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Inter-Subnetwork Interference Minimization in Partially Connected Two-Way Relaying Networks

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Abstract*—In this paper, a partially connected network consisting of multiple subnetworks is considered. Each subnetwork includes a single relay and all nodes connected to this relay. The two-way relaying protocol is employed to achieve a bidirectional communication between the nodes of a communication pair. Only relays which have a connection to both nodes of a communication pair can assist the communication. Throughout the paper, it is assumed that all nodes are served by at least one relay. If a single node of a communication pair is in addition connected to a relay which cannot assist the communication, this node receives only interference and no useful signal from this relay. Such a node suffers from inter-subnetwork interference. In this paper, a closed form algorithm is proposed that minimizes the inter-subnetwork interference power in the whole network. The nodes which suffer from inter-subnetwork interference have to design their transmit filters in order to minimize the inter-subnetwork interference and to achieve a reliable communication with their intended partner node. The simulation results show that the proposed algorithm is able to minimize the inter-subnetwork interference and therefore to improve the sum rate. Under certain conditions, which are derived in this paper, the proposed algorithm achieves an interference free communication and maximizes the degrees of freedom.

I. Introduction

In wireless multi-user communication systems, interference is and remains a fundamental issue [1]. Interference alignment (IA) is a promising technique to mitigate interference. This technique results in a sum rate that scales linearly with the number of nodes at high signal to noise ratio (SNR) [1]. The key idea of IA is to design the transmit filters in such a way that at each receiver, the interference signals are aligned in a subspace of a dimension being smaller than the number of interferers. This subspace is called interference subspace (IS). The interference subspace has to be linearly independent of the useful subspace (US), which contains only the useful signal. Hence, IA can maximize the achievable degrees of freedom (DoF).

The impact of relays in interference networks has been investigated, e.g., in [2], [3]. It has been shown that the use of relays does not increase the DoF. However, relays can help to perform IA which maximizes the DoF [2]. The two-way relaying protocol allows a bidirectional pair-wise exchange of data in two phases, the multiple access (MAC) phase and the broadcast (BC) phase [4]. Relay aided IA considering two-way relaying has been investigated in [3], [5]–[9] and the references therein. All these papers only consider a single relay that assists the communication.

In large communication systems, the received signals have quite different receive power levels due to physical phenomena, e.g., path loss or shadowing [10]. The assumption that sufficiently small channel coefficients of weak links can be approximated by zero leads to a partially connected network. Partially connected two-way relaying networks were considered in some papers, e.g., in [11], [12], where partially connected means that only a subset of the nodes are connected to each relay. Partially connected relaying networks require less channel state information (CSI) to perform IA than fully connected networks. This has been shown in [12] for a two-way relaying network and in [13] for a one way-relaying network, respectively.

In this paper, we investigate a partially connected two-way relaying ad-hoc network. Throughout this paper it is assumed that all nodes are served by at least one relay. Only relays which are connected to both nodes of a communication pair can serve this pair, i.e., they can assist the communication of this pair. Especially in large wireless networks, it may happen that not both nodes of a communication pair are connected to the same relays. If a single node of a communication pair is in addition connected to a relay which, therefore, cannot assist the communication, this node receives only interference and no useful signal from this relay. Such a node suffers from inter-subnetwork interference, due to the connection by an inter-subnetwork link to the additional relay.

One trivial solution to handle this type of interference, is to assume that the relay which does not serve this pair has enough antennas to suppress or align the interference from this single node. An algorithm which follows a similar strategy was proposed in [11]. In the previous studies [5], [12] only the case that both nodes of a communication pair are connected to the same relays was considered. In the present paper, it is assumed that the relay does not have the capability to minimize or align inter-subnetwork interference. Instead, the nodes have to design their filters in order to minimize the inter-subnetwork interference in the whole partially connected network. One advantage, in comparison to [11], of the new algorithm is that the relays which cannot assist the communication can determine their filters without any channel state information (CSI) about the nodes which cannot served by these relays.

The present paper is organized as follows: Section II introduces the system model of the considered partially connected network. In Section III, the proposed closed form algorithm which minimizes the inter-subnetwork interference power is presented. In Section IV, the performance of the proposed inter-subnetwork interference power minimization algorithm is investigated. Section V concludes this paper.

