Receiver-based Protection against Time-varying Noise in xDSL Systems

Dem Fachbereich 18
Elektrotechnik und Informationstechnik
der Technischen Universität Darmstadt
zur Erlangung der Würde eines
Doktor-Ingenieurs (Dr.-Ing.)
vorgelegte Dissertation

von
M.Sc. Wagih Sarhan
geboren am 25.08.1983 in El Giza-Ägypten

Referent:	Prof. Dr.-Ing. Anja Klein
Korreferent:	Prof. Dr.-Ing. habil. Tobias Weber
Tag der Einreichung:	12.12.2014
Tag der mündlichen Prüfung:	...

D 17
Darmstädter Dissertation

Kurzfassung

Heutzutage sind Dienste, wie hochauflösendes Fernsehen (HDTV), dreidimensionales Fernsehen (3DTV), Internet Telefonie (VoIP), Video-Streaming und interaktive Videoanwendungen ein Teil unseres Alltags geworden. Diese Dienste stellen hohe Anforderungen an Datenraten, Latenz und Link-Stabilität. Durch die Entwicklung solcher Dienste haben Digital Subscriber Lines (DSLs) in den letzten Jahren immer mehr an Bedeutung gewonnen. Die ständig steigende Anzahl an DSL-Nutzern und der Versuch, die wachsende Nachfrage an Bandbreite mit der Nutzung hoher Frequenzen zu erfüllen, hat das Fernnebensprechen (FEXT) zwischen Kupferleitungen zum dominanten Wertminderungsfaktor in aktuellen DSL-Systemen gemacht. Das Rauschen, das auf einer DSL-Leitung empfangen wird, kann verschiedene Quellen haben, wobei die Summe aus dem FEXT aller anderen aktiven Leitungen in einem Bündel das größte und dominanteste Rauschen ist. Die Anzahl der aktiven Leitungen hängt stark von der Tageszeit ab und daher ist die Verteilung des Rauschens auf DSL-Leitungen nicht stationär. Weiterhin schwankt das FEXT wenn DSL-Nutzer ihre Betriebszustand ändern. Diese Arbeit untersucht praktische Verfahren zum Schutz vor zeitabhängigem Rauschen in DSL-Systemen, die eine geringe Komplexität aufweisen. Damit unsere vorgeschlagenen Verfahren im Einklang mit den aktuellen DSL-Systemen bleiben, wird in dieser Arbeit angenommen, dass die DSL-Modems unabhängig voneinander und ohne Koordination mit anderen Leitungen betrieben werden. Weiterhin wird davon ausgegangen, dass alle DSL-Modems das gleiche Spektrum zur Übertragung benutzen. In dieser Arbeit werden deswegen nur Empfänger-basierte Verfahren zum Schutz vor zeitabhängigem Rauschen betrachtet.

Ein Ansatz zum Schutz vor zeitvariantem, nicht stationärem Rauschen besteht darin, die Ziel-Datenrate so einzustellen, dass sie auch noch zu Zeiten mit maximalem Rauschpegel erreicht werden kann. Traditionell wird bei der Berechnung der Ziel-Datenrate während der Initialisierung eine feste Signal-zu-Rausch-Verhältnis (SNR) Marge vom gemessenen SNR abgezogen. In praktischen DSL-Systemen, wird der Wert der SNR-Marge unabhängig vom SNR während der Initialisierung und von der Verteilung des Rauschens ad hoc auf 6 dB gesetzt. Die herkömmliche SNR-Marge ist demzufolge in Bezug auf Datenrate und Ausfallwahrscheinlichkeit nicht optimal. Zur Verbesserung des Schutzes vor zeitvariantem Rauschen und zur Erhöhung der Link-Stabilität wurde das Konzept des virtuellen Rauschens (VN) eingeführt. In dieser Arbeit optimieren wir die VN-Maske und die SNR-Marge gemeinsam, um unter der Bedingung, dass eine über 24 Stunden definierte Ziel-Ausfallwahrscheinlichkeit erfüllt wird, die Ziel-Datenrate zu maximieren.

Ein weiteres Problem der herkömmlichen SNR-Marge ist, dass sie unabhängig von den Tönen ist. In praktischen DSL-Systemen kann das empfangene Rauschen nur auf einigen Tönen ansteigen, so dass die pro-Ton-SNR-Margen der betroffenen Töne negativ werden. Um Reinitialisierungen in solchen Fällen zu vermeiden, haben Modems die Fähigkeit, die Bitbelegung der Töne an das SNR so anzupassen, dass die SNR-Margen über allen Tönen ausgeglichen werden. Diese Fähigkeit wird Bit-swapping genannt. Bit-swapping kann jedoch zu langsam sein, um Bits der betroffenen Töne zu verschieben, so dass nicht negative Margen über allen Tönen hergestellt werden. Demzufolge wird die Verbindung unterbrochen. In dieser Arbeit lösen wir dieses Problem, indem wir die VN-Maske und die SNR-Marge so modifizieren, dass, neben der erreichten maximalen Ziel-Datenrate und der erfüllten Ziel-Ausfallwahrscheinlichkeit, die Wahrscheinlichkeit, dass die Leitung in einen Zustand mit negativen pro-Ton-SNR-Margen gerät, verringert wird. Die VN-Maske und die SNR-Marge werden als Funktionen der Ziel-Ausfallwahrscheinlichkeit und der Verteilungen des an jedem Ton über 24 Stunden maximal gemessenen Rauschens ausgedrückt. Der Vergleich mit herkömmlichen Verfahren zeigt, dass das vorgeschlagene Verfahren sowohl die erzielten Datenraten als auch die erfüllte Ausfallwahrscheinlichkeit verbessert.

Ein weiterer Ansatz zum Schutz vor zeitvariantem, nicht stationärem Rauschen besteht darin, die Ziel-Datenrate dem veränderlichen Rauschen anzupassen. Das Konzept der Seamless Rate Adaptation (SRA) wurde eingeführt, um die Ziel-Datenrate nahtlos und ohne Unterbrechung der Dienste an das SNR anzupassen. Interleavers, die in der Übertragung vor Burst-Fehler schützen, geben eine Einschränkung vor, um wie viel die Datenrate in einem SRA-Vorgang geändert werden darf. Im Fall großer Veränderungen des SNRs müssen mehrere aufeinander folgende SRA-Vorgänge ausgeführt werden, um die Ziel-Datenrate zu erreichen. Darüber hinaus ist es sehr wahrscheinlich, dass die Bitbelegung aller Töne geändert werden muss, um die gleiche Fehlerwahrscheinlichkeit über alle Töne nach jedem SRA-Vorgang zu erzielen. Folglich, ist das herkömmliche SRA zu langsam und stellt somit nur ein Mittel zur Anpassung an langsame Veränderungen im SNR dar. Des Weiteren wird das herkömmliche SRA bei großen, schnellen Veränderungen im FEXT keine Reinitialisierungen verhindern können. In dieser Arbeit werden zwei neue Verfahren, die die Datenrate dem SNR anpassen, präsentiert. In dem Fall großer Veränderungen im SNR halten beide Verfahren die Einschränkungen, die von den Interleavers vorgegeben werden, ein. Weiterhin erzielen beide Verfahren die gleiche Fehlerwahrscheinlichkeit über alle Töne erst nach dem letzten Vorgang wenn die Ziel-Bitbelegung aller Töne erreicht wurde. Das erste Verfahren modifiziert die Töne zu deren Ziel-Bitbelegungen in der Reihenfolge, die zum schnellsten Abfall der mittleren Bit-Fehlerwahrscheinlichkeit führt. Das zweite Verfahren modifiziert die Töne in Gruppen und findet die gruppenweise konstante Bitverringerungen, die zum schnellsten Abfall der mittleren Bit-Fehlerwahrscheinlichkeit führen. Darüber hinaus werden praktische Varianten der vorgeschlagenen Verfahren präsentiert. Diese Varianten weisen eine geringe Komplexität auf und können in der Praxis in Modems mit begrenzter Rechenleistung eingesetzt werden. Der Vergleich mit herkömmlichem SRA zeigt, dass die vorgeschlagenen Verfahren die Adaptionszeit enorm verkürzen und die Anzahl der aufgetretenen Fehler während der Adaption verringern.

DSL-Nutzer benutzen oft verschiedene Dienste, die verschiedene Anforderungen an Dienstqualität in Bezug auf Ausfallwahrscheinlichkeit stellen. In solchen Fällen schlagen wir ein iteratives, hybrides VN-SRA-Verfahren vor. Das hybride VN-SRA weist dem vorgeschlagenen Verfahren, das die VN-Maske und die SNR-Marge gemeinsam optimiert, Töne zu, für Dienste, die eine Bit-Fehlerwahrscheinlichkeit, die kleiner oder gleich groß ist als die Ziel-Bit-Fehlerwahrscheinlichkeit, mit einer Ziel-Ausfallwahrscheinlichkeit, die kleiner oder gleich groß ist als die Ziel-Ausfallwahrscheinlichkeit, anfordern. Des Weiteren weist das hybride VN-SRA den vorgeschlagenen Verfahren, die die Ziel-Datenrate an das SNR anzupassen, Töne zu, für Dienste, die temporäre Erhöhungen der Fehlerwahrscheinlichkeit über die Ziel-Fehlerwahrscheinlichkeit während der Adaptionszeit tolerieren können. Simulationsergebnisse der Datenraten zeigen Gewinne gegenüber dem Fall, bei dem über alle Töne eine Bit-Fehlerwahrscheinlichkeit herrscht, die kleiner oder gleich groß als die Ziel-Bit-Fehlerwahrscheinlichkeit ist, mit einer Ziel-Ausfallwahrscheinlichkeit, die kleiner oder gleich groß als die Ziel-Ausfallwahrscheinlichkeit ist.

Die in dieser Arbeit vorgeschlagenen Verfahren können die heute bestehenden DSL-Systeme verbessern. Weiterhin muss für die Implementierung dieser Verfahren der aktuelle DSL-Standard minimal geändert werden.

Abstract

Digital Subscriber Lines (DSLs) have gained importance over the last years as services such as high-definition television (HDTV), three-dimensional television (3DTV), Voice over Internet Protocol (VoIP), video streaming and interactive video applications, which pose high requirements on data rates, latency and line stability, have become part of our every day life. The constantly increasing number of DSL users and trying to match the growing bandwidth demand by using high frequencies have made far-end crosstalk (FEXT) between copper wires the dominant impairment in current DSL systems. The noise perceived on a DSL line is comprised of noises from several sources, but the largest and most dominant noise on DSL lines is the summation of FEXT from all other active lines in the bundle. The number of active lines is strongly daytime dependent and consequently, the distribution of noise on DSL lines is not stationary. Furthermore, FEXT fluctuates when DSL users change their operational state. This work investigates low-complex and practical approaches for the protection against time-varying noise in DSL systems. In order to be in compliance with current DSL systems, in this work, the DSL modems operate independently without any coordination with other lines. Furthermore, the same transmit spectra are assumed for all the modems, and only receiver-based approaches are considered.

One approach to protect against time-varying non-stationary noise is to set the target bit rate such that it can still be achieved at times with peak noise levels. Traditionally, a fixed signal-to-noise-ratio (SNR) margin is subtracted from the measured SNR during initialization when calculating the target data rate. In practical DSL systems, the value of the SNR margin is set ad-hoc to 6 dB and is independent of the SNR at initialization and of the noise distribution, and therefore, the state of the art SNR margin is not optimal in terms of data rate and outage probability. To improve the protection against time-varying noise and increase link stability, the concept of Virtual Noise (VN) has been introduced. In this work, we jointly optimize the VN mask and the SNR margin in order to maximize the target data rate while satisfying a target outage probability defined over 24 hours. Another problem with the state of the art SNR margin is that it is tone-independent. In practical DSL systems, the received noise might increase only at some tones such that the per-tone SNR margins of the affected tones become negative. To prevent re-initializations in such cases, modems have the ability to equalize the SNR margin across tones by adapting their bit-loading to slowly changing SNR by using bit-swapping. However, bit-swapping might be too slow to move bits from the affected tones such that non-negative per tone margins are restored and the connection is interrupted. In this work

we solve this problem by adjusting the VN mask and SNR margin such that in addition to achieving the maximum target data rate while satisfying a target outage probability, the probability of the line going to a state with negative per-tone SNR margins is reduced. The VN mask and the SNR margin are expressed as functions of the target outage probability and the distributions of the maximum noise power measured over 24 hours at each tone. Comparison with state of the art approaches reveals that the proposed approach improves the data rate and outage probability performance.

Another approach to protect against time-varying non-stationary noise is to adapt the target data rate to the varying noise. The concept of Seamless Rate Adaptation (SRA) was introduced to seamlessly adapt the data rate of a connection to the changing SNR while in operation without any service interruption. Interleavers used in transmission for the protection against burst errors impose a constraint on how much the data rate that can be changed by an SRA procedure. Moreover, to keep an equal error probability over all tones, it is very likely that the bit-loadings of all tones have to be modified in each SRA procedure. Hence, the state of the art SRA is too slow and provides means of adaptation only to slowly changing SNRs and will not prevent reinitializations under certain conditions such as large rapid changes in FEXT. In this work, two adaptation approaches are proposed. In the case where multiple adaptation procedures are required to modify the bit-loadings of the tones to the target bit-loadings that correspond to the new SNR, both approaches satisfy the data rate constraint imposed by interleavers. Furthermore, both approaches equalize the error probability over all tones only after the last procedure when the target bit-loadings is achieved at all tones. The first approach modifies the tones to their target bit-loadings in the order that leads to the fastest decrease in the average bit error rate (BER). The second one modifies the bit-loadings of the tones in groups and finds the per group constant bit reductions that decrease the average BER as rapidly as possible. Moreover, low complex versions of the proposed approaches are presented such that they can be used in practice by DSL modems with limited computational power. Performance results reveal that all the proposed approaches shorten the adaptation time tremendously and decrease the number of errors that occur during the adaptation when compared to the state of the art SRA.

DSL users often use services with different requirements on quality of service in terms of outage probability. In such cases, we propose in this work a hybrid VN-SRA approach that iteratively allocates tones to be used by the proposed approach where the VN mask and the SNR margin are jointly optimized for services that require a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability. Moreover, the hybrid VN-SRA approach allocates tones to be used by the proposed adaptive data rate approaches for services that can tolerate temporary increases in the BER above the the target BER during the adaptation to the varying noise. Performance results reveal data rate gains when compared to the case where the data rate achieved over all tones is guaranteed at a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability.

The approaches presented in this work can improve the performance of today's DSL systems. Furthermore, the implementation of those approaches requires only minimal changes of the current DSL standard.

Contents

Chapter 1

Introduction

1.1 xDSL Systems

Ever since Alexander Graham Bell invented the telephone in 1875, the telephone network has continuously grown worldwide and today consists of around 700 million lines [PS99]. This existing infrastructure motivated telephone companies around the world to search for technical solutions that allow the existing telephone network to be used for digital communication. The first digital subscriber line (DSL) system was introduced in 1976 and was the basic integrated services digital network (ISDN) [PS99, ANS92]. The ISDN connection provided a bit rate of 128 kbit/s. Over the years, a number of different xDSL techniques, such as high-bit-rate digital subscriber line (HDSL), asymmetric digital subscriber line (ADSL) and ADSL2, have evolved gradually offering higher data rates [PS99]. The very-high-bit-rate digital subscriber line 2 (VDSL2) is the latest DSL standard and was standardized in 2005 [IT11]. VDSL2 offers data rates of up to 100 Mbit/s. Although other technologies such as wireless communications and fiber optics have also evolved over the recent years, DSL still accounts, with over 350 million DSL lines, for over 65% of the total broadband access market [PS12].

DSL employs physical layer technologies for digital transmission over the local loop of the traditional telephone network [PS99]. The term local loop refers to the twisted-pair telephone line from the edge of a telecommunications service provider's network to the customer premises (CP). In literature, in the context of DSL, the local loop is also referred to as the last mile of the telecommunications network [KBP+13, DQH+06, PFG07]. The physical link of the local loop consists of conventional shielded twisted-pair copper wires of which several run together in a multi-pair cable binder over several hundreds of meters or even kilometers. On the provider's end, the DSL line is terminated by a DSL Access Multiplexer (DSLAM), a networking device serving as the interface between multiple DSLs and a high-speed internet backbone [PFG07]. A DSLAM contains several line-cards which feature a number of ports to which lines are connected. The majority of DSLAMs are located at the Central Office (CO) of the telecommunications service provider. For customers in areas distant from a CO, the DSLAM can be located at an outdoor cabinet (OC) that is close to the customer's premises. The link between the outdoor cabinet and the CO is usually optical fiber [SCGC02]. Figure 1.1 depicts the two mentioned common VDSL/VDSL2 scenarios where a DSLAM is either located at a CO or at an OC. Throughout this dissertation, a copper pair running from the CP to either the CO or the OC will be denoted DSL line.

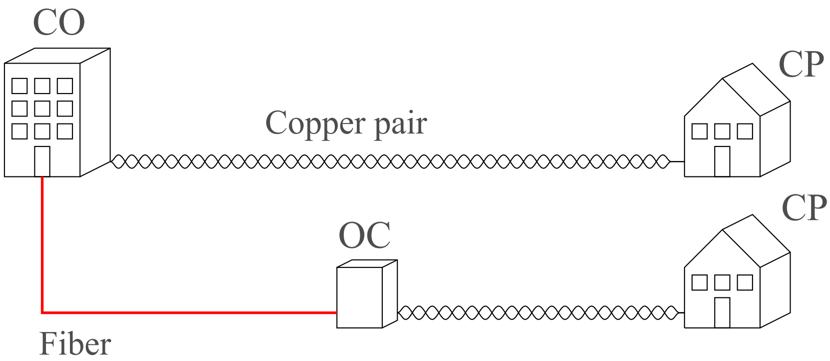


Figure 1.1: Two common xDSL deployment scenarios.

The improvement of the DSL technology led to the evolvement of the services used by DSL users. Figure 1.2 shows a modern home network that is being fed by a DSL access network. As shown in Figure 1.2, the deployment of services such as high-definition television (HDTV), three-dimensional television (3DTV), Voice over Internet Protocol (VoIP), video streaming and interactive video applications has become more popular in recent years. In the early years of DSL, end users were more likely to tolerate interruption of service when internet access was used mainly for basic services such as email or internet browsing. With the evolution of the services deployed over DSL, the expectations of end users with respect to Quality of Service (QoS) has increased [SBB13]. Today's DSL user is more concerned about data rate fluctuations, re-initializations, error rate and delay-jitter than ever before since the services deployed over DSL have become part of his every day life [Jag08]. In order for DSL providers to compete with other providers that use other technologies such as terrestrial TV, coaxial cable and fiber to home for the delivery of such mentioned services, highest levels of line rate, signal quality and service stability have to be guaranteed [PS12]. Furthermore, DSL users often use different combination of services. Real-time services such as HDTV, 3DTV, video conferencing and VoIP pose stricter constraints on QoS than services such as video on demand, streaming and downloading which can tolerate temporary increases in bit-error-rate (BER). Hence, DSL users have specific requirements on QoS depending on the package of services they wish to use.

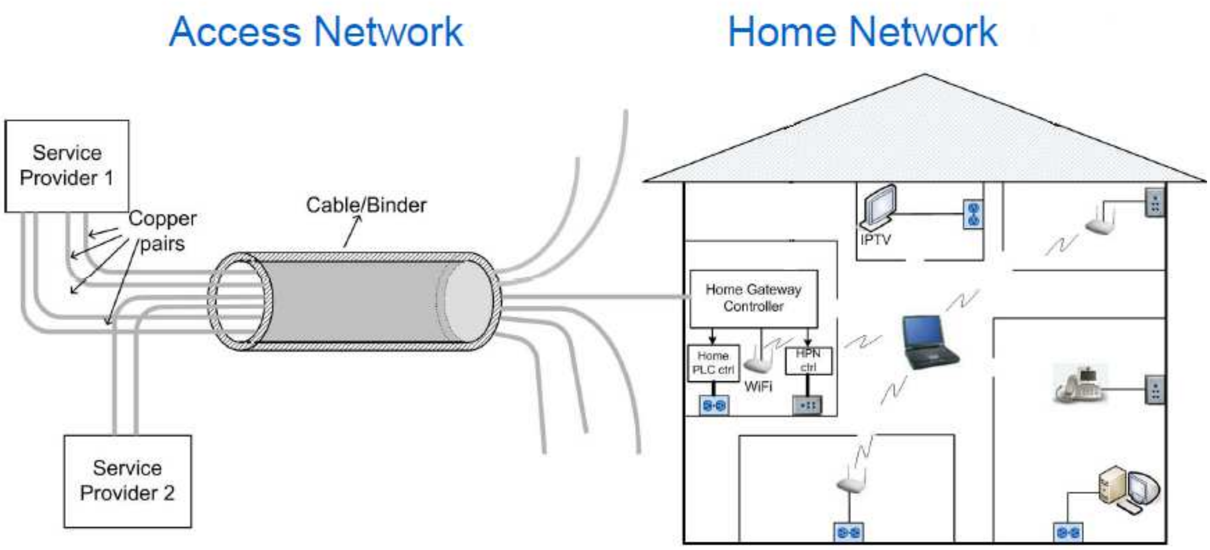


Figure 1.2: Today's home and access networks [Jag08].

1.2 Time-varying Crosstalk in xDSL Systems

In practical DSL systems, a DSL line is distorted by several kinds of noise such as radio frequency interference from nearby electrical circuits, impulse noise and background white noise, but the most common and largest noise in DSL is interference from other lines [HGC+06, GVK01, WAK+08, CFM+03, RWM+06]. In this section, the interference between multiple DSL lines is discussed.

As already mentioned in the previous Section 1.1, and shown in Figure 1.2, DSL lines share the same cable binder at the connection between the CP and either the CO or the OC. The electrical signal in a DSL line generates an electromagnetic field, which surrounds the line and induces an electrical signal into nearby DSL lines [PS99, HGC+06]. This behavior is referred to as inductive coupling between DSL lines. Moreover, in the late nineteenth century it was discovered that the inductive coupling between DSL lines

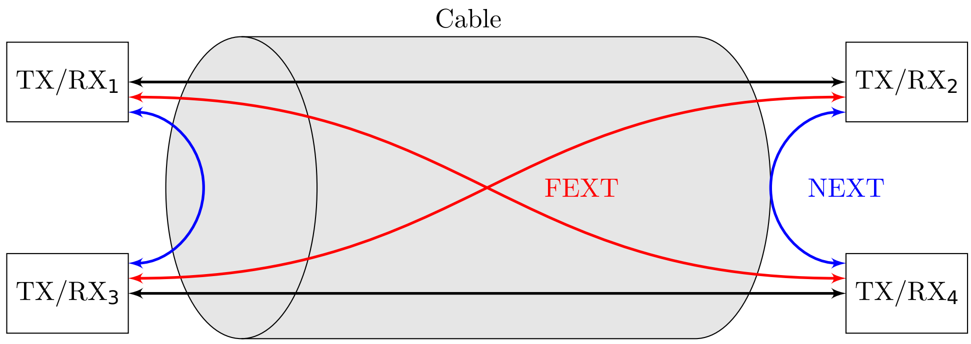


Figure 1.3: NEXT and FEXT in a DSL cable binder.

can be reduced by twisting the individually isolated wires of a line together [GVK01]. With a sufficiently short space between twists, the inductive coupling of energy over a small segment of wire is canceled by the out-of-phase energy coupled on the next segment of wire [PS99]. The twisting of the wire pairs reduces the inductive coupling between lines, but some induced signal leakage remains, also known as crosstalk [GVK01,RWM+06].

Crosstalk is strongest between adjacent lines in a cable binder and it becomes weaker the larger the physical distance between the lines in the binder is. In addition, the crosstalk between lines in different binders is much smaller than the crosstalk between lines sharing the same binder. Furthermore, for any two lines in a cable section, the coupling function does not usually change appreciably over time, and it is symmetrical in that the same coupling function is observed between two ends when measured in either direction [GDJ05]. Coupling functions generally increase with increasing frequency [PS99, HGC+06]. Nevertheless they are a consequence of the construction and deployment of the lines. Thus, coupling functions usually show dramatic nulls and peaks in arbitrary frequencies [GDJ05].

As illustrated in Figure 1.3, one distinguishes between near-end crosstalk (NEXT) where the crosstalk originates from the same end of the loop where the signal is received and far-end crosstalk (FEXT) where it originates from the opposite end of the loop [WTN99]. In Figure 1.3, TX and RX denote the transmitter and receiver, respectively. In VDSL2, upstream transmission from CP to DSLAM and downstream transmission from DSLAM to CP are assigned different frequency bands, so that transmitters do not cause crosstalk on the frequency band used by receivers on the same end of the loop. Therefore, the issue of NEXT has basically vanished and all considerations in this dissertation concentrate on FEXT. In the recent years, DSL frequencies have been extended into the megahertz region in order to increase the achievable data rates. Since FEXT generally increases with increasing frequency [PS99, HGC+06], it has become the dominant impairment in current DSL systems [GDJ05, MTM11, GVK01].

Since the noise perceived by a DSL line is dominated by the summation of FEXT from all other active lines in the binder, it fluctuates when DSL users turn on/off their modems or enter/exit the low power state. The Low Power (L2) state is an operational state that has been standardized for ADSL2 and ADSL2+ and in which the transmitted power can be scaled down by up to 31 dB. The transmitted power is scaled down if there is little or no need for transmission of information on the link from transceiver unit at the CO to the transceiver unit at the CP [Gin05, BGL+11]. Furthermore, the received noise power on a DSL line can be described by the noise power spectral density (PSD). In ADSL and VDSL2 systems, the transmission channel is divided into multiple sub-channels using the discrete multi-tone modulation (DMT) [SSMS03, Cho98, TTPH94]. In the context of DSL, the sub-channels are denoted by tones. For the bit-loading of the used tones, the noise PSD needs to be known. Therefore, for the calculation of the target data rate, the DSL modem measures the noise PSD during initialization. Initialization is a phase that modems have to go through in order to enter the operational state, referred to as showtime. During initialization, the modems at both ends measure the channel and exchange parameters to be used in showtime. In case the noise PSD increases during showtime, e.g., due to a disturber modem being switched on, such that a transmission with the data rate and the error probability calculated during initialization is not possible, the connection is interrupted and the user is considered to be in outage.

In order to provide a reliable service, it is important to be robust against the fluctuating noise. The next section gives an overview on the state of the art measures that can be taken to protect each DSL line from neighboring interfering lines.

1.3 State of the Art Receiver-based Protection against Time-varying Noise

1.3.1 Introduction

This section presents an overview on the state of the art measures that can be taken at the receiver side to protect from time-varying noise. First, the receiver-based protection is motivated. Then, the receiver-based state of the art measures for the protection from time-varying crosstalk are presented and discussed.

The challenge of maximizing the target data rate with a certain guarantee on QoS has been the aim of many researchers over the last years. In dynamic spectrum management (DSM), DSL transmit spectra are reshaped to reduce crosstalk between lines. Finding optimum spectra for all lines, such that the overall data rate is maximized, is a very complex optimization problem and its solution is regarded as a theoretical upper bound [HLLN12, CCLJ08, FMMG11]. Sub-optimal spectra can be found using iterative approaches, where each line iteratively optimizes its spectrum, when other users' spectra are assumed to be fixed, until convergence is achieved [CCLJ08, Jag08, WKK12]. The problem is that such iterative approaches require a very long time until convergence is achieved, since modems cannot change their spectra during operation and have to wait until they are switched off and then on again to transmit with the new spectra. Moreover, changing the spectrum or the data rate requirements of only one line, after an optimum is achieved, would start the whole iterative optimization again. Clearly, this can be the cause for network instability. Moreover, DSM approaches assume all direct and crosstalk channels and the noise distributions on all lines to be available at a central coordination unit called spectrum management center (SMC) [HLLN12, CCLJ08, FMMG11]. However, the estimation of crosstalk channels is not supported or required by current DSL standards [LLD+08].

Another problem with DSM is that lines in a binder might belong to different DSL providers as shown in Figure 1.1. DSL providers in practical DSL systems do not coordinate in the operation of their lines [AADEB12]. This fact severely degrades the performance of DSM [AADEB12]. Furthermore, it makes DSM even more complex, and therefore, less practical.

This dissertation investigates low-complex and practical approaches for the protection against fluctuating noise. In order to be in compliance with current DSL standard, here, the following is assumed. Firstly, the DSL modems operate independently without any coordination with other lines. Secondly, the same transmit spectra are assumed for all the modems and only receiver-based approaches are considered.

The receiver-based approaches can be categorized in two groups. The approaches in the first group aim at setting the target data rate at initialization such that it does not have to be changed during operation, referred to as fixed data rate approaches. The approaches in the second group allow the target data rate to be changed during operation, referred to as adaptive data rate approaches. In Sections 1.3.2 and 1.3.3, the state of the art receiver-based fixed and adaptive approaches are presented and discussed, respectively.

1.3.2 Fixed Data Rate Approaches

One approach to protect against time-varying noise is to set the target bit rate such that it can still be achieved at a BER lower than or equal to the target BER at times with peak noise levels. The most common and widely-used approach to do so, is to subtract a fixed tone-independent SNR margin from the measured SNR during initialization, when calculating the bit-loading of the DMT modulated tones [Gin05, SKK12]. The SNR margin is the decrease in the SNR that can be withstood by the system at the same target data rate and error probability [Jag08]. Traditionally, the value of the SNR margin is set to 6 dB [Jag08, Gin05, ST00]. Consequently, on the one hand, if a user switches his modem on during low noise times and the noise increases during operation due to disturber modems being switched on or exiting the L2 state, the 6 dB SNR margin assigned during initialization might not be enough to sustain the connection and the user might have to reinitialize at a lower data rate. On the other hand, if a user switches his modem on during peak noise times, the 6 dB SNR margin would be unnecessary, and therefore, achievable data rate is wasted. This problem will be denoted here by the "first and last user problem".

Another problem with the state of the art SNR margin is that it is tone-independent. It was agreed upon to use one SNR margin value for all tones, in order to satisfy the same target error probabilities at all tones and in order to reduce the control overhead [PS99, ANS01, JHC08a]. In [HH06, HHvD+10, Has10] it was proposed to use un-equal target error probabilities at different tones in order to achieve different error protection for data with different importance levels. In this dissertation, in order to be in compliance with the current VDSL2 standard, we restrict the analysis to DSL systems with equal target error probabilities at all tones. In practical DSL systems, the received noise power is tone-dependent and the noise power might increase at some tones such that the per tone SNR margins of the affected tones become negative. To prevent re-initializations in such cases, modems have the ability to adapt their bit-loading to slowly changing channels by applying bit-swapping if given enough time. Bit-swapping procedures equalize the SNR margin across tones by moving bits from tones with smaller per tone SNR margins to those with larger per tone SNR margins [Cio98, Cio08]. In case of abrupt noise increase at some tones, the per tone SNR margins of the affected tones might become negative and bit-swapping might be too slow to move bits from the affected tones such that non-negative per tone margins are restored [JHC08a]. In such cases, if the minimum of the per tone SNR margins remains negative for a certain period of time, the connection is interrupted and modems will have to reinitialize at a lower data rate. This problem will be denoted here by the "unequal per-tone noise protection problem".

In [Jag08, JHC08a], it was proposed to solve the "first and last user problem" by setting the SNR margin in relation to the SNR during initialization. Moreover, it was proposed to solve the "unequal per-tone noise protection problem" by distributing the SNR margin unequally between tones, assigning larger per-tone SNR margins to tones where the noise power is more likely to increase and smaller per-tone SNR margins to tones where the noise power is less likely to increase [Jag08, JHC08a]. However, this leads to increased overhead, since one SNR value per tone has to be communicated between the transceivers, as well as very complex bit-swapping algorithms that rely on the noise statistics of all used tones in every bit-swap.

The concept of the Virtual Noise (VN) was introduced to improve the protection against fluctuating noise and increase link stability, by solving the "first and last user problem" and the "unequal per-tone noise protection problem" in a low complex manner. The VN is a tone-dependent noise PSD specified in the central office management information base and communicated to the transceivers during initialization [IT11]. The reference noise PSD, which is the maximum of the VN PSD and the measured noise PSD, is used for calculating the bit-loading on each tone during initialization and showtime [VB05]. An SNR margin much smaller than 6 dB is assigned during initialization to compensate for any increase of the reference noise PSD during operation.

Figure 1.4 demonstrates the basic concept of the VN approach. In Figure 1.4, the noise power received on a victim line and the state of the art SNR margin that is being added to it when calculating the bit-loading during initialization are depicted for some tones. The area between the SNR margin and the received signal power exemplifies the target data rate calculated at initialization.

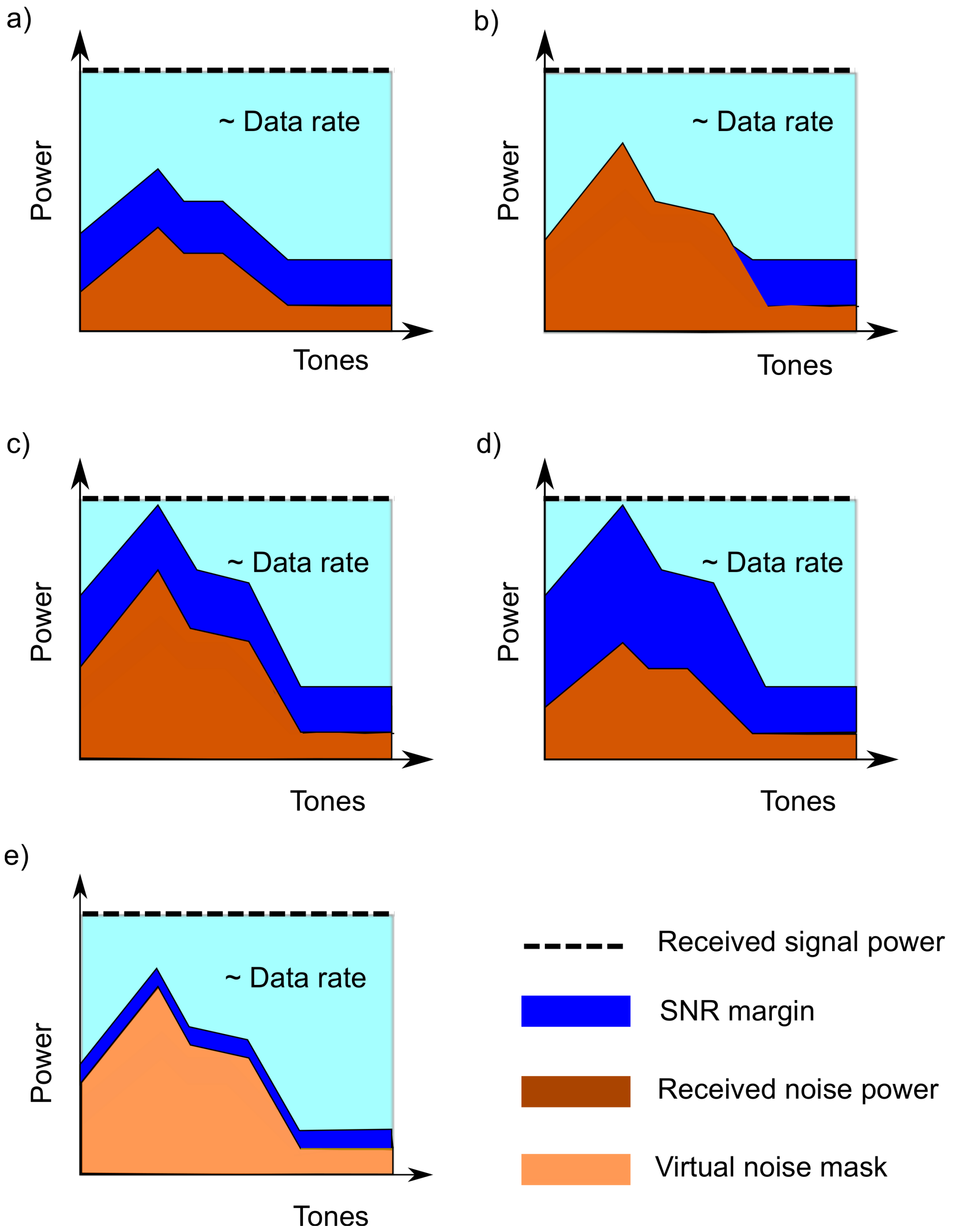


Figure 1.4: Basic concept of the Virtual Noise approach

In Figure 1.4a, the victim line is initialized when the noise power is low. The area between the SNR margin and the received signal power is therefore large which indicates a high target data rate. Figure 1.4b illustrates the busiest time of the day when a lot of DSL lines are switched on. Obviously, the increase in the noise power exceeds the specified SNR margin and the victim line has to be reinitialized at a lower target data rate. In Figure 1.4c, the same SNR margin used for calculating the target data rate in Figure 1.4a is added to the received noise power. The resulting target data rate is therefore smaller than in Figure 1.4a. Figure 1.4d demonstrates the case where the received noise power decreases again. The victim line still transmits with the target data rate calculated in Figure 1.4c. Figures 1.4a-d demonstrate the "first and last user problem" explained above. Furthermore, in this example, the variation in the noise power is higher at lower tones but the used state of the art SNR margin is equal at all tones. Therefore, higher tones have better protection from noise than lower ones. As already mentioned this problem is referred to as "unequal per-tone noise protection problem". Figure 1.4e illustrates how the VN approach conceptually solves the "first and last user problem" and the "unequal per-tone noise protection problem". The target data in Figure 1.4e is calculated from the VN mask instead of the received noise power as in Figures 1.4a and 1.4c. The VN mask is calculated based on long-term noise statistics such that the victim modem is prevented from calculating an overly optimistic target data rate as in 1.4a or an overly pessimistic target data rate as in Figure 1.4c. Consequently, the "first and last user problem" is solved. Furthermore, the VN mask is high at lower tones where the variation in the noise power is high and it is low at higher tones where the variation in the noise power is low. Thus, the "unequal per-tone noise protection problem" explained above is solved.

Although the VN concept has been standardized for VDSL2, there has been, to the best of the authers' knowledge, no solutions published on how to determine the VN PSD and the initialization SNR margin such that a target outage probability is achieved while the data rate is maximized. In [VB05, JHC08a] it was suggested that a worst-case noise PSD shall be used as a VN PSD, but no approaches were shown neither on how to determine the worst-case noise PSD nor on how to determine the initialization SNR margin.

1.3.3 Adaptive Data Rate Approaches

Another approach to protect against time-varying non-stationary noise is to adapt the target data rate to the varying noise. ADSL2/ADSL2plus and VDSL2 address this problem by including the ability to seamlessly adapt the data rate on-line [PS12]. This ability, called Seamless Rate Adaptation (SRA), enables a DSL system to change the data rate of the connection while in operation without any service interruption [PS12]. After an SRA procedure, modems transmit with a new bit-loading where the number of bits transmitted in on the DMT modulated tones is different than before the procedure. Furthermore, an equal SNR margin, and consequently, an equal error probability is restored over all tones. SRA uses the sophisticated online reconfiguration (OLR) procedures to coordinate the data rate change between the transmitter and receiver. According to [IT11], one OLR request can modify up to 128 tones. In order to equalize the SNR margin at all tones after the noise has changed, it is very likely that more than 128 tones have to be modified in one SRA procedure. In that case, consecutive OLR requests are sent. The transmission with the new bit-loading begins after the last OLR request has been acknowledged.

Care must be taken when SRA is implemented in communication paths that include interleavers. Interleavers impose a constraint on the difference in data rate that can be changed by an SRA procedure. In the case of large changes in noise, multiple consecutive SRA procedures have to be executed [IT11].

Considering the above, in the case of large changes in noise, e.g. if a strong VDSL2 interferer is switched on, the state of the art SRA procedure will require a long time to adapt to the new SNR and it is very likely that the connection will be interrupted and that modems will have to reinitialize at a lower data rate. Hence, the state of the art SRA is too slow and provides means of adaptation only to slowly changing noise and will not prevent re-initializations under certain conditions such as large rapid changes in noise [PS12,GY08].

