously useful for the pair of receivers of Rx_i and Rx_j, $1 \leq i, j \leq 3, i \neq j$. In the first transmission block of phase 2, Tx_1 and Tx_2 are scheduled to transmit order-2 symbol vectors $\mathbf{u}_{1|1,2} \in \mathbb{C}^{b_1^{(2)} \times 1}$ and $\mathbf{u}_{2|1,2} \in \mathbb{C}^{b_2^{(2)} \times 1}$, respectively, where $b_1^{(2)} = MT^{(2)}$ and $b_2^{(2)} \geq NT^{(2,1)}$.

Part 1: In part 1 of the transmission block, Tx_1 and Tx_2 transmit the order-2 symbol vectors $\mathbf{u}_{1|1,2}^{(1)} \in \mathbb{C}^{MT^{(2,1)} \times 1}$ and $\mathbf{u}_{2|1,2}$, respectively, where $\mathbf{u}_{1|1,2}^{(1)}$ denotes a vector of the first $MT^{(2,1)}$ elements of $\mathbf{u}_{1|1,2}$. The signals transmitted by Tx_1 and Tx_2 are evaluated as $\mathbf{x}_1^{(2,1)} = \mathbf{u}_{1|1,2}^{(1)}$ and $\mathbf{x}_2^{(2,1)} = \mathbf{C}_{2|1,2}^{(1)} \mathbf{u}_{2|1,2}$, where $\mathbf{C}_{2|1,2}^{(1)} \in \mathbb{C}^{MT^{(2,1)} \times b_2^{(2)}}$ is the concatenation of the random precoding matrices used by Tx_2. In part 1 of the transmission block, Rx_j receives

$$\mathbf{y}_j^{(2,1)} = \mathbf{H}_{j1}^{(2,1)} \mathbf{u}_{1|1,2}^{(1)} + \mathbf{H}_{j2}^{(2,1)} \mathbf{C}_{2|1,2}^{(1)} \mathbf{u}_{2|1,2} + \mathbf{n}_j^{(2,1)}, \quad (16)$$

where $\mathbf{n}_j^{(2,1)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(2,1)}}\right)$.

Part 2: In part 2 of the transmission block, Tx_1 transmits order-2 symbol vector $\mathbf{u}_{1|1,2}^{(2)} \in \mathbb{C}^{MT^{(2,2)} \times 1}$, which is comprised of the last $MT^{(2,2)}$ elements of $\mathbf{u}_{1|1,2}$, where the transmitted signal reads as $\mathbf{x}_1^{(2,2)} = \mathbf{u}_{1|1,2}^{(2)}$. Tx_2 retransmits the interference it produced at Rx_3 in part 1 of the transmission block, which ensures the size of the space occupied by Tx_2 at Rx_3 does not increase during part 2 of the transmission block. The signal transmitted by Tx_2 is evaluated as $\mathbf{x}_2^{(2,2)} = \mathbf{C}_{2|1,2}^{(2)} \mathbf{H}_{32}^{(2,1)} \mathbf{C}_{2|1,2}^{(1)} \mathbf{u}_{2|1,2}$, where $\mathbf{C}_{2|1,2}^{(2)} \in \mathbb{C}^{MT^{(2,2)} \times NT^{(2,1)}}$ is the concatenation of the random precoding matrices. In part 2 of the transmission block, Rx_j receives

$$\mathbf{y}_j^{(2,2)} = \mathbf{H}_{j1}^{(2,2)} \mathbf{u}_{1|1,2}^{(2)} + \mathbf{H}_{j2}^{(2,2)} \mathbf{C}_{2|1,2}^{(2)} \mathbf{H}_{32}^{(2,1)} \mathbf{C}_{2|1,2}^{(1)} \mathbf{u}_{2|1,2} + \mathbf{n}_j^{(2,2)}, \quad (17)$$

where $\mathbf{n}_j^{(2,2)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(2,2)}}\right)$.

Generation of overheard interference terms: To generate the terms to be transmitted in phase 3 of the transmission scheme, Rx_3 cancels the signal of Tx_2 from the received signal and obtains the remaining interference of Tx_1.

Let $\mathbf{y}_j^{(2)} = \left[\mathbf{y}_j^{(2,1)\mathrm{T}} \quad \mathbf{y}_j^{(2,2)\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{C}^{NT^{(2)} \times 1}$ denote the concatenation of the signal vectors received by Rx_j during the transmission block of phase 2. We write the signal received by Rx_3 in a form of

$$\mathbf{y}_3^{(2)} = \mathbf{H}_{31}^{(2)} \mathbf{u}_{1|1,2} + \mathbf{H}_{32}^{(2)} \mathbf{u}_{2|1,2} + \mathbf{n}_j^{(2)}, \quad (18)$$

where $\mathbf{H}_{31}^{(2)}$ and $\mathbf{H}_{32}^{(2)}$ are the effective channel matrices and $\mathbf{n}_2^{(2)} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{I}_{NT^{(2)}}\right)$. $\mathbf{H}_{32}^{(2)}$ can be written as