Notation: In the following, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $\mathbb{C}$ represents the set of complex numbers. $(.)^{*}$, $(.)^{\mathrm{T}}$, $(.)^{\mathrm{H}}$, $(.)^{-1}$, $(.)^{\dagger}$, denote the complex conjugate, transpose, the complex conjugate transpose, the inverse and the pseudo inverse of the element inside the brackets, respectively. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix. The Frobenious norm of $\mathbf{A}$ is denoted by $\|\mathbf{A}\|_{\mathrm{F}} = \sqrt{\mathrm{Tr}\,(\mathbf{A}^{\mathrm{H}}\mathbf{A})}$. The trace of a matrix is denoted by $\mathrm{Tr}(.)$. $\mathbb{E}[.]$ denotes the expectation of the element inside the brackets. The null space of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is given by $\mathrm{null}(\mathbf{A}) = \{\mathbf{x} \in \mathbb{C}^m : \mathbf{Ax} = \mathbf{0}\}$. The span of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is denoted by $\mathrm{span}(\mathbf{A}) = \{\mathbf{Ax} : \mathbf{x} \in \mathbb{C}^m\}$. $\chi_{\min,d}(.)$ denotes a matrix containing the eigenvectors corresponding to the d smallest eigenvalues of the matrix within the brackets, as its columns. $\perp$ denotes the orthogonality.

II. System Model

In this paper, we consider a partially connected network consisting of Q subnetworks. Each of the Q subnetworks contains a single amplify-and-forward half-duplex relay, i.e., the number of subnetworks is equal to the number of relays. The K multi-antenna communication pairs are distributed over the Q subnetworks. An example for $K = 8$ and $Q = 3$ is shown in Figure 1. Let $q \in \mathcal{Q} = \{1, ..., Q\}$ denote the relay index or the subnetwork index, respectively. The q-th relay in the q-th subnetwork is equipped with $R_q \geq 1$ antennas. Let (j, k), $j, k \in \mathcal{K} = \{1, ..., 2K\}$ denote a communication pair, where the communication partner index is given by

$$k = \begin{cases} j + K, & \forall j \leq K, \\ j - K, & \forall j > K. \end{cases} \tag{1}$$

Each node $k \in \mathcal{K}$ of the $2K$ nodes in the whole network is equipped with N_k antennas and wants to transmit $d_k \leq N_k$ data streams to its intended communication partner. To guarantee that node j can receive the d_k data streams transmitted from node k, it is assumed that for all communication pairs (j, k) $N_j \geq \max(d_j, d_k)$, where k is given by (1). In the following, it is assumed that both nodes of a communication pair transmit the same number of data streams i.e., $d_k = d_j$. The set of nodes which are connected to relay q is denoted by $\mathcal{K}(q)$, and $\mathcal{R}(k)$ is the set of relays which are connected to node k. For the scenario in Figure 1, two example sets are $\mathcal{K}(1) = \{1, 2, 3, 9, 10, 11\}$ and $\mathcal{R}(11) = \{1, 2\}$. Nodes which are connected to multiple relays belong to multiple subnetworks and are located in the so-called intersection area.

To exchange information between the two nodes of a communication pair in a bidirectional manner, the two-way relaying protocol [4] is exploited. This bidirectional communication is carried out in two phases, called MAC phase and BC phase. In the MAC phase, all nodes simultaneously transmit to all connected relays. Consequently, during the MAC phase,

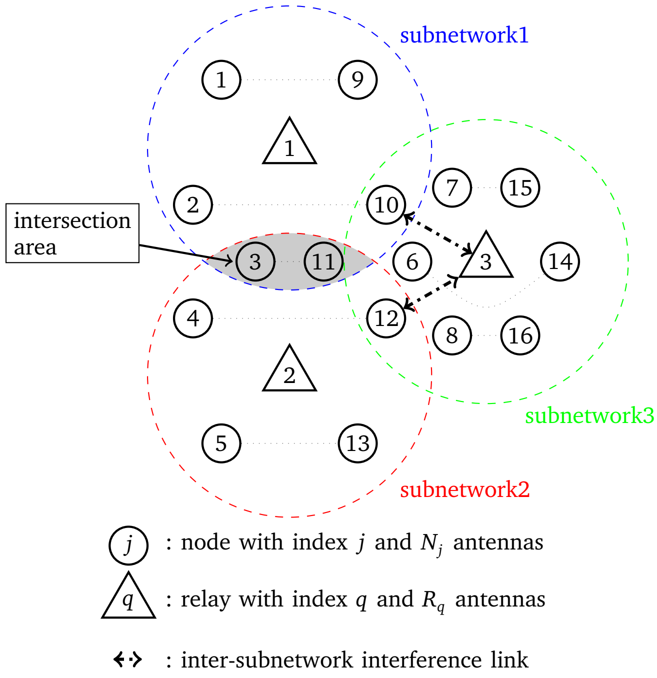

Fig. 1. Partially connected network consisting of $Q = 3$ subnetworks and $K = 8$ communication pairs. The nodes 10 and 12 suffer from inter-subnetwork interference from subnetwork 3.

each relay receives signals from all nodes in its subnetwork. In the BC phase, the relays retransmit a linearly processed version of the received signals back to the nodes. The direct links between the half-duplex nodes are irrelevant, because all nodes are transmitting or receiving simultaneously.