Furthermore, in the VDSL2 standard [IT11], an emergency online-reconfiguration procedure called save our showtime (SOS) was proposed to rapidly adapt to the varying noise by performing group-wise bit-loading reduction where the bit reduction is constant for every group. In contrast to SRA, SOS uses a short procedure for the coordination between transmitter and receiver. In order to keep the procedure short, the DMT modulated tones are divided into tone groups during initialization and the bit-loading reduction in each tone group is constant. In case of persistent performance degradation over a certain period of time, an SOS procedure is triggered as a last resort before reinitializing the modems at a lower target data rate. The SOS approach is therefore not meant for continuous adaptation to the varying noise, but mainly for preventing a session from being interrupted. Hence, the SOS approach is not constrained by the difference in data rate that can be changed by a procedure and the adaptation is therefore not done in a seamless manner.

1.4 Open Issues

In this section, open issues coming from the review of existing literature regarding the receiver-based protection against time-varying noise in xDSL systems are summarized. Receiver-based protection is categorized into fixed data rate approaches and adaptive data rate approaches. First, the open issues regarding the fixed data rate approaches are presented, then, the open issues regarding the adaptive data rate approaches are presented. Finally, open issues regarding the combination of both approaches into one DSL system are presented.

Although the use of a VN mask for the protection against time-varying noise has been standardized for VDSL2, the full scope of the possibilities and functionalities that can be achieved by using both, a VN mask and an SNR margin, has not been explored. Thus, the following questions arise:

1. How can the outage probability of DSL users be defined in terms of the VN mask and the SNR margin in a non-stationary noise environment?

2. How can the VN mask and the SNR margin be used together in practical DSL systems such that the target data rate is maximized while a target outage probability is satisfied?

3. In situations where the noise power increases at some tones and the per-tone SNR margins become negative, bit-swapping procedures might be too slow to restore a positive SNR margin over all tones and the connection is interrupted. How can the VN mask and SNR margin be adjusted such that the occurrence probability of such situations is minimized, and thus, the reliance on bit-swapping to restore a positive SNR margin is reduced?

As explained in Section 1.3.3, the current DSL standard allows changing the target data rate during operation using two procedures, SRA and SOS. SRA procedures require a long time to adapt to the new SNR in case of large sudden changes in noise and it is very likely that the connection will be interrupted. SOS reacts to a persistent degradation of the SNR by rapidly performing group-wise bit-loading reduction where the bit reduction is constant for every group. However, SOS is an emergency online-reconfiguration procedure and is not meant for continuous adaptation to the varying noise but mainly for preventing a session from being interrupted. The following questions arise regarding the optimization of the seamless rate adaptation procedures in xDSL systems:

4. How can the SRA procedures be optimized such that the adaptation time is shortened, the errors occurring during the adaptation are reduced while the adaptation is still done in a seamless manner?
5. How can the SRA procedures make use of the short messages of the SOS procedure but still adapt to the varying noise in a seamless manner?
6. In practice, the computational power of DSL modems is limited, and therefore, low complexity algorithms are needed. How can low complexity versions of the proposed adaptation approaches be developed?

DSL users often use services with different requirements on QoS in terms of outage probability. In that regard, the following question arises:

7. How can the line stability of the VN approach in terms of outage probability and data rate efficiency of the SRA approach be utilized together in one DSL system such that user-specific requirements on QoS are satisfied?

1.5 Contributions and Thesis Overview

This section gives an overview of the thesis and summarizes the main contributions addressing the open problems introduced in Section 1.4. In the following, the contents along with the main contributions of each chapter are briefly described.

In Chapter 2, the system model is introduced. First, the considered DMT transmission model is provided. Then, a novel statistical model that describes the DSL user activity over 24 hours is introduced. The status of a DSL user is modeled by a three-state Markov chain and covers hereby the three operational modes online, L2 mode and offline and the transition probabilities between the states are determined based on statistics from practical DSL systems. Finally, a time-varying noise model that combines the user activity model with FEXT measurements of interferer lines is introduced. The introduced time-varying noise model will be used throughout this dissertation for the generation of time-varying noise for the evaluation of the proposed approaches.

Chapter 3 addresses the optimization of fixed data rate approaches for the protection against time-varying noise and gives answers to Questions 1-3 stated in Section 1.4:

1. The outage probability is defined for a non-stationary noise environment over 24 hours. It is shown that in order to express the outage probability in terms of the VN mask and the SNR margin, the adaptation functionality, in terms of margin equalization carried out by bit-swapping, of the DSL modems has to be considered. The outage probability is then derived as a function of the VN mask, the SNR margin and the maximum noise power measured over 24 hours at each tone while assuming the SNR margin that would result after the margin equalization.

2. From the derived expression of the outage probability, the VN mask and the SNR margin that maximize the target data rate while satisfying a target outage probability are derived as functions of the distributions of the maximum noise power measured over 24 hours at each tone.

3. In order to reduce the reliance on bit-swapping to restore a positive SNR margin in situations, where the noise power increases at some tones and the per-tone SNR margins become negative, the VN mask and SNR margin have to be modified such that the occurrence probability of such situations is minimized. It is shown that this can be achieved by equalizing the per-tone noise protection. The per-tone outage probability is used as a metric for the per-tone noise protection and it is shown that by equalizing the per-tone outage probability during initialization and minimizing the SNR margin, the probability of the line going to an unstable state with negative per-tone SNR margins is minimized.

At the end of Chapter 3, performance results of the target data rate, the outage probability and the per-tone outage probabilities when using the proposed approaches and when using the state of the art approaches are shown. The performance results are shown for DSL systems where the L2 mode is not implemented and for DSL systems where the L2 mode is implemented.

Chapter 4 addresses the optimization of the seamless adaptation procedures in xDSL systems. First, the constraint on the difference in data rate allowed to be changed in an adaptation procedure by an adaptive data rate approach such that the approach is considered seamless is stated. Then, the performance measures that are considered in the optimization of the state of the art adaptive data rate approaches are presented. Finally, adaptation approaches are proposed for the case where multiple adaptation procedures are required to modify the bit-loadings of the tones to the target bit-loadings that correspond to the new channel SNR while the stated data rate constraint is satisfied. The proposed approaches give answers to Questions 4-6 stated in Section 1.4:

4. An approach is proposed that modifies the bit-loadings of the tones to their target bit-loadings in the order that leads to the fastest decrease in the average BER while fulfilling the stated data rate constraint.
5. An approach that makes use of the short messages of the SOS procedure but still adapts to the varying noise in a seamless manner is proposed. The proposed approach modifies the bit-loadings of the tones in groups and finds the per group bit reduction that decreases the average BER as rapidly as possible while fulfilling the stated data rate constraint.
6. Low complex versions of both proposed approaches are presented.

At the end of Chapter 4, performance results of the average BER during the adaptation and of the number of errors that occur during the adaptation are shown for the proposed approaches and the state of the art SRA.

Chapter 5 proposes to combine the fixed and adaptive data rate approaches according to the internet services that are used by the DSL user and gives hereby the answer to Questions 7 stated in Section 1.4:

7. A hybrid VN-SRA approach is proposed that improves the data rate performance by combining the fixed and adaptive data rate approaches in one DSL system. The

hybrid VN-SRA approach iteratively allocates tones to be used by fixed data rate approaches for services that require a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability. Moreover, it allocates tones to be used by adaptive data rate approaches for services that can tolerate temporary increases of the BER above the target BER during the adaptation to the varying noise.

At the end of Chapter 5, performance results of the average data rate achieved when using the hybrid VN-SRA approach are presented. The performance results are shown for DSL systems where the L2 mode is not implemented and for DSL systems where the L2 is implemented.

Finally, the main conclusions of the thesis are summarized in Chapter 6.

Chapter 2

System Model

2.1 Introduction

This chapter gives the system model used throughout this dissertation. Furthermore, a statistical model for the time-varying is introduced. The chapter is organized as follows. Section 2.2 presents the transmission model of a DSL system. The transmission model includes the part of the communication chain between the modems at both ends of a victim line that is application independent. Therefore, the input and output of the transmission model consists of only one binary bit-stream. In Section 2.3, first, the use of a statistical model to emulate the DSL user activity in practical DSL systems is motivated, then, the user activity is modeled by a Markov model using statistics from practical DSL systems. Finally, in Section 2.4, a model for the time-varying noise, consisting of background noise and crosstalk from neighboring lines, received on the victim line is presented. The model combines the user activity model of Section 2.3 with single FEXT measurements of single interferer lines to generate time-varying noise. Moreover, the single FEXT measurements used are from a downstream scenario and therefore, the presented model models the time-varying noise in a downstream scenario. Given measurements from an upstream scenario, the presented time-varying noise model can also be used in an upstream scenario. The time-varying noise model is used to evaluate the performance of the approaches presented in this dissertation.

2.2 Transmission Model

This section presents the transmission model used throughout this dissertation. The functional blocks of the transmission model are shown in Figure 2.1. In Figure 2.1, the input bit-stream originating from the applications used by a DSL user is encoded by a forward error correction (FEC) encoder. FEC provides protection against random and burst errors by adding redundancies in each generated code word which enables the FEC decoder to correct a limited amount of errors in each code word [IT11, CMGG98]. The code words at the output of the FEC encoder enter then an interleaver. Interleaving protects against

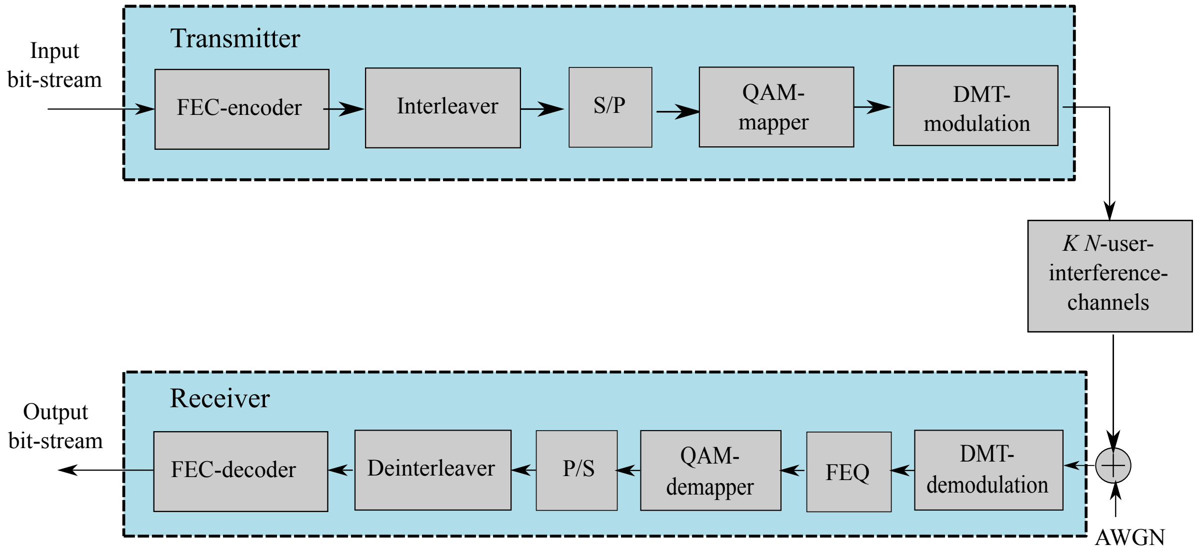


Figure 2.1: Functional blocks of the transmission model.

burst errors by spreading the errors over a number of code words [IT11, Yse04, WKHN10]. One coded and interleaved bit-stream leaves the interleaver and is divided into parallel bit streams at the serial-to-parallel converter (S/P). Each single stream is mapped to a Quadrature Amplitude Modulation (QAM) symbol at the QAM-mapper.

The QAM symbols enter then the multi-carrier modulation stage. xDSL employs a multi-carrier modulation scheme called Discrete Multi-Tone (DMT) [Cio91, PS99], where the QAM symbols are distributed over K mutually orthogonal sub-carriers (tones) with frequencies $f_k = k\Delta f$, where Δf is the frequency spacing and with $k = 1, \dots, K$ using an inverse discrete Fourier transform (IDFT) [Yse04, CMGG98, Cio91]. Thus, the communication channel is effectively divided into K independent sub-channels, each corresponding to a tone k. Moreover, the channel frequency response is assumed to be flat within the bandwidth Δf of a sub-channel. This allows the channel to be described by a single complex channel coefficient per tone. In order to ensure that there is no inter-symbol-interference (ISI) between consecutive DMT symbols, the DMT symbols are extended by a cyclic prefix which has to be at least as long as the channel impulse response [Yse04, CMGG98, Cio91].

The K QAM symbols at the output of the DMT-modulator in Figure 2.1 are then sent over the K tones. In an N-user DSL binder, each tone can be used by users $n = 1, \dots, N$. Thus, the K tones can be modeled as K independent N-user-interference channels, where crosstalk from other users is treated as noise [CMGG98, Cio91]. In order to use the multiple-input-multiple-output (MIMO) system model presented in [RWM+06, CG02] for each of the K N-user-interference channels, let

$$\boldsymbol{\theta}_{\mathrm{tr},k} = [\theta_{\mathrm{tr},k}^{(1)}, \ldots, \theta_{\mathrm{tr},k}^{(N)}]^\top \tag{2.1}$$

and

$$\boldsymbol{\theta}_{\mathrm{rec},k} = [\theta_{\mathrm{rec},k}^{(1)}, \ldots, \theta_{\mathrm{rec},k}^{(N)}]^\top \tag{2.2}$$

denote the vector of transmitted and received QAM symbols of the N users transmitting on tone k, respectively. Now, let a diagonal element $h_k^{(n,n)}$ describe the transfer functions of the k^{th} tone between the output of the DMT-modulation stage and the input of the DMT-demodulation stage of user n, also known as the direct channel. Furthermore, let an off-diagonal element $h_k^{(n,m)}$ $(n \neq m)$ describe the transfer functions of the k^{th} tone between the output of the DMT-modulation stage of user m and the input of the DMT-demodulation stage of user n, also known as FEXT channel. The channel matrix for the k^{th} tone is given by

$$\mathbf{H}_k = \begin{bmatrix} h_k^{(1,1)} & \cdots & h_k^{(1,N)} \\ \vdots & \ddots & \vdots \\ h_k^{(N,1)} & \cdots & h_k^{(N,N)} \end{bmatrix}. \tag{2.3}$$

According to [WKK12, CMGG98, Cio91], the background noise at the receiver can be assumed white and Gaussian. Let $z_k^{(n)}$ with variance $(\sigma^{(n)})^2$ model the white Gaussian noise at the receiver of user n at tone k. Furthermore, let

$$\mathbf{z}_k = [z_k^{(1)}, \ldots, z_k^{(N)}]^\top \tag{2.4}$$

be a vector of white Gaussian noise at the receivers of the N users transmitting on the k^{th} tone. Then, the MIMO system model for the k^{th} N-user-interference channel is given by

$$\boldsymbol{\theta}_{\mathrm{rec},k} = \mathbf{H}_k \boldsymbol{\theta}_{\mathrm{tr},k} + \mathbf{z}_k. \tag{2.5}$$

At the DMT-demodulation stage at the receiver side in Figure 2.1, the cyclic prefix is removed and a discrete Fourier transform (DFT) is performed. The frequency-domain equalizer (FEQ) performs then a one-tap equalization per tone to invert the flat channel frequency response. The QAM demapper followed by a parallel-to-serial converter (P/S) convert the received estimates of the transmitted QAM symbols on each tone back into a bit-stream. The bit-stream is then brought to the right order at the deinterleaver and the reassembled code words enter the FEC-decoder where a limited amount of erroneous bits is corrected and the redundancies are removed, yielding the output bit-stream.

2.3 DSL User Activity Model

2.3.1 DSL User Activity

As shown in Section 2.2, a DSL user does not receive only his transmitted QAM symbol on a tone k, but also crosstalk from every active DSL line that utilizes tone k. Since crosstalk from neighboring DSL lines is treated as noise, the distribution of the noise on a DSL line depends strongly on the DSL user activity. In this section, the aspects that influence the activity of a DSL user are discussed.

The user activity depends on many factors that are very difficult to express mathematically. For example, the online time of a DSL user per day is dependent on the working and sleeping hours of the user, moreover is it dependent on the internet applications that are used by the user. In [MFPA09] statistics measured from 20.000 DSL lines were presented and discussed. According to [MFPA09], the number of DSL active users is strongly daytime dependent.

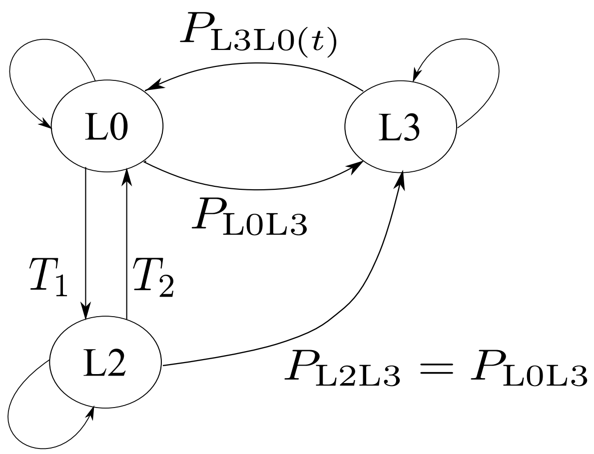
Furthermore, in addition to the ON/OFF behavior of a DSL user, an active DSL user can enter the L2 state. During the time in the L2 state, the downstream data rate can be reduced, which allows the average transmitted power to be scaled down. The effect of lowering the transmitted power is to potentially reduce the power consumed by the chip sets, as well as lower its thermal dissipation. Note that the power savings apply only for the TU-CO. The reason for this is that central office chip sets generally have tighter constraints on power consumption and thermal dissipation, especially when deployed in the outside plant (e.g. DSLAM at OC) [Gin05].

The most serious concern about L2 is that when a modem exits the L2 state and enters the operational state, also referred to as L0, the transmitted power will suddenly change by as much as tens of dB. Such a transition would have an impact on neighboring lines that is comparable to the impact of an interfering modem being switched from off to on [Gin05]. In addition, transitions from L2 to L0 depend on the users traffic, that is according to [MFPA09] strongly daytime dependent, and are expected to happen more frequently than modem turning on events. Since transitions from L2 to L0 depend on the users traffic, the number of users in L2 is daytime dependent.

Another issue with Low Power is that bit-swapping operations cannot be performed while the link is in the L2 state. If a bit-swap operation must take place, then a transition to L0 must take place. This increases the number of transitions from L2 to L0. Furthermore, SNR margin equalization becomes more difficult in the case of abrupt changes in the line conditions, since the modems must first exit from L2 mode to L0 mode and then start performing bit-swapping operations. For all the reasons mentioned above, the variation in crosstalk increases tremendously when L2 is implemented. It was therefore agreed upon to abandon L2 mode for VDSL2 [Gin05]. Later on, in Chapter 3, we will show how the use of the Virtual Noise approach for the protection against time-varying crosstalk can make the implementation of L2 mode practicable in VDSL2 systems.

2.3.2 Markov Model

In this section, we propose a Markov model that describes the DSL user activity over 24 hours is introduced. The status of a DSL user is modeled by a three-state Markov chain. A user can be either connected, in showtime (L0 state) or in Low Power mode (L2 state), or off the Internet (L3 state). The model is shown in Figure 2.2.



$$T_1 = P_{\mathrm{L0L2}}(t) \cdot (1 - P_{\mathrm{L0L3}})$$
$$T_2 = P_{\mathrm{L2L0}} \cdot (1 - P_{\mathrm{L2L3}})$$

Figure 2.2: Modeling user activity with a three-state Markov chain.

The probabilities $P_{\mathrm{L3L0}}(t)$, $P_{\mathrm{L0L2}}(t)$, P_{L0L3}, P_{L2L0} and P_{L2L3} are the transition probabilities between the states L0, L2 and L3. The daytime dependency of the state of the DSL user, explained in the previous section, is modeled by the daytime dependent transition probabilities $P_{\mathrm{L3L0}}(t)$ and $P_{\mathrm{L0L2}}(t)$ between the states L3 and L0 and the states L0 and L2. Furthermore, since most of the internet services are available throughout the day, we assume that the time a DSL user spends in the L0 state, also referred to as L0 session duration, and the time a DSL user spends in the L2 state, also referred to as L2 session duration are independent of the daytime. The probabilities P_{L0L3}, P_{L2L0} and P_{L2L3} are therefore time invariant. For the determination of the transition probabilities, statistics from [MFPA09] were used. In [MFPA09], the statistics were calculated from 20000 DSL line. The following statistics were used:

- Average fraction $N_{\mathrm{ONrel}}(t)$ of active DSL lines over a 24h period. It was shown that 40% of the DSL users are always connected. Moreover, the maximum average fraction $N_{\mathrm{ONrel,max}}$ of active DSL lines occurs between 19:00 and 20:00 o'clock and is slightly over 50%.

- Probability density function (PDF) of the online (L0+L2) session duration per DSL line. The PDF exhibited two strong modes around 20-30 minutes and around 24 hours, partitioning the DSL lines into two large groups: permanently connected lines, and lines that only connect on demand and disconnect shortly after, independent of the daytime.

- Average fraction of active lines using at least 10% of the target data rate assigned to them at initialization over a 24h period. It was shown that at peak hours, when many users are connected, roughly 30% of the online users use less than 10% of the target data rate assigned to them at initialization, and at quiet hours, when fewer users are connected compared to the peak hours, roughly 90% use less than 10% of the target data rate assigned to them at initialization. For the determination of the transition probabilities, it was assumed that the average fraction $N_{\mathrm{L2rel}}(t)$ of active DSL users in L2 state over 24 h is given by the fraction of active lines using less than 10% of their available target data rate.

In order to model the 40% of the DSL users permanently connected, the probabilities $P_{\mathrm{L0L3}} = P_{\mathrm{L2L3}}$ are set equal to zero. For the determination of the transition probability $P_{\mathrm{L3L0}}(t)$ of the remaining 60% of the DSL users, we define the probabilities $P_{\mathrm{L3L0,min}}$ and $P_{\mathrm{L3L0,max}}$, which are the transition probabilities from state L3 to L0 at the day time where the minimum average fraction $N_{\mathrm{ONrel,min}}$ of active users occur and at the day time where the maximum average fraction $N_{\mathrm{ONrel,max}}$ of active users occur, respectively. Using the gradient of the progression of average fraction of active DSL lines $N_{\mathrm{ONrel}}(t)$ over a 24h period shown in [MFPA09], $P_{\mathrm{L3L0}}(t)$ can be expressed as

$$P_{\mathrm{L3L0}}(t) = P_{\mathrm{L3L0,min}} + \frac{N_{\mathrm{ONrel}}(t)}{N_{\mathrm{ONrel,max}} - N_{\mathrm{ONrel,min}}}(P_{\mathrm{L3L0,max}} - P_{\mathrm{L3L0,min}}). \tag{2.6}$$

Similarly, for determination of the transition probability $P_{\mathrm{L0L2}}(t)$, we define the probabilities $P_{\mathrm{L0L2,min}}$ and $P_{\mathrm{L0L2,max}}$, which are the transition probabilities from state L0 to L2 at the day time where the minimum average fraction $N_{\mathrm{L2rel,min}}$ of active DSL users in L2 state occur and at the day time where the maximum average fraction $N_{\mathrm{L2rel,max}}$ of active DSL users in L2 state occur, respectively. Using the gradient of the progression of average fraction of active lines using at least 10% of the target data rate assigned to them at initialization over a 24h period shown in [MFPA09], $P_{\mathrm{L3L0}}(t)$ can be expressed as

$$P_{\mathrm{L0L2}}(t) = P_{\mathrm{L0L2,min}} + \frac{N_{\mathrm{L2rel}}(t)}{N_{\mathrm{L2rel,max}} - N_{\mathrm{L2rel,min}}}(P_{\mathrm{L0L2,max}} - P_{\mathrm{L0L2,min}}). \tag{2.7}$$

The parameters in Table 2.1 were adjusted such that the simulation of $N_{\mathrm{ONrel}}(t)$ over a 24h period is consistent with the average fraction of active DSL lines $N_{\mathrm{ONrel}}(t)$ over a 24h period shown in [MFPA09], and such that $N_{\mathrm{L2rel}}(t)$ is consistent with the average fraction of active lines using at least 10% of the target data rate assigned to them at initialization over a 24h period shown in [MFPA09], and such that the average session duration of the 60% of DSL users, not permanently connected, is around 20-30 minutes, as suggested by [MFPA09].

Number of users	40
Minimum time in state	30s
$P_{\mathrm{L3L0,min}}$	$4 \cdot 10^{-5}$
$P_{\mathrm{L3L0,max}}$	$3.5 \cdot 10^{-3}$
P_{L0L3}	$1.75 \cdot 10^{-2}$
$P_{\mathrm{L0L2,min}}$	$5.5 \cdot 10^{-3}$
$P_{\mathrm{L0L2,max}}$	$1 \cdot 10^{-2}$

Table 2.1: Parameters of the user activity model

Figures 2.3 and 2.4 show simulations of the fraction of active DSL lines over a 24h period and the fraction of lines in L2 state over a 24h period, respectively, when modeling the activity of 40 users by using the presented Markov model with the parameters in Table 2.1.

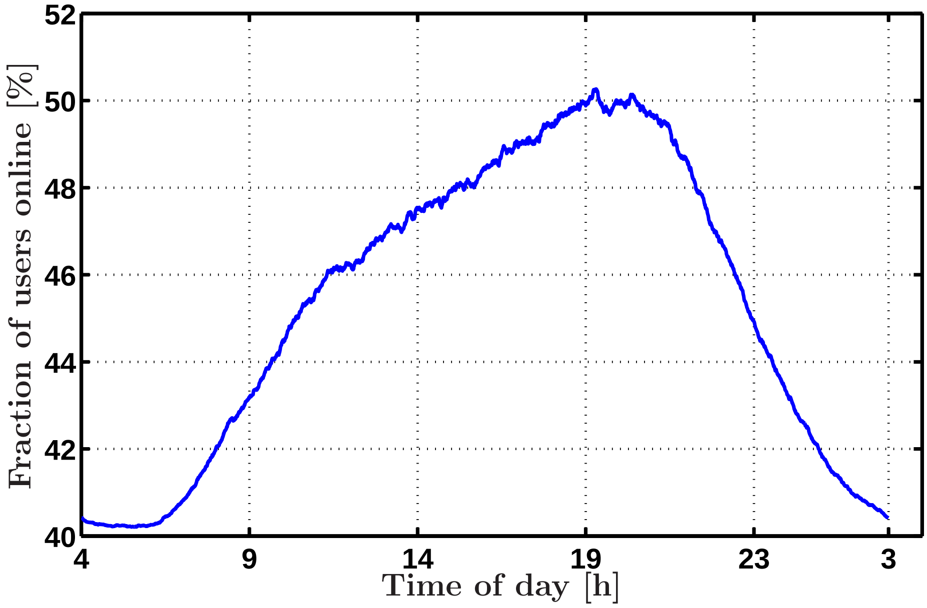


Figure 2.3: Fraction of active DSL lines over a 24h period, determined by simulation using the Markov model in Figure 2.2 with the parameters in Table 2.1.

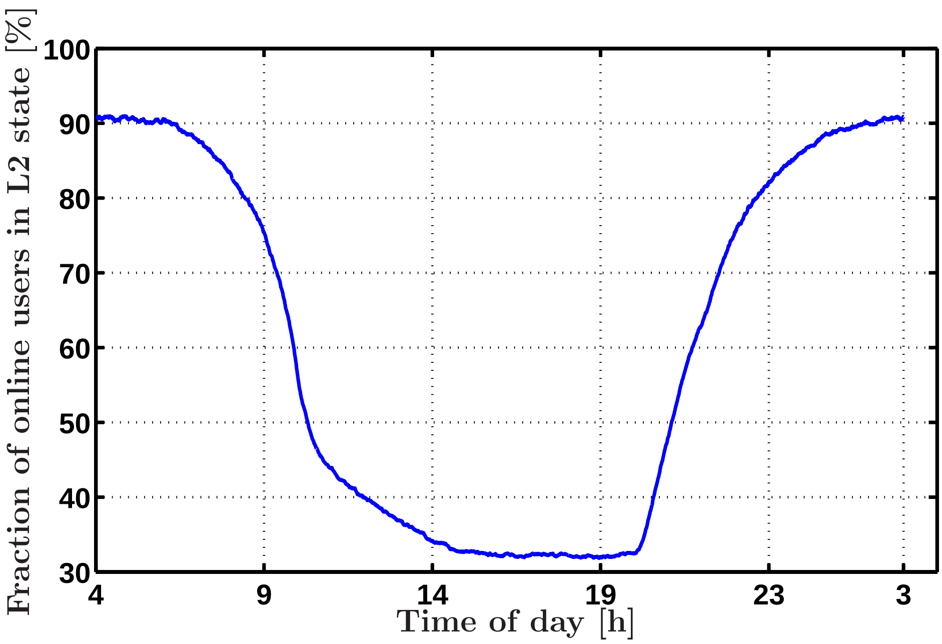


Figure 2.4: Fraction of lines in L2 state over a 24h period, determined by simulation using the Markov model in Figure 2.2 with the parameters in Table 2.1.

Note that throughout the dissertation, the axis describing the time of day will start at 4 am in order to reflect the daily rhythm of the sleep-wake cycle.

The simulated average durations of the online and the L2 sessions of the 40% of users permanently connected and the 60% of users connecting on demand are shown in Table 2.2. There were no statistics in literature for the average L2 session durations and their values in Table 2.2 are therefore assumptions. Note that the value of the average durations of the L2 sessions of the 60% of users connecting on demand is chosen to be smaller than the average durations of the online sessions since a user has to be first online in order to enter the L2 state. Moreover, the value of the average durations of the L2 sessions of the 40% of the users permanently connected is chosen to be larger than this of the 60% of users connecting on demand since it is more likely for users that are always connected to spend more time without internet activity than users that only connect on demand.

Sessions of 40% of users permanently connected	**Average duration**
Online (L0+L2) sessions	24 h
L2 sessions	50 min
Sessions of 60% of users connecting on demand	**Average duration**
Online (L0+L2) sessions	28 min
L2 sessions	18 min

Table 2.2: Average durations of online and L2 sessions.

2.4 Time-varying Noise Model

In this section, we present a model for the time-varying noise in xDSL systems. In order to emulate the noise that occurs in practical DSL systems, we use measurements for the generation of noise. The time-varying noise is generated in two separate stages.

- In the first stage, a measurement of the background noise on a VDSL2 victim line and single disturber FEXT measurements are taken in a downstream scenario at

ADTRAN's DSL lab. The cable length of the victim line is 500m and the cable length of the 39 disturber lines vary between 200m and 800m. All lines share the same binder and use the cable type DTAG 50x2x0,5. The transmit PSDs used are according to VDSL2 band plan 998ADE17M2xB [IT11]. The measurement of the background noise is taken when all disturber lines are switched off. Then, one further measurement is taken when each of the disturbers is active alone and the measured background noise is subtracted from it. Figure 2.5 shows the resulting FEXT experienced by the victim line when each disturber is active alone. In Figure 2.5, the FEXT experienced by the victim line from each disturber is given a different color.

- In the second stage, non-stationary noise is generated by using the user activity model introduced in Section 2.3 combined with the background noise and the FEXT noise measured in the first stage. For example, if at any point in time the states calculated by the user activity model dictate that only lines 5 and 7 are active, then the noise seen by the victim line at that point in time is given by the addition of the FEXT noise measured when line 5 is active alone, the FEXT noise measured when line 7 is active alone and the background noise. Since L2 mode was not included in the VDSL2 standard, it was not possible to operate the modems in L2 mode, and therefore, a modem in the L2 state was treated as if it was turned off.

In conclusion, since the user activity model is based on statistics from practical DSL systems and the FEXT noise is generated from measurements, the resulting time-varying noise is a good emulation of the noise that occurs in practical DSL systems. The time-varying noise model is used in this dissertation to evaluate the performance of the proposed approaches.

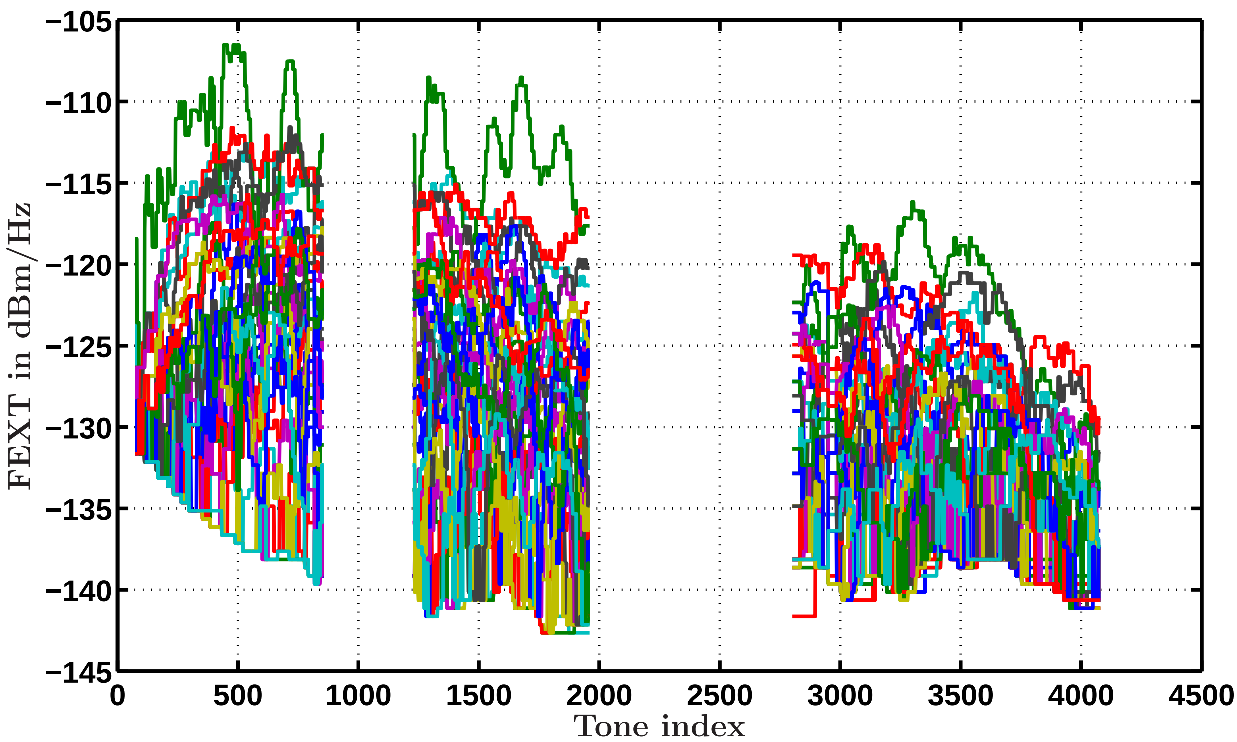


Figure 2.5: FEXT noise experienced by the victim line when each disturber is active alone.

Chapter 3
Fixed Data Rate Approaches for Improved Protection against Time-varying Noise

3.1 Introduction

This chapter addresses the optimization of fixed data rate approaches for the protection against time-varying noise. As explained in Section 1.3.2, the current DSL standard [IT11] specifies two approaches for the protection against crosstalk, the SNR margin and VN approach. The state of the art 6 dB SNR margin is not optimal in terms of data rate and QoS, and therefore, the "first and last user problem" and the and the "unequal per-tone noise protection problem" arouse, as explained in Section 1.3.2. Furthermore, no approaches can be found in literature on how to set the VN mask such that the mentioned problems are solved while the data rate is maximized for a certain QoS. In this chapter, we jointly optimize the VN mask and the SNR margin such that the "first and last user problem" and the "unequal per-tone noise protection problem" are solved, while the target data rate is maximized for a certain QoS.

First, in Section 3.2, the outage probability is defined as metric for QoS for fixed data rate approaches. Then, in Section 3.3, the target data rates that can be achieved when using the SNR approach and the VN approach for the protection against time-varying noise are presented. Sections 3.4 and 3.5 contain the main contributions of this chapter. In Section 3.4, the VN mask and the SNR margin are optimized such that the outage probability described in Section 3.2 is satisfied while the target data rate presented in Section 3.3 is maximized. Furthermore, in Section 3.4, only the equal SNR margin that would result after bit-swapping has equalized the per-tone SNR margins was considered in optimization of the VN mask and the SNR margin. In Section 3.5, the per-tone outage probability is used as a metric for the per-tone noise protection and the VN mask and SNR margin are modified such that all tones have an equal per-tone outage probability while the achieved target data rate and the satisfied target outage probability in Section 3.4 are maintained. Finally, in Section 3.6, the performance of the proposed approaches is compared to the performance of the state of the art approaches presented in Section 1.3. Some of the contributions presented in this Chapter are published by the author of this thesis in [SKK12] and [SKK13].

3.2 Quality of Service for Fixed Data Rate Approaches

In this section, the term Quality of Service is defined on the physical layer for fixed data rate approaches.

The target data rate for fixed data rate approaches is calculated during initialization. For the calculation of the data rate, the bit-loading process matches the constellation size of the QAM symbol transmitted on each tone to the channel SNR such that the error probability is smaller than a target error probability, assuming AWGN at the receiver. The way to guarantee a certain QoS for fixed data rate approaches is to take into account future noise variations when calculating the target data rate such that the future error probability does not exceed the target error probability. As explained in Section 1.3.2, techniques like the SNR margin and the Virtual Noise were invented for that purpose. Fixed data rate approaches are suitable for services that require a fixed data rate and cannot tolerate any temporary degradation in signal quality. For example, to watch one channel of HDTV, 12 Mbits/s have to be accessible by the IPTV application. Furthermore, to guarantee an HD quality, the error probability should not exceed the target error probability even for a short time.

For fixed target data rate approaches, the outage probability is the metric for ensuring a certain QoS on the physical layer. As already explained in Section 2.3, the distribution of noise in practical DSL systems depends on the DSL user activity and is therefore strongly daytime dependent. With this observation, and by assuming that the distribution of the number of active DSL users is cyclostationary with the period of one day, we define the outage probability as:
The outage probability $P_{\mathrm{out}}^{(n)}$ is the probability that the error probability at any tone exceeds the target error probability for a user n who is connected for 24 hours and operating at the target data rate.
The intuitive approach of setting the target data rate so low such that the outage probability described above is always satisfied is clearly not data rate efficient. Therefore, new approaches that satisfy the outage probability described above and are data rate efficient are needed.

In the next section, it will be shown how the target data rate is calculated for fixed data rate approaches when using an SNR margin and when using a VN mask for the protection against future noise variations. Then, in Sections 3.4 and 3.5, it will be shown how the SNR margin and the VN mask can be optimized such that the outage probability described above is satisfied while the target data rate is maximized.

3.3 Data Rate Calculations for Fixed Data Rate Approaches

3.3.1 SNR Margin Approach

In this section it is shown how the target data rate is calculated when an SNR margin is used for the protection against time-varying noise. First, a formula for the target data rate is derived. Then, an adaptation functionality used in practical DSL systems by the DSL modems in combination with the SNR margin called bit-swapping is explained.

Let s_k^n be the average transmit power of user n on tone k. The SNR seen at the receiver of user n at tone k is given by

$$SNR_k^{(n)} = \frac{|h_k^{(n,n)}|^2 s_k^{(n)}}{\sum_{m \neq n} |h_k^{(n,m)}|^2 s_k^{(m)} + (\sigma^{(n)})^2}, \tag{3.1}$$

where $h_k^{(n,n)}$ and $h_k^{(n,m)}$ are defined in (2.3) and $(\sigma^{(n)})^2$ is defined in (2.4).

For the calculation of the target data rate of user n, the DSL modem measures $SNR_k^{(n)},\ k = 1, \cdots, K$ during initialization. In case the SNR decreases during operation, e.g. due to a disturber modem being switched on, the transmission with the data rate and the error probability calculated during initialization will not be possible. To overcome this problem, the SNR margin was introduced to give the DSL modem the ability to withstand impairments of the channel. The SNR margin is defined as the decrease in the SNR that can be withstood by the system at the same target data rate and an error probability smaller than or equal to the target error probability [JHC08a]. Moreover, the state of the art SNR margin specified during initialization is equal at all tones [IT11].