$$\mathbf{H}_{32}^{(2)} = \begin{bmatrix} \mathbf{I}_{NT^{(2,1)}} \\ \mathbf{H}_{32}^{(2,2)} \mathbf{C}_{2|1,2}^{(2)} \end{bmatrix} \mathbf{H}_{32}^{(2,1)} \mathbf{C}_{2|1,2}^{(1)}, \quad (19)$$

which almost surely has a rank of $NT^{(2,1)}$ with $\left(NT^{(2,2)}\right)$-dimensional left null space. Hence, there exists a full rank matrix $\mathbf{W}_3 \in \mathbb{C}^{NT^{(2,2)} \times NT^{(2)}}$, with $\mathbf{W}_3 \mathbf{H}_{32}^{(2)} = \mathbf{0}_{NT^{(2,2)} \times b_2^{(2)}}$.

By multiplying $\mathbf{y}_3^{(2)}$ with $\mathbf{W}_3$, Rx_3 cancels the signal of Tx_2 and obtains the remaining interference term of Tx_1 of $\mathbf{W}_3 \mathbf{y}_3^{(2)} = \mathbf{W}_3 \mathbf{H}_{31}^{(2)} \mathbf{u}_{1|1,2}$, which is simultaneously useful for both Rx_1 and Rx_2, where the noise term has been omitted. Further, we use the notation of order-(2,1) symbols, where an order-(2,1) symbol is a term desired by two receivers and overheard at the third unintended receiver. We denote by $\mathbf{u}_{l|i_1,i_2;j} \in \mathbb{C}^{q \times 1}$ a vector of $q \in \mathbb{N}$ order-(2,1) symbols, which is desired by Rx_{i_1} and Rx_{i_2}, available at Tx_l, and is known at Rx_j, $1 \leq i_1, i_2, l, j \leq 3, i_1 \neq i_2 \neq j$, $l \in \{i_1, i_2\}$. The remaining interference term of Tx_1 is hence denoted as order-(2,1) symbol vector $\mathbf{u}_{1|1,2;3} = \mathbf{W}_3 \mathbf{H}_{31}^{(2)} \mathbf{u}_{1|1,2} \in \mathbb{C}^{NT^{(2,2)} \times 1}$, with $q^{(2)} = NT^{(2,2)}$.

Choice of $T^{(2,1)}$, $T^{(2)}$ and $b_2^{(2)}$: The parameters are chosen to maximize the normalized number of the transmitted order-2 symbols $\frac{b_\Sigma^{(2)}}{T^{(2)}}$, while ensuring the transmitted order-2 symbols can be decoded given all order-(2,1) symbols are provided to the receivers which desire them. Since $b_1^{(2)} = MT^{(2)}$, maximizing $\frac{b_\Sigma^{(2)}}{T^{(2)}}$ is equivalent to maximizing $\frac{b_2^{(2)}}{T^{(2)}}$.

Let us consider the decodability of $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{2|1,2}$ at Rx_1, where the identical decodability condition holds for Rx_2 due to symmetry. $\mathbf{y}_1^{(2)}$ and $\mathbf{u}_{1|1,2;3}$ provide in total $N\left(T^{(2)} + T^{(2,2)}\right)$ linear combinations of $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{2|1,2}$ to Rx_1. To ensure the decodability of $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{2|1,2}$, we require the number of the available linear combinations to be equal to the number of the unknowns $N\left(T^{(2)} + T^{(2,2)}\right) = MT^{(2)} + b_2^{(2)}$, which can be used to express $\frac{b_2^{(2)}}{T^{(2)}}$ as

$$\frac{b_2^{(2)}}{T^{(2)}} = (2N - M) - N\frac{T^{(2,1)}}{T^{(2)}}, \quad (20)$$

which is inversely proportional to $\frac{T^{(2,1)}}{T^{(2)}}$.

$b_2^{(2)}$ is restricted as $b_2^{(2)} \leq MT^{(2,1)}$, which can be rewritten in terms of $\frac{T^{(2,1)}}{T^{(2)}}$ using (20) as

$$\frac{T^{(2,1)}}{T^{(2)}} \geq \frac{2N - M}{M + N}. \quad (21)$$

A further restriction on $\frac{T^{(2,1)}}{T^{(2)}}$ follows from Theorem 2.

Theorem 2: $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{2|1,2}$ are decodable only if

$$\frac{T^{(2,1)}}{T^{(2)}} \geq \frac{2N - M}{4N - M}. \quad (22)$$

Proof: The statement can be proven by showing linear dependency of the linear combinations of $\mathbf{u}_{1|1,2}$ and $\mathbf{u}_{2|1,2}$ obtained by Rx_1 and Rx_2 when (22) does not hold by using the approach of the proof of Theorem 1 in [9]. The details of the proof are omitted due to space limitation. ■

Since $\frac{b_2^{(2)}}{T^{(2)}}$ is inversely proportional to $\frac{T^{(2,1)}}{T^{(2)}}$, to maximize $\frac{b_2^{(2)}}{T^{(2)}}$, $\frac{T^{(2,1)}}{T^{(2)}}$ has to be chosen as a minimum satisfying (21) and (22). Next, we distinguish two regions of antenna configurations, where either (21) or (22) override each other.