Each node which belongs to a single subnetwork receives, besides its useful signal and self-interference, only interference from this subnetwork, e.g., node 1 in Figure 1 receives interference only from a single relay, namely relay 1. Nodes which belong to multiple subnetworks receive besides self-interference, interference from several subnetworks, e.g., node 11 in Figure 1 receives interference from multiple relays, namely relays 1 and 2.

Due to the considered two-way relaying protocol, communication pairs can only be served by relays which are connected to both nodes of a communication pair. A relay which is only connected to a single node j of the communication pair (j, k) cannot assist the communication of the communication pair (j, k). Hence, nodes which are connected to multiple relays can receive the useful signal from these relays if and only if both nodes of the communication pair are connected to these relays. If a single node of a communication pair is connected to multiple relays, this node receives inter-subnetwork interference from all relays which cannot serve the pair to which this single node belongs, e.g., node 10 in Figure 1 receives inter-subnetwork interference from relay 3.

In this paper, it is assumed that all nodes are served by at least one relay and that single nodes of a communication pair can have an additional connection to at most one relay, which cannot assist the communication. The set $\mathcal{K}^{\wedge}(q)$ denotes the nodes which are only connected to relay q, e.g., $\mathcal{K}^{\wedge}(1) = \{1, 2, 9\}$. $\mathcal{R}^{\cap}(j, k) = \mathcal{R}(j) \cap \mathcal{R}(k)$ denotes the set of relays which are connected to the communication pair (j, k). All relays in this set are able to serve the communication pair (j, k), e.g., $\mathcal{R}^{\cap}(3, 11) = \{1, 2\}$.

Let $\mathbf{H}^{\mathrm{sr}}_{j,q} \in \mathbb{C}^{R_q \times N_j}$ and $\mathbf{H}^{\mathrm{rd}}_{q,j} \in \mathbb{C}^{N_j \times R_q}$ denote the frequency-flat, quasi-static channel matrices for the MAC phase and the BC phase, respectively. It is assumed that the entries of the channel matrices are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables. Thus, the channel matrices are of full rank with probability 1. Further, $\mathbf{V}_j \in \mathbb{C}^{N_j \times d_j}$ denotes the linear precoding matrix and $\mathbf{d}_j \in \mathbb{C}^{d_j \times 1}$ the data vector of node j. It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_j \mathbf{d}_j^{\mathrm{H}}] = \mathbf{I}_{d_j}, \forall j \in \mathcal{K}$ holds. The $2K$ nodes transmit independent data, i.e., $\mathbb{E}[\mathbf{d}_k \mathbf{d}_j^{\mathrm{H}}] = \mathbf{0}, \forall k \neq j$. Each of the $2K$ nodes has a maximum transmit power denoted by $P_{\mathrm{n,max}}$. To satisfy the maximum transmit power constraint, the precoders are normalized, i.e., $\|\mathbf{V}_j\|_F^2 \leq P_{\mathrm{n,max}}$. Let $\mathbf{n}_{\mathrm{r},q} = \mathcal{CN}(0, \sigma^2_{\mathrm{r},q}) \in \mathbb{C}^{R_q \times 1}$ denote the noise at relay q and $\mathbf{n}_{\mathrm{n},k} = \mathcal{CN}(0, \sigma^2_{\mathrm{n},k}) \in \mathbb{C}^{N_k \times 1}$ denote the noise at node k. The components of the two noise vectors $\mathbf{n}_{\mathrm{r},q}$ and $\mathbf{n}_{\mathrm{n},k}$ are i.i.d. complex Gaussian random variables.

In the MAC phase, all $2K$ nodes transmit their signals to the relays simultaneously. The received signal at relay q is given by

$$\mathbf{r}_q = \sum_{k \in \mathcal{K}(q)} \mathbf{H}^{\mathrm{sr}}_{k,q} \mathbf{V}_k \mathbf{d}_k + \mathbf{n}_{\mathrm{r},q}. \tag{2}$$

Before relay q retransmits the received signal to all connected nodes, the relay processes this signal. The processing matrix of relay q is denoted by $\mathbf{G}_q$ and is normalized such that the maximum transmit power constraint is fulfilled, i.e., $\|\mathbf{G}_q \mathbf{r}_q\|_F^2 \leq P_{\mathrm{r,max}}$, where $P_{\mathrm{r,max}}$ denotes the maximum transmit power of relay q. It is assumed that all relays have the same maximum transmit power.