Let us define $\gamma_{\text{init}}^{(n)}$ as the SNR margin assigned to all tones during initialization and let Γ_c denote the SNR gap of the code, which includes the coding gain, γ_c, i.e., $\Gamma_c = \Gamma/\gamma_c$ where Γ is the uncoded SNR gap to the Shannon capacity [Jag08, PS99]. Furthermore, let $SNR^{(n)}_{\text{init},k}$ denote the SNR during initialization at tone k. With these definitions and by using the Shannon approximation [Jag08, PS99], the bit-loading calculated during initialization is given by

$$b^{(n)}_{\text{init},k} = \log_2\left(1 + \frac{1}{\Gamma_c \gamma^{(n)}_{\text{init}}} SNR^{(n)}_{\text{init},k}\right). \tag{3.2}$$

The bit-loading in (3.2) provides the number of bits on tone k that can be sustained at least at the desired target error probability, implicitly specified by the choice of Γ_c, if $SNR^{(n)}_{\text{init},k}$ decreases by at most $\gamma^{(n)}_{\text{init}}$ [Jag08]. Now, with f_s being the DMT symbol rate, the target data rate is given by

$$R^{(n)}_{\text{init}} = f_s \sum_k b^{(n)}_{\text{init},k} \tag{3.3}$$

[Cio91, WKK12]. The SNR margin $\gamma^{(n)}_{\text{init}}$ used in (3.2) is equal for all tones. In practical DSL systems, the SNR might decrease during showtime due to an increase in the noise. Furthermore, the noise increase might be localized only at some tones. As mentioned earlier in this section, if this localized noise increase is equal to the SNR margin specified during initialization, the same amount of bits can still be transmitted on these tones at an error probability equal to the target error probability. The problem now is that these tones will no longer have immunity to further noise increase, like additional crosstalk appearing, and any further disturbance may result in the reinitialization of the modem. To prevent this from happening, modems have the ability to vary their bit distribution on the tones using bit-swapping. In any situation, when the SNR margin at some tones becomes negative due to a sudden noise change, bit-swapping can help restore a positive margin and, thus, prevent the system from unnecessary reinitializations [SSMS03]. Basically, bit-swapping provides SNR margin equalization by changing the bit-loadings of the tones. It takes away the vulnerability of a single tone to localized noise increase by converting peak loss in the margin to an average loss over all tones [VB05]. Because the target data rate cannot be changed with bit-swapping, the average SNR margin will decrease in case the average noise over all tones is increased.

3.3.2 Virtual Noise Approach

In this section it is shown how the VN mask is integrated in the target data rate calculations given in the previous Section 3.3.1.

As shown in (3.2), the achievable bit-loading on a tone is a function of the SNR and hence, a function of the received noise PSD on that tone. A measurement of the noise PSD during initialization could be overly optimistic or overly pessimistic, as it only represents a snapshot in time, not taking into account future increase or decreases in noise PSD, e.g., due to DSL lines being switched on or off. The Virtual Noise is used to prevent the DSL modem's bit-loading algorithm from assigning an overly optimistic or an overly pessimistic number of bits to a tone.

In order to derive a formula for the target data rate, first the VN mask has to be incorporated in the SNR calculation. Therefore, we define $VN_k^{(n)}$ as the value of the VN PSD at the receiver of user n on tone k. The VN PSD is communicated to the DSL modems during initialization and remains constant during operation. The virtual-signal-to-noise-ratio (VSNR) seen at the receiver of user n at tone k is given in [SKK12, IT11] by

$$VSNR_k^{(n)} = \frac{|h_k^{(n,n)}|^2 s_k^{(n)}}{\max\{VN_k^{(n)}, \sum_{m \neq n} |h_k^{(n,m)}|^2 s_k^{(m)} + (\sigma^{(n)})^2\}}. \tag{3.4}$$

As mentioned in Section 3.2, the target data rate for fixed data rate approaches is specified during initialization. For the calculation of the target data rate, the DSL modem calculates the VSNR during initialization. By defining the noise PSD $x_{\text{init},k}^{(n)}$, experienced by user n on tone k during initialization, the VSNR during initialization is given by

$$VSNR_{\text{init},k}^{(n)} = \frac{|h_k^{(n,n)}|^2 s_k^{(n)}}{\max\{VN_k^{(n)}, x_{\text{init},k}^{(n)}\}}. \tag{3.5}$$

Similar to the traditional SNR margin approach, an equal SNR margin $\gamma_{\text{VNinit}}^{(n)}$ is assigned to all tones during initialization to withstand decreases in the $VSNR_{\text{init},k}^{(n)}$, $k = 1, \ldots, K$ that might happen during operation. The bit-loading calculated during initialization is then given by

$$b_{\text{VNinit},k}^{(n)} = \log_2\left(1 + \frac{VSNR_{\text{init},k}^{(n)}}{\Gamma_c \gamma_{\text{VNinit}}^{(n)}}\right). \tag{3.6}$$

The SNR margin $\gamma_{\text{VNinit}}^{(n)}$ is typically much smaller than $\gamma_{\text{init}}^{(n)}$. Furthermore, bit-swapping equalizes $\gamma_{\text{VNinit}}^{(n)}$ during operation.

Consider the VSNR $VSNR_{\text{init},k}^{(n)}$ in (3.5). In case the noise PSD $x_{\text{init},k}^{(n)}$ is smaller than $VN_k^{(n)}$ at all $k = 1, \ldots, K$, then the bit-loading in (3.6) is defined, for a given direct channel and transmit PSD, only by the VN mask and the SNR margin. But in case the noise PSD $x^{(n)}_{\text{init},k}$ is larger than $VN^{(n)}_k$ at any $k = 1, \ldots, K$, the bit-loading in (3.6) is also dependent on $x^{(n)}_{\text{init},k}$, $k = 1, \cdots, K$. To be able to calculate a target data rate that can be achieved on a DSL line regardless of the noise PSD during initialization, in this dissertation, the bit-loading at initialization will be determined only by the VN mask and the SNR margin. In order to differentiate between the VSNR in (3.5) and the VSNR used in this dissertation for calculating the bit-loading at initialization and between the SNR margin used for calculating the bit-loading in (3.6) and the SNR margin used in this dissertation for calculating the bit-loading at initialization, we denote the VSNR and the SNR margin used in this dissertation for calculating the bit-loading at initialization by $VSNR^{(n)}_{\text{init-bl},k}$ and $\gamma_{\text{VNinit-bl}}$, respectively. $VSNR^{(n)}_{\text{init-bl},k}$ is given by

$$VSNR^{(n)}_{\text{init-bl},k} = \frac{|h^{(n,n)}_k|^2 s^{(n)}_k}{VN^{(n)}_k}. \tag{3.7}$$

Now, with $VSNR^{(n)}_{\text{init-bl},k}$ and $\gamma_{\text{VNinit-bl}}$, the bit-loading calculated in (3.6) becomes

$$b^{(n)}_{\text{VNinit},k} = \log_2\left(1 + \frac{VSNR^{(n)}_{\text{init-bl},k}}{\Gamma_c \gamma^{(n)}_{\text{VNinit-bl}}}\right). \tag{3.8}$$

Note that in case $x^{(n)}_{\text{init},k}$ is smaller than the $VN^{(n)}_k$ at all tones, the VSNR $VSNR^{(n)}_{\text{init},k}$ and the SNR margin $\gamma^{(n)}_{\text{VNinit}}$ at initialization are identical to $VSNR^{(n)}_{\text{init-bl},k}$ and $\gamma_{\text{VNinit-bl}}$, respectively. However, in case $x^{(n)}_{\text{init},k}$ is larger than the $VN^{(n)}_k$ at any $k = 1, \ldots, K$, the VSNR $VSNR^{(n)}_{\text{init},k}$ and the SNR margin $\gamma^{(n)}_{\text{VNinit}}$ at initialization will differ from $VSNR^{(n)}_{\text{init-bl},k}$ and $\gamma_{\text{VNinit-bl}}$. In that case, the SNR margin $\gamma^{(n)}_{\text{VNinit}}$ that will result at initialization after calculating the bit-loading according to (3.8) is calculated by

$$\gamma^{(n)}_{\text{VNinit}} = \frac{K\gamma^{(n)}_{\text{VNinit-bl}} + \sum_k VN^{(n)}_k - \sum_k \max\{VN^{(n)}_k, x^{(n)}_{\text{init},k}\}}{K}. \tag{3.9}$$

In (3.9), the amount of noise exceeding the VN mask during initialization is subtracted from the SNR margin $\gamma_{\text{VNinit-bl}}$.

Finally, the target data rate is given by

$$R^{(n)}_{\text{VNinit}} = f_s \sum_k b^{(n)}_{\text{VNinit},k}. \tag{3.10}$$

In this section, a formula for the target data rate for a DSL system applying VN has been derived. The derived target data rate is a function of the direct channel, the transmit

PSD, the VN mask and the SNR margin $\gamma_{\text{VNinit-bl}}$. In Sections 3.4 and 3.5, it will be shown how the SNR margin $\gamma_{\text{VNinit-bl}}$ and the VN mask can be optimized such that the outage probability described in Section 3.2 is satisfied while the target data rate $R^{(n)}_{\text{VNinit}}$ is maximized.

3.4 Joint Optimization of the VN Mask and SNR Margin for Long-term Stability

3.4.1 Introduction

In this section, we first show, in Section 3.4.2, how the outage probability $P^{(n)}_{\text{out}}$, defined in Section 3.2, can be expressed in terms of the VN mask and the SNR margin. Then, we jointly optimize the VN mask and the SNR margin such that the target data rate is maximized and the outage probability defined in Section 3.4.2 is smaller than a given target outage probability.

In order to derive an expression for the outage probability, the adaptation functionality of the system has to be considered. In practice, as already explained in Section 3.3.1, modems have the ability to equalize the SNR margin over all tones every time the noise PSD changes by using bit-swapping. The long-term stability is not concerned with the temporary decrease of the SNR margin at single tones before bit-swapping restores an equal SNR margin over all tones, and is only concerned with the long-term outage probability constraint that describes the probability of an outage event over 24 hours assuming an equalized SNR margin. In contrast to the long-term stability, the short term stability is concerned with the temporary decreases of the SNR margins at single tones before bit-swapping restores an equal SNR margin over all tones.

Bit-swapping algorithms allow the DSL modems to achieve higher target data rates than the target data rate achieved by modems that do not have bit-swapping capabilities [SSMS03]. Therefore, the outage probability and the target data rate are expressed with the equalized SNR margin assumption [JHC08b]. Since in this section, the target data rate is being maximized for a given target outage probability, only long-term stability is considered.

Since only one victim user is considered, for the remainder of the dissertation the index (n) will be omitted from the parameters. Furthermore, the index dB will be attached to a parameter when the parameter is given in logarithmic scale.

3.4.2 Outage Probability as a Function of VN Mask and SNR Margin

In this section, we show how the outage probability P_{out}, defined in Section 3.2, can be expressed in terms of the VN mask and the SNR margin when considering only the long-term stability.

As previously explained in Section 3.3.2, the SNR margin used for calculating the bit-loading during initialization is equal for all tones. The noise perceived during operation is tone dependent in practical DSL systems. Hence, with $VSNR_k^{\text{dB}}(\tau_0)$ being the value of $VSNR_k^{\text{dB}}$ at time τ_0, the per-tone SNR margin at tone k, before the bit-swapping procedures start equalizing the margin, is given in dB scale by

$$\gamma_{\text{VN},k}^{\text{dB}}(\tau_0) = \gamma_{\text{VNinit-bl}}^{\text{dB}} - \left(VSNR_{\text{init-bl},k}^{\text{dB}} - VSNR_k^{\text{dB}}(\tau_0)\right). \tag{3.11}$$

Obviously, when $VSNR_k^{\text{dB}}$ deviates from $VSNR_{\text{bl\_init},k}^{\text{dB}}$ used for calculating the bit-loading during initialization, the per-tone SNR margin $\gamma_{\text{VN},k}^{\text{dB}}$ changes. Moreover, a negative $\gamma_{\text{VN},k}^{\text{dB}}$ indicates that the error probability is larger than the target error probability on tone k.

In DSL, the direct channel can be assumed static [Cio91]. Therefore, the average transmitted power s_k stays constant for a fixed number of bits being transmitted on tone k. Thus, the variation in $VSNR_k^{\text{dB}}$ is a consequence of the variation in the received noise power. Let us define $X_k(t),\ t \in 24\text{h}$ as a time dependent random variable modeling the non-stationary noise power experienced on tone k over 24 hours. The variation in noise power results, as explained in Section 2.3, from disturbers changing their operational mode. Furthermore, let $\Upsilon_{\text{VN},k}(t)$ be a random variable describing the variation in the per-tone SNR margin $\gamma_{\text{VN},k}$ caused by the variation in $X_k(t)$. With these definitions, $\Upsilon_{\text{VN},k}(t)$ is given in dB scale by

$$\Upsilon_{\text{VN},k}^{\text{dB}}(t) = \gamma_{\text{VNinit-bl}}^{\text{dB}} - \left(\max\{VN_k^{\text{dB}}, X_k^{\text{dB}}(t)\} - VN_k^{\text{dB}}\right). \tag{3.12}$$

In order to incorporate the per-tone SNR margins in the outage probability calculation, the adaptation functionality of the system has to be considered. In practice, as explained

in Section 3.3.1, modems have the ability to equalize the SNR margin over all tones using bit-swapping. With this motivation the equivalent SNR margin was defined in [JHC08a] to be:
For a given channel, noise spectrum and transmit PSD, the equivalent SNR margin is the SNR margin that can be applied equally to all tones, while maintaining the data rate obtained using the unequal per-tone SNR margins.

The equivalent SNR margin expresses the equal SNR margin that would result if bit-swapping procedures would equalize the per-tone SNR margins. Now, let us define the random variable $\tilde{\Upsilon}_{\mathrm{VN}}(t)$ modeling the variation in the equivalent SNR margin. The variation in $\tilde{\Upsilon}_{\mathrm{VN}}(t)$ is caused by the variation in the random variables $\Upsilon_{\mathrm{VN},k}(t),\ k = 1, \cdots, K$. Applying the above definition for the equivalent SNR margin then implies

$$\sum_k \log_2\left(1+\frac{1}{\Gamma_c \Upsilon_{\mathrm{VN},k}(t)}\cdot\frac{|h_k|^2 s_k}{\max\{VN_k, X_k(t)\}}\right)=\sum_k \log_2\left(1+\frac{1}{\Gamma_c \tilde{\Upsilon}_{\mathrm{VN}}(t)}\cdot\frac{|h_k|^2 s_k}{\max\{VN_k, X_k(t)\}}\right). \tag{3.13}$$

Since high SNR is typical in DSL systems [JHC08a], the addition of 1 in the logarithm on both sides can be neglected. Then, (3.13) is given in linear scale by

$$\prod_k \left(\frac{1}{\Gamma_c \Upsilon_{\mathrm{VN},k}(t)}\cdot\frac{|h_k|^2 s_k}{\max\{VN_k, X_k(t)\}}\right)\approx\prod_k \left(\frac{1}{\Gamma_c \tilde{\Upsilon}_{\mathrm{VN}}(t)}\cdot\frac{|h_k|^2 s_k}{\max\{VN_k, X_k(t)\}}\right). \tag{3.14}$$

In log scale, (3.14) results in

$$\tilde{\Upsilon}^{\mathrm{dB}}_{\mathrm{VN}}(t) \approx \frac{1}{K}\sum_k \Upsilon^{\mathrm{dB}}_{\mathrm{VN},k}(t). \tag{3.15}$$

The random variable $\tilde{\Upsilon}^{\mathrm{dB}}_{\mathrm{VN}}(t)$ is time dependent. Hence, the probability $\Pr\{\tilde{\Upsilon}^{\mathrm{dB}}_{\mathrm{VN}}(t) < 0\}$ is also time dependent and does not express the probability of an outage event over a period of 24 hours, as stated by the definition of the outage probability in Section 3.2.
To be able to express P_{out} as defined in Section 3.2, an expression for the equivalent SNR margin that is stationary over a period of 24 hours is needed. We therefore express the outage probability by the minimum equivalent SNR margin over 24 hours, yielding

$$P_{\mathrm{out}} = \Pr\left\{\min_{\{24\mathrm{h}\}}\{\tilde{\Upsilon}^{\mathrm{dB}}_{\mathrm{VN}}(t)\} < 0\right\}. \tag{3.16}$$

Inserting (3.12) and (3.15) into (3.16) and rearranging leads to

$$P_{\mathrm{out}} = \Pr\left\{\max_{\{24\mathrm{h}\}}\left\{\sum_k \max\{VN^{\mathrm{dB}}_k, X^{\mathrm{dB}}_k(t)\} - VN^{\mathrm{dB}}_k\right\} > K\gamma^{\mathrm{dB}}_{\mathrm{VNinit\text{-}bl}}\right\}. \tag{3.17}$$

As mentioned in Section 3.3.2, it is our objective to maximize the target data rate with $VN_k^{\text{dB}},\ k = 1, \cdots, K$ and $\gamma_{\text{VNinit-bl}}^{\text{dB}}$ as optimization parameters. In that context, (3.17) shall be used to ensure that the outage probability obtained by the optimization parameters satisfies a certain target outage probability. Ideally, the relationship between the optimization parameters and the outage probability is expressed by the cumulative distribution function (CDF) of the noise power as shown in [Jag08], where only an SNR margin is used for the protection against noise. In that case, only the parameters of the CDF, such as mean and variance, have to be estimated from noise measurements and the optimization parameters can be found for a given target outage probability. The second maximum operator in (3.17) makes expressing the relationship between the optimization parameters and the outage probability by the CDFs of $X_k^{\text{dB}}(t)\ k = 1, \cdots, K$ not possible, and therefore, the outage probability in (3.17) can only be computed empirically for every $VN_k^{\text{dB}},\ k = 1, \cdots, K$ and $\gamma_{\text{VNinit-bl}}^{\text{dB}}$ from many noise measurements. Furthermore, the random variables $X_k^{\text{dB}}(t),\ k = 1, \cdots, K$, are time dependent. Thus, finding $VN_k^{\text{dB}},\ k = 1, \cdots, K$, and $\gamma_{\text{VNinit-bl}}^{\text{dB}}$ that maximize the target data rate while the outage probability obtained empirically from (3.17) satisfies a certain target outage probability would require the recording of the noise spectrum over many days, which is not practicable.

To overcome this problem, we express the outage probability in (3.17) by the maximum noises experienced on a victim line on all tones over 24 hours. In that case, only one value per-tone per day has to be stored in practical DSL systems. Throughout the dissertation, the maximum noise power experienced on a tone over 24 hours will be referred to as noise day maximum. By defining the random variable $Y_{\max,k}^{\text{dB}}$ modeling the noise day maximum on tone k and $F_{Y_{\max,k}^{\text{dB}}}$ as its CDF, the outage probability in (3.17) can be expressed by

$$P_{\text{out}} = \Pr\{\sum_k \max\{VN_k^{\text{dB}}, Y_{\max,k}^{\text{dB}}\} - VN_k^{\text{dB}} > K\gamma_{\text{VNinit-bl}}^{\text{dB}}\}. \tag{3.18}$$

Note that in order to be able to express the outage probability according to (3.18), it is assumed that the noise day maxima on all tones occur at the same time. In practical DSL systems, the noise day maxima do not always occur at the exact same time. Later on in Section 3.6, it will be shown that due to the high dependency between the noise at different tones, the effect of this assumption on the target data rate is very small.

In conclusion, in this section, an expression for the outage probability defined in

Section 3.2 has been derived. The outage probability in (3.18) captures the intuition that for a desired outage probability, the VN mask and the SNR margin have to be determined in relation to the distribution of the noise day maxima.

3.4.3 Joint Optimization of the VN Mask and the SNR Margin

In this section, we jointly optimize the VN mask and the SNR margin such that the target data rate R_{VNinit}, given in (3.10) is maximized and the outage probability P_{out} defined in (3.18) is smaller than a given target outage probability.

By defining the vector $\mathbf{VN}$ containing the values $VN_k,\ k = 1, \cdots, K$, and by defining $P_{\text{out}}^{\text{target}}$ as the target outage probability, the problem of maximizing the data rate can be formulated as follows:

$$\begin{aligned} \underset{\mathbf{VN}, \gamma_{\text{VNinit-bl}}}{\text{maximize}} \quad & \sum_k \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\text{VNinit-bl}} VN_k} \right) \\ \text{subject to} \quad & P_{\text{out}} \leq P_{\text{out}}^{\text{target}}. \end{aligned} \tag{3.19}$$

In order to clarify the relationship between the optimization parameters, the objective function and the constraint in (3.19), let us assume that $\mathbf{VN}^{\text{opt}}$, with indices VN_k^{opt}, $k = 1, \cdots, K$, and $\gamma_{\text{VNinit-bl}}^{\text{opt}}$ solve problem (3.19) optimally and achieve the data rate $R_{\text{VNinit}}^{\text{opt}}$. Now, for any $\mathbf{VN}$ and $\gamma_{\text{VNinit-bl}}$, $R_{\text{VNinit}}^{\text{opt}}$ is achieved if

$$\sum_k \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\text{VNinit-bl}} VN_k} \right) = \sum_k \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\text{VNinit-bl}}^{\text{opt}} VN_k^{\text{opt}}} \right). \tag{3.20}$$

Again, by neglecting the addition of one in the logarithm, the following is obtained in log scale:

$$K \gamma_{\text{VNinit-bl}}^{\text{dB}} + \sum_k VN_k^{\text{dB}} = K \gamma_{\text{VNinit-bl}}^{\text{opt dB}} + \sum_k VN_k^{\text{opt dB}}. \tag{3.21}$$

The left hand side of (3.21) shows that for every VN mask, $VN_k^{\text{dB}},\ k = 1, \cdots, K$, there exists an SNR margin, $\gamma_{\text{VNinit-bl}}^{\text{dB}}$, such that $R_{\text{VNinit}}^{\text{opt}}$ is achieved. Furthermore, if the outage probability constraint in problem (3.19) is satisfied with $VN_k^{\text{dB}},\ k = 1, \cdots, K$ and $\gamma_{\text{VNinit-bl}}$, then $VN_k^{\text{dB}},\ k = 1, \cdots, K$ and $\gamma_{\text{VNinit-bl}}$ are also optimum solutions for (3.19).

With this motivation, in the following, the SNR margin is determined for a fixed VN mask such that the target data rate is maximized and the outage probability $P_{\text{out}}^{\text{target}}$ is satisfied.

Let us take a look at the outage probability defined in (3.18). The maximum operator $\max\{VN_k^{\mathtt{dB},n}, Y_{\mathtt{max},k}^{\mathtt{dB},n}\}$ makes the derivation of a closed form expression for P_{out} not possible. By approximating the smallest possible noise day maximum at tone k by the 0,1% percentile of the CDF of $Y_{\text{max},k}^{\text{dB}}$ and by denoting the value of the CDF at the 0,1% percentile as $P_{0,1}$, the VN mask is given by

$$VN_k^{\text{lt dB}} = F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(P_{0,1}),\ k = 1, \cdots, K. \tag{3.22}$$

The index lt in $VN_k^{\text{lt dB}}$ indicates that the long-term stability is considered. Now, with $VN_k^{\text{lt dB}}$ and by defining the random variable $J = \sum_k Y_{\text{max},k}^{\text{dB}}$ and F_J as its CDF, a closed form for the outage probability in (3.18), that puts P_{out} in a one to one relationship with the VN mask and the SNR margin, is given by

$$P_{\text{out}} = 1 - F_J(K\gamma_{\text{VNinit-bl}}^{\text{dB}} + \sum_k VN_k^{\text{lt dB}}), \tag{3.23}$$

Moreover, the optimization problem in (3.19) becomes

$$\begin{aligned} \underset{\gamma_{\text{VNinit-bl}}}{\text{maximize}} \quad & \sum_k \log_2\left(1 + \frac{|h_k^{(n,n)}|^2 s_k}{\Gamma\gamma_{\text{VNinit-bl}} VN_k^{\text{lt}}}\right) \\ \text{subject to} \quad & P_{\text{out}} \leq P_{\text{out}}^{\text{target}}. \end{aligned} \tag{3.24}$$

Since the data rate is a strictly decreasing function of the SNR margin $\gamma_{\text{VNinit-bl}}$, the maximum data rate $R_{\text{VNinit}}^{\text{opt}}$ is achieved for the minimum SNR margin that satisfies the outage probability constraint.

The minimum SNR margin $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$ that solves the optimization problem (3.24) is then finally given by

$$\gamma_{\text{VNinit-bl}}^{\text{lt dB}} = \frac{F_J^{-1}(1 - P_{\text{out}}^{\text{target}}) - \sum_k VN_k^{\text{lt dB}}}{K}. \tag{3.25}$$

With the VN mask defined in (3.22) and with the SNR margin defined in (3.25), the target data rate in (3.10) is maximized while the outage probability in (3.18) is smaller than the target outage probability $P_{\text{out}}^{\text{target}}$. For the calculation of the target data rate in (3.10), the VN mask in (3.22) and the SNR margin in (3.25) have to be estimated from measurements of noise day maxima. In practical DSL systems, a DSL line can be monitored over a long time by the SMC. The SMC can then approximate the CDFs $F_{Y^{\text{dB}}_{\text{max},k}}, k = 1, \cdots, K$ and F_J as we have shown in [SKK12]. The parameters of the CDFs, such as mean and variance, can then be estimated from measurements of noise day maxima and the VN mask in (3.22) and the SNR margin in (3.25) can be calculated. With the VN mask and SNR margin, the SMC can finally calculate the maximum target data rate according to (3.10).

3.5 Modification of the VN Mask and SNR Margin for Additional Short-term Stability

3.5.1 Introduction

In practical DSL systems, the received noise power is frequency-dependent and the noise power might increase at some tones such that the per-tone SNR margins of the affected tones become negative. In Section 3.4, only the equal SNR margin that would result after bit-swapping has equalized the per-tone SNR margins was considered in optimization of the VN mask $VN_k^{\text{lt dB}}, k = 1, \cdots, K$ and the SNR margin $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$. Unfortunately, in practice, bit-swapping might be too slow to move bits from the affected tones such that non-negative per-tone margins are restored. In such cases, if the minimum of the per-tone SNR margins remains negative for a certain period of time, the connection is interrupted and modems will have to reinitialize at a lower data rate [JHC08a]. To prevent this from happening and thereby increase the short-term stability, the noise protection at tones with large noise variances should be increased such that the occurrence probability of negative per-tone margins is reduced while the noise protection at tones with small noise variances should be decreased such that the target data rate achieved with $VN_k^{\text{lt dB}}, k = 1, \cdots, K$ and $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$ stays unchanged.

In this section, the per-tone outage probability is used as a metric for the per-tone noise protection, and thus, equalizing the per-tone noise protection implies equalizing the per-tone outage probabilities. In Section 3.5.2, the per-tone outage probability is defined and a formula for the per-tone outage probability is derived. In Section 3.5.3, the problem of equalizing the per-tone outage probabilities in (3.30) with the VN mask and the SNR margin as optimization parameters is formulated. Then, in Sections 3.5.4 and 3.5.5, the VN mask in (3.22) and the SNR margin in (3.25) are modified such that the optimization problem stated in Section 3.5.3 is solved. Finally, in Section 3.5.6, an approximation that can be used in practical DSL systems for the estimation of the SNR margin derived in Section 3.5.5 is presented.

3.5.2 Per-tone Outage Probability

In this section, the per-tone outage probability is defined and a formula for the per-tone outage probability is derived.

As explained in Section 3.3.1, the per-tone SNR margin on a tone k is the decrease in the future SNR that can be withstood at tone k such that the same number of bits can be transmitted at an error probability that is at most equal to the target error probability. Therefore, both, the number of bits transmitted on tone k and the value of the SNR margin on tone k have a direct impact on the per-tone level of protection against future noise variations. Furthermore, after every bit-swapping procedure, the value of both, the number of bits transmitted on tone k and the SNR margin can change, and thus, the per-tone level of protection against future noise variation can also change. We therefore define the per-tone outage probabilities for given per-tone SNR margins and bit-loadings. The per-tone outage probabilities are defined for the bit-loadings that will result after bit-swapping has restored an equal SNR margin over all tones. With this motivation, the per-tone outage probability $P_{\text{out},k}$ is defined as follows:
Given the bit-loadings that lead to equal per-tone SNR margins, the per-tone outage probability probability $P_{\text{out},k}$ is the probability that the error probability exceeds the target error probability on tone k for a user n who is connected for 24 hours.

In order to derive a formula for the per-tone outage probability probability $P_{\text{out},k}$ defined above, let us first determine the SNR margin that results, after bit-swapping has equalized the per-tone SNR margins. With $x_k(\tau_0)$, $k = 1, \cdots, K$, being the noise perceived at the time τ_0, the SNR margin that results, after bit-swapping has equalized the margin, is approximated, according to (3.15), by

$$\tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0) \approx \frac{1}{K} \sum_k \gamma_{\text{VNinit-bl}}^{\text{dB}} - \left(\max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\} - VN_k^{\text{dB}} \right). \tag{3.26}$$

Now, the random variable $\Upsilon_{\text{VNmin},k}^{\text{dB}}(\tau_0)$, describing the minimum per-tone SNR margin that could be experienced over 24 hours after τ_0 at tone k assuming the bit-loadings that lead to the equal SNR margin $\tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0)$, can be expressed by

$$\Upsilon_{\text{VNmin},k}^{\text{dB}}(\tau_0) = \tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0) - \left(\max\{\max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\}, Y_{\max,k}^{\text{dB}}\} - \max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\}\right). \tag{3.27}$$

The per-tone outage probability at tone k defined above is then given by

$$P_{\text{out},k}(\tau_0) = \Pr\{\Upsilon_{\text{VNmin},k}^{\text{dB}}(\tau_0) < 0\} \tag{3.28}$$

Now inserting (3.27) in (3.28) yields

$$P_{\text{out},k}(\tau_0) = \Pr\{\tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0) + \max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\} < \max\{\max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\}, Y_{\max,k}^{\text{dB}}\}\} \tag{3.29}$$

Since $\tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0) + \max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\}$ is larger than $\max\{VN_k^{\text{dB}}, x_k^{\text{dB}}(\tau_0)\}$ for $\tilde{\gamma}_{\text{VN}}^{\text{dB}}(\tau_0) > 0$, $P_{\text{out},k}$ can be expressed by the CDF $F_{Y_{\max,k}^{\text{dB}}}$. Furthermore, (3.26) can be inserted in (3.29) and $P_{\text{out},k}$ can be expressed more generally, by using by the random variable $X_k^{\text{dB}}(t)$ instead of the noise power $x_k^{\text{dB}}(\tau_0)$ at time τ_0. $P_{\text{out},k}$ is finally given by

$$P_{\text{out},k}(t) = 1 - F_{Y_{\max,k}^{\text{dB}}}\left(\max\{VN_k^{\text{dB}}, X_k^{\text{dB}}(t)\} + \gamma_{\text{VNinit-bl}}^{\text{dB}} - \frac{1}{K}\sum_k \left(\max\{VN_k^{\text{dB}}, X_k^{\text{dB}}(t)\} - VN_k^{\text{dB}}\right)\right). \tag{3.30}$$

The per-tone outage probabilities in (3.30) are functions of the actual noise power modeled by the random variables $X_k^{\text{dB}}(t)$, $k = 1, \cdots, K$ (3.30). Therefore, they will change every time the actual noise power exceeds the VN mask at any tone. Moreover, the variation in the noise power and the variation in the maximum noise power measured over 24 hours can be different from one tone to another. Therefore, $X_k^{\text{dB}}(t)$, $k = 1, \cdots, K$ modeling the variation in the noise power and the CDFs $F_{Y_{\max,k}}$, $k = 1, \cdots, K$ of the random variables modeling the variation in the maximum noise power measured over 24 hours can be different from one tone to another. Since $X_k^{\text{dB}}(t)$, $k = 1, \cdots, K$ and the CDFs $F_{Y_{\max,k}}$, $k = 1, \cdots, K$ are tone dependent, so can the per-tone outage probabilities in (3.30) be different from one tone to another. In Section 3.5.3, the problem of equalizing the per-tone outage probabilities in (3.30) is discussed and at the end of the section an optimization problem is formulated. Then, in Sections 3.5.4 and 3.5.5, the VN mask in (3.22) and the SNR margin in (3.25) are modified such that the optimization problem stated in Section 3.5.3 is solved.

3.5.3 Problem Formulation

In this section, the problem of equalizing the per-tone outage probabilities in (3.30) with the VN mask and the SNR margin as optimization parameters is formulated. Moreover, the constraints of the problem are to maintain the maximum data rate $R_{\text{VNinit}}^{\text{opt}}$ achieved with the VN mask in (3.22) and the SNR margin in (3.25) derived in Section 3.4 and to achieve an outage probability at most equal to the target outage probability when the optimization parameters are inserted in (3.18).

The per-tone outage probabilities in (3.30) are time dependent. In order to clarify this time dependency, let us assume that the VN mask and the SNR margin have been modified such that the per-tone outage probabilities $P_{\text{out},k}^{n}$, $k = 1 \cdots K$ are equalized given the noise power at initialization. During showtime, the actual noise power might exceed the VN mask only at some tones. Bit-swapping will distribute the available SNR margin equally on all tones according to (3.26). Consequently, tones that were distorted by the noise power increase will have larger per-tone SNR margins and therefore, a lower per-tone outage probability than before the bit-swap procedure and tones that were not distorted by the noise power increase will have smaller per-tone SNR margins and therefore, a higher per-tone outage probability than before the bit-swap procedure. Thus, in order to equalize the per-tone outage probabilities every time the actual noise power exceeds the VN mask at any tone, the VN mask would have to be modified. Unfortunately, the VN mask is set only once during initialization and cannot be modified during showtime. Thus, the per-tone outage probabilities in (3.30) can only be equalized at initialization, but the VN mask VN_k^{dB}, $k = 1 \cdots K$ and the SNR margin $\gamma_{\text{VNinit-bl}}^{\text{dB}}$ can be set such that the maximum per-tone deviation of the per-tone outage probability during showtime from the equalized per-tone outage probability during initialization is minimized.

As already mentioned in Section 3.3.2, the bit-loadings during initialization are calculated with the VN mask VN_k^{dB}, $k = 1, \cdots, K$, and the SNR margin $\gamma_{\text{VNinit-bl}}^{\text{dB}}$. The per-tone outage probabilities are then given by

$$P_{\text{out},k}^{\text{init-bl}} = 1 - F_{Y_{\max,k}^{\text{dB}}}\left(VN_k^{\text{dB}} + \gamma_{\text{VNinit-bl}}^{\text{dB}}\right), \quad k = 1 \cdots K. \tag{3.31}$$

Now, the problem of minimizing the maximum per-tone deviation of the per-tone outage probability during showtime from the equalized per-tone outage probability during initialization, while the target data rate $R_{\text{VNinit}}^{\text{opt}}$ achieved with $VN_k^{\text{lt dB}}$, $k = 1 \cdots K$ and $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$ is maintained and the outage probability (3.29) is at most equal to the target outage probability $P_{\text{out}}^{\text{target}}$, can be formulated as

$$
\begin{aligned}
\underset{\mathbf{VN},\gamma_{\text{VNinit-bl}}}{\text{minimize}} \quad & \max_{\{k_1,\cdots,k_K\}} \left(P_{\text{out},k}(t) - P_{\text{out},k}^{\text{init-bl}}\right) \\
\text{subject to} \quad & P_{\text{out},k_1}^{\text{init-bl}} = P_{\text{out},k_2}^{\text{init-bl}},\ k_1, k_2 = 1 \cdots K, \\
& R_{\text{VNinit}} = R_{\text{VNinit}}^{\text{opt}}, \\
& P_{\text{out}} \leq P_{\text{out}}^{\text{target}}.
\end{aligned}
\tag{3.32}
$$

According to (3.21), there exists a $\gamma_{\text{VNinit-bl}}$ for every $\mathbf{VN}$ such that $R_{\text{VNinit}}^{\text{opt}}$ is achieved. Thus, the problem in (3.32) can be solved in two steps. First, in Section 3.5.4, the shape of the VN mask $VN_k^{\text{lt dB}}$, $k = 1, \cdots, K$ is modified such that the per-tone outage probabilities are equalized when calculating the bit-loading during initialization, while the data rate $R_{\text{VNinit}}^{\text{opt}}$, achieved with $VN_k^{\text{lt dB}}$, $k = 1 \cdots K$ and $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$, is maintained. The modified VN mask will be given as a function of the SNR margin. Then, in Section 3.5.5, the SNR margin is found that minimizes the maximum deviation of the per-tone outage probabilities during showtime from the equalized per-tone outage probabilities during initialization, while the outage probability (3.29) is at most equal to the target outage probability $P_{\text{out}}^{\text{target}}$.

3.5.4 Equalization of the Per-tone Outage Probabilities at Initialization

In this section, the VN mask $VN_k^{\text{lt dB}}$, $k = 1, \cdots, K$ is modified such that the per-tone outage probabilities $P_{\text{out},k}^{\text{init-bl}}$ are equalized while the data rate $R_{\text{VNinit}}^{\text{opt}}$, achieved with $VN_k^{\text{lt dB}}$, $k = 1 \cdots K$ and $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$, is maintained.

Consider equation (3.31). As explained in Section 3.5.2, the parameters of the CDFs $F_{Y_{\max,k}^{\text{dB}}}$, $k = 1, \cdots, K$, such as mean and variance, can differ from one tone to another. Furthermore, $VN_k^{\text{lt dB}}$ in (3.22) is set equal to the 0,1% percentile of the CDFs $F_{Y_{\max,k}^{\text{dB}}}$, $k = 1, \cdots, K$ and the SNR margin $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$ in (3.25) is equal for all tones. Thus, the per-tone outage probability $P_{\text{out},k}$ will vary from one tone to another when calculated from $VN_k^{\text{lt dB}}$ and $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$. In the following, the shape of the VN mask $VN_k^{\text{lt dB}}$, $k = 1, \cdots, K$, is modified such that the per-tone outage probabilities $P_{\text{out},k}^{\text{init-bl}}$, $k = 1, \cdots, K$, are equalized, while the data rate $R_{\text{VNinit}}^{\text{opt}}$, achieved with $VN_k^{\text{lt dB}}$, $k = 1 \cdots K$ and $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$, is maintained.

The problem described above can be formulated as

$$
\begin{aligned}
\underset{\mathbf{VN}}{\text{minimize}} \quad & \max_{\{k_1,\cdots,k_K\}} P_{\text{out},k}^{\text{init-bl}} \\
\text{subject to} \quad & K\gamma_{\text{VNinit-bl}}^{\text{dB}} + \sum_k VN_k^{\text{dB}} = K\gamma_{\text{VNinit-bl}}^{\text{lt dB}} + \sum_k VN_k^{\text{lt dB}}.
\end{aligned}
\tag{3.33}
$$

Note that the constraint in (3.33) ensures that $R^{\text{opt}}_{\text{VNinit}}$ is achieved. Furthermore, the left hand side of the constraint in (3.33) can be expressed by rearranging (3.31). Problem (3.33) then becomes

$$\begin{aligned} &\underset{\mathbf{VN}}{\text{minimize}} && \max_{\{k_1,\cdots,k_K\}} P^{\text{init-bl}}_{\text{out},k} \\ &\text{subject to} && \sum_k F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{init-bl}}_{\text{out},k}) = K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_k VN^{\text{lt dB}}_k. \end{aligned} \tag{3.34}$$

Since $P^{\text{init-bl}}_{\text{out},k}$, defined in (3.31), is a decreasing function of the optimization parameter VN^{dB}_k, problem (3.34) can be solved by finding the minimum equal per-tone outage probability that satisfies the constraint in (3.34) and then expressing VN^{dB}_k, as a function of the minimum equal per-tone outage probability. By defining the probability P as an equal per-tone outage probability, problem (3.34) becomes

$$\begin{aligned} &\underset{P}{\text{minimize}} && P \\ &\text{subject to} && \sum_k F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P) = K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_k VN^{\text{lt dB}}_k. \end{aligned} \tag{3.35}$$

Since the inverse CDFs $F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P)$, $k = 1 \cdots K$ are monotonically decreasing with P, the optimal solution of (3.37) can be obtained using bisection on the scalar P, yielding $P^{\text{eq}}_{\text{out-pt}}$, as shown in Algorithm 1.