Region 2.1: $M/N \leq 3/2$, (21) overrides (22). To ensure (21), we choose

$$T^{(2,1)} = 2N - M, \; T^{(2)} = M + N. \quad (23)$$

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TABLE I
CALCULATION OF NUMBERS OF TRANSMISSION BLOCKS

	Phase 1			Phase 2			Phase 3	
M/N	$b_\Sigma^{(1)}$	$q^{(1)}$	k_1	$b_\Sigma^{(2)}$	$q^{(2)}$	k_2	$b_\Sigma^{(3)}$	k_3
$1 < \frac{M}{N} < \frac{3}{2}$	$12MN$	$6MN$	3	$3MN$	$N(2M-N)$	6	$6N$	$2M-N$
$\frac{3}{2} < \frac{M}{N} \leq \frac{5}{3}$	$12MN$	$6MN$	$6N-M$	$N(6N-M)$	$2N^2$	$6M$	$6N$	$2MN$
$\frac{5}{3} < \frac{M}{N} \leq 2$	$4N(6M^2 - 15MN+10N^2)$	$2N(6M^2 - 15MN+10N^2)$	$3(6N - M)$	$N(6N-M)$	$2N^2$	$6(6M^2 - 15MN+10N^2)$	$6N$	$2N(6M^2 - 15MN+10N^2)$

Region 2.2: $M/N > 3/2$, (22) overrides (21). To ensure (22), we choose

$$T^{(2,1)} = 2N - M,\ T^{(2)} = 4N - M. \tag{24}$$

D. Transmission in Phase 3

In phase 3, all transmitters are scheduled to transmit simultaneously, where during the transmission block of $T^{(3)} = 4N$ time slots, each transmitter transmits $b_1^{(3)} = b_2^{(3)} = b_3^{(3)} = 2N$ order-(2,1) symbols. The transmission block of phase 3 is identical to the transmission block of phase 3 of the transmission scheme of [6], where the description of the transmission block is omitted due to lack of space.

IV. Achieved Number of DoF

In this section, the number of DoF achieved by the proposed transmission scheme is evaluated. The numbers of the blocks of each phase are chosen according to (2), where the calculations are summarized in Table I. Using $d = \frac{3b}{\sum_{i=1}^{3} k_i T^{(i)}}$, the achieved number of DoF is evaluated as

$$d = \begin{cases} \frac{36MN}{17M+14N}, & \text{if } 1 < \frac{M}{N} \leq \frac{3}{2}, \\ \frac{12MN(6N-M)}{-7M^2+34MN+24N^2}, & \text{if } \frac{3}{2} < \frac{M}{N} \leq \frac{5}{3}, \\ \frac{12N(6N-M)(6M^2-15MN+10N^2)}{-42M^3+321M^2N-552MN^2+284N^3}, & \text{if } \frac{5}{3} < \frac{M}{N} < 2. \end{cases} \tag{25}$$

As shown in Fig. 3, the normalized number of DoF $\frac{d}{3N}$ of the proposed transmission scheme is greater than that of the transmission schemes of [7] and [10]. As compared to [10], the proposed transmission scheme has a more effective transmission in phase 2 and in phase 1 for the antenna configurations of $3/5 < M/N < 2$, where the difference in phase 1 has a greater impact on the performance difference.

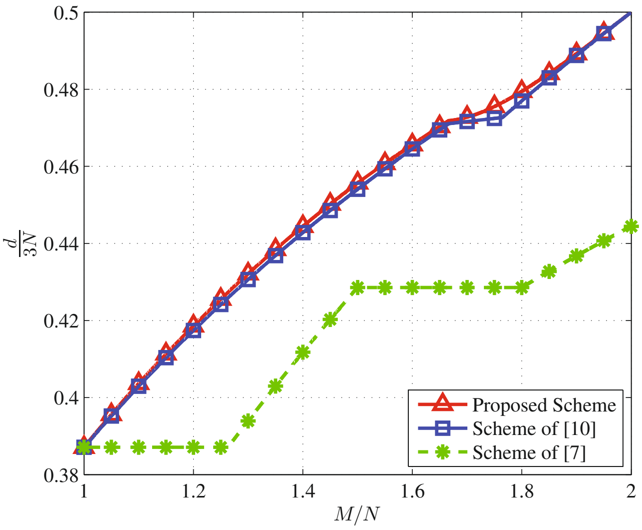

Fig. 3. Achievable number of DoF of 3-user MIMO IC with delayed CSIT

V. Conclusion

The three-user MIMO IC with delayed CSIT has been considered, where a new transmission scheme which achieves a number of DoF greater than that known in literature for the region of antenna configurations of $1 < M/N < 2$ has been proposed. The proposed scheme has a three-phase transmission structure, where in phases 1 and 2 a novel approach of using the delayed CSIT is employed.

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