In the BC phase, node k receives the signal

$$\begin{aligned}\mathbf{y}_k = &\sum_{q \in \mathcal{R}^{\cap}(k,j)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{j,q} \mathbf{V}_j \mathbf{d}_j + \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{k,q} \mathbf{V}_k \mathbf{d}_k \\ &+ \sum_{q \in \mathcal{R}^{\cap}(k,j)} \sum_{\substack{i \in \mathcal{K}(q),\\ i \neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i \mathbf{d}_i \\ &+ \sum_{q \in \mathcal{R}(j) \setminus \mathcal{R}(k)} \sum_{\substack{i \in \mathcal{K}(q),\\ i \neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i \mathbf{d}_i \\ &+ \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k},\end{aligned} \tag{3}$$

where nodes j and k are communication partners.

The first and the second term of (3) are the useful signal and the self-interference signal, respectively. The third and the fourth term of (3) represent the intra-subnetwork and inter-subnetwork interference, respectively. The last two terms represent the effective noise at node k.

Since node k knows its precoding matrix $\mathbf{V}_k$ and its data vector $\mathbf{d}_{jk}$, node k can subtract the backpropagated self-interference from the received signal $\mathbf{y}_k$, if the backpropagation channel $\mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{k,q} \forall q \in \mathcal{R}(k)$ is also known at node k [4]. In absence of perfect backpropagation channel information, pilot sequences can be used to estimate the backpropagation channel. Throughout the paper, it is assumed that the self-interference can be perfectly canceled. Let $\mathbf{U}_k^{\mathrm{H}} \in \mathbb{C}^{d_k \times N_k}$ denote the receive zero-forcing filter at node k. The estimated data vector at node k is given by

$$\hat{\mathbf{d}}_j = \mathbf{U}_k^{\mathrm{H}} \mathbf{y}_k. \tag{4}$$

To achieve IA, it is necessary that the unknown interference signal in the IS and the useful signal in the US are linearly independent at each receiver. The self-interference can be in the US or the IS. This results in the following IA conditions:

$$\begin{aligned}\mathbf{\Lambda}_i &= \mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i, && (5)\\ \mathbf{\Lambda}_i &= \mathbf{0}, \quad \forall i \in \{i \in \mathcal{K}(q) : i \neq k, j\}, && (6)\\ \mathrm{rank}(\mathbf{\Lambda}_i) &= d_j, \quad \forall i \in \{i \in \mathcal{K}(q) : i = j\}. && (7)\end{aligned}$$

III. Proposed algorithm

A. Introduction of the Proposed Algorithm

This section presents an algorithm to minimize the received inter-subnetwork interference power in the whole partially connected network. Communication pairs which are only connected to a single relay can perform signal alignment (SA) and channel alignment (CA) at this single relay, as proposed in [5] to achieve IA. If both nodes of a communication pair are connected to the same multiple relays, this pair can perform simultaneous signal alignment (SSA) and simultaneous channel alignment (SCA), as proposed in [12] to achieve IA.

If a single node of a communication pair is in addition connected to a relay which cannot assist the communication, this node receives only interference and no useful signal from this relay. Such a node suffers from inter-subnetwork interference. One trivial solution to handle this interference, is to assume that the relay which does not serve this pair has enough antennas to suppress the interference from this single node. In the present paper, it is assumed that the relay does not have the capability to minimize or align this inter-subnetwork interference. In the new algorithm proposed in this paper, the nodes themselves have to design their transmit filters in order to minimize the inter-subnetwork interference. It is worth to mention that the relays which have in addition connections to single nodes of communication pairs can determine their filters without any CSI of these single nodes.