Algorithm 1 Bisection method to find $P^{\text{eq}}_{\text{out-pt}}$

1: Initialize $P_{\text{min}} = 0$ and $P_{\text{max}} = 1$
2: Choose a tolerance $\epsilon > 0$
3: **repeat**
4: $\quad P = (P_{\text{min}} + P_{\text{max}})/2$
5: $\quad$ **if** $\sum_k F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P) \geq K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_k VN^{\text{lt dB}}_k$ **then**
6: $\qquad P = P_{\text{max}}$
7: $\quad$ **else**
8: $\qquad P = P_{\text{min}}$
9: $\quad$ **end if**
10: **until** $|\sum_k F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P) - \left(K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_k VN^{\text{lt dB}}_k\right)| \leq \epsilon$
11: $P^{\text{eq}}_{\text{out-pt}} = P$

Now, the VN mask that equalizes the per-tone outage probabilities $P^{\text{init-bl}}_{\text{out},k}$, $k = 1, \cdots, K$, while the data rate $R^{\text{opt}}_{\text{VNinit}}$, achieved with $VN^{\text{lt dB}}_k$, $k = 1 \cdots K$ and $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$, is maintained, is given as a function of the SNR margin $\gamma^{\text{dB}}_{\text{VNinit-bl}}$ by

$$VN^{\text{dB}}_k = F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}},\ k = 1 \cdots K. \tag{3.36}$$

3.5.5 Minimizing the SNR Margin

In the previous section, the per-tone outage probabilities $P_{\text{out},k}^{\text{init-bl}},\ k = 1, \cdots, K$, were equalized during initialization such that the target data rate $R_{\text{VNinit}}^{\text{opt}}$ is achieved. With the equal per-tone outage probability $P_{\text{out-pt}}^{\text{eq}}$, the VN mask was determined as a function of the SNR margin in (3.36). In this section, the SNR margin is found that minimizes the maximum deviation of the per-tone outage probabilities $P_{\text{out},k}(t),\ k = 1, \cdots, K$, given in (3.30), during showtime from the equal per-tone outage probability $P_{\text{out-pt}}^{\text{eq}}$ achieved during initialization, while the resulting outage probability P_{out}, given in (3.18), satisfies the target outage probability $P_{\text{out}}^{\text{target}}$. First, the described problem is formulated. Then, the relationships between the SNR margin and the per-tone outage probabilities $P_{\text{out},k}(t),\ k = 1, \cdots, K$, during showtime and between the SNR margin and the outage probability P_{out} are analyzed. Finally, the SNR margin that solves the described problem is derived.

Since the equal probability $P_{\text{out-pt}}^{\text{eq}}$, determined in Algorithm 1, should not change regardless of the value of the SNR margin $\gamma_{\text{VNinit-bl}}^{\text{dB}}$, minimizing the maximum deviation of the per-tone outage probabilities $P_{\text{out},k}(t),\ k = 1, \cdots, K$, from the equal per-tone outage probability $P_{\text{out-pt}}^{\text{eq}}$ is equivalent to minimizing the maximum per-tone outage probability while ensuring the equal per-tone outage probability $P_{\text{out-pt}}^{\text{eq}}$ during inialization. The problem described above can now be formulated as

$$\begin{aligned} \underset{\gamma_{\text{VNinit-bl}}}{\text{minimize}} \quad & \max_{\{k_1, \cdots, k_K\}} P_{\text{out},k}(t) \\ \text{subject to} \quad & VN_k^{\text{dB}} = F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(1 - P_{\text{out-pt}}^{\text{eq}}) - \gamma_{\text{VNinit-bl}}^{\text{dB}},\ k = 1 \cdots K \\ & P_{\text{out}} \leq P_{\text{out}}^{\text{target}}. \end{aligned} \tag{3.37}$$

To understand the relationship between the per-tone outage probabilities $P_{\text{out},k}(t),\ k = 1, \cdots, K$ and the optimization parameter $\gamma_{\text{VNinit-bl}}^{\text{dB}}$ in (3.37), consider Figure 3.1.

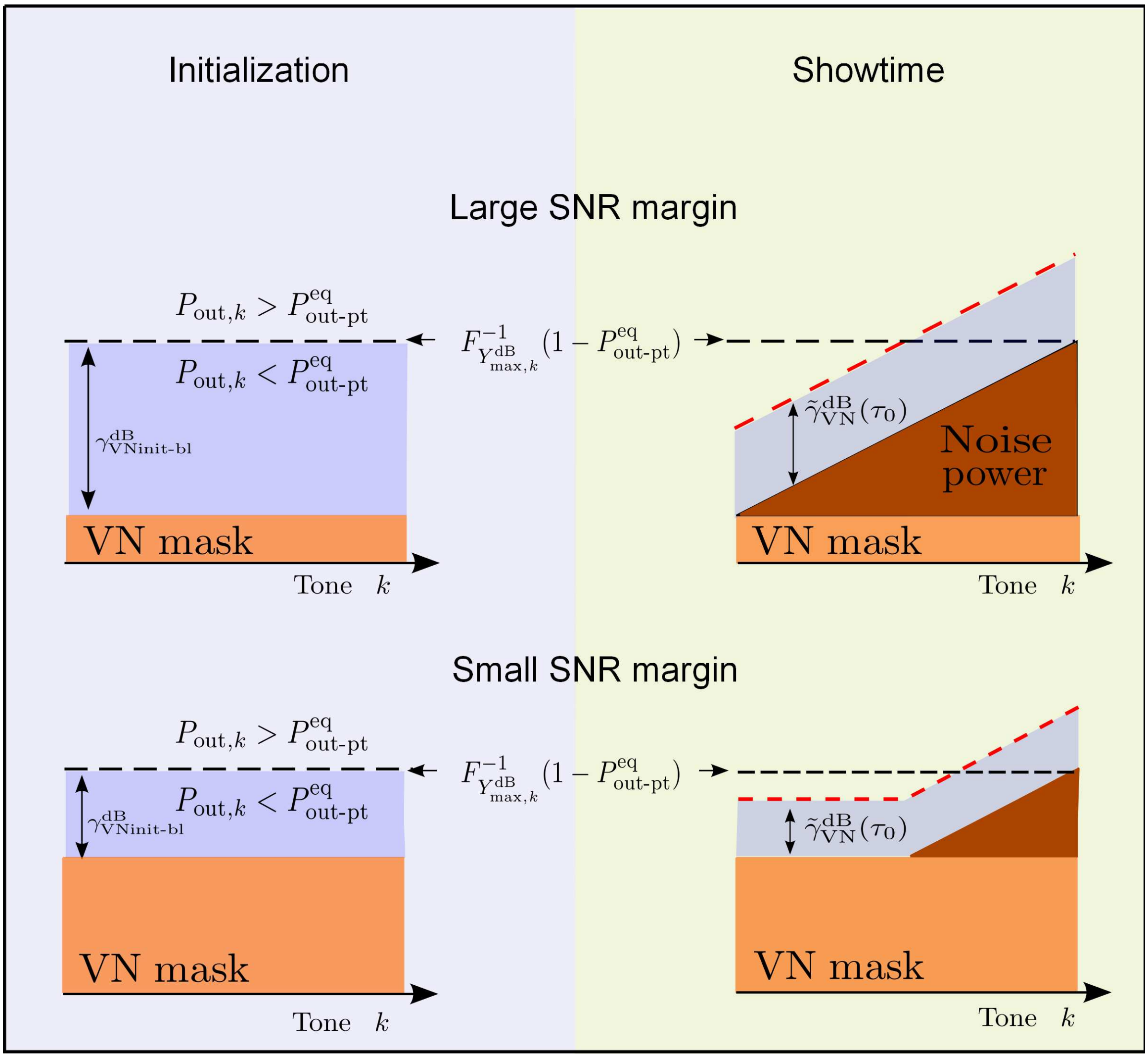


Figure 3.1: Illustration of the relationship between the per-tone outage probabilities during showtime and the SNR margin.

The left part of Figure 3.1 illustrates the VN mask and the SNR margin used for calculating the bit-loading for some tones during initialization. The right part of the Figure illustrates the situation during showtime at time τ_0 when the noise power exceeds the VN mask and the bit-swapping operations distribute the remaining SNR margin equally over all tones. The resulting SNR margin $\tilde{\gamma}^{\text{dB}}_{\text{VN}}(\tau_0)$ is defined in (3.26). Furthermore, two scenarios are shown in Figure 3.1. In the upper part of the figure, the scenario when a large SNR margin is used is depicted and in the lower part the scenario when a small one is used is depicted. In both scenarios, the per-tone outage probabilities have been equalized during initialization and have the same per-tone outage probability $P^{\text{eq}}_{\text{out-pt}}$. The black dashed line illustrates the first constraint of problem (3.37) that the VN mask and the SNR margin have to satisfy during initialization. Note that the black dashed line and the VN mask in Figure 3.1 are diagrams and do not represent real values. The values of $F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}})$ and the VN mask at a tone are determined by the inverse CDF of the noise day maxima at that tone, according to (3.36), and can therefore be different from one tone to another. In order to simplify the visualization, the values of both, $F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}})$ and the VN mask, at different tones are assumed to be equal in Figure 3.1.

The red dashed line illustrates the argument of the CDFs in (3.30) for the noise power at time τ_0. Since the per-tone outage probabilities in (3.30) decrease with increasing arguments of the CDFs, the red dashed line exceeding the black one at any tone indicates a per-tone outage probability smaller than $P^{\text{eq}}_{\text{out-pt}}$, and vice versa. Furthermore, the larger the deviation between the two dashed lines is at a tone, the larger is the deviation between the per-tone outage probability at time τ_0 and the probability $P^{\text{eq}}_{\text{out-pt}}$ at that tone.

When the SNR margin is equalized after the noise power has increased during showtime at some tones more than the others, the amount of noise power that exceeded the VN mask at those tones is subtracted evenly from the SNR margin of all tones according to (3.26). Thus, tones that where less distorted by the noise power end up with per-tone outage probabilities larger than $P^{\text{eq}}_{\text{out-pt}}$ and vice versa. This behavior is demonstrated by the deviation between the black and the red dashed lines in the right part of Figure 3.1. When the VN mask is increased and the SNR margin is therefore decreased to satisfy the constraint in problem (3.37), as shown in the lower part of Figure 3.1, the amount of noise power exceeding the VN mask and subtracted evenly from the SNR margin of all tones is decreased and therefore, tones that where less distorted by the noise power end up with per-tone outage probabilities smaller than the ones achieved with a large SNR margin. Thus, minimizing the maximum per-tone outage probability in problem (3.37) implies minimizing the SNR margin $\gamma^{\text{dB}}_{\text{VNinit-bl}}$.

With this motivation, problem (3.37) becomes

$$\begin{aligned} \underset{\gamma_{\text{VNinit-bl}}}{\text{minimize}} \quad & \gamma^{\text{dB}}_{\text{VNinit-bl}} \\ \text{subject to} \quad & VN^{\text{dB}}_k = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}},\ k = 1 \cdots K \\ & P_{\text{out}} \leq P^{\text{target}}_{\text{out}}. \end{aligned} \tag{3.38}$$

Consider problem (3.38). The first constraint in (3.38) is satisfied for any $\gamma^{\text{dB}}_{\text{VNinit-bl}}$. It is the second one that constrains the minimization of $\gamma^{\text{dB}}_{\text{VNinit-bl}}$. In (3.22), the VN mask $VN^{\text{lt dB}}_k$, $k = 1 \cdots K$ was set equal to the smallest possible realizations of $Y^{\text{dB}}_{\max,k}, k = 1 \cdots K$ approximated by the 0,1% percentile of the CDFs $F_{Y_{\max,k}}, k = 1, \cdots, K$, such that the outage probability in (3.18) can be expressed in a closed form as a function of F_J, as in (3.23), and the SNR margin in (3.25) can therefore be calculated for a certain target outage probability. Now, in (3.36), the shape of the VN mask has been changed. To be able to express the outage probability in a closed form similar to (3.23), the SNR margin has to be set such that the resulting VN mask in (3.36) is smaller than or equal to $VN^{\text{lt dB}}_k$, $k = 1 \cdots K$. This will lead to a large SNR margin that will not minimize the maximum the per-tone outage probabilities during showtime as shown in Figure 3.1. Thus, the solution of problem (3.38) cannot be given in a closed form.

In order to better understand (3.38), in the following the relationship between the optimization parameter $\gamma^{\text{dB}}_{\text{VNinit-bl}}$ and the outage probability P_{out} is analyzed.

The outage probability P_{out} in the second constraint of problem (3.38) is defined in (3.18) as a function of the VN mask and the SNR margin. By constraining the VN mask and the SNR margin by the first constraint of problem (3.38), the second constraint of problem (3.38) is given by

$$\Pr\{\sum_k \max\{VN^{\text{dB}}_k, Y^{\text{dB}}_{\max,k}\} > \sum_k F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}})\} = P^{\text{target}}_{\text{out}}. \tag{3.39}$$

Furthermore, in Algorithm 1, the probability $P^{\text{eq}}_{\text{out-pt}}$ was determined such that $\sum_k F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}}) = K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_k VN^{\text{lt dB}}_k$ and therefore, according to (3.25), $\sum_k F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}}) = F^{-1}_J(1 - P^{\text{target}}_{\text{out}})$. By using this result and rearranging (3.39), problem (3.38) is now given by

$$\begin{aligned}
\underset{\gamma_{\text{VNinit-bl}}}{\text{minimize}} \quad & \gamma^{\text{dB}}_{\text{VNinit-bl}} \\
\text{subject to} \quad & VN^{\text{dB}}_{k} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}}, \; k = 1 \cdots K \\
& \Pr\{\sum_{k} \max\{VN^{\text{dB}}_{k}, Y^{\text{dB}}_{\max,k}\} \leq F^{-1}_{J}(1 - P^{\text{target}}_{\text{out}})\} = 1 - P^{\text{target}}_{\text{out}}.
\end{aligned} \tag{3.40}$$

In order to clarify the relationship between the optimization parameter $\gamma^{\text{dB}}_{\text{VNinit-bl}}$ and the target outage probability $P^{\text{target}}_{\text{out}}$ in (3.40), let $\mathcal{Y}1^{\text{dB}}_{\max,k}, k = 1, \cdots, K$ be realizations of the random variables $Y^{\text{dB}}_{\max,k}, k = 1, \cdots, K$, for which the following applies:

$$\sum_{k} \mathcal{Y}1^{\text{dB}}_{\max,k} = F^{-1}_{J}(1 - P^{\text{target}}_{\text{out}}) = K\gamma^{\text{lt dB}}_{\text{VNinit-bl}} + \sum_{k} VN^{\text{lt dB}}_{k}. \tag{3.41}$$

The sum $\sum_{k} \mathcal{Y}1^{\text{dB}}_{\max,k}$ is the maximum sum of noise power that can be tolerated by a DSL system using the VN mask $VN^{\text{lt dB}}_{k}, k = 1, \cdots, K$ and the SNR margin $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$ and operating at the target data rate $R^{\text{opt}}_{\text{VNinit}}$ with target outage probability $P^{\text{target}}_{\text{out}}$, assuming an equalized SNR margin every time the noise powers exceeds the VN mask. In order to use a diffrent SNR margin and VN mask but still operate at the same target data rate and outage probability, any noise that was withsood when using $VN^{\text{lt dB}}_{k}, k = 1, \cdots, K$ and $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$ has to be also withstood when using a diffrent SNR margin and VN mask. Thus, the following must hold:

$$\sum_{k} \max\left\{VN^{\text{dB}}_{k}, \mathcal{Y}1^{\text{dB}}_{\max,k}\right\} \leq F^{-1}_{J}(1 - P^{\text{target}}_{\text{out}}), \tag{3.42}$$

and consequently,

$$F_{J}\left(\sum_{k} \max\left\{VN^{\text{dB}}_{k}, \mathcal{Y}1^{\text{dB}}_{\max,k}\right\}\right) \leq 1 - P^{\text{target}}_{\text{out}}. \tag{3.43}$$

Equation (3.43) is only satisfied when

$$VN^{\text{dB}}_{k} \leq \mathcal{Y}1^{\text{dB}}_{\max,k}, \quad k = 1, \cdots, K. \tag{3.44}$$

Considering the first constraint in problem (3.40) that relates the SNR margin to the VN mask, the SNR margin has to be set large enough such that the VN mask satisfies (3.44). Moreover, for another realization $\mathcal{Y}2^{\text{dB}}_{\max,k}, k = 1, \cdots, K$ of the noise day maxima, for which $F_{J}(\sum_{k} \mathcal{Y}2^{\text{dB}}_{\max,k}) = 1 - P^{\text{target}}_{\text{out}}$ holds, a different VN mask will result according to (3.44) and therefore a different SNR margin. Setting the SNR margin too small will result in an outage probability that is larger than the target outage probability.

In general, the random variable $Y_{\max,k}^{\mathrm{dB}}$ can have different values for each realization of $J = \sum_k Y_{\max,k}^{\mathrm{dB}}$. The randomness in $Y_{\max,k}^{\mathrm{dB}}$ for any J is described by the conditional distribution of $Y_{\max,k}^{\mathrm{dB}}$ given J and will be denoted here by $Y_{\max,k}|J$. Moreover, the randomness in $Y_{\max,k}^{\mathrm{dB}}$ given $J = \mathcal{J}$ is denoted by $Y_{\max,k}^{\mathrm{dB}}|\mathcal{J}$.

By using $Y_{\max,k}|J$ instead of $\mathcal{Y}1_{\max,k}^{\mathrm{dB}}$ in (3.43), we can formulate (3.43) for all possible realizations of $Y_{\max,k}^{\mathrm{dB}}, k = 1, \cdots, K$ that satisfy $J = \sum_k Y_{\max,k}^{\mathrm{dB}} \leq F_J^{-1}(1 - P_{\mathrm{out}}^{\mathrm{target}})$, yielding:

$$F_J\left(\sum_k \max\left\{VN_k^{\mathrm{dB}}, Y_{\max,k}|J\right\}\right) \leq 1 - P_{\mathrm{out}}^{\mathrm{target}}, \quad J \leq F_J^{-1}(1 - P_{\mathrm{out}}^{\mathrm{target}}). \tag{3.45}$$

By taking the inverse CDF of (3.45), problem (3.38) can be rewritten as

$$\begin{aligned}
\underset{\gamma_{\mathrm{VNinit\text{-}bl}}}{\text{minimize}} \quad & \gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{dB}} \\
\text{subject to} \quad & VN_k^{\mathrm{dB}} = F^{-1}_{Y_{\max,k}^{\mathrm{dB}}}(1 - P_{\mathrm{out\text{-}pt}}^{\mathrm{eq}}) - \gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{dB}}, \; k = 1 \cdots K \\
& \sum_k \max\left\{VN_k^{\mathrm{dB}}, Y_{\max,k}|J\right\} \\
& \leq F_J^{-1}\left(1 - P_{\mathrm{out}}^{\mathrm{target}}\right), \quad J \leq F_J^{-1}(1 - P_{\mathrm{out}}^{\mathrm{target}}).
\end{aligned} \tag{3.46}$$

Problem (3.46) can be solved iteratively by setting $\mathcal{J} = F_J^{-1}(1 - P_{\mathrm{out}}^{\mathrm{target}})$ and $\gamma_{\mathrm{VNinit\text{-}bl}} = 0$ and increasing $\gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{dB}}$ gradually with the granularity ζ until the constraint in problem (3.46) is satisfied for all possible realizations of $Y_{\max,k}|\mathcal{J}$, then decreasing $\mathcal{J}$ with the step size ξ and continuing increasing $\gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{dB}}$ until the constraint in problem (3.46) is satisfied for all possible realizations of $Y_{\max,k}|(\mathcal{J} - \xi)$, finally terminating when $\mathcal{J} \leq \xi$, as shown in Algorithm 2.

The SNR margin $\gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{st\ dB}}$ is the minimum SNR margin that satisfies the constraint in problem (3.46). The index st in $\gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{st\ dB}}$ indicates that the maximum per-tone outage probability is minimized (short-term stability). Finally, the VN mask in (3.36) becomes

$$VN_k^{\mathrm{st\ dB}} = F^{-1}_{Y_{\max,k}^{\mathrm{dB}}}(1 - P_{\mathrm{out\text{-}pt}}^{\mathrm{eq}}) - \gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{st\ dB}}, \; k = 1 \cdots K. \tag{3.47}$$

With $VN_k^{\mathrm{st\ dB}}$ and $\gamma_{\mathrm{VNinit\text{-}bl}}^{\mathrm{st\ dB}}$, the maximum per-tone deviation of the per-tone outage probability during showtime from the equalized per-tone outage probability during initialization

Algorithm 2 Iterative algorithm to find the minimum $\gamma^{\text{dB}}_{\text{VNinit-bl}}$ that solves (3.46)

1: Initialize $\mathcal{J} = F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$, $\gamma^{\text{dB}}_{\text{VNinit-bl}} = 0$
2: choose a step size ξ and a granularity ζ
3: **repeat**
4: $\quad \nu = \max\left\{\sum_k \max\left\{F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}}, Y_{\max,k}|\mathcal{J}\right\}\right\}$
5: $\quad$ **if** $\nu \leq F_J\left(1 - P^{\text{target}}_{\text{out}}\right)$ **then**
6: $\quad\quad \mathcal{J} = \mathcal{J} - \xi$
7: $\quad$ **else**
8: $\quad\quad \gamma^{\text{dB}}_{\text{VNinit-bl}} = \gamma^{\text{dB}}_{\text{VNinit-bl}} + \zeta$
9: $\quad$ **end if**
10: **until** $\mathcal{J} \leq \xi$
11: $\gamma^{\text{st dB}}_{\text{VNinit-bl}} = \gamma^{\text{dB}}_{\text{VNinit-bl}}$

is minimized while the data rate $R^{\text{opt}}_{\text{VNinit}}$ achieved with $VN_k^{\text{lt dB}}$, $k = 1, \cdots, K$ and $\gamma^{\text{lt dB}}_{\text{VNinit-bl}}$ is maintained and the outage probability in (3.29) is at most equal to the target outage probability $P^{\text{target}}_{\text{out}}$, when calculated from $VN_k^{\text{st dB}}$ and $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$.

To calculate the maximum operator in row 4 of Algorithm 2, enough measurements of noise day maxima have to be available for each value of $\mathcal{J}$. Estimating the conditional distributions of $Y_{\max,k}|\mathcal{J}$ only from measurements of noise day maxima will require an enormous amount of measurements since for each value of $\mathcal{J}$ many measurements of noise day maxima have to be available. Thus, it is very difficult to estimate the conditional distributions of $Y_{\max,k}|\mathcal{J}$ only from measurements in practical systems. In the next section, a model for the random variable $Y_{\max,k}|\mathcal{J}$ is derived. By using the model, an approximation for the minimum SNR margin $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$ can be found.

3.5.6 Approximation of the Minimum SNR Margin

As already explained in the previous section, it is not practicable to iteratively solve problem (3.46) because an enormous amount of realizations of $Y_{\max,k}|\mathcal{J}$, $\mathcal{J} = \xi, \xi + \xi, \cdots, F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$, have to be available. In this section, first, a model for the random variable $Y_{\max,k}|\mathcal{J}$ is derived, then, an approximation for the minimum SNR margin $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$ is found.

In DSL systems, users are allowed to transmit over the entire frequency band prescribed in the band plan [IT11]. Consequently, the crosstalk caused by an active user covers the entire frequency band. If a tone is experiencing high crosstalk levels, it is very likely that other tones are also experiencing high crosstalk levels. The random variables $Y^{\mathrm{dB}}_{\max,k}$ $k = 1 \cdots K$ are therefore highly correlated. Moreover, as a consequence, the random variable $J = \sum_k Y^{\mathrm{dB}}_{\max,k}$ is also correlated to each of the random variables $Y^{\mathrm{dB}}_{\max,k}$ $k = 1 \cdots K$. Furthermore, let the correlation between the random variable J and the random variable $Y^{\mathrm{dB}}_{\max,k}$ be described by the correlation coefficient $\rho_{Y^{\mathrm{dB}}_{\max,k},J}$.

By defining $Cov(Y^{\mathrm{dB}}_{\max,k}, J)$ as the covariance between $Y^{\mathrm{dB}}_{\max,k}$ and J and σ_J and $\sigma_{Y^{\mathrm{dB}}_{\max,k}}$ as the standard deviations of J and $Y^{\mathrm{dB}}_{\max,k}$ respectively, the correlation coefficient $\rho_{Y^{\mathrm{dB}}_{\max,k},J}$ is given by

$$\rho_{Y^{\mathrm{dB}}_{\max,k},J} = \frac{Cov(Y^{\mathrm{dB}}_{\max,k}, J)}{\sigma_{Y^{\mathrm{dB}}_{\max,k}} \sigma_J} \tag{3.48}$$

By assuming that $\rho_{Y^{\mathrm{dB}}_{\max,k},J}$, $k = 1 \cdots K$, take values that are close to 1, the relationship between each random variable $Y_{\max,k}|J$ and the random variable J can be modeled by the linear regression equation

$$Y^{\mathrm{dB}}_{\max,k}|J \quad = \alpha_k + \beta_k J + \epsilon_k, \quad k = 1 \cdots K, \tag{3.49}$$

[BF10], where α_k and β_k are constants for each k and ϵ_k is a zero mean, independent and identically distributed random variable that models the error and that is also independent of J.

Note that in practical DSL systems, the correlation between the sum of noise day maxima, modeled in this chapter by the random variable J, and noise maximum at each tone, modeled in this chapter by the random variables $Y^{\mathrm{dB}}_{\max,k}, k = 1, \cdots, K$, is strong, however, the correlation coefficients $\rho_{Y^{\mathrm{dB}}_{\max,k},J}, k = 1 \cdots K$ are not necessarily close to 1 for all K tones. Later on in Section 3.6, the effect of assuming that $\rho_{Y^{\mathrm{dB}}_{\max,k},J}$, $k = 1 \cdots K$, have values that are close to 1 on the derived approximated minimum SNR margin is shown.

Now, by defining $E(\cdot)$ as the expected value operation and $E(Y^{\mathrm{dB}}_{\max,k}|J)$ as the expected value of $Y^{\mathrm{dB}}_{\max,k}$ given J, the conditional expectations $E(Y^{\mathrm{dB}}_{\max,k}|J)$, $k = 1 \cdots K$ are given by [BF10]

$$E(Y^{\mathrm{dB}}_{\max,k}|J) = \alpha_k + \beta_k J, \quad k = 1 \cdots K. \tag{3.50}$$

Furthermore, the error term ϵ_k is given by

$$\epsilon_k = Y^{\text{dB}}_{\text{max},k}|J - (\alpha_k + \beta_k J), \quad k = 1 \cdots K. \tag{3.51}$$

Now, the parameters α_k and β_k that minimize the sum of squares of ϵ_k, given different realizations of J, are given according to [BF10] by

$$\beta_k = \frac{\rho_{Y^{\text{dB}}_{\text{max},k},J}\,\sigma_{Y^{\text{dB}}_{\text{max},k}}}{\sigma_J}, \qquad \alpha_k = E(Y^{\text{dB}}_{\text{max},k}) - \beta_k E(J), \quad k = 1 \cdots K. \tag{3.52}$$

As explained in the beginning of the section, the objective of the SMC in practice is to find a model for the random variables $Y^{\text{dB}}_{\text{max},k}$ $k = 1, \cdots, K$ given a certain $\mathcal{J}$. To do so, the SMC can measure the noise day maxima over D days. With the noise day maxima of each day, one measurement of the sum of noise day maxima is calculated. From the sample containing D measured noise day maxima and D measurements of the sum of noise day maxima, estimates of α_k and β_k can be calculated according to (3.52). By using the estimates of α_k and β_k in (3.50), estimates $\hat{\mu}_{Y_{\text{max},k}|\mathcal{J}}$, $k = 1 \cdots K$ of the expected values of $Y_{\text{max},k}|\mathcal{J}$, $k = 1, \cdots, K$ can be calculated. Finally, the model for $Y_{\text{max},k}|\mathcal{J}$ is given by

$$\hat{Y}^{\text{dB}}_{\text{max},k}|\mathcal{J} \;\; = \;\; \hat{\mu}_{Y_{\text{max},k}|\mathcal{J}} \;\; + \;\; \epsilon_k, \qquad k = 1 \cdots K. \tag{3.53}$$

With this result, problem (3.46) is given by

$$\begin{aligned} &\underset{\gamma_{\text{VNinit-bl}}}{\text{minimize}} && \gamma_{\text{VNinit-bl}} \\ &\text{subject to} && \sum_k \max\left\{F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}}, \hat{\mu}_{Y_{\text{max},k}|\mathcal{J}} + \epsilon_k\right\} \\ &&& \leq F_J^{-1}\left(1 - P^{\text{target}}_{\text{out}}\right), \;\; \mathcal{J} = F_J^{-1}(1 - P^{\text{target}}_{\text{out}}). \end{aligned} \tag{3.54}$$

Note that since it is assumed in (3.49) that the random variable $Y_{\text{max},k}|J$ changes linearly with J, the left hand side of the constraint in problem (3.54) is non-decreasing with $\mu_{Y_{\text{max},k}|J}$. Hence, problem (3.54) has to be solved only for $\mathcal{J} = F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$.

Similar to (3.44), but now using $F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}}$ instead of VN^{dB}_k and $\hat{Y}^{\text{dB}}_{\text{max},k}|F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$ instead of $\mathcal{Y}1^{\text{dB}}_{\text{max},k}$, the constraint in problem (3.54) can be written as

$$F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - \gamma^{\text{dB}}_{\text{VNinit-bl}} \leq \mu_{Y_{\text{max},k}|F_J^{-1}(1-P^{\text{target}}_{\text{out}})} + \epsilon_k, \quad k = 1, \cdots, K. \tag{3.55}$$

The SNR margin $\gamma^{\text{dB}}_{\text{VNinit-bl}}$ required to satisfy the constraint (3.55) is dependent on $\epsilon_k, k = 1, \cdots, K$. For every realization of $\epsilon_k, k = 1, \cdots, K$, another SNR margin results. By denoting the CDF of $\hat{Y}^{\text{dB}}_{\text{max},k}|\mathcal{J}$ as $F_{\hat{Y}^{\text{dB}}_{\text{max},k}|\mathcal{J}}$ and by approximating the minimum noise day maximum on tone k given that $\mathcal{J} = F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$ by the value of the CDF $F_{\hat{Y}^{\text{dB}}_{\text{max},k}|F_J^{-1}(1-P^{\text{target}}_{\text{out}})}$ at the 0,1% percentile and using $P_{0,1}$ from Section 3.4.3, an approximation of $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$ is given by

$$\tilde{\gamma}^{\text{st dB}}_{\text{VNinit-bl}} = \max_{\{k_1,\cdots,k_K\}} \left\{ F^{-1}_{Y^{\text{dB}}_{\text{max},k}}(1 - P^{\text{eq}}_{\text{out-pt}}) - F^{-1}_{\hat{Y}^{\text{dB}}_{\text{max},k}|F_J^{-1}(1-P^{\text{target}}_{\text{out}})}(P_{0,1}) \right\}. \tag{3.56}$$

For better understanding of equation (3.56), let us assume that the sample containing D measured noise day maxima and D measurements of the sum of noise day maxima is available at the SMC and that the CDFs $F^{-1}_{Y^{\text{dB}}_{\text{max},k}}, k = 1, \cdots, K$ have already been estimated as explained in Section 3.4.3. Furthermore, as explained earlier in the section, let us assume that α_k, β_k and $\mu_{Y_{\text{max},k}|F_J^{-1}(1-P^{\text{target}}_{\text{out}})}$, $k = 1, \cdots, K$ have already been estimated. Now with every measurement of the noise day maxima, the SMC can calculate an error term according to (3.51), and consequently, can calculate a realization of $\hat{Y}^{\text{dB}}_{\text{max},k}|F_J^{-1}(1 - P^{\text{target}}_{\text{out}})$ at each tone according to (3.53). From the realizations of $\hat{Y}^{\text{dB}}_{\text{max},k}|\mathcal{J}, k = 1, \cdots, K$, the CDFs $F_{\hat{Y}^{\text{dB}}_{\text{max},k}|F_J^{-1}(1-P^{\text{target}}_{\text{out}})}, k = 1, \cdots, K$ can be estimated, and finally, an approximation of the SNR $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$ derived in Section 3.5.5 can be calculated according to (3.56).

3.6 Performance Analysis

3.6.1 Introduction

In this section, the performance of the proposed approaches is compared to the performance of the state of the art approaches presented in Section 1.3. Furthermore, simulation results will show the benefits of using the proposed approaches.

First, in Section 3.6.2, an overview of the approaches that are being compared is presented. Then, in Section 3.6.3, the simulation parameters are stated. In Section 3.6.4, simulation results are shown.

3.6.2 Overview of Compared Approaches

This section gives an overview on the approaches that are being compared.

- **Optimal Reference Approach.** In [JHC08b, JHC08a], the problem of finding the SNR margin that maximizes the target data rate while satisfying a certain target outage probability was solved optimally. However, in order to be able to use the Optimal Reference Approach, the current DSL standard has to be changed. Moreover, complex bit-swapping algorithms have to be developed.

  In [JHC08b], the "first and last user problem" was solved by setting the SNR margin during initialization in relation to the noise spectrum. If the noise spectrum during initialization is high compared to long-time observations of the noise spectrum, then a small SNR margin is used, and vice versa. In [JHC08b], for the case where the noise perceived by a victim line was assumed to be stationary, the outage probability was given by

$$P_{\text{out}} = \Pr\{\sum_k X_k^{\text{dB}} > \sum_k \gamma_{\text{init}}^{\text{dB}} + x_{\text{init},k}^{\text{dB}}\}, \tag{3.57}$$

  where $\gamma_{\text{init}}^{\text{dB}}$, $x_{\text{init},k}^{\text{dB}}$ and X_k^{dB} are defined in (3.2), (3.5) and (3.12), respectively. As shown in [MFPA09], the assumption of stationarity is not valid in practical DSL systems. In order to satisfy the outage probability defined in Section 3.2, we extend the definition of outage probability in (3.57) for the non-stationary case by expressing the outage probability by the maximum $\sum_k X_k^{\text{dB}}(t)$ over 24 hours, yielding

$$P_{\text{out}} = \Pr\{\max_{\{24\text{h}\}} \left\{ \sum_k X_k^{\text{dB}}(t) \right\} > \sum_k \gamma_{\text{init}}^{\text{dB}} + x_{\text{init},k}^{\text{dB}}\}. \tag{3.58}$$

  By denoting the CDF of $\max_{\{24\text{h}\}} \left\{ \sum_k X_k^{\text{dB}}(t) \right\}$ as $F_{\sum}$, $\gamma_{\text{init,opt}}^{\text{dB}}$ is given by

$$\gamma_{\text{init,opt}}^{\text{dB}} = \frac{F_{\sum}^{-1}(1 - P_{\text{out}}^{\text{target}}) - \sum_k x_{\text{init},k}^{\text{dB}}}{K}. \tag{3.59}$$

  Moreover, the "unequal per-tone noise protection problem" was solved by distributing the remaining SNR margin every time the noise spectrum changes unevenly on the tones such that all tones end up with equal per-tone outage probabilities. Applying this approach with the definition of per-tone outage probability in (3.30), the per-tone SNR margins $\gamma^{\text{dB}}_{\text{opt},k}$ were determined every time the noise spectrum changes in [JHC08a] such that

$$P_{\text{out},k_1}(t) = P_{\text{out},k_2}(t), \quad k_1, k_2 = 1, \cdots K, \tag{3.60}$$

with

$$P_{\text{out},k}(t) = 1 - F_{Y^{\text{dB}}_{\text{max},k}}\left(X^{\text{dB}}_k(t) + \gamma^{\text{dB}}_{\text{opt},k}\right), \tag{3.61}$$

while

$$\sum_k \gamma^{\text{dB}}_{\text{opt},k} = \gamma^{\text{dB}}_{\text{init,opt}} - \sum_k X^{\text{dB}}_k(t) - x^{\text{dB}}_{\text{init},k}. \tag{3.62}$$

It is very important to point out that the Optimal Reference Approach is not compliant with the current DSL standard. Neither setting the SNR margin during initialization in relation to the noise spectrum nor distributing the SNR margin every time the noise spectrum changes unevenly on the tones are possible with the current standard. Moreover, distributing the SNR margin unevenly on the tones every time the noise spectrum changes requires new complex bit-swapping algorithms that will increase the overhead traffic [JHC08a].

- **6 dB SNR Margin Approach**. The value of the SNR margin is set ad-hoc to 6 dB and is independent of the SNR at initialization. Therefore, the state of the art SNR margin is not optimal in terms of data rate and outage probability.

- **Adjusted SNR Margin Approach** As shown in (3.25), when Virtual Noise is used, and in (3.59), when the the Optimal Reference Approach is used, the smaller the target outage probability is, the larger the SNR margin becomes, and hence, the smaller the target data rate becomes. In order to have a fair comparison between the data rate performance of the state of the art SNR margin approach and the other approaches, it is necessary to compare them at the same outage probability.

  The outage probability is dependent on the initialization time when using the state of the art SNR margin approach. Hence, for the Adjusted SNR Margin Approach, the minimum SNR margin that satisfies the target outage probability for a victim line that is initialized at the time with the lowest noise spectrum is determined. The minimum SNR margin that satisfies the target outage probability for a victim line that is initialized at the time with the lowest noise spectrum is chosen because by using this SNR margin when the victim line is initialized at any other time, the outage probability that will result will be smaller than the target outage probability. Hence, if the target outage probability is satisfied with the SNR margin when a victim line is initialized at the time with the lowest noise spectrum, it is definitely satisfied with the same SNR margin when the victim line is initialized at any other time.

By defining the lowest noise spectrum $x_{\min,k}^{\text{dB}} \quad k = 1, \cdots, K$ as the the noise spectrum that satisfies $\sum_k x_{\min,k}^{\text{dB}} = \min\limits_{\{24\text{h}\}} \left\{\sum_k X_k^{\text{dB}}(t)\right\}$, the minimum SNR margin that guarantees an outage probability smaller than or equal to the target outage probability for the victim line regardless of the initialization time is given by

$$\gamma_{\text{init,adj}}^{\text{dB}} = \frac{F_{\sum}^{-1}(1 - P_{\text{out}}^{\text{target}}) - \sum_k x_{\min,k}^{\text{dB}}}{K}. \tag{3.63}$$

Note that the SNR margin of the Adjusted SNR Margin Approach is set such that the target outage probability is satisfied when the victim line is initialized at the time with the lowest noise spectrum. Thus, when the victim line is actually initialized at the time with the lowest noise spectrum, the achieved target data rate and outage probability will be equal to those achieved by the Optimal Reference Approach. However, when initialized at any other time, the SNR margin will be too large and achievable data rate is lost since the victim line will be operating at a target outage probability below the target outage probability.

- **Trivial Virtual Noise Approach**. In [VB05] it was suggested that the VN mask should be set to the worst case noise mask and thus, to the maximum noise power that can occur. By defining $x_{\max,k}^{\text{dB}}$ as the maximum noise power that can occur on tone k, the VN mask is given by

$$VN_k^{\text{tr dB}} = x_{\max,k}^{\text{dB}}, \ k = 1, \cdots, K. \tag{3.64}$$

In that case, there is no need for an SNR margin and it is set therefore to zero.