B. Review of Signal Alignment and Channel Alignment

In this section, we shortly review the principle of SA and CA proposed in [5] to make this paper self-contained. Since these transmission techniques are able to maximize the degrees of freedom in a two-way relaying network [8], all K communication pairs shall perform SA and CA at all relays which can serve these nodes. To avoid inter-pair interference, each node designs its transmit filter in such a way that the signals of a communication pair are pairwise aligned in a subspace of the entire signal space at relay $q \in \mathcal{Q}$ [12]. These aligned signals have to be linearly independent of the signals of each other node pair at relay $q \in \mathcal{Q}$. The SA condition to align the signals from communication pair (j,k); $\forall j \neq k$; $j,k \in \mathcal{K}(q)$, at relay q is given by

$$\text{span}\left(\mathbf{H}^{\text{sr}}_{j,q}\mathbf{V}_j\right) = \text{span}\left(\mathbf{H}^{\text{sr}}_{k,q}\mathbf{V}_k\right), \tag{8}$$

as proposed in [5]. The entire solution space $\mathbf{A}$ of (8) is determined by taking the null space of $\mathbf{H}^{\text{ss}}_{j,k,q} = \begin{bmatrix}\mathbf{H}^{\text{sr}}_{j,q} & -\mathbf{H}^{\text{sr}}_{k,q}\end{bmatrix}$, given by

$$\underbrace{\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix}}_{\mathbf{A}} = \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,q}\right). \tag{9}$$

The transmit filters $\mathbf{V}_j$ and $\mathbf{V}_k$ of the communication pair are a subset of $\mathbf{A}$, given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} \subseteq \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,q}\right). \tag{10}$$

CA proposed in [5] is performed in the BC phase and is a dual problem to SA. The channel alignment condition for the communication pair (j,k); $\forall j \neq k$; $j,k \in \mathcal{K}(q)$, at relay q is given by

$$\text{span}\left(\mathbf{H}^{\text{rdH}}_{q,j}\mathbf{U}_j\right) = \text{span}\left(\mathbf{H}^{\text{rdH}}_{q,k}\mathbf{U}_k\right), \tag{11}$$

as proposed in [5].

Algorithms to perform SA and CA at multiple relays simultaneously were proposed in [12] and are termed SSA and SCA, respectively.

C. Properness Condition to Perform SA and CA

The number of required antennas at each relay to perform SA and CA was derived in [12] and has to be reformulated in order to handle subnetworks with an odd number of nodes

$$R_q = \frac{1}{2} \sum_{i \in \{\mathcal{K}_q : (i,j) \in \mathcal{K}_q\}} d_i. \tag{12}$$

The required number of antennas at each node to perform SA and CA was derived in [5] and is given by

$$N_k \geq \frac{R_q + d_k}{2}, \quad \forall k \in \mathcal{K}^\wedge(q). \tag{13}$$

D. Inter-Subnetwork Interference Power Minimization

In this section, an algorithm to minimize the inter-subnetwork interference power at a relay which receives signals from a single node of a communication pair, i.e., signals which do not contain any useful information for other nodes connected to this relay, is proposed. Communication pairs can be served by multiple relays in general. In the following, node j of the communication pair (j,k) shall be the node which suffers from inter-subnetwork interference. For simplicity of the notation, we assume an intersection of two subnetworks, where $\bar{q}$ denotes the relay which is connected to only node j of the communication pair (j,k) and $\breve{q}$ denotes the relay which serves this pair, i.e., which has a connection to both nodes of the communication pair (j,k).

In general, the power which relay $q \in \mathcal{Q}$ receives from node $i \in \mathcal{K}(q)$ in the MAC phase, is given by

$$\left\|\mathbf{H}^{\text{sr}}_{i,q}\mathbf{V}_i\right\|^2_{\text{F}} = \text{Tr}\left(\mathbf{V}^{\text{H}}_i\mathbf{H}^{\text{srH}}_{i,q}\mathbf{H}^{\text{sr}}_{i,q}\mathbf{V}_i\right). \tag{14}$$

This leads to the following optimization problem given by

$$\begin{aligned} \underset{\substack{\mathbf{V}_j \in \mathbb{C}^{N_j \times d_j} \\ \mathbf{V}^{\text{H}}_j\mathbf{V}_j = \text{I}}}{\text{minimize}} \quad & \text{Tr}\left(\mathbf{V}^{\text{H}}_j\mathbf{H}^{\text{srH}}_{j,\bar{q}}\mathbf{H}^{\text{sr}}_{j,\bar{q}}\mathbf{V}_j\right) \\ \text{subject to} \quad & \|\mathbf{V}_j\|^2_F \leq P_{\text{n,max}}. \end{aligned} \tag{15}$$

By applying the Lagrangian method and the property that the eigenvalues of the Hermitian matrix $\mathbf{H}^{\text{srH}}_{j,\bar{q}}\mathbf{H}^{\text{sr}}_{j,\bar{q}}$ are real [14], the optimum $\mathbf{V}_j$ which minimizes the interference power of the d_j data streams is given by

$$\mathbf{V}_{j,\min} = \chi_{\min,d_j}\left(\mathbf{H}^{\text{srH}}_{j,\bar{q}}\mathbf{H}^{\text{sr}}_{j,\bar{q}}\right), \tag{16}$$

where $\chi_{\min,d}(.)$ denotes a matrix containing the eigenvectors corresponding to the d smallest eigenvalues of the matrix within the brackets as its columns. To avoid inter-pair interference, all communication pairs (j,k) perform SA at the relays which serve these pairs, therefore the SA condition (8) has to be fulfilled. By fixing $\mathbf{V}_{j,\min}$, one loses $N_j d_j$ variables to solve (8). Rewriting (8) leads to a inhomogeneous system of linear equations, given by

$$\mathbf{v}^{(l)}_{k,\min}\mathbf{H}^{\text{sr}}_{k,\breve{q}} = \mathbf{H}^{\text{sr}}_{j,\breve{q}}\mathbf{v}^{(l)}_{j,\min}, \tag{17}$$

where $\mathbf{v}^{(l)}_{j,\min}$ is the l^{th} column of $\mathbf{V}_{j,\min}$. Such a system of equations can have no, one or infinitely many solutions for a fixed $\mathbf{v}^{(l)}_{j,\min}$. If $\text{rank}\left(\mathbf{H}^{\text{sr}}_{k,\breve{q}}\right) = R$, (17) has one particular solution and $N - R$ special solutions in the null space of $\mathbf{H}^{\text{sr}}_{k,\breve{q}}$. If (17) is overdetermined and has therefore no solution, $\mathbf{v}^{(l)}_{k,\min}$ is given by the least squares solution

$$\mathbf{v}^{(l)}_{k,\min} = \mathbf{H}^{\text{sr}\dagger}_{k,\breve{q}}\mathbf{H}^{\text{sr}}_{j,\breve{q}}\mathbf{v}^{(l)}_{j,\min}. \tag{18}$$

To guarantee that the transmit filters $\mathbf{V}_{k,\min}$ and $\mathbf{V}_{j,\min}$ are in the column space of the SA solution space $\mathbf{A} = \text{null}\left(\begin{bmatrix}\mathbf{H}^{\text{sr}}_{j,\breve{q}} & -\mathbf{H}^{\text{sr}}_{k,\breve{q}}\end{bmatrix}\right)$, one has to project the transmit filters $\mathbf{V}_{k,\min}$ and $\mathbf{V}_{j,\min}$ onto $\text{span}(\mathbf{A})$ along $\text{span}(\mathbf{A})\perp$. This operation is denoted by the projection matrix $\mathbf{P}$ and is given by

$$\mathbf{P} = \mathbf{A}\left(\mathbf{A}^{\text{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\text{H}}, \tag{19}$$

where $\left(\mathbf{A}^{\text{H}}\mathbf{A}\right) = \mathbf{I}$.

The transmit filters which are in the SA solution space and which minimize the inter-subnetwork interference power are given by

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \alpha \cdot \mathbf{P}\begin{bmatrix}\mathbf{V}_{j,\min} \\ \mathbf{V}_{k,\min}\end{bmatrix}, \tag{20}$$

where α is determined such that the node transmit power constraint is fulfilled.

Based on (13), one can define four different ranges depending on N_j. In the first range I., it is not possible to perform

TABLE I
REQUIRED NUMBER OF ANTENNAS

No optimization II.	Optimization III.	Orthogonal IV.
$N_j = \dfrac{R_{\breve{q}} + d_j}{2}$	$\dfrac{R_{\breve{q}} + d_j}{2} < N_j < R_{\bar{q}} + d_j$	$N_j \geq R_{\bar{q}} + d_j$

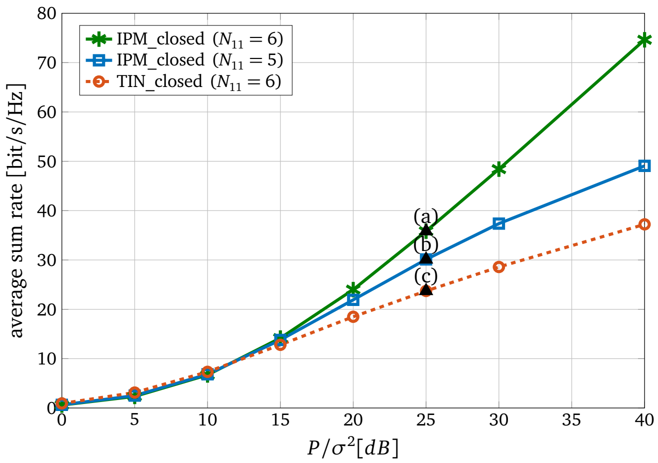

Fig. 2. Sum rate performance of the interference power minimization algorithm in comparison with an algorithm treating the interference as noise, $K = 8$, $Q = 2$, $R_1 = 4$, $R_2 = 5$, $d = 1$