- **Proposed Virtual Noise Approach**. In Section 3.4.3, only the equal SNR margin that would result after bit-swapping has equalized the per-tone SNR margins was considered in the optimization of the VN mask and SNR margin. Furthermore, the SNR margin and the VN mask were jointly optimized such that the overall data rate is maximized while a certain target outage probability is guaranteed. The SNR margin $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$ in (3.25) and the VN mask $VN_k^{\text{lt dB}}, \ k = 1, \cdots K$ in (3.22) were derived. In this section, when $\gamma_{\text{VNinit-bl}}^{\text{lt dB}}$ and $VN_k^{\text{lt dB}}, \ k = 1, \cdots K$ are used, the approach is denoted **Long-term Stability (LTS) Virtual Noise Approach**.

In Section 3.5, the SNR margin $\gamma_{\text{VNinit-bl}}^{\text{st dB}}$ in Algorithm 2 and the VN mask $VN_k^{\text{st dB}},\ k = 1, \cdots K$ in (3.47) were derived such that the per-tone outage probabilities are equalized at initialization and such that the maximum per-tone deviation of the per-tone outage probability during showtime from the equalized per-tone outage probability during initialization is minimized while the achieved data rate and satisfied target outage probability in Section 3.4.3 are maintained. In this section, when $\gamma_{\text{VNinit-bl}}^{\text{st dB}}$ and $VN_k^{\text{st dB}},\ k = 1, \cdots K$ are used, the approach is denoted **Song-term Stability (STS) Virtual Noise Approach**. Furthermore, when $\gamma_{\text{VNinit-bl}}^{\text{st dB}}$ is approximated by the SNR margin $\tilde{\gamma}_{\text{VNinit-bl}}^{\text{st dB}}$ in (3.56), as shown in Section 3.5.6, the approach is denoted **STS Approximated Virtual Noise Approach**

The SNR margins and VN masks used by all the compared approaches are summarized in Table 3.1.

Approach	SNR margin at initialization	Virtual noise mask
Optimal Reference	$\gamma_{\text{init,opt}}^{\text{dB}} = \frac{F_{\sum}^{-1}(1-P_{\text{out}}^{\text{target}})-\sum_k x_{\text{init},k}^{\text{dB}}}{K}$	–
6 dB SNR Margin	$\gamma_{\text{init,opt}}^{\text{dB}} = 6$ dB	–
Adjusted SNR Margin	$\gamma_{\text{init,adj}}^{\text{dB}} = \frac{F_{\sum}^{-1}(1-P_{\text{out}}^{\text{target}})-\sum_k x_{\text{min},k}^{\text{dB}}}{K}$	–
Trivial VN	–	$VN_k^{\text{tr dB}} = x_{\text{max},k}^{\text{dB}},\ k = 1,\cdots,K$
LTS VN	$\gamma_{\text{VNinit-bl}}^{\text{lt dB}} = \frac{F_J^{-1}(1-P_{\text{out}}^{\text{target}})-\sum_k VN_k^{\text{lt dB}}}{K}$	$VN_k^{\text{lt dB}} = F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(P_{0,1}),\ k = 1,\cdots,K$
STS VN	$\gamma_{\text{VNinit-bl}}^{\text{st dB}}$: Iteratively according to Algorithm 2	$VN_k^{\text{st dB}} = F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(1-P_{\text{out-pt}}^{\text{eq}}) - \gamma_{\text{VNinit-bl}}^{\text{st dB}},\ k = 1,\cdots,K$
STS Approximated VN	$\tilde{\gamma}_{\text{VNinit-bl}}^{\text{st dB}} = \max_{\{k_1,\cdots,k_K\}} \left\{ F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(1-P_{\text{out-pt}}^{\text{eq}}) - F_{\hat{Y}_{\text{max},k}^{\text{dB}} \mid F_J^{-1}(1-P_{\text{out}}^{\text{target}})}^{-1}(P_{0,1}) \right\}$	$\tilde{V}N_k^{\text{st dB}} = F_{Y_{\text{max},k}^{\text{dB}}}^{-1}(1-P_{\text{out-pt}}^{\text{eq}}) - \tilde{\gamma}_{\text{VNinit-bl}}^{\text{st dB}},\ k = 1,\cdots,K$

Table 3.1: Overview of the SNR margins and VN masks used by the approaches stated in Section 3.6.2

3.6.3 Simulation Parameters

For the following simulations, the transmission and noise models presented in Chapter 2 are assumed. Furthermore, noise measurements of 10^5 days were generated with the noise model, emulating the measurements of the noise that occurs in practical DSL systems. All the values of the inverse CDFs in Table 3.1 were determined empirically from the generated measurements. Moreover, the parameters in Table 3.2 are assumed. The values for the Symbol rate f_s and the number K of downstream tones parameters in Table 3.2 are typical values for VDSL2 systems. Moreover, the values for the SNR gap and the coding gain are typical approximations used when simulating DSL systems [WKK12, Jag08, Cio91].

Table 3.2: System parameters

Symbol rate f_s	4kHz
Number K of downstream tones	2800
SNR gap Γ	9.8 dB
Coding gain γ_c^{dB}	3 dB

3.6.4 Simulation Results

3.6.4.1 Data rate and Outage probability

In this section, we show results concerning the data rate and outage probability performance of the approaches stated in Section 3.6.2. The results of the proposed VN approaches are obtained by solving the long-term stability problem where an equal SNR margin that would result after bit-swapping has equalized the per-tone SNR margins every time the channel changes is assumed. Since the LTS VN Approach, the STS VN Approach and the STS Approximated VN Approach solve the long-term stability problem and since in this section the results only concern data rate and outage probability that are obtained by solving the long-term stability problem, the notation "Proposed Virtual Noise Approach" encloses the LTS VN Approach, the STS VN Approach and the STS

Approximated VN Approach.

As a first result, in Figure 3.2, we show the trade-off between target outage probability and target data-rate when using the approaches stated in Section 3.6.2 in a scenario where L2 mode is not implemented.

According to equations (3.2) and (3.3), when using an SNR margin, the target data rate is dependent on the SNR at initialization and is therefore, dependent on the initialization time. In order to compare the data rate performance of the SNR margin approaches with VN approaches that achieve a target data rate regardless of the initialization time, the target data rate results of the SNR margin approaches are average of the target data rates achieved on the victim line when initialized at different times.

For the Proposed VN Approach and the Optimal Reference Approach, the x-axis describes the target data rate that can be achieved such that the target outage probability on the y-axis can be satisfied. Furthermore, for the Adjusted SNR Margin Approach, the x-axis describes the average target data rate that can be achieved on the victim line when initialized at different times while the target outage probability on the y-axis can be satisfied even when the victim line is initialized at the time with the lowest noise spectrum.

The average target data rate in Figure 3.2 achieved by the 6 dB SNR Margin Approach, illustrated in by the dashed red line, and the target data rate achieved by the Trivial VN Approach illustrated by the dashed brown line, are not functions of the target outage probability, thus, the dashed red line and the dashed brown line do not show trade-offs between outage probability and target data rate and serve as reference data rates. In order to illustrate in Figure 3.2 that the data rates achieved by both approaches are not functions of the target outage probability on the y-axis, the data rates are illustrated by dashed lines.

A target outage probability of 1 in Figure 3.2 indicates that even after equalizing the SNR margin every time the channel varies, the bit-error rate on the victim will exceed the target bit error rate at least one time during 24 hours. An outage probability of 0 indicates that after equalizing the SNR margin every time the channel varies, the bit-error rate on the victim line will never exceed the target bit error rate during 24 hours. Values between 0 and 1 on the y-axis are the outage probabilities that can be satisfied for the data rates on the y-axis while assuming the equal SNR margin that would result after bit-swapping has equalized the per-tone SNR margins every time the channel changes.

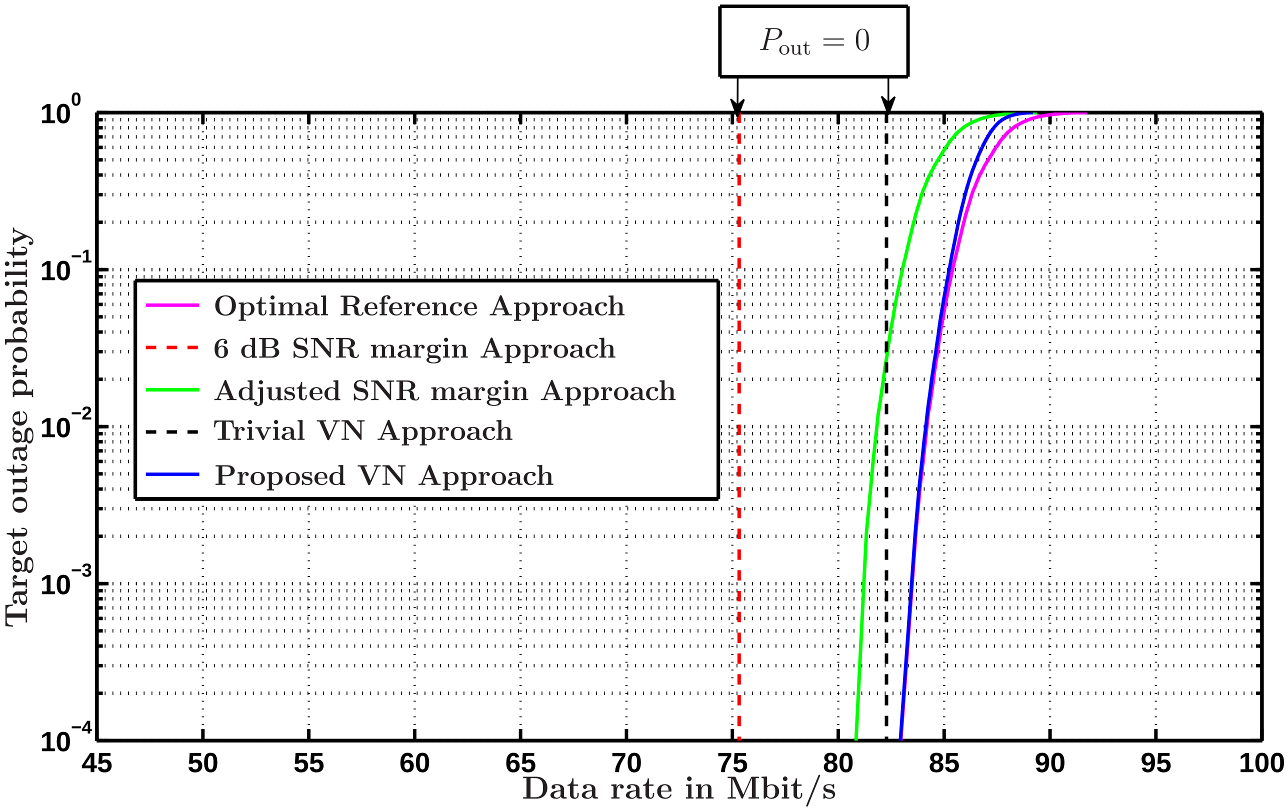


Figure 3.2: Trade-off between data rate and outage probability in a scenario where L2 mode is not implemented. Note that the dashed lines of the State of art 6 dB SNR margin and the Trivial VN Approach do not show trade-offs between outage probability and target data rate and serve as reference data rates

The plots in Figure 3.2 can be used for determining the service points for each line in the network. When an SNR margin approach is used, a service point is the average data rate that can be achieved on the victim line when initialized at different times while a certain target outage probability is satisfied even when the victim line is initialized at the time with the lowest noise spectrum. Similarly, when a VN approach is used, a service point is the target data rate that can be achieved on the victim line while a certain target outage probability is satisfied.

The green curve shows the service points achieved by the Adjusted SNR Margin Approach. When a fixed SNR margin is used, any service point left of the green curve indicates that the used SNR margin is too large and that the outage probability of the victim line when initialized at the time with lowest noise spectrum is smaller than the target outage probability, and therefore, achievable data rate is wasted. Similarly, any service point right of the green curve indicates that the used SNR margin is too small and that the outage probability of the victim line when initialized at the time with lowest noise spectrum is larger than the target outage probability. Obviously, the 6 dB SNR margin is too large for this scenario and the outage probability is therefore equal to 0 as depicted in Figure 3.2.

The Optimal Reference Approach sets the SNR margin in relation to the noise spectrum during initialization as shown in Section 3.6.2. This explains the better performance compared to the SNR margin approaches that use the same SNR margin regardless of the noise spectrum during initialization.

When the Trivial VN Approach is used, the VN mask is given by a worst case noise mask. Hence, the outage probability is always equal to 0 as depicted in Figure 3.2. The blue curve represents the service points that can be achieved when the Proposed VN Approach is used. Any service points left of the blue curve indicate that achievable data rate is wasted and any service point right of the blue curve indicates that the target outage probability is not satisfied. Obviously, the Proposed VN Approach outperforms the Trivial VN Approach .

The Proposed VN Approach sets the VN mask and the SNR margin such the target data rate is maximized while the target outage probability is satisfied even if the noise day maxima at all tones occur at the same time. In practical DSL systems, the noise day maxima do not always occur at the exact same time. This explains the difference in performance between the Proposed VN Approach and the Optimal Reference Approach. Furthermore, the smaller the target outage probability is, the smaller is the difference between the target data rate achieved by the Proposed VN Approach and the target data rates achieved by the Optimal Reference Approach since the target data rates achieved by both approaches converge with a decreasing target outage probability towards the target data rate achieved by the Trivial VN Approach that assumes a worst case. For example, at the target outage probability of $7.3 \cdot 10^{-3}$, which is a typical value in practical DSL systems [SKK12], the difference between the target data rate achieved by the Proposed VN Approach and the target data rates achieved by the Optimal Reference Approach is roughly 0.1% of the target data rate achieved by the Proposed VN Approach at this target outage probability.

In Figure 3.3, the data rate and outage probability performance of the approaches stated in Section 3.6.2 is compared in a scenario where L2 mode is implemented.

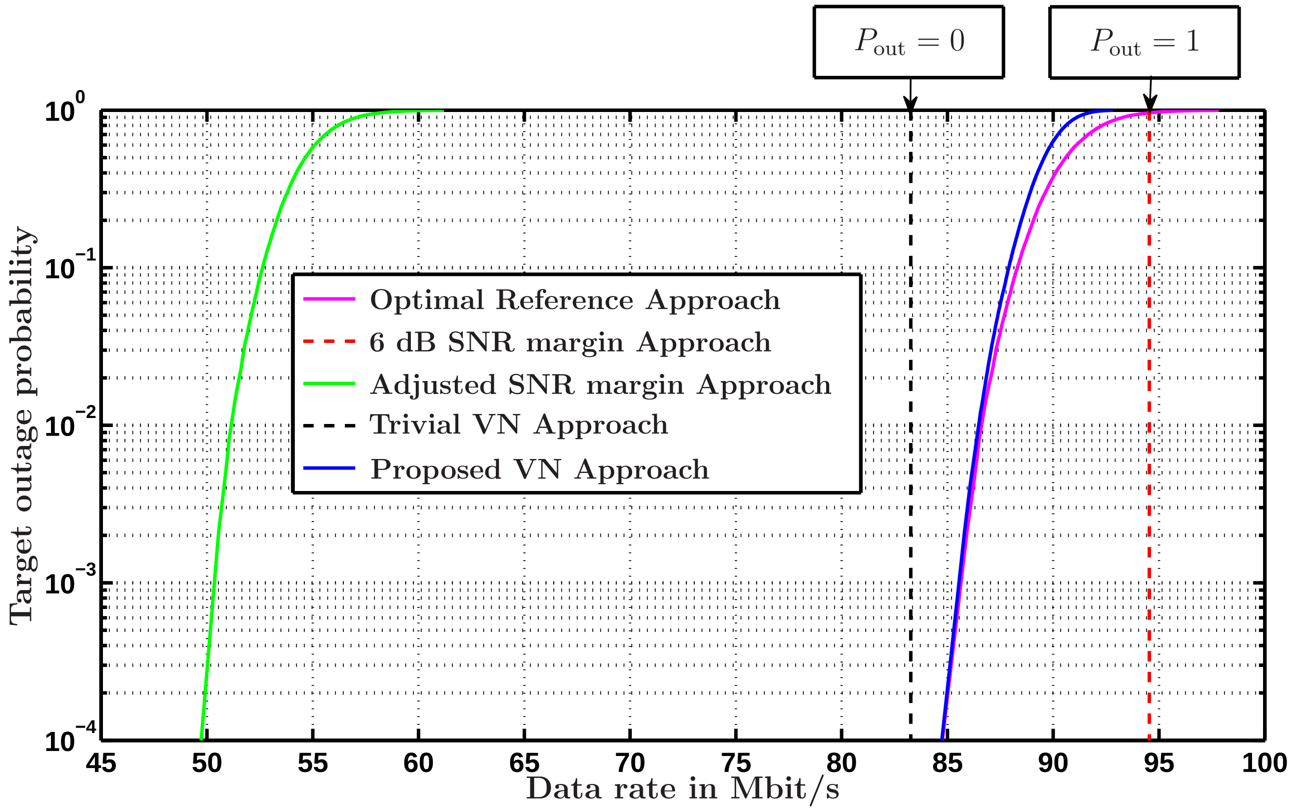


Figure 3.3: Trade-off between data rate and outage probability in a scenario where L2 mode is implemented. Note that the dashed lines of the State of art 6 dB SNR margin and the Trivial VN Approach do not show trade-offs between outage probability and target data rate and serve as reference data rates

As already explained in Section 2, when L2 mode is implemented, the variance of FEXT increases tremendously. Thus, the average data rate achieved by the state of the art 6 dB SNR margin approach is now right from the green curve, and hence, the 6 dB SNR margin fails to protect the victim line from the varying FEXT and the resulting outage probability is therefore equal to 1 as depicted in Figure 3.3. Moreover, the average data rate achieved by the Adjusted SNR Margin Approach is decreased tremendously when compared to Figure 3.2. This is explained by the huge SNR margin that is necessary to protect from the time varying FEXT. This result is the reason why L2 mode is abandoned from the current VDSL2 standard.

The target data rate achieved by the Proposed VN Approach is increased when compared to Figure 3.2. When L2 mode is implemented, the FEXT received on a victim line from an interferer that is in the L2 state is much smaller than the received FEXT when the interferer is transmitting with the regular transmit PSD. Thus, when L2 mode is implemented, the received FEXT is always smaller than or equal to the received FEXT when L2 mode is not implemented. This explains the increase of the target data rate when compared to Figure 3.2. Moreover, this result proves that, in contrast to the state of the art SNR margin, the Proposed VN Approach makes the implementation of the L2 mode in VDSL2 systems beneficial.

The difference in the data rate performance between the Proposed VN Approach and the Optimal Reference Approach is increased when compared to Figure 3.2. This is explained by the increased FEXT variance. The Optimal Reference Approach protects the victim line with a certain outage probability from the highest noise spectrum, given according to (3.58) by $\max_{\{24\mathrm{h}\}} \left\{ \sum_k X_k^{\mathrm{dB}}(t) \right\}$. When L2 mode is implemented, the difference between the actual noise power at the time of the highest noise spectrum and the maximum noise power experienced over 24 hours, assumed by the Proposed VN Approach, at the tones that are not experiencing their maximum noise power is due to the increased FEXT variance more likely to be larger than when L2 mode is not implemented. The difference in the achieved data rate between the Proposed VN Approach and the Optimal Reference Approach is, however, still very small. For example, at the target outage probability of $7.3 \cdot 10^{-3}$ the difference in data rate is roughly 0.2% of the target data rate achieved by the Proposed VN Approach at this target outage probability.

In conclusion, the Proposed VN Approach calculates, in contrast to the Trivial VN Approach , the target data rate as a function of the desired target outage probability and achieves therefore higher target data rates. Moreover, the Proposed VN Approach solves the "first and last user problem" and increases therefore the achievable target data rate when compared to the Adjusted SNR Margin Approach. This increase is tremendous in scenarios where L2 mode is implemented. Furthermore, the performance of the Proposed VN Approach is very close to the Optimal Reference Approach. However, in contrast to the Optimal Reference Approach, the Proposed VN Approach can be realized with the current VDSL2 standard.

3.6.4.2 Per-tone Outage Probabilities

In this section, the per-tone outage probabilities achieved with the SNR margin approaches, the VN approaches and the Optimum Reference Approach are compared. In order to compare results of the per-tone outage probabilities belonging to different approaches, the approaches must be operating at the same target outage probability. In the following results, the target outage probability of $7.3 \cdot 10^{-3}$ is chosen, which is a typical value in practical DSL systems [SKK12]. As shown in the previously in Section 3.6.4.1, the target outage probability of $7.3 \cdot 10^{-3}$ cannot be realized by the 6 dB SNR Margin Approach and by the Trivial VN Approach. Furthermore, as explained in Section 3.6.2, when the Adjusted SNR Margin Approach is used and the victim line is initialized at any time but the time with the lowest noise spectrum, the victim line will be operating at a target outage probability below the target outage probability. In order to be able to compare the Adjusted SNR Margin Approach with the other approaches, in this section we assume that the victim line is always initialized at the time with the lowest noise spectrum. Note that since the Optimal reference Approach and the VN approaches are able to calculate a target data rate that is independent of the initialization time, this assumption does not affect the target data rate and outage probability performance of the Optimal Reference Approach, the LTS VN Approach, the STS VN Approach and the STS Approximated VN Approach.

With this assumption and for the target outage probability of $7.3 \cdot 10^{-3}$, the SNR margins and VN masks used by the Optimal Reference Approach, the Adjusted SNR Margin Approach, the LTS VN Approach, the STS VN Approach and the Approximated STS VN Approach are given in Table 4.3 for a scenario where L2 mode is not implemented and in Table 4.4 for a scenario where L2 mode is implemented.

Obviously, due to the increased FEXT variation when L2 mode is implemented, the SNR margin required by the Optimal Reference Approach and the Adjusted SNR Margin Approach at the outage probability of $7.3 \cdot 10^{-3}$ increase tremendously when compared to SNR margins required at the same outage probability in a scenario where L2 mode is not implemented.

The SNR margin used by the VN approach is calculated from noise day maxima. Furthermore, noise day maxima are more likely to occur at times when neighboring lines are not operated in L2 mode, and are therefore not strongly affected by the implementation

Approach	**SNR margin at initialization**	**Virtual noise mask**
Optimal Reference	$\gamma_{\text{init,opt}}^{\text{dB}} = 4.2$	–
Adjusted SNR margin	$\gamma_{\text{init,adj}}^{\text{dB}} = 4.2$	–
LTS VN	$\gamma_{\text{VNinit-bl}}^{\text{lt dB}} = 1.6$	$VN_k^{\text{lt dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(P_{0,1}),\ k = 1, \cdots, K$
STS VN	$\gamma_{\text{VNinit-bl}}^{\text{st dB}} = 0.7$	$VN_k^{\text{st dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - 1.72 \cdot 10^{-2}) - 0.7,\ \ k = 1, \cdots, K$
STS approximated VN	$\tilde{\gamma}_{\text{VNinit-bl}}^{\text{st dB}} = 1.2$	$\tilde{V}N_k^{\text{st dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - 1.72 \cdot 10^{-2}) - 1.2,\ \ k = 1, \cdots, K$

Table 3.3: SNR margins and VN masks at a target outage probability of $7.3 \cdot 10^{-3}$ for a victim line when initialized at the time with the lowest noise spectrum in a scenario where L2 mode is not implemented.

Approach	**SNR margin at initialization**	**Virtual noise mask**
Optimal Reference	$\gamma_{\text{init,opt}}^{\text{dB}} = 18.6$	–
Adjusted SNR margin	$\gamma_{\text{init,adj}}^{\text{dB}} = 18.6$	–
LTS VN	$\gamma_{\text{VNinit-bl}}^{\text{lt dB}} = 2.3$	$VN_k^{\text{lt dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(P_{0,1}),\ k = 1, \cdots, K$
STS VN	$\gamma_{\text{VNinit-bl}}^{\text{st dB}} = 1.1$	$VN_k^{\text{st dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - 1.98 \cdot 10^{-2}) - 1.1,\ \ k = 1, \cdots, K$
STS approximated VN	$\tilde{\gamma}_{\text{VNinit-bl}}^{\text{st dB}} = 1.7$	$\tilde{V}N_k^{\text{st dB}} = F^{-1}_{Y^{\text{dB}}_{\max,k}}(1 - 1.98 \cdot 10^{-2}) - 1.7,\ \ k = 1, \cdots, K$

Table 3.4: SNR margins and VN masks at a target outage probability of $7.3 \cdot 10^{-3}$ for a victim line when initialized at the time with the lowest noise spectrum in a scenario where L2 mode is implemented

of the L2 mode. Thus, the SNR margin required by the VN approaches are not as strongly affected by the implementation of the L2 mode as the SNR margins required by the Optimal Reference Approach and the Adjusted SNR Margin Approach.

The SNR margin $\tilde{\gamma}^{\text{st dB}}_{\text{VNinit-bl}}$ of the Approximated STS VN Approach, given in (3.56), is larger than the SNR margins $\gamma^{\text{st dB}}_{\text{VNinit-bl}}$ of the STS VN Approach, given in Algorithm 2, in both scenarios, where L2 mode is not implemented and when L2 mode is implemented. In the derivation of $\tilde{\gamma}^{\text{st dB}}_{\text{VNinit-bl}}$ in Section 3.5.6, the relationship between the sum of noise day maxima in dB scale, expressed by the random variable $J = \sum_k Y^{\text{dB}}_{\text{max},k}$ defined before (3.23), and each of the noise day maxima, expressed by the random variables $Y^{\text{dB}}_{\text{max},k}$ $k = 1 \cdots K$ defined before (3.18), was assumed to be linear. Figure 3.4 shows simulations of the correlation coefficients between the random variable J and each of the random variables $Y^{\text{dB}}_{\text{max},k}$ $k = 1 \cdots K$. In the upper figure, for a scenario where L2 mode is not implemented and in the lower one, for a scenario where L2 mode is implemented. The correlation coefficients are given for all the 2800 tones of the downstream band. At the tones where the correlation coefficient is low, the relationship between the random variables is not linear and therefore, the use of the linear regression in (3.50) will lead to larger error terms in (3.51), and consequently, to larger SNR margins in (3.56). Furthermore, the stronger the correlation between the random variable J and each of the random variables $Y^{\text{dB}}_{\text{max},k}$ $k = 1 \cdots K$, the better is the approximation of the SNR margin, and vice versa. In the following, the effect of the approximation error on the per-tone outage probabilities is shown.

Figures 3.5 and 3.6 show the per-tone outage probabilities at initialization when using the Optimal Reference Approach, the Adjusted SNR Margin Approach, the LTS VN Approach, the STS VN Approach and the Approximated STS VN Approach for a scenario where L2 mode is not implemented and for a scenario where L2 mode is implemented, respectively. The per-tone outage probabilities are illustrated on the y-axis for the tone indices on the x-axis. A per-tone outage probability of 1 for a tone in Figures 3.5 and 3.6 indicates that given the bit-loading that will lead to an equal SNR margin for the noise power at initialization, the bit-error rate will exceed the target bit-error rate at that tone at least one time during 24 hours. Moreover, a probability of 0 indicates that the bit-error rate will never exceed the target bit-error rate at that tone during 24 hours. Note that the missing per-tone outage probabilities in Figures 3.5 and 3.6 are equal to 0.

The Adjusted SNR Margin Approach adds the SNR margin given in Table 3.3,

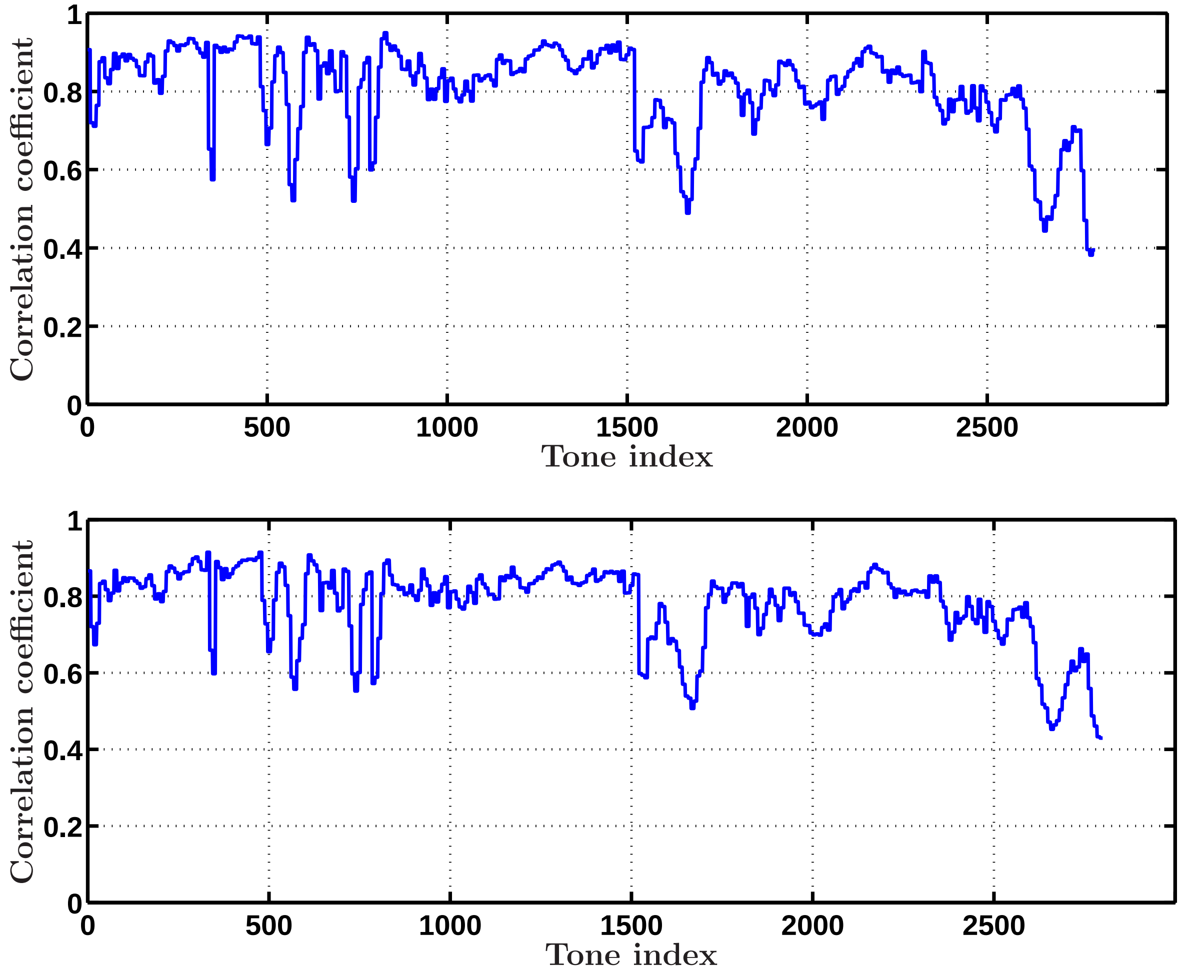


Figure 3.4: Correlation coefficients between the random variable J and each of the random variables $Y^{\mathrm{dB}}_{\max,k}$ $k = 1 \cdots K$. Upper figure: scenario where L2 mode is not implemented, lower figure: scenario where L2 mode is implemented.

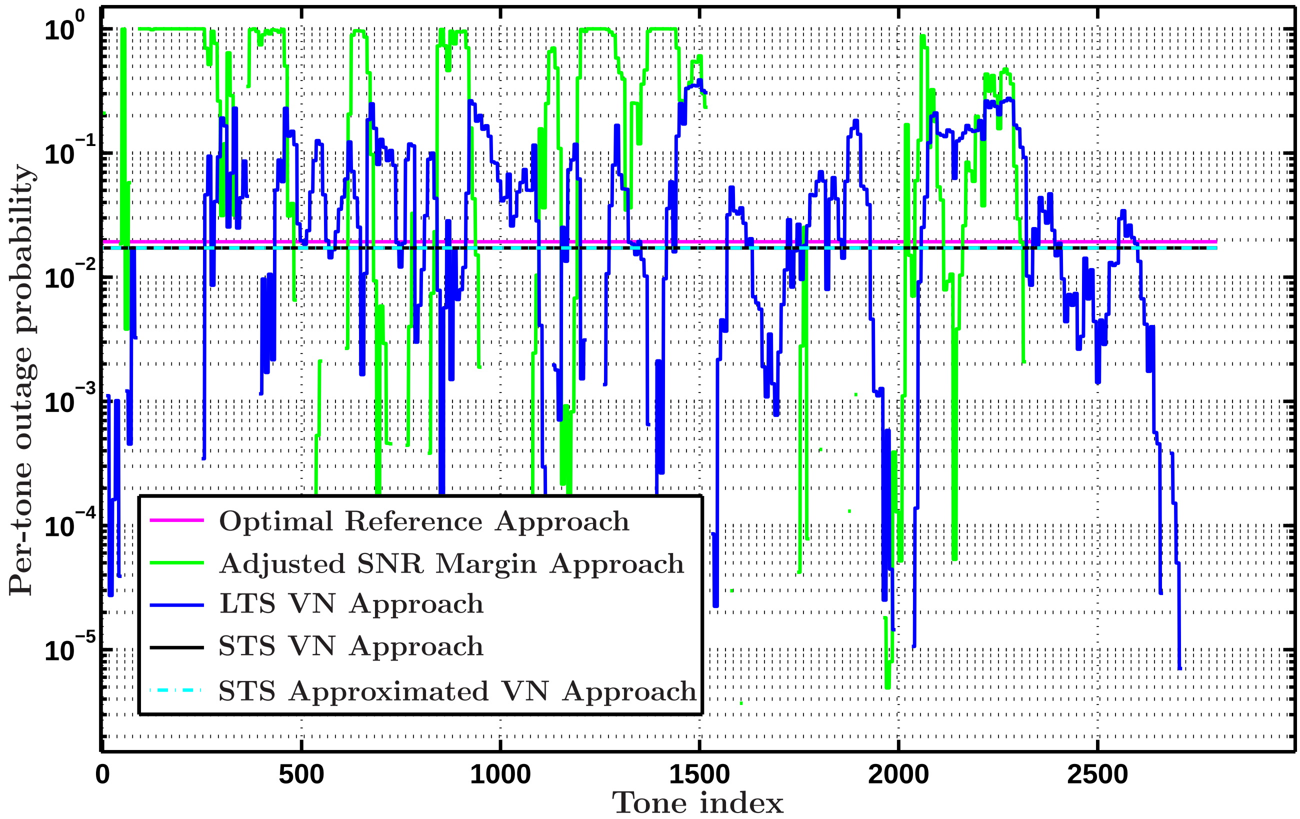


Figure 3.5: Per-tone outage probabilities at initialization in a scenario where L2 mode is not implemented.

when L2 mode is not implemented, and the SNR margin given in Table 3.4, when L2 mode is implemented, to the noise measured during initialization when calculating the bit-loading. The noise spectrum during initialization is only a snapshot in time, and therefore, can be unevenly distributed between tones. Since the SNR margin used by the Adjusted SNR Margin Approach is equal for all tones, the resulting per-tone outage probabilities are not equal and vary from one tone to another. Furthermore, as shown in Tables 3.3 and 3.4, the Optimal Reference Approach, when initialized at the time with the lowest noise spectrum, uses the same SNR margin as the Adjusted SNR Margin Approach. However, the Optimal Reference Approach redistributes the SNR margin unequally on the tones such that the per-tone outage probabilities are equalized as shown in Figures 3.5 and 3.6.

When L2 mode is not implemented, the LTS VN Approach adds the SNR margin

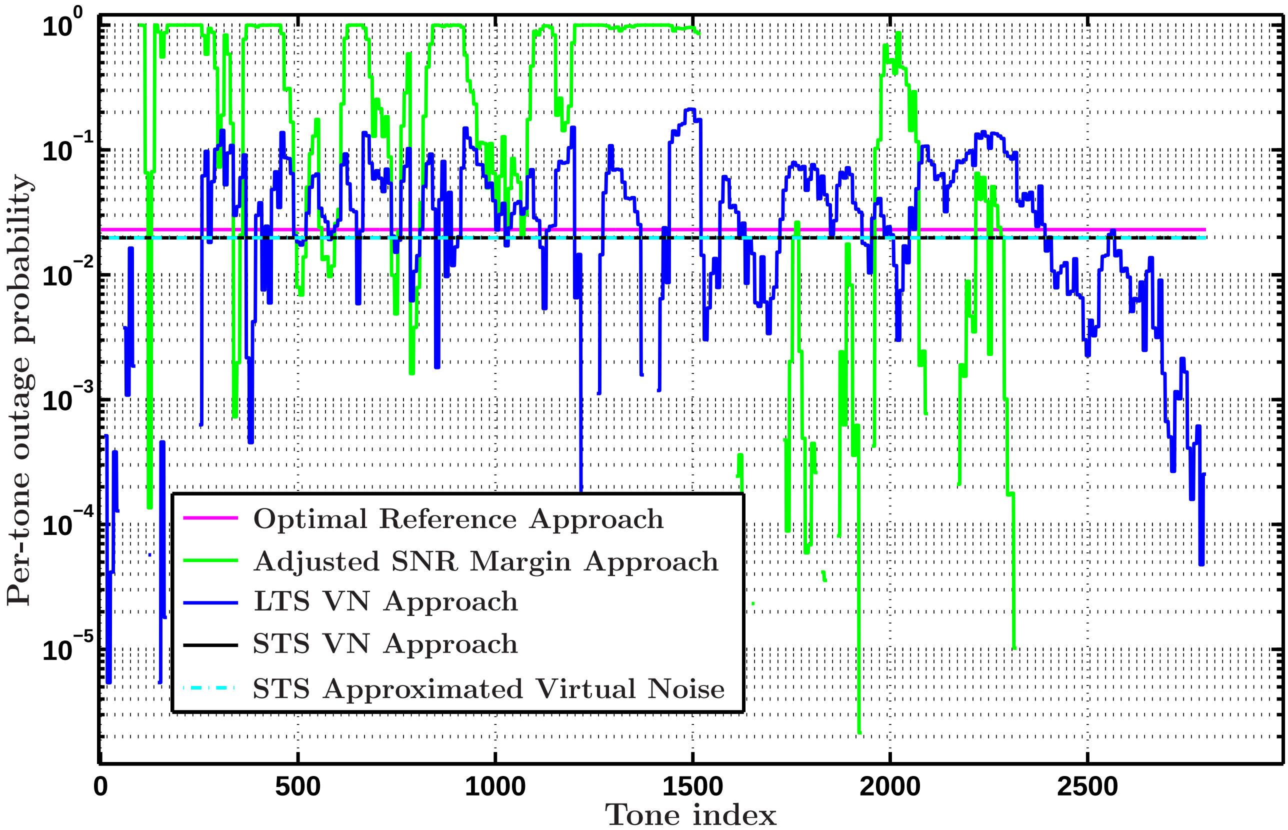


Figure 3.6: Per-tone outage probabilities at initialization in a scenario where L2 mode is implemented.

given in Table 3.3 to the VN mask given in the same table, and, when L2 mode is not implemented, the SNR margin given in Table 3.4 to the VN mask given in the same table, when calculating the bit-loading during initialization. The VN mask provides a minimum per-tone noise protection and the variation of the per-tone outage probabilities is therefore decreased when compared to per-tone outage probabilities achieved by the Adjusted SNR Margin Approach.

The SNR margins and VN masks used by the STS VN Approach and the Approximated STS VN Approach and given in Table 3.3, when L2 mode is not implemented, and Table 3.4, when L2 mode is not implemented, have been modified such that the resulting bit-loading leads to equal per-tone outage probabilities at least as long as the noise spectrum is below the VN mask. Since in Figures 3.5 and 3.6, it is assumed that the victim line is initialized at the time with the lowest noise spectrum, the VN masks used by both approaches are larger than the lowest noise spectrum at every tone. Hence, the resulting per-tone outage probabilities achieved by both approaches are equal for all tones, as shown in Figures 3.5 and 3.6.

As shown in the previous Section 3.6.4.1, the data rate achieved by the Optimal Reference Approach is slightly higher than the data rate achieved by any of the VN approaches. This small difference in the achieved data rate explains the small difference in the equal per-tone outage probability achieved by the Optimal Reference Approach and by the STS VN Approach and the Approximated STS VN Approach. The STS VN Approach and the Approximated STS VN Approach provide protection for less bits per symbol than the Optimal Reference Approach and can therefore achieve smaller per-tone outage probabilities. The difference, however, is very small.

In Figures 3.5 and 3.6, the per-tone outage probabilities were compared given the noise spectrum during initialization. For the same victim line, Figures 3.7 and 3.7 show the maximum per-tone outage probability over 24 hours, when L2 mode is not implemented and when L2 mode is implemented, respectively. The x-axis represents the time of the day and the y-axis represents the average of the maximum per-tone outage probability at each point in time during the day.

Obviously, the Adjusted SNR Margin Approach shows the highest maximum per-tone outage probabilities and relies therefore the most on bit-swapping to move bits away from the tones that are experiencing negative per-tone SNR margins. Furthermore, the Optimal Reference Approach achieves much lower maximum per-tone outage probabilities than the Adjusted SNR Margin Approach by equalizing the per-tone outage probabilities every time the channel changes.

The VN mask of the LTS VN Approach provides a minimum per-tone noise protection and decreases therefore the maximum per-tone outage probabilities when compared to the Adjusted SNR Margin Approach. However, the maximum per-tone outage probabilities are much higher than those of the Optimal Reference Approach. Moreover, by equalizing the per-tone outage probabilities at initialization and minimizing the SNR margin, the STS VN Approach and the STS Approximated VN Approach achieve much lower maximum per-tone outage probabilities than the LTS VN Approach. Furthermore, at the early and the late hours of the day where fewer neighboring lines are active compared to peak hours, the noise power experienced by the victim line is smaller than the VN masks of STS VN Approach and the STS Approximated VN Approach, hence the maximum per-tone outage probability is the same as the equal per-tone outage probability shown in Figures 3.5 and 3.6. At peak hours, the noise power experienced by the victim line exceeds the VN masks and the remaining SNR margin is distributed equally among all tones, which leads to unequal per-tone outage probabilities, and therefore, to an increase of the maximum per-tone outage probability. Furthermore, the amount of noise power exceeding the VN mask and distributed equally among tones increases the larger the SNR margin is, which explains the difference in the maximum per-tone outage probabilities between the STS VN Approach and the STS Approximated VN Approach.

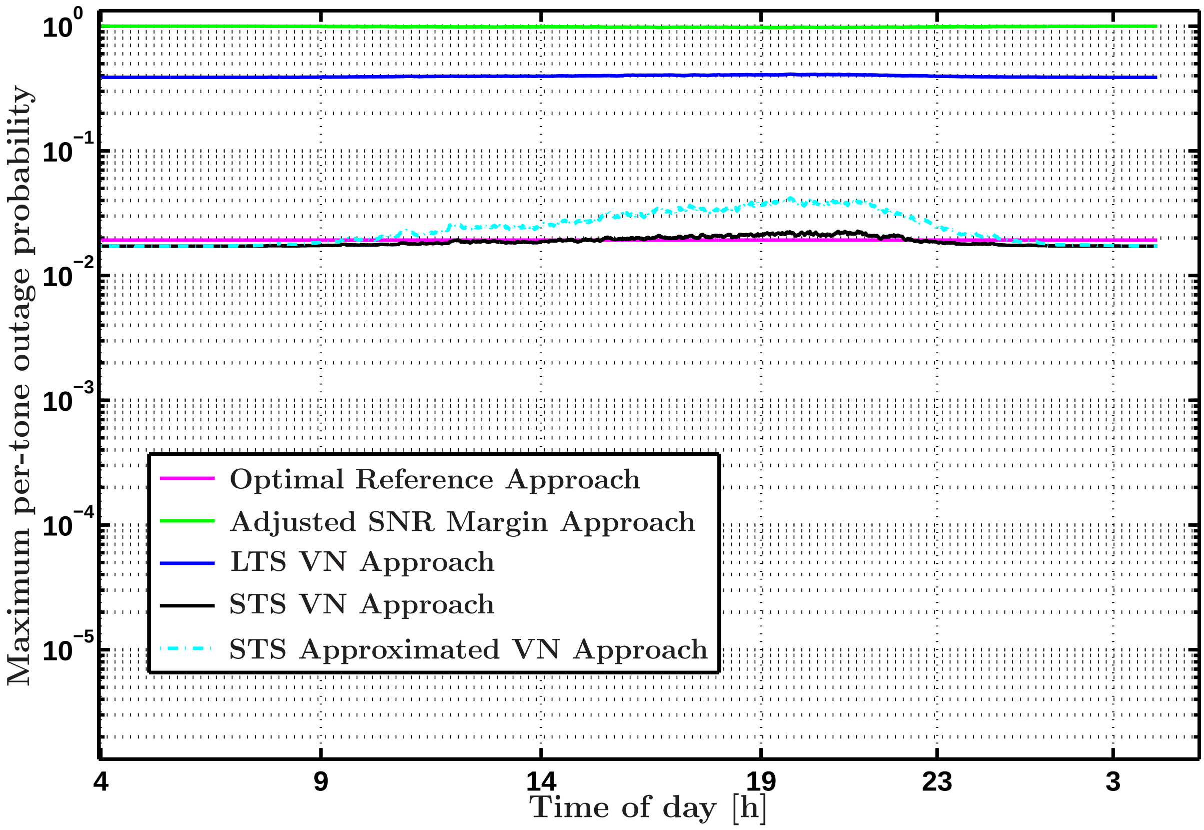


Figure 3.7: Maximum per-tone outage probability over 24 hours in a scenario where L2 mode is not implemented.

In conclusion, the Adjusted SNR margin Approach fails to prevent the line from going

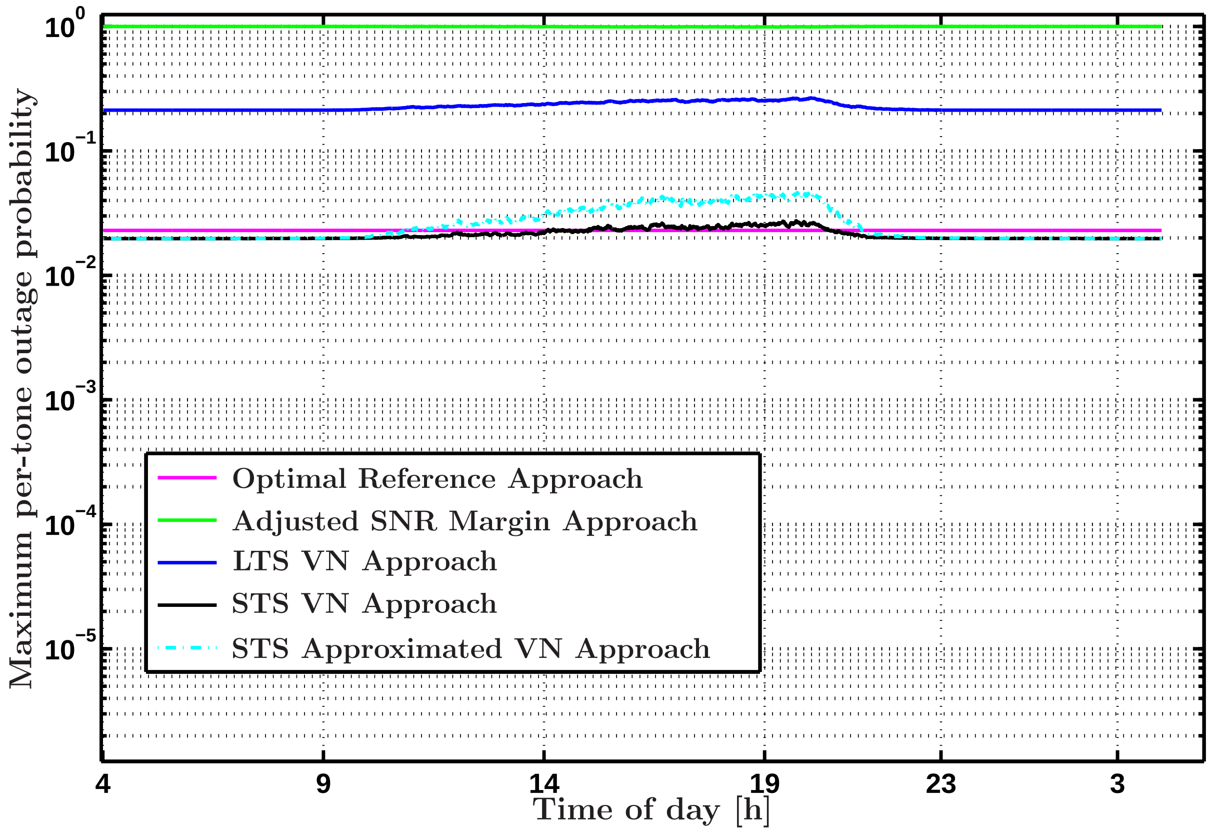


Figure 3.8: Maximum per-tone outage probability over 24 hours in a scenario where L2 mode is implemented.

to the unwanted state where some tones have negative margins. The optimal Reference Approach redistributes the same SNR margin used by the Adjusted SNR Margin Approach unequally among the tones, and therefore, the probability of the line going to the state with negative per-tone SNR margins is reduced. The optimal Reference Approach is however not standard compliant and redistributing the SNR margin unequally among tones will require new bit-swapping algorithms with increased overhead. All the presented VN approaches in this section outperform the Adjusted SNR Margin Approach and achieve smaller per-tone outage probabilities, and therefore, improve the per-tone noise protection. Furthermore, by modifying the VN mask and SNR margin of the LTS VN Approach, the probability of the line going to a state with negative per-tone SNR margins of the STS and STS Approximated VN Approaches is smaller than the one of the LTS VN Approach, and is closer to the one achieved by the optimal Reference Approach. Moreover, by approximating the SNR margin used by the STS VN Approach, the STS Approximated VN Approach has higher per-tone outage probabilities compared to the STS VN Approach. However, the difference in performance is small. In addition to that, the VN mask and SNR margin used by the STS Approximated VN Approach can be easily estimated from measurements of noise day maxima. Moreover, the STS Approximated VN Approach is compliant with the current VDSL2 standard. Thus, the STS Approximated VN Approach is a good candidate to be used for the protection against time-varying noise in today's DSL systems.

Chapter 4

Adaptive Data Rate Approaches for Improved Protection against Time-varying Noise

4.1 Introduction

This Chapter addresses the optimization of adaptive data rate approaches. As explained in Section 1.3.3, the current DSL standard [IT11] allows changing the target data rate during operation using two procedures, SRA and SOS. SRA adapts the bit-loading of the used tones to the varying SNR in a seamless manner. However, in case of large abrupt changes in noise, multiple consecutive adaptation procedures will have to be executed, and thus, the state of the art SRA procedure will require a long time to adapt to the SNR and it is very likely that the connection will be interrupted and that modems will have to reinitialize. SOS reacts to a persistent degradation of the SNR by rapidly performing group-wise bit-loading reduction where the bit reduction is constant for every group. However, SOS is not constrained by the amount of data rate that can be changed by a procedure and the adaptation is therefore not done in a seamless manner. In this Chapter, new approaches for seamless adaptation of the data rate to the varying noise are presented. The presented approaches show significant improvement in performance when compared to the state of the art ones.

First, in Section 4.2, the QoS is defined for adaptive data rate approaches. Then, in Section 4.3, the functionality of the state of the art SRA and SOS approaches is explained and it is shown how SRA and SOS adapt their target data rate to the channel SNR. In Section 4.4, we motivate the use of the average BER and the time required for the rate adaptation as performance measures in the optimization of the state of the art approaches. In Sections 4.5 and 4.6, two new approaches that seamlessly adapt to the varying noise are presented. In the case of multiple adaptation procedures, instead of equalizing the error rate at all tones after each adaptation procedure which requires the modification of all tones within each adaptation procedure, both approaches decrease the average bit-error rate as rapidly as possible after each adaptation procedure while the QoS constraint presented in Section 4.2 is satisfied. The Tone-by-Tone SRA in Section 4.5

modifies each tone only once throughout all multiple adaptation procedures. This is achieved by modifying each tone directly to its target bit-loading. The Group SRA in Section 4.6 modifies the bit-loadings of the tones in groups and makes use of the short messages of the SOS procedure. Moreover, in Sections 4.5 and 4.6, low complex versions of the proposed approaches are presented. The low complex versions do not require high computational power and can therefore be used by DSL modems in practice. Finally, in Section 4.7, the performance of the discussed approaches is analyzed and simulation results are shown. Some of the contributions presented in this chapter are published in [SK13] and [SK14b].

4.2 Quality of Service for Adaptive Data Rate Approaches

In this section, the term QoS is defined for adaptive data rate approaches.

The strict definition of QoS for fixed data rate approaches cannot be used for adaptive target data rate approaches. The target data rate is allowed to be changed during operation and is therefore not constrained by the target data rate calculated during initialization. Rate adaptive approaches do not take into account future FEXT variations when calculating the target data rate which allow them to operate at the maximum achievable data rate. However, rate adaptive approaches cannot foresee a change in the noise and can only react to a change. In the case of adapting the target data rate to a decreased SNR, an error rate larger than the target error rate has to be tolerated until the adaptation is complete. Thus, adaptive data rate approaches are more suitable for services that can tolerate such a temporary degradation of the signal quality. For example, video on demand applications are able to buffer data locally such that the temporary degradation of the signal quality during the data rate adaptation procedure stays unnoticed by the end user.

The biggest concern for adaptive target data rate approaches is the seamlessness of the adaptation, especially when there are interleavers in the communication path. Interleavers operate on blocks of data. An output cannot be generated until several input blocks have been received. The time required to receive the input blocks causes an interleaver delay d_{int}, which increases when data rate is reduced and vice versa. Similarly, the degree of impulse noise protection decreases when the data rate is increased since more bits will be corrupted by an impulse noise over a period of time, and vice versa. The way to achieve true seamless behavior when an interleaver is enabled is to change the interleaver depth in proportion to the data rate so that the overall delay and the degree of impulse noise protection remain constant [XSL08, MC06]. Changing the interleaver depth introduces an offset in the transmission since it requires emptying the interleaver and filling it up again according to the new depth. Dummy bytes insertion methods like those given in [XSL08, MC06] can minimize this offset in transmission to the time required for the filling up of the interleaver with the smaller depth until the larger depth is reached, in case of an increase of the interleaver depth, or the time required for the emptying of the interleaver with the larger depth until the smaller depth is reached, in case of a decrease of the interleaver depth. During this time, in case of an increase of the interleaver depth, blocks of data enter the interleaver but no blocks leave it, and in case of a decrease of the interleaver depth, blocks of data leave the interleaver but no blocks enter it. In both cases, an offset in transmission is unavoidable [MC06]. Thus, in the VDSL2 standard [IT11], the parameter maximum delay variation $DV_{\max}$ defines the maximum allowed time required to fill up or empty the interleaver according to the new depth.

In any rate adaptation procedure, let us define the higher and the lower data rates in the procedure R_{high} and R_{low}, respectively. Furthermore, let us define $\Delta R = R_{\text{high}} - R_{\text{low}}$. Now, the amount of data rate allowed to be changed in an adaptation procedure such that the maximum delay variation is not exceeded is given by

$$\Delta R \leq \frac{DV_{\max} R_{\text{high}}}{d_{\text{int}}} = \Delta R_{\max}, \tag{4.1}$$

[IT11]. For adaptive target data rate approaches, the metric for QoS is the maximum delay variation $DV_{\max}$. To guarantee seamless adaptation, $DV_{\max}$ has to be so small that the offset in the transmission is not recognized by the end user. However, a small $DV_{\max}$ will lead to a small $\Delta R_{\max}$. In the case of large changes in noise, $\Delta R_{\max}$ might not be enough to adapt to the noise, and multiple consecutive adaptation procedures have to be executed. Multiple adaptation procedures will consume more adaptation time, during which the error rate is higher than the target error rate. Furthermore, in case the error rate remains higher than the target error rate for a certain period of time, the connection is interrupted and modems have to reinitialize at a lower data rate.

It is important to point out that SRA is the only state of the art adaptation approach that is constrained by (4.1), and hence, SRA is the only state of the art adaptation approach that adapts to the varying noise seamlessly. In contrast to SRA, when SOS is used, the reduction in the bit-loading is not constrained by the constraint in (4.1). Hence, the state of the art SOS does not provide means for the adaptation to the varying noise in a seamless manner.

In this section, a constraint on the amount of data rate allowed to be changed in an adaptation procedure by an adaptive data rate approach such that the approach is considered seamless has been derived. In the next Section 4.3, it will be shown how the state of the art SRA and SOS approaches adapt their target data rate to the varying noise. Then, in Section 4.4, the performance measures we consider in the optimization of the state of the art approaches are presented. In Sections 4.5 and 4.6, we then optimize the state of the art approaches such that the performance measures defined in Section 4.4 are improved while the constraint in (4.1) satisfied.

4.3 Data Rate Calculations for Adaptive Data Rate Approaches

4.3.1 Seamless Rate Adaptation

In this section, it is shown how the state of the art SRA adapts the target data rate to the varying noise.

When SRA is used, the target data rate can be adapted to changes in the SNR. By defining γ_{SRA} as the SNR margin used by such an approach, the bit-loading is given by

$$b_k = \log_2\left(1 + \frac{1}{\Gamma_c \gamma_{\text{SRA}}} SNR_k\right). \tag{4.2}$$

The bit-loading in (4.2) provides the number of bits on tone k that can be sustained at least at the desired target error probability, implicitly specified by the choice of Γ_c, if SNR_k decreases by at most γ_{SRA} [Jag08]. Furthermore, γ_{SRA} is equal at all tones, and therefore, the same error probability results at all tones. The target data rate is given by

$$R_{\text{SRA}} = f_s \sum_k b_k. \tag{4.3}$$

To adapt to the changing noise conditions, an adaptation procedure is executed every time the minimum SNR margin over all K tones reaches a threshold value. Obviously, γ_{SRA} can be much smaller than γ_{init} of Section 3.3 since it is not meant for the protection against the varying noise but mainly for triggering the adaptation procedures. Furthermore, if the decrease in the channel SNR at some tones is smaller than the threshold value, the SNR margin is equalized over all tones using bit-swapping.

In the case of large changes in the SNR, $\Delta R_{\max}$ defined in (4.1) might not be enough to reach the bit-loading in (4.2) and multiple consecutive adaptation procedures have to be executed. After each of those procedures, the bit-loadings are determined such that the SNR margin, and therefore, the error probability are equal at each tone. Only after the last procedure, the SNR margin is equal to γ_{SRA} and the bit-loading in (4.2) is achieved.

4.3.2 Save Our Session

In this section, it is shown how the state of the art SOS adapts the target data rate to the varying noise.

The SOS approach, defined in [IT11], provides emergency online reconfiguration of the DSL link to adapt to conditions of sudden large increases in noise. With an SOS request, a short and reliable message is used to request the new bit-loading. In order to keep the request short, the K tones are divided into N_{TG} tone groups during initialization and the bit-loading reduction Δb_g in each tone group g is constant [PS12].
For $\Delta b_g,\ g = 1, \cdots, N_{\text{TG}}$ being the per group bit reductions, G_g the number of tones in group g and $b_{g,v}$ being the bit-loading of the vth tone in group g before the SOS procedure, the bit-loading after the SOS procedure is given by

$$b_{\text{SOS},g,v} = b_{g,v} - \Delta b_g. \tag{4.4}$$

The target data rate after the SOS procedure is given by

$$R_{\text{SOS}} = f_s \sum_g \sum_v b^{(n)}_{\text{SOS},g,v}. \tag{4.5}$$

Note, that bit-loading in (4.4) does not match the number of bits on a tone to the SNR at that tone as in (4.2). As the name of the approach suggests, the SOS is not meant to adapt the bit-loading of a single tone to the changing SNR, but mainly for saving a session from reinitialization. According to [IT11], an SRA procedure should be executed after an SOS procedure such that bit-loading in (4.2) is achieved.

4.4 Performance Measures

4.4.1 Introduction

In this section, the performance measures we consider in the optimization of the state of the art adaptive data rate approaches are presented. First, in Section 4.4.2.1, the adaptation time required by the state of the art procedures is derived. Then, in Section 4.4.3, the average BER during an adaptation is introduced as a performance measure.

4.4.2 Adaptation Time

4.4.2.1 Seamless Rate Adaptation

In this section, the adaptation time required by the state of the art SRA is derived.

SRA uses the online reconfiguration (OLR) procedure to coordinate the change in the transmission parameters that leads to the new data rate [IT11]. Before the first OLR request is sent, the receiver modem measures the channel over a time period T_{meas} and calculates the new bit-loading that will result after the procedure is complete consuming the calculation time T_{cal}. The OLR request carrying the new bit loading parameters for up to 128 tones is then sent to the transmitter consuming the transmission time T_{SRAreq}. After the reception of the request at the transmitter side, the requested change in the transmission parameters is processed consuming the time T_{pr}. The transmitter can either reject or acknowledge the request. In the second case, an acknowledgment specifying the time when the transmission with the new parameters should take place is sent back to the receiver modem. The transmission time of the acknowledgment is denoted by T_{ack} and the time between receiving the acknowledgment and the beginning of transmission with the new parameters is denoted by T_{syn}.

In the case where more than 128 tones have to be modified in an SRA procedure i, consecutive OLR requests are executed, where a request l can only be sent when the acknowledgment of the previous request $l-1$ has been received. Moreover, the transmission with the new parameters begins T_{syn} after the reception of the last acknowledgment [IT11].

Assuming all the overhead traffic is transmitted over a separate robust overhead channel as described in [IT11], one can assume that all the transmitted messages will be received correctly at the other end. By defining $N_{\text{T},i}$ as the number of tones modified by SRA procedure i and $\lceil\cdot\rceil$ as the ceiling function, the time needed by SRA procedure i is given by

$$T_{\text{sra},N_{\text{T},i}} = T_{\text{meas}} + T_{\text{cal}} + \sum_{l=1}^{\lceil\frac{N_{\text{T},i}}{128}\rceil} T_{\text{SRAreq},l}\lceil\frac{N_{\text{T},i}}{128}\rceil\,(T_{\text{pr}} + T_{\text{ack}}) + T_{\text{syn}}. \tag{4.6}$$

Note that $T_{\text{req},l}$ increases linearly with the number of tones in OLR request l. Now, by defining the number N_{sra} of SRA procedures needed to fully adapt to the channel SNR while each procedure fulfills the data rate constraint in (4.1), the total adaptation time is given by

$$T_{\text{adapt}} = \sum_{i=1}^{N_{\text{sra}}} T_{\text{sra},N_{\text{T},i}}. \tag{4.7}$$

In the case of large changes in noise, e.g. if a strong VDSL2 interferer is switched on, the state of the art SRA procedure will have to modify all K tones which requires $\lceil\frac{K}{128}\rceil$ OLR requests in order to equalize the SNR margin in every procedure i. Obviously, the adaptation time will be very long.

4.4.2.2 Save Our Session

In this section, the adaptation time required by the state of the art SOS is derived.

The SOS approach, defined in [IT11], provides emergency Online Reconfiguration of the DSL link to adapt to conditions of sudden large increases in noise. With an SOS request, a short and reliable message is used to request the new bit-loading. In order to keep the request short, the K tones are divided into N_{TG} tone groups during initialization and the bit-loading reduction Δb_g in each tone group g is constant [PS12].

Before an SOS request is sent, the receiver modem measures the channel over a time period T_{meas} and calculates the bit loading reduction per group consuming the calculation time T_{cal}. The SOS request is then sent to the transmitter over the transmission time T_{SOSreq}. After the reception of the request at the transmitter side, the requested change in the transmission parameters is processed consuming the time T_{pr}. The transmitter can either reject or acknowledge the request. In the second case, an acknowledgment specifying the time when the transmission with the new parameters should take place is sent back to the receiver modem. The transmission time of the acknowledgment is denoted by T_{ack} and the time between receiving the acknowledgment and the beginning of transmission with the new bit-loading of the first group is denoted by T_{syn}. The change for subsequent groups shall be done T_{ss} after the execution of the previous group [IT11]. The times T_{meas}, T_{cal}, T_{pr}, T_{ack} and T_{syn} are the same as those defined in Section 4.4.2.1 for the SRA procedure.

Again assuming all the overhead traffic is transmitted over a separate robust overhead channel, one can assume that all the transmitted messages will be received correctly at the other end. By defining n_{sos} as the number of groups modified by an SOS procedure, the time needed by an SOS procedure is given by

$$T_{\mathrm{sos},n_{\mathrm{sos}}} = T_{\mathrm{meas}} + T_{\mathrm{cal}} + T_{\mathrm{SOSreq}} + T_{\mathrm{pr}} + T_{\mathrm{ack}} + T_{\mathrm{syn}} + (n_{\mathrm{sos}} - 1) \cdot T_{\mathrm{ss}}. \tag{4.8}$$

4.4.3 Average Bit-Error Rate

In this section, the average BER over K tones during an adaptation is introduced as a performance measure. The average BER over K tones is derived from the mean BER at each tone. Furthermore, the mean BER at each tone is derived from the mean symbol error rate.

Let us define M_k as the constellation size at tone k of the M-QAM symbol assumed in Section 2.2 and $Q(\cdot)$ as the Gaussian Q function. According to [Pop98], the mean symbol error rate expected in the transmission of the M-QAM symbols is given by the symbol error probability. The mean symbol error rate can then be approximated by

$$SER_k \approx 4Q\left(\sqrt{\frac{3\gamma_c}{M_k - 1} SNR_k}\right), \tag{4.9}$$

[Cio91, Jag08]. As explained in Section 2.2, the FEC code is applied to the input bit-stream before it is mapped to QAM-symbols in DSL systems. The code can therefore be interpreted as coding across tones [Jag08]. Only in the case when the QAM symbols at all tones have the same error probability, the tones are equally susceptible to errors thus allowing the coding gain to be interpreted as an amplification factor of the SNR at each tone as in (4.9). Thus, the multiplication of γ_c with SNR_k in the approximation of the mean symbol error rate in (4.9) is only valid when the symbols at all tones have the same error probability, which is the case in current DSL systems thanks to bit-loading that matches the constellation size on each tone to the SNR such that the same symbol error probability results at all tones.

According to [Pop98], the mean bit error rate expected in the transmission on tone k is given by the bit error probability. With the bit-loading b_k defined in (4.2), the mean bit error rate can then be approximated by

$$BER_k \approx \frac{2^{b_k-1}}{2^{b_k}-1} SER_k, \tag{4.10}$$

[Pro89,VVdBB04]. The approximation of BER_k in (4.10) is accurate for high SNR, which is typical for DSL systems [JHC08a]. The term before SER_k in (4.10) converges to 0.5 for large constellation sizes. Since high SNR is typical for DSL systems [JHC08a], BER_k can be viewed as a scaled version of SER_k. Furthermore, BER_k in (4.10) is degraded when SNR_k in (4.9) becomes smaller than the SNR used for calculating the constellation size M_k and vice versa. The average bit error rate over K tones is given by

$$BER_{\mathrm{avg}} = \frac{\sum_k BER_k b_k}{\sum_k b_k}, \tag{4.11}$$

[WLK05,BDAB12]. As explained in Section 4.2, in order to guarantee seamless adaptation of the data rate to the SNR, the data rate constraint in (4.1) has to be satisfied by each adaptation procedure, and as explained in Section 4.3.1, in the case of large changes in the SNR, $\Delta R_{\max}$ defined in (4.1) might not be enough to fully adapt to the new SNR and multiple consecutive adaptation procedures have to be executed. Furthermore, in order to equalize the symbol error probability at each tone after every adaptation procedure, the bit-loading of all K tones will have to be modified. Modifying K tones will in turn require $\lceil \frac{K}{128} \rceil$ OLR requests as explained in Subsection 4.4.2.1. Obviously, the adaptation time will be very long. Moreover, equalizing the symbol error probabilities prevents from reinitializations only if the resulting equal error probability is smaller than the target error probability. As discussed in Section 4.3.1, this is not the case when multiple adaptation procedures are executed.

In the case of multiple adaptation procedures, in order to prevent modifying all K tones after each adaptation procedure, which is required for equalizing the symbol error probabilities, we propose to minimize the average BER after every adaptation procedure, which allows us to put a constraint on the number of tones to be modified by each procedure, and thus, shorten the adaptation time. Only after the last adaptation procedure, the SNR margin, and consequently, the symbol error probabilities are equalized at all tones. In contrast to the state of the art SRA, with this new approach, the number of tones modified with each procedure can be viewed as a new optimization parameter.

The multiplication of γ_c with SNR_k in the approximation of the mean symbol error rate in (4.9) is only valid when the symbols at all tones have the same error probability. If we choose not to equalize the symbol error probabilities after each adaptation procedure, in the case of multiple adaptation procedures, the approximation in (4.9) will not be accurate and our performance measure, the average BER, will therefore also not be accurate. Thus, in this chapter we use the average BER at the input of the FEC decoder in Figure 2.1 as a performance measure. The average BER at the input of the FEC decoder describes the average BER before any bits are corrected.

The mean symbol error rate at the input of the FEC decoder is given by

$$\overline{SER}_k \approx 4Q\left(\sqrt{\frac{3}{M_k - 1} SNR_k}\right), \tag{4.12}$$

[Cio91, Jag08]. Note that the constellation size M_k in (4.12) is the constellation size that results when tone k has the bit-loading b_k of (4.2) where the SNR is amplified by the coding gain γ_c. In (4.2), γ_c is included in Γ_c, i.e., $\Gamma_c = \Gamma/\gamma_c$. The BER at the input of the FEC decoder is given by

$$\overline{BER}_k = \frac{2^{b_k - 1}}{2^{b_k} - 1} \overline{SER}_k, \tag{4.13}$$

[Pro89, VVdBB04]. Finally, the average BER at the input of the FEC decoder is given by

$$\overline{BER}_{\text{avg}} = \frac{\sum_k \overline{BER}_k b_k}{\sum_k b_k}. \tag{4.14}$$

At the output of the FEC decoder in Figure 2.1, $\overline{BER}_{\text{avg}}$ describes the average BER in case the decoder fails to correct any erroneous bits. $\overline{BER}_{\text{avg}}$ can therefore be viewed as an upper bound for the average BER. For the remainder of the dissertation, a line over

BER represents the BER at the input of the decoder.

In the next two sections, two new modified versions of SRA and SOS that seamlessly adapt to the varying noise are presented. Both new approaches minimize $\overline{BER}_{\text{avg}}$ as rapidly as possible in each adaptation step.

4.5 Tone-by-Tone SRA

4.5.1 Introduction

As previously explained, in the case of multiple adaptation procedures $i = 1, \cdots, N_{\text{sra}}$, in order to equalize the symbol error probabilities after every procedure i, all K tones have to be modified. Thus, after the last adaptation procedure, each tone will have been modified N_{sra} times. For the purpose of accelerating the adaptation procedure, we propose that each tone is modified only once throughout the N_{sra} adaptation procedures. This can be achieved if each tone is modified to its target bit-loading in (4.2) and only after the last adaptation procedure, all tones will have reached their target bit-loadings and therefore, an equal symbol error probability over all tones is restored. The crux here is to choose the tones to be modified by every adaptation procedure such that $\overline{BER}_{\text{avg}}$ is decreased as rapidly as possible while fulfilling the data rate constraint in (4.1).

In Section 4.5.2, the problem of choosing the tones to be modified by an adaption procedure is formulated as a binary integer minimization problem. Then, in Section 4.5.3, the stated minimization problem is solved optimally, however with very high complexity. In Section 4.5.4, a sub-optimum low complex approach is presented

4.5.2 Problem Formulation

In this section, the problem of choosing the tones to be modified by an adaption procedure is formulated as a binary integer minimization problem.

Assuming the SNR of the channel has changed due to an interferer being switched on and assuming multiple adaptation procedures have to be executed in order to adapt the bit-loading from the bit-loading that corresponds to the old SNR to the bit-loading that corresponds to the new SNR while fulfilling the data rate constraint in (4.1), let us define $\mathbb{U}$ as the set of indices of the U tones that have already been modified during the adaptation to the new SNR by a previous adaptation procedure. Furthermore, let us define b_u and $\overline{BER}_u$, $u = 1, \cdots, U$ as the target bit-loading and the target $\overline{\text{BER}}$ at the tone with the uth index in $\mathbb{U}$ that result with the new SNR according to (4.2) and (4.13), respectively. $\overline{BER}_u$ is the BER at the input of the decoder when the tone with the uth index has the bit-loading b_u that results with the new SNR and with the coding gain γ_c. Furthermore, let us define $\mathbb{V}$ as the set of indices of the V tones that have not yet been modified during the adaptation and $b_v^\star$ and $\overline{BER}_v^\star$ as the bit-loading and the $\overline{\text{BER}}$ at the tone with the vth index in $\mathbb{V}$ before the adaptation to the new SNR, respectively. $\overline{BER}_v^\star$ is the BER at the input of the decoder after the SNR has changed and when the tone with the vth index still has the bit-loading $b_v^\star$ that results with the old SNR and with the coding gain γ_c. Furthermore, let us define b_v and $\overline{BER}_v$ as the target bit-loading and the target $\overline{\text{BER}}$ at the tone with the vth index in $\mathbb{V}$ that would result with the new SNR according to (4.2) and (4.13), respectively. $\overline{BER}_v$ is the BER that would result at the input of the decoder if the tone with the vth index has the bit-loading b_u that would result with the new SNR and with the coding gain γ_c.

Let $\mathbf{x}$ be a vector with V binary integer variables, where $x_v = 1$ and $x_v = 0$ indicate that the tone with the vth index in $\mathbb{V}$ will be modified and will not be modified by the adaptation procedure, respectively. By denoting the average $\overline{\text{BER}}$ before an adaptation procedure as $\overline{BER}_{\text{avg0}}$, the problem of choosing the tones to be modified by every adaptation procedure such that $\overline{BER}_{\text{avg}}$ is decreased as rapidly as possible while fulfilling the data rate constraint in (4.1) can then be formulated as the following binary integer minimization problem with $\mathbf{x}$ as an optimization parameter:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \frac{\frac{\sum_v b_v^\star \overline{BER}_v^\star (1-x_v) + \sum_v b_v \overline{BER}_v x_v + \sum_u b_u \overline{BER}_u}{\sum_v b_v^\star (1-x_v) + \sum_v b_v x_v + \sum_u b_u} - \overline{BER}_{\text{avg0}}}{T_{\text{sra}, \sum_v x_v}} \\ \text{subject to} \quad & f_s \cdot \sum_v (b_v^\star - b_v) x_v \leq \Delta R_{\max}, \end{aligned} \tag{4.15}$$

where f_s and $\Delta R_{\max}$ are defined in (3.3) and (4.1), respectively. The nominator of the objective function in (4.15) describes the difference between $\overline{BER}_{\text{avg}}$ after and before the adaptation procedure in which the tones allocated by the vector $\mathbf{x}$ will be modified to their target bit-loading. The denominator of the objective function is the adaptation time as defined in (4.6) required to modify the bit-loadings of the tones allocated by the vector $\mathbf{x}$. A negative objective function in (4.15) indicates that $\overline{BER}_{\text{avg}}$ has decreased after modifying the bit-loadings of the tones allocated by the vector $\mathbf{x}$ and vice versa. Note that at the beginning of the adaptation to the new SNR, the set $\mathbb{U}$ is empty and the set $\mathbb{V}$ contains the indices of all the tones and after the last adaptation procedure, the set $\mathbb{V}$ is empty and the set $\mathbb{U}$ contains the indices of all the tones.

4.5.3 Optimal Solution

In this section, the binary integer minimization problem in (4.15) is solved optimally.

The adaptation time in the denominator of the minimization problem in (4.15) is a function of the number of tones to be modified in an adaptation procedure. In order to solve the minimization problem in (4.15), the problem is split into Z optimization problems, where $z = 1, \cdots, Z$ is the number of tones being modified by an adaptation procedure. By putting a constraint on the number of tones being modified, the adaptation time in (4.15) becomes constant for every z and the minimization problem (4.15) splits into Z minimization problems. The zth minimization problem is given, after rearranging the objective function, by

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & \frac{\sum_v (b_v \overline{BER}_v - b_v^\star \overline{BER}_v^\star) x_v + \sum_v b_v^\star \overline{BER}_v^\star + \sum_u b_u \overline{BER}_u}{\sum_v (b_v - b_v^\star) x_v + \sum_v b_v^\star + \sum_u b_u} \\ \text{subject to} \quad & f_s \cdot \sum_v (b_v^\star - b_v) x_v \leq \Delta R_{\max}, \\ & \sum_v x_v = z. \end{aligned} \tag{4.16}$$

The constraints in (4.16) are linear. Furthermore, both, the nominator and the denominator are linear equations. Thus, by setting the objective function smaller than or equal to an upper bound UP_b, the objective function is transformed into a further linear constraint and similarly, by setting the objective function larger than or equal to a lower bound LOW_b, a further linear constraint is added to the constraints. The resulting linear integer optimization problem can be solved by any Linear Programming approach [Van01]. The distance between the upper and lower bound is then decreased in bisection fashion until the optimum solution is reached.

By defining the vector $\mathbf{a}$ containing $b_v\overline{BER}_v - b_v^\star\overline{BER}_v^\star$ and the vector $\mathbf{q}$ containing $b_v - b_v^\star$ for $v = 1, \cdots, V$ and by defining $\overline{BER}_{\text{avg,target}}$ as the average $\overline{\text{BER}}$ that would result when all tones have been modified to their target bit-loadings and with $\overline{BER}_{\text{avg0}}$ of (4.15), the bisection method on the objective function to find the optimum $\mathbf{x}$ is given in Algorithm 3.

Algorithm 3 Bisection method to find the optimum $\mathbf{x}$

Initialize $UP_b = \overline{BER}_{\text{avg0}}$, $LOW_b = \overline{BER}_{\text{avg,target}}$
Choose a tolerance $\epsilon \geq 0$
repeat
    Solve the following linear integer problem for vector $\tilde{\mathbf{x}}$, with $\tilde{x}_v,\ v = 1, \cdots, V$
    $\mathbf{a}^{\mathrm{T}}\tilde{\mathbf{x}} + \sum_v b_v^\star\overline{BER}_v^\star + \sum_u b_u\overline{BER}_u \leq UP_b(\mathbf{q}^{\mathrm{T}}\tilde{\mathbf{x}} + \sum_v b_v^\star + \sum_u b_u)$
    $\mathbf{a}^{\mathrm{T}}\tilde{\mathbf{x}} + \sum_v b_v^\star\overline{BER}_v^\star + \sum_u b_u\overline{BER}_u \geq LOW_b(\mathbf{q}^{\mathrm{T}}\tilde{\mathbf{x}} + \sum_v b_v^\star + \sum_u b_u)$
    $f_s \cdot \sum_v (b_v^\star - b_v)\tilde{x}_v \leq \Delta R_{\max}$
    $\sum_v \tilde{x}_v = z$
    if $\tilde{\mathbf{x}}$ exists between UP_b and LOW_b **then**
        $UP_b = \frac{UP_b + LOW_b}{2}$
    else
        $\delta = LOW_b,\quad LOW_b = UP_b$
        $UP_b = UP_b + \frac{UP_b - \delta}{2}$
    end if
until $UP_b - LOW_b \leq \epsilon$
$\mathbf{x} = \tilde{\mathbf{x}}$

The Z minimization problems in (4.16) yield the solution vectors $\mathbf{x}_z,\ z = 1, \cdots, Z$. The solution vector that corresponds to the smallest value of the objective function in (4.15) is the optimal choice of tones to be modified by an adaptation procedure such that the average $\overline{\text{BER}}$ is decreased as rapidly as possible while fulfilling the data rate constraint in (4.1).

Solving each of the Z minimization problems in (4.16) with Algorithm 3 requires high computational power, since in every bisection iteration in Algorithm 3 a linear integer problem has to be solved. In practice, the computational power of modems is limited and, therefore, low complexity algorithms are needed. In the following Section 4.5.4, a low complex rate adaptation algorithm is presented.

4.5.4 Low Complex Tone-by-Tone SRA

In this section, a low complex rate adaptation algorithm is presented. The Low Complex Tone-by-Tone SRA first modifies the tones that will lead to the largest decrease in the average $\overline{\text{BER}}$.