SA, because (13) is not fulfilled. In the second range II., the number N_j of antennas corresponds to the minimum required number, i.e., a minimization of the interference power at $\bar{q}$ is not possible. The third range III., represents a range in which minimization of the interference power at $\bar{q}$ is possible. In the forth range IV., a further increase of the number N_j of antennas has no influence on the performances, since relay $\bar{q}$ receives already no interference from node j. Table I shows the required number of antennas of node j for the different ranges. The first range I. is not shown in this table, because it is not possible to perform SA.

E. Transceive Zero Forcing

The q relays perform transceive zero forcing as described in [12]. Let $\mathbf{G}_q^{\mathrm{RX\,H}}$ and $\mathbf{G}_q^{\mathrm{TX}}$ denote the receive and transmit zero forcing matrices, respectively. The effective channels in the MAC and BC phase are given by (21) and (22), respectively. These two square matrices are non-singular with probability one [12]. The relay processing matrix is given by

$$\mathbf{G}_q = \beta \cdot \mathbf{G}_q^{\mathrm{TX}} \cdot \mathbf{G}_q^{\mathrm{RX\,H}} = \beta \cdot \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}} \cdot \mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}\right)^{-1}, \tag{23}$$

where β is determined such that the relay transmit power constraint is fulfilled.

IV. Performance Analysis

In this section, the sum rate performance of the proposed algorithm is analyzed. We consider a scenario with $K = 8$ communication pairs distributed over $Q = 2$ subnetworks. The two connection sets are given by $\mathcal{K}(1) = \{1, 2, 3, 4, 9, 10, 11, 12\}$ and $\mathcal{K}(2) = \{4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16\}$. Relay 1 and

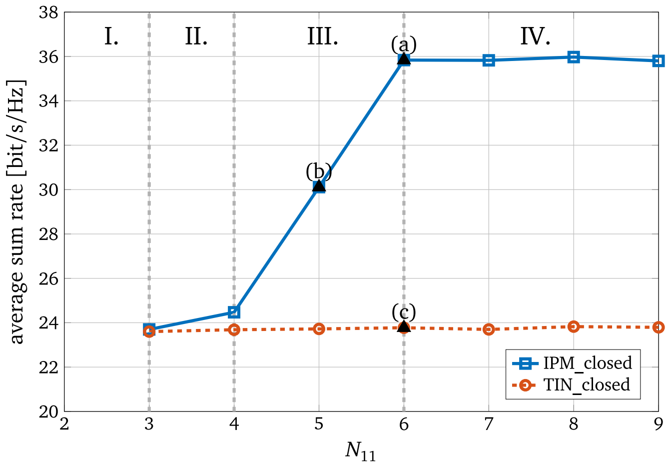

Fig. 3. Average sum rate versus the number of antennas N_{11} of the node which is in addition connected to a relay which cannot assist the communication. $K = 8$, $Q = 2$, $R_1 = 4$, $R_2 = 5$, $d = 1$, $\frac{P}{\sigma^2} = 25$dB. For I. - IV. see Table I

relay 2 are equipped with $R_1 = 4$ and $R_2 = 5$ antennas, respectively. Let N_{11} be the number of antennas of node 11 which suffers from inter-subnetwork interference. All other nodes are equipped with with the minimum required number of antennas to perform SA, given by (13). Each of the $2K$ nodes wants to transmit $d = 1$ data stream to its communication partner. For the simulations, it is assumed that the channels between the nodes and the relays are random i.i.d. Rayleigh fading channels. The channel matrices are normalized such that the average received signal power is the same as the average transmit signal power. Furthermore, we assume channel reciprocity and that the channel coefficients are constant during the MAC and BC phase. The noise power at each node and at each relay is assumed to be the same for the simulation, i.e., $\sigma^2 = \sigma_k^2 = \sigma_q^2, \quad \forall k \in \mathcal{K}, \forall q \in \mathcal{Q}$.

As a reference algorithm we chose an algorithm in which the relay which is in addition connected to a single node of a communication pair treats the signals from this node as interference. The relay is not able to suppress this interference. In the following, we will show two different types of results that are first briefly described.