By using the set $\mathbb{V}$ defined in Section 4.5.2 and containing the indices of the V tones that have not yet been modified during the adaptation, let $\overline{BER}_{\text{avg},v}$ be the value of the objective function of problem (4.16) when only the tone with the vth index is modified to the target bit loading. Furthermore, let $\overline{\mathbf{BER}}_{\text{avg}}$ be the vector with entries $\overline{BER}_{\text{avg},v}$ $v = 1, \cdots, V$. The Low Complex Tone-by-Tone SRA sorts $\overline{\mathbf{BER}}_{\text{avg}}$ and modifies the tones to their target bit-loadings in their sorting order. The Low Complex Tone-by-Tone SRA terminates if the modification of an extra tone to its target bit-loading would hurt the data rate constraint in (4.1).

By using the same notation $\mathbf{x}$ for the solution vector as in Section 4.5.2 and the same notation $b_v^\star$ and b_v for the bit-loading at tone with the vth index in $\mathbb{V}$ before and after the adaptation to the new SNR, respectively, as in Section 4.5.2, the Low Complex Tone-by-Tone SRA is given in Algorithm 4.

Algorithm 4 Low Complex tone-by-tone SRA to find a sub-optimal $\mathbf{x}$

1: Compute the vector $\overline{\mathbf{BER}}_{\text{avg}}$, with entries $\overline{BER}_{\text{avg},v}$ $v = 1, \cdots, V$
2: Sort $\overline{\mathbf{BER}}_{\text{avg}}$ in ascending order yielding $\overline{\mathbf{BER}}_{\text{avg,sorted}}$
3: Compute the permutation vector $\mathbf{W}$ containing the indices of $\overline{\mathbf{BER}}_{\text{avg}}$ w_p, $p = 1, \cdots, V$ in their order in $\overline{\mathbf{BER}}_{\text{avg,sorted}}$
4: Initialize $b_{w_p}^\star = b_v^\star$, $b_{w_p} = b_v$, $x_{w_p} = x_v$, for $w_p = v$
5: Initialize $y = 1$
6: **while** $\sum_{p=1}^{y} b_{w_p}^\star - b_{w_p} \leq \Delta R_{\max}$ **do**
7: $\quad y = y + 1$
8: **end while**
9: Solution vector: $\mathbf{x}$ with $x_{w_p} = 1,\ p = 1, \cdots, y - 1$

4.6 Group SRA

4.6.1 Introduction

In this section, an adaptation procedure that makes use of the short messages of the SOS procedure but still adapts to the varying noise in a seamless manner is proposed. Again it is assumed that the change in the SNR is so large such that multiple adaptation procedures are needed to adapt the target data rate to the new SNR while satisfying the constraint in (4.1). Similar to the SOS approach, the K tones are divided into N_{TG} tone groups during initialization. Then the per group bit reductions $\Delta b_g,\ g = 1, \cdots, N_{\mathrm{TG}}$ that minimize the average $\overline{\mathrm{BER}}$ while fulfilling the data rate constraint in (4.1) is found. The N_{TG} tone groups can contain, according to [IT11], from 256 and up to 1024 tones. Since only $\Delta b_g,\ g = 1, \cdots, N_{\mathrm{TG}}$ has to be communicated between the receiver and transmitter in every adaptation step, the data rate reduction, and therefore, also the average BER reduction are very fast. However, after rapidly decreasing the data rate and the average BER, the bit-loadings of each single tone have to be modified to the target bit-loading according to (4.2). Therefore, one final SRA procedure is executed.

In Section 4.6.2, the problem of finding the per group bit reductions $\Delta b_g,\ g = 1, \cdots, N_{\mathrm{TG}}$ that minimize the average $\overline{\mathrm{BER}}$ while fulfilling the data rate constraint in (4.1) is formulated as a discrete minimization problem. Then, in Section 4.6.3, comments on the optimal solution of the stated problem are made and an exhaustive search is proposed for the determination of the optimal solution. In Section 4.6.4, a sub-optimum low complexity approach is presented.

4.6.2 Problem Formulation

In this section, the problem of finding the per group bit reductions $\Delta b_g,\ g = 1, \cdots, N_{\mathrm{TG}}$ that minimize the average $\overline{\mathrm{BER}}$ while fulfilling the data rate constraint in (4.1) is formulated as a discrete minimization problem.

Let G_g be the number of tones in group g. Normally, all N_{TG} tone groups would have the same size, here G_g has a group index g in order to cover the case when the number of all tones K is not a multiple of the group size and the last group must contain fewer tones. Moreover, before any of the multiple adaptation procedures needed for the adaptation to the new SNR, let $b_{g,l}$ be the number of bits on the lth tone in group g and $\overline{BER}_{g,l}(b_{g,l})$ be the bit-error rate defined in (4.13) for the lth tone in group g when it is carrying $b_{g,l}$ bits, and $\overline{BER}_{\text{avg0}}$ be the average $\overline{\text{BER}}$ as defined in Section 4.5.2. Furthermore, let us define

$$T_{\text{const}} = T_{\text{meas}} + T_{\text{cal}} + T_{\text{req}} + T_{\text{pr}} + T_{\text{ack}} + T_{\text{syn}}. \tag{4.17}$$

T_{const} is the part of the adaptation time that is independent of the per group bit reductions $\Delta b_g,\ g = 1, \cdots, N_{\text{TG}}$. By defining $\mathbf{\Delta b}$ as the vector with entries $\Delta b_g,\ g = 1, \cdots, N_{\text{TG}}$ and $\text{sgn}(\cdot)$ as the sign function, the problem of finding the per group bit reductions $\Delta b_g,\ g = 1, \cdots, N_{\text{TG}}$ that minimize the average $\overline{\text{BER}}$ while fulfilling the data rate constraint in (4.1) can then be formulated as the following discrete minimization problem with $\mathbf{\Delta b}$ as an optimization parameter:

$$\begin{aligned} &\underset{\mathbf{\Delta b}}{\text{minimize}} && \frac{\frac{\sum_{g=1}^{N_{\text{TG}}} \sum_{v=1}^{G_g} (b_{g,v} - \Delta b_g) \overline{BER}_{g,v}(b_{g,v} - \Delta b_g)}{\sum_{g=1}^{N_{\text{TG}}} \sum_{v=1}^{G_g} b_{g,v} - \Delta b_g} - \overline{BER}_{\text{avg0}}}{T_{\text{const}} + (\sum_g \text{sgn}(\Delta b_g) - 1) \cdot T_{\text{ss}}} \\ &\text{subject to} && f_s \cdot \sum_g G_g \cdot \Delta b_g \le \Delta R_{\max}, \end{aligned} \tag{4.18}$$

where f_s and $\Delta R_{\max}$ are defined in (3.3) and (4.1), respectively. The nominator of the objective function in (4.18) describes the difference between $\overline{BER}_{\text{avg}}$ after and before the adaptation procedure in which the tone groups are modified with the per group bit reductions in $\mathbf{\Delta b}$. The denominator of the objective function is the adaptation time as defined in (4.8) required to modify the tone groups with the per group bit reductions in $\mathbf{\Delta b}$. A negative objective function in (4.18) indicates that $\overline{BER}_{\text{avg}}$ has decreased after modifying the bit-loadings of the tone groups allocated by the vector $\mathbf{\Delta b}$ and vice versa.

4.6.3 Optimal Solution

Consider the objective function in (4.18). The terms $\overline{BER}_{g,v}(b_{g,v} - \Delta b_g)$ and $\text{sgn}(\Delta b_g))$ are non-linear functions of the optimization parameter $\mathbf{\Delta b}$. Thus, problem (4.18) cannot be transformed into multiple linear optimization problems similar to (4.16) and therefore cannot be solved with a linear programming approach [Van01]. Fortunately, since the number of tone groups N_{TG} is much smaller than the number of tones K, an exhaustive search can be used for the determination of the optimal $\mathbf{\Delta b}$.

4.6.4 Low Complex Group SRA

Finding the optimal $\mathbf{\Delta b}$ that solves problem (4.18) requires high computational power. As already mentioned, in practice, the computational power of modems is limited and, therefore, low complex algorithms are needed. In this section, a low complexity rate adaptation algorithm is presented.

The proposed Low Complex Group SRA is an iterative algorithm that reduces the bit-loading at every iteration by one bit from the group that will lead to the largest decrease in the average $\overline{\text{BER}}$. The Low Complex Group SRA terminates if the constraint in (4.18) is not going to be satisfied after any further reduction in the bit-loading.

Let $\overline{BER}'_{\text{avg},g}$ be the average BER when only the bit-loadings of the tones in group g are reduced by one bit and $\overline{\mathbf{BER}}'_{\text{avg}}$ be the vector with entries $\overline{BER}'_{\text{avg},g}$, $g = 1, \cdots, N_{\text{TG}}$. The Low Complex Group SRA is given in Algorithm 5.

Algorithm 5 Low Complex Group SRA to find a sub-optimal $\mathbf{\Delta b}$

```
Initialize Δb_g = 0, g = 1, ··· , N_TG, const_1 = true
while const = true do
    Compute the vector BER'_avg with entries BER'_avg,g, g = 1, ··· , N_TG
    Find w such that BER'_avg,w = min{BER'_avg}
    Δb_g = Δb_g + 1, with g = w
    if f_s · Σ_g G_g · Δb_g ≤ ΔR_max then
        b_g,l = b_g,l − 1, with l = 1, ··· , G_g, g = w
        const = true
    else
        Δb_g = Δb_g − 1, with g = w
        const_1 = false
    end if
end while
Solution vector: Δb with entries Δb_g, g = 1, ··· , N_TG
```

4.7 Performance Analysis

4.7.1 Introduction

In this section, the performance of the proposed approaches is analyzed and simulation results are shown. First, in Subsection 4.7.2, an overview of the approaches that are being compared is presented. Then, in Subsection 4.7.3, the simulation scenario is explained and the simulation parameters are stated. In Subsection 4.7.4, simulation results are shown.

4.7.2 Overview of Compared Approaches

This subsection gives an overview of the seamless rate adaptive approaches that are being compared. Since only seamless approaches are being compared, the constraint in (4.1) is satisfied by all approaches.

- **State of the art SRA**. The error probabilities at all tones are equalized after each adaptation procedure.
- **Tone-by-Tone SRA**. The tones that minimize $\overline{BER}_{\text{avg}}$ as rapidly as possible are modified to their target bit-loading as shown Algorithm 3. The Tone-by-Tone SRA solves the optimization problem (4.15) optimally, however, requires high computational power which is not available at DSL modems.
- **Low Complex Tone-by-Tone SRA**. The tones that will lead to the largest decrease in $\overline{BER}_{\text{avg}}$ are modified to their target bit-loading as shown Algorithm 4. The Low Complex Tone-by-Tone SRA does not require high computational power and can therefore be used by DSL modems in practice.
- **Group SRA**. The tone groups are modified with the per group bit reductions that minimize $\overline{BER}_{\text{avg}}$ as rapidly as possible. The Group SRA uses an exhaustive search for the determination of the optimal per group bit reductions for problem (4.18). The Group SRA requires high computational power which is not available in DSL modems in practice.

- **Low Complex Group SRA**. The Low Complex Group SRA is an iterative algorithm that reduces the bit-loading at every iteration by one bit from the group that will lead to the largest decrease in $\overline{BER}_{\text{avg}}$ as shown in Algorithm 5. The Low Complex Group SRA does not require high computational power and can therefore be used by DSL modems in practice.

4.7.3 Simulation Setup

In this section, the scenario used for the simulations is described and simulation parameters are shown.

For the simulations, a downstream scenario with a VDSL2 victim line and a VDSL2 disturber line is assumed. The SNRs used for calculating the bit-loadings are determined according to the model in Section 2.4. For the evaluation of the rate adaptation algorithms, the bit-loading of the victim line is calculated when the disturber is switched off and then again when the disturber is switched on, as shown in Figure 4.1. The curves in Figure 4.1 show the bit-loadings of the tones with the indices on the x-axis that can be sustained at least at the desired target error probability if the SNR at those tones decreases by at most γ_{SRA}. The blue curve shows the bit-loadings calculated by the victim line according to (4.2) with the SNR experienced when no disturbers are active and with the SNR margin γ_{SRA} and with the SNR gap of the code Γ_c which includes the coding gain γ_c, i.e., $\Gamma_c = \Gamma/\gamma_c$. Similarly, the red curve shows the bit-loadings calculated by the victim line according to (4.2) with the SNR experienced when a VDSL2 disturber is active and with the SNR margin γ_{SRA} and with the SNR gap of the code Γ_c which includes the coding gain γ_c. The approaches in Section 4.7.2 are then used to seamlessly decrease the bit-loadings illustrated by the blue curve in Figure 4.1 until the bit-loadings illustrated by the red curve in the same figure is reached.

Beside the parameters in Table 3.2, the parameters in Table 4.1 are assumed. The values for the maximum delay variation DV_{max}, the interleaver delay d_{int} and the data rate for the overhead channel are within the allowed range specified in the VDSL2 standard [IT11]. Moreover, DV_{max} is set equal to only 1ms such that the offset in transmission while changing the interleaver depth stays unnoticed by the end user. The SNR margin $\gamma_{\text{SRA}}^{\text{dB}}$ is set to only 1 dB since it not meant for the protection against the varying noise but mainly for triggering the adaptation procedures. The values for the remaining parameters in Table 4.1 are set according to the VDSL2 standard [IT11].

Table 4.1: System parameters

Number of tone groups N_{TG}	11
Number of tones in group $G_g, g = 1, \cdots, 10$	256
Number of tones in group G_{11}	240
SNR margin $\gamma_{\mathrm{SRA}}^{\mathrm{dB}}$	1dB
Data rate of overhead channel	256 bits/ms
$DV_{\max}$ of (4.1)	1 ms
d_{int} of (4.1)	20 ms
T_{meas} of (4.6), (4.8)	64 ms
T_{cal} of (4.6), (4.8)	100 ms
$T_{\mathrm{req}}^{\mathrm{SRA}}$ of (4.6)	$8 \cdot (12 + 4 \cdot N_{\mathrm{T}})/256$ ms
$T_{\mathrm{req}}^{\mathrm{SOS}}$ (4.8)	$8 \cdot (11 + N_{\mathrm{TG}}/2)/256$ ms
T_{pr} of (4.6), (4.8)	140 ms
T_{ack} of (4.6), (4.8)	0.1 ms
T_{syn} of (4.6), (4.8)	16.25 ms
T_{ss} of (4.8)	12 ms

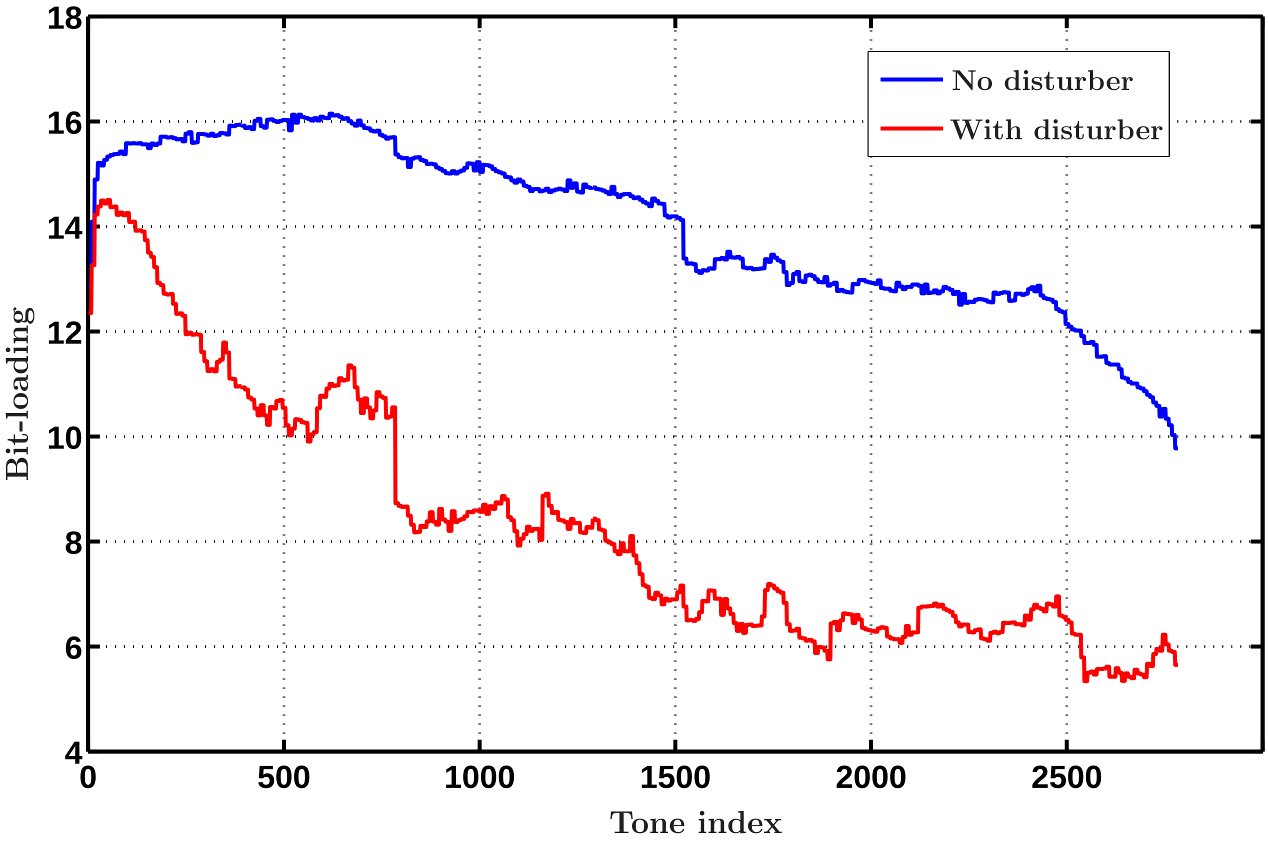


Figure 4.1: The bit-loading of the victim line when the disturber is switched off and then again when the disturber is switched on.

4.7.4 Simulation Results

In this section, we show results of the performance measures explained in Section in 4.4.

Figure 4.2 shows the average BER versus the adaptation time when using the approaches given in Section 4.7.2 to seamlessly decrease the bit-loading illustrated by the blue curve in Figure 4.1 until the target bit-loading illustrated by the red curve is reached. Obviously, due to the constraint in (4.1), multiple adaptation procedures are required. Moreover, the bit-loading used in transmission during the time required by an adaptation procedure is equal to the bit-loading coordinated by the previous adaptation procedure. This explains the step-like progression of the curves in Figure 4.1.

As explained in Section 4.4.3, the state of the art SRA approach equalizes the symbol error probabilities at all tones after every adaptation procedure, and therefore, the symbol error rate approximation in (4.9) can be used and the blue curve in Figure 4.2 illustrates the average BER BER_{avg} at the output of the decoder according to (4.11). After the last adaptation procedure, the average BER is equal to the target average BER $BER_{\text{avg,target}}$. $BER_{\text{avg,target}}$ is the average BER that is achieved at the output of the FEC-decoder when the target bit-loading is reached.

Furthermore, when the Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA, the Group SRA and Low Complex Group SRA are used, the symbol error probability is not equal at all tones after each intermediate adaptation procedure. Thus, the symbol error rate approximation in (4.9) cannot be used for the calculation of the average BER the output of the FEC-decoder and the curves of those approaches in Figure 4.2 describe the average BER $\overline{BER}_{\text{avg}}$ at the input of the FEC-decoder according to (4.14). As explained in Section 4.4.3, $\overline{BER}_{\text{avg}}$ is an upper bound for BER_{avg} that describes the case when the FEC-decoder fails to correct any erroneous bit. After the last adaptation procedure, the target bit-loading is reached by the Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA, the Group SRA and Low Complex Group SRA, and therefore, the same target BER $\overline{BER}_{\text{avg,target}}$ at the input of the FEC-decoder is achieved by all those approaches, as shown in Figure 4.1. $\overline{BER}_{\text{avg,target}}$ is the average BER that is achieved at the input of the FEC-decoder when the target bit-loading is reached. Moreover, only after the last adaptation procedure when the target bit-loading is reached, are the symbol error probabilities at all tones equal and the symbol error rate approximation in (4.9) can be used for the calculation of the average BER at the output of the FEC-decoder. After the last adaptation all approaches reach the same target bit-loading, and therefore, the same target average BER $BER_{\text{avg,target}}$ at the output of the FEC-decoder is reached by all approaches.

The Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA, the Group SRA and Low Complex Group SRA outperform the state of the art SRA approach. In contrast to all the over approaches, the state of the art SRA modifies all K tones after each adaptation procedure and requires therefore long constant adaptation times for each procedure as shown in Figure 4.2. The Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA modify each tone only once through out the adaptation to the new bit-loading and require therefore less adaptation time when compared to the state of the art SRA. The Group SRA and Low Complex Group SRA modify the bit-loadings of the tones in groups and use the short messages of the SOS approach and therefore achieve the fastest decrease in average BER, however, both, the Group SRA and the Low Complex Group SRA, need to modify the bit-loadings of each tone to the target bit-loadings at the final adaptation procedure using the state of the art SRA. This explains the long adaptation time of the last adaptation procedure.

Moreover, as expected, the Tone-by-Tone SRA outperforms the Low Complex Tone-by-Tone SRA and the Group SRA outperforms Low Complex Group SRA. However, the difference in performance between the Tone-by-Tone SRA and the Low Complex Tone-by-Tone SRA and between the Group SRA and the Low Complex Group SRA is very small which makes the Low Complex Tone-by-Tone SRA and the Low Complex Group SRA very good candidates to be used by modems with limited computational power.

Figure 4.3 shows the mean number of erroneous bits that are expected to occur during the adaptation time required to seamlessly decrease the bit-loading illustrated by the blue curve in Figure 4.1 until the target bit-loading illustrated by the red curve is reached when using the approaches in Section 4.7.2 relative to the mean number of erroneous bits that are expected to occur when using the state of the art SRA. For the Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA, the Group SRA and Low Complex Group SRA the BER $\overline{BER}_{\text{avg}}$ at the input of the FEC-decoder, shown in Figure 4.2, is used for the calculation of the mean number of erroneous bits during the adaptation. Although $\overline{BER}_{\text{avg}}$ is an upper bound of the average BER BER_{avg} at the output of the FEC-decoder, the mean number of erroneous bits calculated with $\overline{BER}_{\text{avg}}$ when using the Tone-by-Tone SRA, the Low Complex Tone-by-Tone SRA, the Group SRA and Low Complex Group SRA is significantly smaller than the mean number of erroneous bits expected to occur when using the state of the art SRA. Moreover, although, as shown in Figure 4.2, the Group SRA and the Low Complex Group SRA require a longer adaptation time when compared to the Tone-by-Tone SRA and the Low Complex Tone-by-Tone SRA, due to the final adaptation procedure using the state of the art SRA, the mean number of erroneous bits expected to occur during the adaptation when using those approaches is less than half of the mean number of erroneous bits expected to occur when using the Tone-by-Tone SRA and the Low Complex Tone-by-Tone SRA. However, for the Group SRA approaches to work in practice, the DSL modems must coordinate between two different online reconfiguration procedures, one for the per group bit reductions and one

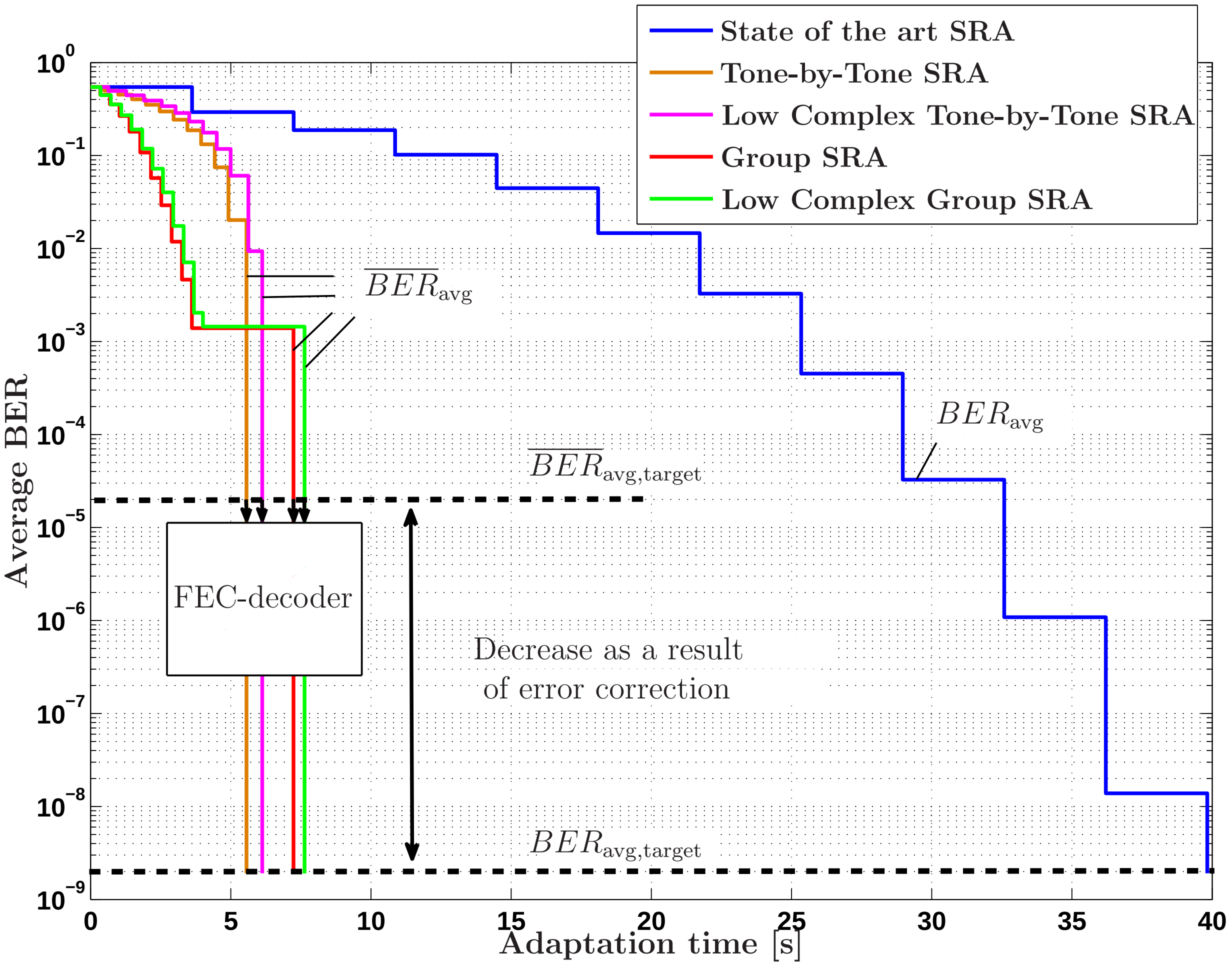


Figure 4.2: Average BER versus the adaptation time when using the approaches given in Section 4.7.2.

in order to achieve the target bit-loading at each tone. In this aspect, the Tone-by-Tone SRA approaches are less complex.

In conclusion, in case of multiple adaptation procedures, minimizing the average bit-error rate after every adaptation procedure allows the tone-by-tone modification of the bit-loadings to the target bit-loadings and the modification of the bit-loadings of the tones in groups, and thus, shortens the adaptation time tremendously. Moreover, the modification of the bit-loadings in groups leads to the fastest decrease in the average BER and therefore to the least occurred errors, however, the bit-loadings of the tones have to modified to

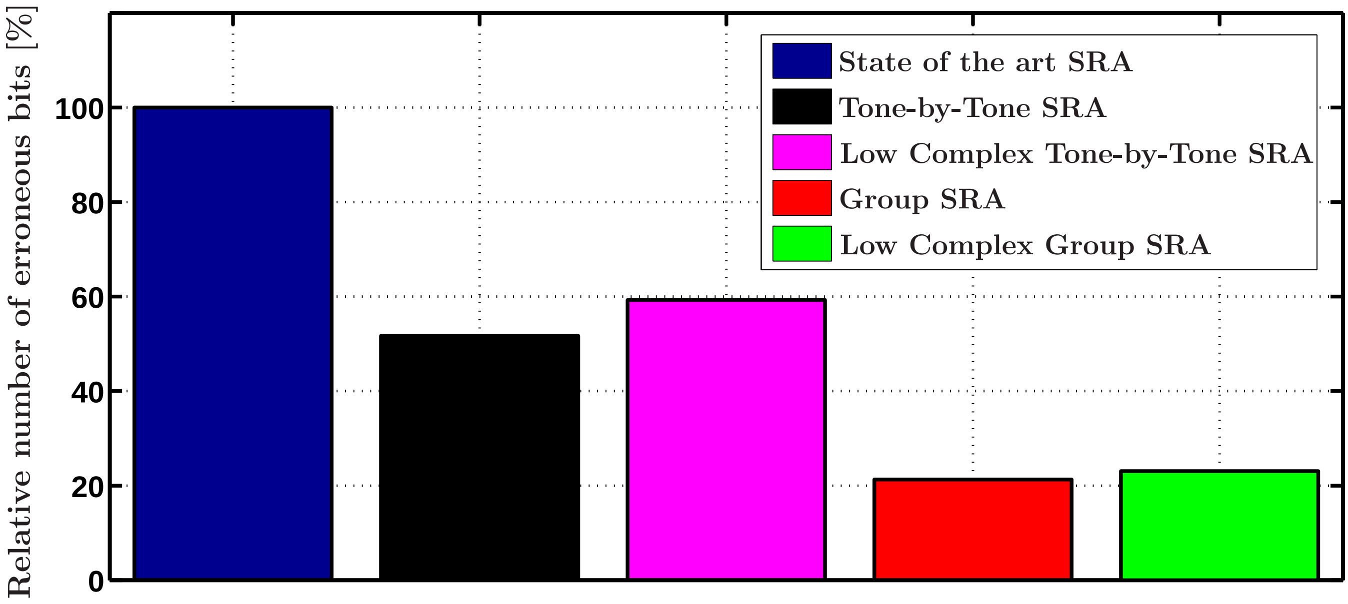


Figure 4.3: Mean number of erroneous bits expected to occur during the adaptation when using the approaches in Section 4.7.2 relative to the mean number of erroneous bits that are expected to occur when using the state of the art SRA.

the target bit-loadings using one final state of the art SRA procedure. Furthermore, the performances of the Low Complex Tone-by-Tone SRA and the Low Complex Group SRA are similar to the performances of the Tone-by-Tone SRA and the Group SRA, respectively, however, they do not require high computational power and can therefore be used by DSL modems in practice.

Chapter 5

Combining Virtual Noise and SRA for User Specific Requirements on QoS

5.1 Introduction

In Chapters 3 and 4 two different approaches were presented for the protection against time-varying crosstalk. In Chapter 3, it was shown how the Virtual Noise approach can guarantee a target BER that is below or equal to the target BER, however, in order to be able to guarantee the data rate at times with high noise levels, achievable data rate is wasted at times with low noise levels. Moreover, the proposed adaptation procedures in Chapter 4 are able to rapidly adapt to the channel and prevent reinitializations from happening, and thus, are able to maximize the achievable data rate. However, a BER smaller than the target BER cannot be guaranteed at least during the adaptation times.

DSL users often use services with different requirements on QoS in terms of outage probability. In order for a provider to offer a DSL user real-time services such as IPTV, VoIP and video over IP, a data rate achieved at a BER that is smaller than the target BER has to be guaranteed with a target outage probability. Where as for services that are not offered in real-time, such as video on demand, file sharing and downloading, the data rate achieved at a BER that is smaller than the target BER is allowed to vary and temporary increases of the BER over the target BER can be tolerated. In the case when DSL users use services with different requirements on QoS, the current VDSL2 standard advises to set the system parameters such that the strictest requirements on outage probability are satisfied. Clearly, this results in a data rate loss. In this chapter, an approach that utilizes the line stability in terms of outage probability of the VN approach and the data rate efficiency of the SRA approach is presented. The hybrid VN-SRA approach iteratively allocates tones to be used by the VN approach and tones to be used by the SRA approach such that the average data rate achieved over 24 hours is maximized while user specific requirements on QoS in terms of outage probability are satisfied. In cases when DSL users use services with different constraints on QoS, the proposed approach will always lead to a data rate performance better than the state of the art approach, especially when L2 mode is implemented.

First, in Section 5.2, the problem of allocating tones to be used by the VN approach and tones to be used by the SRA approach is formulated into a binary optimization problem. In Section 5.3, the Hybrid VN-SRA approach, that solves the binary optimization problem sub-optimally, is presented. In Section 5.4.4, the performance of the proposed approach is analyzed and simulation results are shown. Some of the contributions presented in this chapter are published by the author of this dissertation in [SK14a].

5.2 Problem Formulation

In this section, the problem of allocating tones to be used by the VN approach and tones to be used by the SRA approach is formulated into a binary optimization problem.

DSL users often deploy services with different requirements on quality of service in terms of outage probability. We propose that the VN approach should be used for services that require a data rate R_{req} at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ and that the SRA approach should be used for services that allow the varying of the data rate achieved at a BER that is smaller than the target BER and can tolerate temporary increases of the BER over the target BER. The crux here is to allocate the tones to be used by the VN approach and the tones to be used by the SRA approach such that a data rate R_{req} can be guaranteed with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ at a BER that is smaller than or equal to the target BER while the average data rate achieved over 24 hours at all tones is maximized.

Let us define $\mathbb{K}$ as the set containing the indexes $k = 1, \cdots, K$ of all used tones. Furthermore, let us define $\mathbb{W}$ as a sub-set of $\mathbb{K}$ containing the tone indexes of the tones used by the SRA approach and $\mathbb{Z}$ as a sub-set of $\mathbb{K}$ containing the tone indexes of the tones used by the VN approach. According to Chapter 3, in order to maximize the target data rate while satisfying a certain outage probability on the tones with the indexes in $\mathbb{Z}$, the SNR margin and the VN mask are given as functions of the distributions of the noise day maxima of those tones. To highlight the dependency of the VN mask and the SNR margin on the tones in $\mathbb{Z}$, in this chapter, the set $\mathbb{Z}$ will be added as an index to the SNR margin and VN mask. In the following, the SNR margin and the VN mask are denoted by $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{dB}}$ and $VN_{\mathbb{Z},k}^{\text{dB}}$, respectively. Furthermore, the SNR margins $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{lt dB}}$ and $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{st dB}}$ and their corresponding VN masks $VN_{\mathbb{Z},k}^{\text{lt dB}}$ and $VN_{\mathbb{Z},k}^{\text{st dB}}$, $k = 1, \cdots, K$ derived in Chapter 3 are special cases of $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{dB}}$ and $VN_{\mathbb{Z},k}^{\text{dB}}$, $k = 1, \cdots, K$ and hence, all the derivations and algorithms that apply for $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{dB}}$ and $VN_{\mathbb{Z},k}^{\text{dB}}$ apply for $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{lt dB}}$ and $VN_{\mathbb{Z},k}^{\text{lt dB}}$ and for $\gamma_{\mathbb{Z},\text{init-bl}}^{\text{st dB}}$ and $VN_{\mathbb{Z},k}^{\text{st dB}}$, too.

Furthermore, let us define the average number of bits $\bar{b}_{\text{SRA},k}$ transmitted on tone k over 24 hours when the bit-loading at tone k is adapted to the varying SNR. The average number of bits $\bar{b}_{\text{SRA},k}$ can be estimated by the SMC from showtime measurements and will be assumed to be known.

Now, by defining $\mathbf{x}$ as a vector with K binary integer variables, where $x_k = 1$ and $x_k = 0$ indicate that tone k will be allocated to the sub-sets $\mathbb{Z}$ and $\mathbb{W}$, respectively, the problem of allocating tones to sub-sets $\mathbb{W}$ and $\mathbb{Z}$ such that the average data rate over 24 hours achieved over all tones is maximized while the data rate R_{req} can be guaranteed with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ at a BER that is smaller than or equal to the target BER can be formulated as the following binary integer optimization problem with $\mathbf{x}$ as an optimization parameter:

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \sum_k \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\mathbb{Z},\text{init-bl}} VN_{\mathbb{Z},k}}\right) x_k + \bar{b}_{\text{SRA},k}(1 - x_k) \\ \text{subject to} \quad & f_s \cdot \sum_k \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\mathbb{Z},\text{init-bl}} VN_{\mathbb{Z},k}}\right) x_k \geq R_{\text{req}}. \end{aligned} \tag{5.1}$$

The first term in the summation in the objective function in (5.1) is the number of bits transmitted on the tones allocated to sub-set $\mathbb{Z}$ at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$. The second term in the summation in the objective function in (5.1) is the average number of bits transmitted over 24 hours on the tones allocated to sub-set $\mathbb{W}$ at a BER that is equal to the target BER. The constraint in (5.1) ensures that the data rate achieved on the tones in $\mathbb{Z}$ is at least equal to the data rate required by the user at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$. Note that the value of R_{req} is set by the DSL customer according to the Internet services he uses.

The fact that $\gamma_{\mathbb{Z},\text{init-bl}}$ and $VN_{\mathbb{Z},k}$ are functions of $\mathbf{x}$ makes the integer optimization problem (5.1) non-linear. For every $\mathbf{x}$, there exists a different SNR margin and a different

VN mask. Finding the optimal solution for (5.1) would therefore require trying out all possible combinations, which is not feasible since the number K of variables in the optimization parameter $\mathbf{x}$ is in the order of thousands in practical DSL systems. In the next section, a sub-optimal solution for problem (5.1) is presented.

5.3 Iterative Hybrid VN-SRA Approach

In this section, an iterative algorithm that finds a sub-optimal solution for problem (5.1) is presented. The Iterative Hybrid VN-SRA finds a sub-optimal allocation of the K used tones to the sub-sets $\mathbb{Z}$ and $\mathbb{W}$ such that the data rate is maximized while the constraint in problem (5.1) is satisfied.

The Iterative Hybrid VN-SRA starts with an empty sub-set $\mathbb{Z}$ and a full sub-set $\mathbb{W}$ containing the indexes of all K used tones. At every iteration, it moves only one tone from sub-set $\mathbb{W}$ to sub-set $\mathbb{Z}$ until the constraint in problem (5.1) is satisfied. At every iteration, the Iterative Hybrid VN-SRA calculates the objective function in problem (5.1) when each tone in sub-set $\mathbb{W}$ is moved separately to sub-set $\mathbb{Z}$. The tone that leads to the highest value of the objective function in (5.1) is moved permanently to sub-set $\mathbb{Z}$.

By defining OBJ_k as the value of the objective function in problem (5.1) when tone k is moved separately from sub-set $\mathbb{W}$ to sub-set $\mathbb{Z}$ and $\mathbf{OBJ}$ as the vector containing $\mathrm{OBJ}_k,\ k \in \mathbb{W}$, the Iterative Hybrid VN-SRA is shown in Algorithm 6.

Algorithm 6 Hybrid VN-SRA approach to find a sub-optimal tone allocation to the sub-sets $\mathbb{W}$ and $\mathbb{Z}$

1: Initialize $\mathbb{W} = \mathbb{K}$, $\mathbb{Z} = \emptyset$
2: **repeat**
3: **for** each $k \in \mathbb{W}$ **do**
4: Move k from $\mathbb{W}$ to $\mathbb{Z}$
5: Calculate $\gamma^{\text{dB}}_{\mathbb{Z},\text{init-bl}}$ and $VN^{\text{dB}}_{\mathbb{Z},k}$, $k \in \mathbb{Z}$
6: Calculate OBJ_k
7: Move k from $\mathbb{Z}$ to $\mathbb{W}$
8: **end for**
9: Find $k_{\max}$ such that $\text{OBJ}_{k_{\max}} = \max\limits_{\{k \in \mathbb{W}\}} \{\mathbf{OBJ}\}$
10: Move $k_{\max}$ from $\mathbb{W}$ to $\mathbb{Z}$
11: **until** $f_s \cdot \sum\limits_{k \in \mathbb{Z}} \log_2 \left(1 + \frac{|h_k|^2 s_k}{\Gamma_c \gamma_{\mathbb{Z},\text{init-bl}} VN_{\mathbb{Z},k}}\right) \geq R_{\text{req}}$
12: Sub-optimal allocation is complete with $\mathbb{W}$ and $\mathbb{Z}$

The SNR margin and VN mask in row 5 of Algorithm 6 are calculated from the tones in $\mathbb{Z}$ according to the approaches presented in Chapter 3 such that the data rate achieved over the tones in $\mathbb{Z}$ is maximized while a target outage probability is guaranteed .