Figure 2 shows the sum rate performance of the proposed interference power minimization ("IPM_closed") algorithm for $N_{11} = 6$ antennas and the "IPM_closed" algorithm for $N_{11} = 5$ antennas, in comparison to the reference algorithm treating interference as noise given by "TIN_closed", as a function of $\frac{P}{\sigma^2}$ where $P = P_{\mathrm{n,max}}$ denotes the transmit power of each node. σ^2 is the noise power per antenna at each relay and at each node. The transmit power at each relay is adjusted to $P_{\mathrm{r,max}} = \frac{1}{Q} KP$.

Figure 3 shows the sum rate performance of the proposed "IPM_closed" algorithm, in comparison to the "TIN_closed" reference algorithm, as a function of N_{11}. One can see the different ranges that are listed at Table I. The markers (a) in Figure 2 and Figure 3 represent a point with the same sum rate, N_{11} and $\frac{P}{\sigma^2}$, this is also fulfilled for the markers (b) and

$$\mathbf{H}_{\text{eff}q}^{\text{MAC}} = \begin{bmatrix}\mathbf{H}_{x,q}^{\text{sr}}\mathbf{V}_x & \cdots & \mathbf{H}_{y,q}^{\text{sr}}\mathbf{V}_y\end{bmatrix}, \quad x,y \in \left\{x,y : x,y \in \mathcal{K}(q); x \neq y; \text{span}\left(\mathbf{H}_{x,q}^{\text{sr}}\mathbf{V}_x\right) \neq \text{span}\left(\mathbf{H}_{y,q}^{\text{sr}}\mathbf{V}_y\right)\right\} \tag{21}$$

$$\mathbf{H}_{\text{eff}q}^{\text{BC}} = \begin{bmatrix}\mathbf{U}_x^{\text{H}}\mathbf{H}_{q,x}^{\text{rd}} \\ \vdots \\ \mathbf{U}_y^{\text{H}}\mathbf{H}_{q,y}^{\text{rd}}\end{bmatrix}, \quad x,y \in \left\{x,y : x,y \in \mathcal{K}(q); x \neq y; \text{span}\left(\mathbf{U}_x^{\text{H}}\mathbf{H}_{q,x}^{\text{rd}}\right) \neq \text{span}\left(\mathbf{U}_y^{\text{H}}\mathbf{H}_{q,y}^{\text{rd}}\right)\right\} \tag{22}$$

(c), respectively.

The proposed algorithm "IPM_closed", see Figure 2, achieves an interference free communication of all $2K$ nodes in the whole network, if $N_{11} = 6$. This means that the relay which is in addition connected to a single node of a communication pair receives no interference from this node. The reason for this is that the node connected to the relay which cannot assist the communication is able to transmit orthogonal to the channel to this relay. The DoF are maximized in this case, see Figure 2. Since the reference algorithm selects arbitrarily an SA solution out of the entire solution space, (9) a variation of the number of antennas N_{11}, has no influence on the performance, see Figure 3.

The proposed algorithm "IPM_closed", see Figure 2 and Figure 3, still outperforms the reference algorithm "TIN_closed" if the number of antennas N_{11} is decreased from $N_{11} = 6$ to $N_{11} = 5$. In this case, an interference free communication in the whole network is impossible, but in contrast to the reference algorithm, the proposed algorithm "IPM_closed" is able to minimize the receive interference power at the relay which cannot assist the communication.

The proposed "IPM_closed" algorithm, see Figure 3, outperforms the reference algorithm "TIN_closed", as long as the number of antennas N_{11} is larger than the minimum required number of antennas to perform SA (13), see Table I and Figure 3.

V. Conclusion

In this paper, we consider a partially connected network consisting of Q subnetworks where each subnetwork consists of a single relay and all nodes being connected to this relay. The bidirectional communication between the nodes takes place via intermediate relays, using the two-way relaying protocol. All communication pairs are served by at least one relay. If a single node of a communication pair is connected to an additional relay, this relay cannot assist the communication of this pair to which these node belongs. Such a node suffers therefore from inter-subnetwork interference. A new closed form algorithm which minimizes the inter-subnetwork interference power in the whole partially connected network is proposed. Furthermore, four different ranges of power minimization potential are defined. It is shown that the proposed algorithm can achieve an interference free communication in the whole network, i.e., the degrees of freedom are maximized. It is worth to mention that CSI estimation errors affects the IA performance. The simulation results show that the proposed algorithm is able to minimize the inter-subnetwork interference and therefore to improve the sum rate. Furthermore, the dependency of the performance on the number of antennas at the nodes is investigated.

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