Note that in practical DSL systems, the network provider would have to run Algorithm 6 only once for every value of R_{req} chosen by the customer.

5.4 Performance Analysis

5.4.1 Introduction

In this section, the performance of the Iterative Hybrid VN-SRA is analyzed and simulation results are shown. First, in Section 5.4.2, an overview of the approaches that being compared is presented. Then, in Section 5.4.3, the simulation parameters are stated. In Section 5.4.4, simulation results are shown.

5.4.2 Overview of Compared Approaches

This section gives an overview on the approaches that are being compared.

- **VN approach**. A VN mask and an SNR margin are applied over all tones as explained in Chapter 3. The target data rate achieved by the VN approach is the maximum data rate that can be achieved at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$.
- **SRA approach**. The target data rate is adapted to the channel SNR by using an SRA approach as shown in Chapter 4. During the adaptation, a BER that is smaller than or equal to the target BER cannot be guaranteed.
- **Hybrid VN-SRA approach**. The tones are allocated to sub-sets $\mathbb{W}$ and $\mathbb{Z}$ such that the average data rate achieved over all tones is maximized while the data rate R_{req} can be guaranteed with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ at a BER that is smaller than or equal to the target BER as shown in Algorithm 6.

5.4.3 Simulation Parameters

For the following simulations, the transmission and noise models presented in Chapter 2 are assumed. Furthermore, the average bit-loading $\bar{b}_{\text{SRA},k}$ over 24 hours of the victim line on tone k was determined from noise measurements that were generated with the noise model in Section 2.4. Moreover, the target outage probability $P_{\text{out}}^{\text{target}}$ of $7.3 \cdot 10^{-3}$ is chosen, which is a typical value in practical DSL systems [SKK12] and the parameters in Table 3.2 and Table 4.1 are assumed.

5.4.4 Simulation Results

This section presents simulation results to show the benefits of using the Hybrid VN-SRA approach.

Figures 5.1 and 5.2 show the average data rate achieved when using the Hybrid VN-SRA approach in a scenario where L2 is not implemented and in a scenario where L2 is implemented, respectively. The x-axis describes the data rate required by the DSL user at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$. The value of the x-axis can be set by the DSL user depending on the services he uses. The y-axis shows the average data rate over 24 hours. The black curves in Figures 5.1 and 5.2 are the average data rates achieved when using the Hybrid VN-SRA approach presented in Algorithm 6 where the values of R_{req} are the values on the x-axis. The magenta lines in Figures 5.1 and 5.2 illustrate the part of the average data rates achieved on the tones in $\mathbb{Z}$ using the VN approach, and the difference between the data rates on the black curve and the data rate on the magenta line is the part of the average data rate achieved on the tones in $\mathbb{W}$ using the SRA approach. Furthermore, the blue dashed line is the target data rate that would be achieved if the VN approach is applied over all the tones and the red dashed line is the average data rate that would result if SRA was used on all tones. Note that the data rates illustrated by those lines are not functions of the values on the x-axis and serve as reference data rates. In order to show in Figures 5.1 and 5.2 that the data rates illustrated by those lines are not functions of the values on the x-axis, the data rates are illustrated by dashed lines.

In case the DSL user does not use services that require a data rate at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$, the average data rate achieved by using the Hybrid VN-SRA approach will be equal to the average data rate that would result if SRA was used on all tones. Similarly, in case the DSL user uses only services that require a data rate at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$, the average data rate achieved by using the Hybrid VN-SRA approach will be equal the target data rate that would be achieved if the VN approach is applied over all the tones. Furthermore, the target data rate that would be achieved if the VN approach is applied over all the tones is the maximum data rate that a can be required by a user at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ and is therefore, the maximum value the x-axis can take. In other case, where the DSL user uses only some services that require a data rate at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$, the Hybrid VN-SRA approach will achieve a higher average data rate than the data rate that would be achieved by using the VN approach over all tones. For example, if the only services that the DSL user requires at a BER that is smaller than or equal to the target BER with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ are two HDTV channels requiring 24Mbit/s, the Hybrid VN-SRA approach will increase the data rate that would be achieved by using the VN approach over all tones by 11% when L2 is not implemented and by 26% when L2 is implemented as shown in Figures 5.1 and 5.2.

In conclusion, by combining the SRA approach with the VN approach, the Hybrid VN-SRA approach achieves the data rate R_{req} with the same requirements on QoS as VN approach, however, the overall data rate is increased. Moreover, the DSL user can customize the value of R_{req} according to the services he uses.

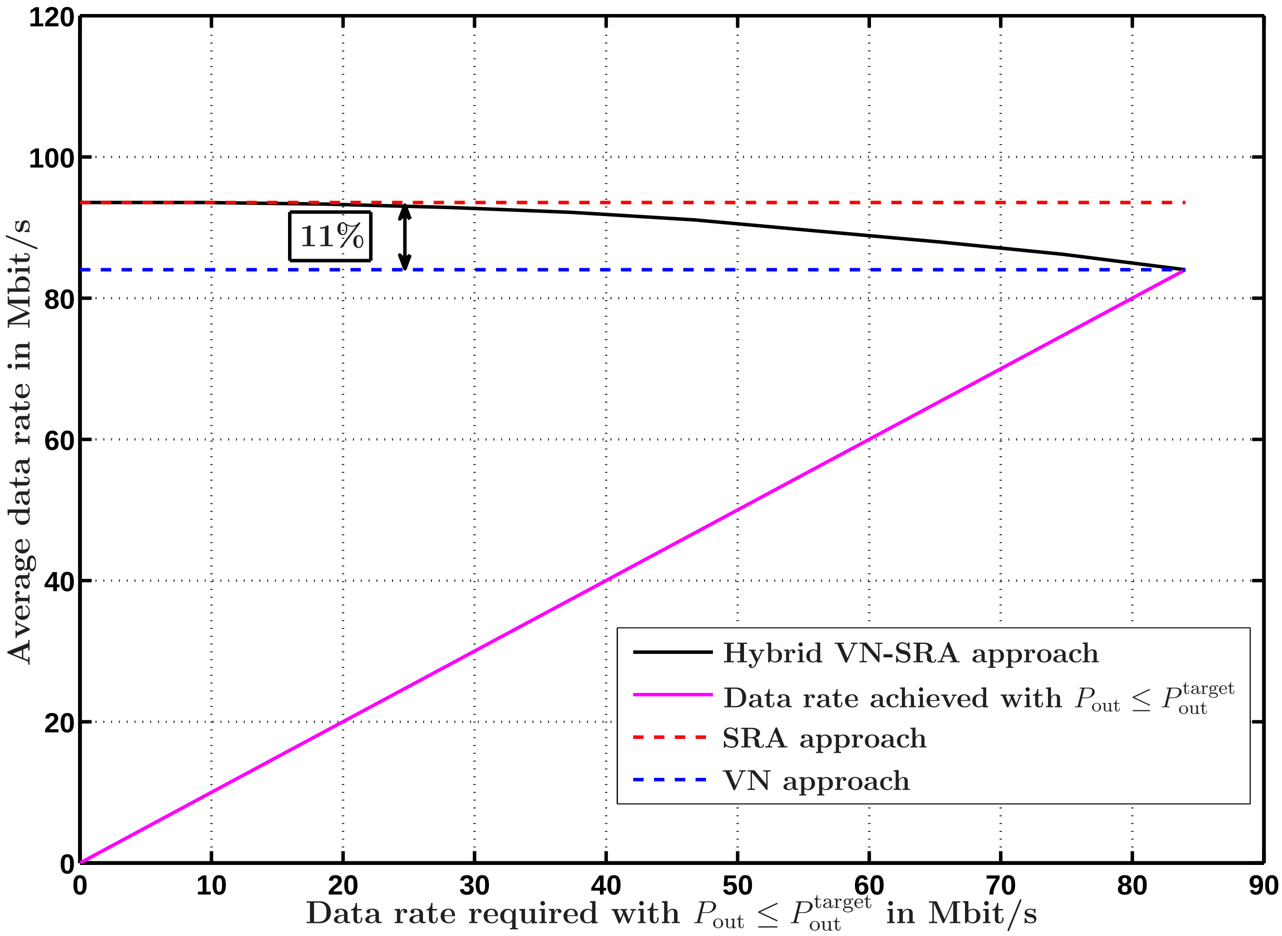


Figure 5.1: Average data rate achieved when using the Hybrid VN-SRA approach compared to the data rate achieved by using the VN approach and the SRA approach in a scenario where L2 is not implemented. Note that the dashed lines of the VN approach and the SRA approach are not functions of the values on the x-axis and serve as reference data rates

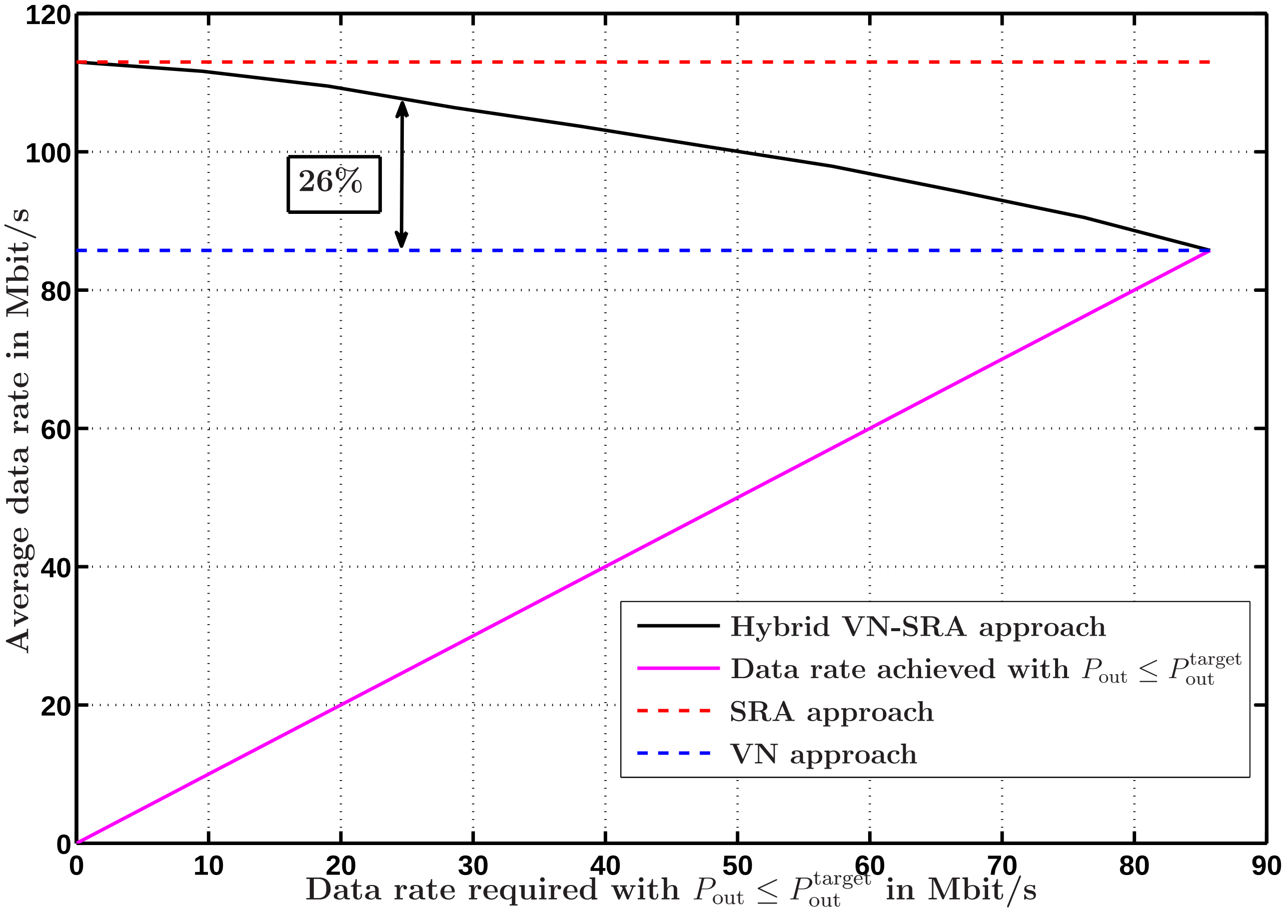


Figure 5.2: Average data rate achieved when using the Hybrid VN-SRA approach compared to the data rate achieved by using the VN approach and the SRA approach in a scenario where L2 is implemented. Note that the dashed lines of the VN approach and the SRA approach are not functions of the values on the x-axis and serve as reference data rates

Chapter 6
Conclusions

This thesis deals with the optimization of the receiver-based protection against time-varying noise in xDSL systems and investigates low-complex and practical approaches that can be easily implemented in today's existing access networks. Receiver-based protection is categorized into fixed data rate approaches and adaptive data rate approaches. The proposed fixed data rate approaches use statistics of noise variance on a DSL line to optimize the VN mask and SNR margin such that the target data rate is maximized while a certain assurance on QoS in terms of outage probability can be guaranteed. The proposed adaptive data rate approaches optimize the state of the art adaptation procedures such that the BER performance during the adaptation is improved while QoS constraints, in terms of the seamlessness of the adaptation, are satisfied.

In Chapter 1, the problem of time-varying noise in xDSL systems is introduced and the receiver-based protection against the time-varying noise is motivated. Then, an overview of current state-of-the-art is presented. Based on that, the open issues are identified and formulated. Finally, the main contributions and an overview of the thesis is provided.

In Chapter 2, first the considered DMT transmission model is introduced. Then, the significance of DSL user activity on the time-varying noise in xDSL systems is explained and a Markov model that describes the DSL user activity over 24 hours is introduced. The status of a DSL user is modeled by a three-state Markov chain and covers hereby the three operational modes L0, L2 and L3. The transition probabilities between the states are determined based on statistics from practical DSL systems. Furthermore, the daytime dependency of the user activity is modeled by daytime dependent transition probabilities to the states L0 and L2. Finally, a model for the time-varying non-stationary noise is presented. The model combines the user activity model with single FEXT measurements of single interferer lines for the generation of time-varying noise. Since the user activity models is based on statistics from practical DSL systems and the FEXT noise is generated from measurements, the resulting time-varying noise is a good emulation of the noise that occurs in practical DSL systems.

Chapter 3 addresses the optimization of fixed data rate approaches and proposes to jointly optimize the VN mask and the SNR margin in order to maximize the target data rate while satisfying a target outage probability. The most common and widely-used state of the art approach is to subtract a fixed SNR margin from the measured SNR during initialization, when calculating the data rate. The state of the art SNR margin does not satisfy the requirements of QoS. This problem is known as the "first and last user problem". In Chapter 3, in order to solve the "first and last user problem", first, the outage probability over 24 hours is defined as metric for QoS. It is then shown that in order to express the outage probability in terms of the VN mask and the SNR margin assigned at initialization, the adaptation functionality, in terms of SNR margin equalization carried out by bit-swapping has to be considered. By assuming the SNR margin that would result after margin equalization every time after the noise power changes, the outage probability is then derived as a function of the VN mask, the SNR margin assigned at initialization and the noise day maxima. From the expression of the outage probability, the VN mask and the SNR margin assigned at initialization that maximize the target data rate while satisfying a target outage probability are derived as functions of the CDFs of the noise day maxima. Moreover, expressing the VN mask and the SNR margin assigned at initialization by the noise day maxima allows the DSL provider to store only one value per tone per day for the estimation of the VN mask and the SNR margin instead of recording the noise spectrum over many days that would be necessary if the VN mask and SNR margin are expressed in terms of the daytime dependent noise power.

Another problem with the state of the art SNR margin is that it is tone-independent. The received noise power in practical DSL systems is tone-dependent and the noise power might increase at some tones such that the per tone SNR margins of the affected tones become negative. Moreover, bit-swapping might be too slow to move bits from the affected tones. This problem is known by the "unequal per-tone noise protection problem". In Chapter 3, the VN mask and SNR margin assigned at initialization are then modified in order to solve the "unequal per-tone noise protection problem" and hereby ensure a better robustness of the DSL modems against abrupt noise changes at single tones while maintaining the maximum target data rate achieved when assuming the SNR margin that would result after margin equalization every time the noise power changes. The per-tone outage probability is used as a metric for the per-tone noise protection. The VN mask and the SNR margin assigned at initialization that minimize the maximum per-tone outage probability are then found and it is shown that by minimizing the maximum per-tone outage probability, the probability of the line going to the state with negative per-tone SNR margins is minimized, and therefore, the reliance on bit-swapping to move away bits from the tones with negative per-tone SNR margins is reduced. Furthermore, an approximation for the SNR margin that can be determined in practical DSL systems only from measurements of noise day maxima is found.

Performance results show that because the proposed approach calculates, in contrast to the Trivial VN Approach that assumes a worst case noise mask, the target data rate as a function of the target outage probability, higher target data rates are achieved when compared to the data rate achieved by the Trivial VN Approach. Moreover, the proposed approach solves the "first and last user problem" and increases therefore the achievable target data rate when compared to the state of the art fixed SNR margin approach. This increase is tremendous in scenarios where L2 is implemented. Furthermore, by modifying the VN mask and SNR margin, the "unequal per-tone noise protection problem" is solved and the probability of the line going to the state with negative per-tone SNR margins is reduced. The performance degradation caused by the approximation of the SNR margin is small. Furthermore, the performance of the proposed approach is very close to the Optimal Reference Approach that requires major changes in the current VDSL2 standard and the development of new complex bit-swapping algorithms. However, in contrast to the Optimal Reference Approach, the proposed approach can be realized with the current VDSL2 standard.

Chapter 4 addresses the optimization of the seamless adaptation procedures in xDSL systems. First, a constraint on the amount of data rate allowed to be changed in an adaptation procedure by an adaptive data rate approach such that the approach is considered seamless is derived. Then, the adaptation time and the average BER are introduced as the performance measures that are considered in the optimization of the state of the art adaptive data rate approaches. It is then proposed that in case multiple adaptation procedures are required to adapt to the channel SNR while the seamlessness constraint is satisfied, instead of equalizing the error probabilities at all tones after each adaptation procedure which requires the modification of all tones within each adaptation procedure as done by the state of the art SRA, the average BER should be decreased as rapidly as possible after each adaptation procedure. Doing so allows the tone-by-tone modification of the bit-loadings to the target bit-loadings and the modification of the bit-loadings of the tones in groups, and thus, shortens the adaptation time tremendously. Furthermore, two new approaches that decrease the average BER as rapidly as possible in each adaptation step are presented. The Tone-by-Tone SRA chooses the tones to be modified to their target bit-loadings such that the average BER is decreased as rapidly as possible. The Group SRA modifies the bit-loadings of the tones in groups and finds the per group bit reduction that decrease the average BER as rapidly as possible. Moreover, low complex versions of the proposed approaches are presented.

Performance results show that all the proposed approaches shorten the adaptation time tremendously and decrease the number of errors that occur during the adaptation when compared to the state of the art SRA. Moreover, it is shown that the modification of the bit-loadings in groups leads to the fastest decrease in the average BER and therefore to the least occurred errors, however, the bit-loadings of the tones have to modified to the target bit-loadings using one final state of the art SRA procedure. Furthermore, the performances of the Low Complex Tone-by-Tone SRA and the Low Complex Group SRA are similar to the performances of the Tone-by-Tone SRA and the Group SRA, respectively, however, they do not require high computational power and can therefore be used by DSL modems in practice.

Chapter 5 proposes to combine the fixed and adaptive data rate approaches according to the Internet services that are used by the DSL user. Furthermore, it is proposed that the a VN approach should be used for services that require a data rate at a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability and that an SRA approach should be used for services that allow the varying of the data rate achieved at a BER that is smaller than the target BER and can tolerate temporary increases of the BER over the target BER. A hybrid VN-SRA approach is presented. The hybrid VN-SRA is an iterative approach that allocates tones to be used by the VN approach and tones to be used by the SRA approach such that the average data rate over 24 hours is maximized while a user specific constraint on the amount of data rate required at a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability is satisfied.

Performance results show that in any case where the DSL user uses only some services that require a data rate at a BER that is smaller than or equal to the target BER with an outage probability that is smaller than or equal to the target outage probability, the Hybrid VN-SRA approach will achieve a higher average data rate than the data rate that would be achieved by using the VN approach over all tones.

In this thesis, low-complex and practical approaches that can be easily implemented in today's existing access networks have been presented. Using the presented approaches will improve the data rate and QoS performance of today's networks.

List of Acronyms

DSL	Digital Subscriber Line
ADSL	Asymmetric Digital Subscriber Line
HDSL	High-bit-rate Digital Subscriber Line
VDSL	Very-high-bit-rate Digital Subscriber Line
ISDN	Integrated Services Digital Network
CP	Customer Premises
FEC	Forward Error Correction
DSLAM	DSL Access Multiplexer
DFT	Discrete Fourier Transform
IDFT	Inverse Discrete Fourier Transform
CO	Central Office
OC	Outdoor Cabinet
HDTV	High-Definition Television
3DTV	Three-Dimensional Television
MIMO	Multiple Input Multiple Output
VoIP	Voice over Internet Protocol
QoS	Quality of Service
VN	Virtual Noise
BER	Bit Error Rate
RFI	Radio Frequency Interference
NEXT	Near-End Crosstalk
FEXT	Far-End Crosstalk

L2	Low Power mode
L0	Operational mode
L3	Stand by mode
PSD	Power Spectral Density
DMT	Discrete Multi-Tone
DSM	Dynamic Spectrum Management
QAM	Quadrature Amplitude Modulation
SMC	Spectrum Management Center
SRA	Seamless Rate Adaptation
OLR	Online Reconfiguration
SOS	Save our Showtime
SNR	Signal-to-Noise Ratio
S/P	Serial to Parallel Converter
P/S	Parallel to Serial Converter
ISI	Inter Symbol Interference
FEQ	Frequency-Domain Equalizer
TU-CO	Transceiver Unit at the Central Office
TU-OC	Transceiver Unit at the Outdoor Cabinet
TU-CP	Transceiver Unit at the Customer Premises
PDF	Probability Density Function
CDF	Cumulative Distribution Function
LTS	Long Term Stability
STS	Short Term Stability
SoA	State of the Art

List of Symbols

K	Number of mutually orthogonal tones
k	Tone index
f_k	Center frequency of tone k
Δf	Tone spacing
N	Number of users whose DSL lines are in the same DSL binder
n	User index
$\boldsymbol{\theta}_{\mathrm{tr},k}$	Vector of N transmitted QAM symbols on tone k
$\theta_{\mathrm{tr},k}$	Transmitted QAM symbol on tone k
$\boldsymbol{\theta}_{\mathrm{rec},k}$	Vector of N received QAM symbols on tone k
$\theta_{\mathrm{rec},k}$	Received QAM symbol on tone k
$\mathbf{H}_k$	N-user-interference channel on tone k
$h_k^{(n,n)}$	Direct channel of user n on tone k
$h_k^{(n,m)}$	FEXT channel from user m to user n on tone k
$\mathbf{z}_k$	Vector of the background noises at the receivers of the N users
$(\sigma^{(n)})^2$	Variance of the background noise at the receivers of user n
$P_{\mathrm{L3L0}}(t)$	Transition probability between the states L3 and L0 over 24 hours
$P_{\mathrm{L0L2}}(t)$	Transition probability between the states L0 and L2 over 24 hours
P_{L0L3}	Transition probability between the states L0 and L3
P_{L2L0}	Transition probability between the states L2 and L0
P_{L2L3}	Transition probability between the states L2 and L3
$N_{\mathrm{ONrel}}(t)$	Average relative number of active DSL lines over a 24 hours
$N_{\mathrm{ONrel,max}}$	Maximum average relative number of active DSL lines
$N_{\mathrm{ONrel,min}}$	Minimum average relative number of active DSL lines
$N_{\mathrm{L2rel}}(t)$	Average fraction of active DSL users in L2 state over 24 hours
$N_{\mathrm{L2rel,max}}$	Maximum average fraction of active DSL users in L2 state
$N_{\mathrm{L2rel,min}}$	Minimum average fraction of active DSL users in L2 state
$P_{\mathrm{L3L0,max}}$	Transition probabilities from state L0 to L2 at the day time where $N_{\mathrm{ONrel,max}}$ occur
$P_{\mathrm{L3L0,min}}$	Transition probabilities from state L0 to L2 at the day time where $N_{\mathrm{ONrel,min}}$ occur

$P_{\text{L0L2,max}}$	Transition probabilities from state L0 to L2 at the day time where $N_{\text{L2rel,max}}$ occur
$P_{\text{L0L2,min}}$	Transition probabilities from state L0 to L2 at the day time where $N_{\text{L2rel,max}}$ occur
SNR_k	SNR on tone k
$SNR_{\text{init},k}$	SNR during initialization on tone k
γ_{init}	SNR margin assigned to all tones during initialization when only an SNR margin is used for the protection against noise
γ_c	Coding gain
Γ_c	SNR gap of the code
Γ	Uncoded SNR gap to the Shannon capacity
$b_{\text{init},k}$	Bit-loading on tone k during initialization when only an SNR margin is used for the protection against noise
M_k	Constellation size on tone k
f_s	DMT symbol rate
R_{init}	Target data rate when only an SNR margin is used for the protection against noise
VN_k	Value of the VN PSD at the receiver on tone k
$VSNR_k$	Virtual SNR on tone k
$VSNR_{\text{init},k}$	Virtual SNR during initialization on tone k
$VSNR_{\text{init-bl},k}$	Virtual SNR used for calculating the bit-loading during initialization on tone k
$VSNR_k^{\text{dB}}(t)$	Virtual SNR on tone k at time t
γ_{VNinit}	SNR margin assigned to all tones during initialization when a VN mask and an SNR margin are used for the protection against noise
$\gamma_{\text{VNinit-bl}}$	SNR margin used for calculating the bit-loading during initialization when a VN mask and an SNR margin are used for the protection against noise
$\gamma_{\text{VN},k}^{\text{dB}}(t)$	Per-tone SNR margin on tone k, before bit-swapping procedures start equalizing the margin at time t, when a VN mask and an SNR margin are used for the protection against noise
$\tilde{\gamma}_{\text{VN}}^{\text{dB}}(t)$	SNR margin that results after bit-swapping has equalized the margin at time t when a VN mask and an SNR margin are used for the protection against noise
$b_{\text{VNinit},k}$	Bit-loading on tone k during initialization when a VN mask and an SNR margin are used for the protection against noise

$x_{\text{init},k}$	Noise power experienced on tone k during initialization
$x_k(t)$	Noise power experienced on tone k at time t
R_{VNinit}	Target data rate when when a VN mask and an SNR margin are used for the protection against noise
$X_k(t)$	Time dependent random variable modeling the non-stationary noise power experienced on tone k over 24 hours
$\Upsilon_{\text{VN},k}(t)$	Time dependent random variable describing the variation in the per-tone SNR margin over 24 hours, when a VN mask and an SNR margin are used for the protection against noise
$\tilde{\Upsilon}_{\text{VN}}(t)$	Time dependent random variable describing variation in the equivalent SNR margin over 24 hours, when a VN mask and an SNR margin are used for the protection against noise
P_{out}	Outage probability
$P_{\text{out}}^{\text{target}}$	Target outage probability
$Y_{\max,k}^{\text{dB}}$	Random variable modeling the maximum noise power experienced on tone k over 24 hours
$F_{Y_{\max,k}^{\text{dB}}}$	CDF of $Y_{\max,k}^{\text{dB}}$
$F_{Y_{\max,k}^{\text{dB}}}^{-1}$	Inverse of the CDF of $Y_{\max,k}^{\text{dB}}$
$\mathcal{Y}_{\max,k}^{\text{dB}}$	Realization of $Y_{\max,k}^{\text{dB}}$
J	Random variable modeling the sum of $Y_{\max,k}^{\text{dB}}$ $k = 1, \cdots, K$
F_J	CDF of J
F_J^{-1}	Inverse of the CDF of J
$\mathcal{J}$	Realization of J
$P_{0,1}$	Value of the CDF at the 0,1% percentile (=0.001)
$P_{\text{out},k}(t)$	Per-tone outage probability at tone k at time t
$P_{\text{out},k}^{\text{init-bl}}$	Per-tone outage probability at tone k during initialization
$Y_{\max,k}\|J$	Random variable describing the randomness in $Y_{\max,k}^{\text{dB}}$ given J
$Cov(Y_{\max,k}^{\text{dB}}, J)$	Covariance between $Y_{\max,k}^{\text{dB}}$ and J
$\sigma_{Y_{\max,k}^{\text{dB}}}$	Standard deviation $Y_{\max,k}^{\text{dB}}$
σ_J	Standard deviation of J
$\rho_{Y_{\max,k}^{\text{dB}},J}$	Correlation coefficient between $Y_{\max,k}^{\text{dB}}$ and J
α_k	Parameter of the linear model for $Y_{\max,k}\|J$ on tone k
β_k	Parameter of the linear model for $Y_{\max,k}\|J$ on tone k

ϵ_k	Random variable that models the error of the linear model for $Y_{\max,k}\|J$ on tone k
$DV_{\max}$	Maximum delay variation
R_{high}	Higher data rates in the rate adaption procedure
R_{high}	Higher data rates in the rate adaption procedure
ΔR	$R_{\text{high}} - R_{\text{low}}$
$\Delta R_{\max}$	ΔR allowed to be changed in a rate adaptation procedure such that $DV_{\max}$ is not exceeded
γ_{SRA}	SNR margin applied when a seamless rate adaptation procedure is used
b_k	bit-loading on tone k at the end of the adaptation to the new SNR
$b_k^\star$	bit-loading on tone k at the beginning of the adaptation to the new SNR
R_{SRA}	Target data rate after the adaptation to the new SNR
N_{TG}	Number of tone groups
g	Group index
G_g	Number of tones in group g
Δb_g	Bit-loading reduction of the tones in g
v	Tone index within a Tone group
$b_{\text{SOS},g,v}$	Bit-loading of the vth tone in group g after an SOS procedure
R_{SOS}	Target data rate after an SOS procedure
i	SRA procedure index
N_{sra}	Number of SRA procedures required to adapt to the new SNR
$Q(\cdot)$	Gaussian Q function
$\lceil\cdot\rceil$	Ceiling function
$\text{sgn}(\cdot)$	Sign function
$N_{\text{T},i}$	Number of tones modified by SRA procedure i
n_{sos}	Number of groups modified by an SOS procedure
T_{meas}	Measurement time
T_{cal}	Calculation time
T_{pr}	Processing Time
T_{syn}	Time between receiving the acknowledgment and the beginning of transmission with the new parameters
T_{SRAreq}	Transmission time of an SRA request

T_{SOSreq}	Transmission time of an SOS request
T_{ack}	Transmission time of an acknowledgment
$T_{\text{sra},N_{\text{T},i}}$	Adaptation time of SRA procedure i
T_{adapt}	Adaptation time to the new SNR
T_{ss}	Time between modification of subsequent tine groups
$T_{\text{sos},n_{\text{sos}}}$	Adaptation time of an SOS procedure modifying n_{sos} tone groups
SER_k	Symbol error rate at tone k
BER_k	Bit error rate at tone k
$\overline{\text{SER}}_k$	Symbol error rate at tone k before the FEC decoder
$\overline{\text{BER}}_k$	Bit error rate at tone k before the FEC decoder
$\overline{\text{BER}}_{\text{avg}}$	Average bit error rate at tone k before the FEC decoder
$\overline{\text{BER}}_{\text{avg0}}$	Average bit error rate at tone k before the FEC decoder before a rate adaptation procedure
$\overline{\text{BER}}_v^{\star}$	Bit error rate at tone k before the FEC decoder before a rate adaptation procedure
$\bar{b}_{\text{SRA},k}$	Average bit-loading over 24 hours on tone k
R_{req}	Data rate that can be guaranteed with an outage probability $P_{\text{out}} \leq P_{\text{out}}^{\text{target}}$ at a BER that is smaller than or equal to the target BER
$\mathbb{K}$	Set of all tone indexes
$\mathbb{U}$	Set of indexes of tones that have already been modified during the adaptation to the new SNR
$\mathbb{V}$	Set of indexes of tones that have not yet been modified during the adaptation to the new SNR
$\mathbb{Z}$	Set of tone indexes of the tones used by the VN approach
$\mathbb{W}$	Set of tone indexes of the tones used by the SRA approach

Bibliography

[AADEB12] A. Awasthi, N. Al-Dhahir, O. E. Eliezer, and P. T. Balsara, "Alien crosstalk mitigation in vectored DSL systems for backhaul applications," in *Proc. IEEE International Conference on Communications (ICC)*, June 2012.

[ANS92] ANSI, "Integrated Services Digital Network (ISDN)- basic access interface for use on metallic loops for application on the network side of NT," American National Standards Institute (ANSI), Tech. Rep. T1. 601, 1992.

[ANS01] ——, "Spectrum Management for Loop Transmission Systems," American National Standards Institute (ANSI), Tech. Rep. T1. 417, 2001.

[BDAB12] E. Bedeer, O. Dobre, M. Ahmed, and K. Baddour, "Joint optimization of bit and power allocation for multicarrier systems with average BER constraint," in *Proc. IEEE Vehicular Technology Conference (VTC Fall)*, September 2012.

[BF10] N. H. Bingham and J. M. Fry, *Regression: Linear Models in Statistics*. Springer, 2010.

[BGL+11] C. Bianco, G. Griffa, N. Lee, P. Gemma, and Z. Bin, "The energy saving evaluation of green DSL while in-field mass implementation," in *Proc. IEEE 33rd International Telecommunications Energy Conference (INTELEC)*, Oct 2011.

[CCLJ08] H. Cioffi, J .M and Zou, A. Chowdhery, W. Lee, and S. Jagannathan, "Greener copper with dynamic spectrum management," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, 2008.

[CFM+03] N. Caouras, M. Freda, F. Monfet, V. Aldea, O. Naeem, T. Le-Ngoc, and B. Champagne, "Performance evaluation platform for xDSL deployment in a complex multi-segment environment," in *Proc. IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)*, May 2003.

[CG02] J. M. Cioffi and G. Ginis, "Vectored transmission for digital subscriber line systems," *Journal on Selected Areas in Communications*, pp. 1085–1104, 2002.

[Cho98] J. Chow, "Superframe-based bit allocation algorithms for DMT systems," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, vol. 5, 1998.

[Cio91] J. M. Cioffi, "A multicarrier primer," American National Standards Institute (ANSI), Tech. Rep. T1E1.4, 1991.

[Cio98] ——, "The essential merit of bit-swapping," American National Standards Institute (ANSI), Tech. Rep. T1E1.4/98-318R1, 1998.

[Cio08] ——, "EE379 course reader [online]," Stanford University, Tech. Rep., 2008.

[CMGG98] M. Colin, C. Modlin, M. Gharbi, and M. Gazalet, “QoS considerations for DMT-based ADSL and VDSL systems,” in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, May 1998.

[DQH+06] S. E. M. Dudley, T. Quinlan, I. Henning, S. D. Walker, R. Davey, A. Wallace, I. Boyd, and D. PAYNE, “Low-power optical feeder for VDSL over twisted pair for last mile access networks,” *Journal of Lightwave Technology*, vol. 24, pp. 65–70, January 2006.

[FMMG11] A. Forouzan, M. Moonen, J. Maes, and M. Guenach, “Joint level 2 and 3 dynamic spectrum management for upstream VDSL,” *IEEE Transactions on Communications*, vol. 59, no. 10, pp. 2851–2861, 2011.

[GDJ05] P. Golden, H. Dedieu, and K. Jacobsen, *Fundamentaly of DSL Technology*. Auerbach Publications, 2005.

[Gin05] G. Ginis, “Low-power Modes for ADSL2 and ADSL2+,” Texas Instruments, Tech. Rep., 2005.

[GVK01] S. Galli, C. Valenti, and K. Kerpez, “A frequency-domain approach to crosstalk identification in xDSL systems,” *IEEE Journal on Selected Areas in Communications*, vol. 19, no. 8, pp. 1497–1506, August 2001.

[GY08] D. Z. G. Young, “ADSL2plus configuration guidelines for IPTV,” The Broadband Forum, Tech. Rep. TR-176, 2008.

[Has10] K. S. Hassan, “Unequal error protection adaptive modulation in multicarrier systems,” Ph.D. dissertation, Jacobs University Bremen, Shaker Verlag, November 2010.

[HGC+06] J. Homer, M. Gujrathi, R. Cendrillon, I. V. L. Clarkson, and M. Moonen, “Adaptive nlms partial crosstalk cancellation in digital subscriber lines,” in *Proc. IEEE Asilomar Conference on Signals, Systems and Computers (ACSSC)*, October 2006.

[HH06] K. Hassan and W. Henkel, “OFDM (DMT) bit and power loading for unequal error protection,” in *Proc. IEEE OFDM-Workshop*, August 2006.

[HHvD+10] W. Henkel, K. Hassan, N. von Deetzen, S. Sandberg, L. Sassatelli, and D. Declercq, “UEP concepts in modulation and coding,” *Hindawi, Advances in Multimedia*, June 2010.

[HLLN12] S. Huberman, C. Leung, and T. Le-Ngoc, “Dynamic spectrum management (dsm) algorithms for multi-user xDSL,” *IEEE Communications Surveys Tutorials*, vol. 14, 2012.

[IT11] ITU-T, “Very high speed digital subscriber line transceivers 2 (VDSL2),” International Telecommunication Union-Telecommunication (ITU-T), Tech. Rep. G.993.2, 2011.

[Jag08] S. Jagannathan, "Interference and outage optimization in multi-user multi-carrier communication systems," Ph.D. dissertation, Stanford University, 2008.

[JHC08a] S. Jagannathan, C. S. Hwang, and J. M. Cioffi, "Per-tone margin optimization in multi-carrier communication systems," *in Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2008.

[JHC08b] S. Jagannathan, C.-S. Hwang, and J. Cioffi, "Margin optimization in digital subscriber lines employing level-1 dynamic spectrum management," in *Proc. IEEE International Conference on Communications (ICC)*, May 2008.

[KBP+13] I. Kateeb, L. Burton, M. Peluso, P. Chopade, and K. Alotaibi, "Copper cables: Should they be phased out as last mile broadband solutions?" in *Proc. IEEE Southeastcon*, April 2013.

[LLD+08] N. Lindqvist, F. Lindqvist, B. Dortschy, E. Pelaes, and A. Klautau, "Impact of crosstalk estimation on the dynamic spectrum management performance," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, 2008.

[MC06] C. S. Modlin and C. Chase, "Dynamic interleaver depth change for general convolutional interleaver," United States Patent, Tech. Rep. US 2006/0153285 A1, 2006.

[MFPA09] G. Maier, A. Feldmann, V. Paxson, and M. Allman, "On dominant characteristics of residential broadband internet traffic," *Intelligent Interactive Assistance and Mobile Multimedia Computing*, 2009.

[MTM11] R. Moraes, P. Tsiaflakis, and M. Moonen, "Dynamic spectrum management in DSL with asynchronous crosstalk," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2011.

[PFG07] K. Patel, S. Francois, and M. Gagnaire, "Characterization of a new copper cable for next generation dsl systems," in *Proc. IEEE. International Symposium on Advanced Networks and Telecommunication Systems (ANTS)*, December 2007.

[Pop98] A. Popovici, "Fast measurement of bit error rate and error probability estimation in digital communication systems," in *Proc. IEEE Global Telecommunications Conference. The Bridge to Global Integration*, 1998.

[Pro89] J. G. Proakis, *Digital Communications.* McGraw-Hill, 1989.

[PS99] J. G. Proakis and M. Salehi, *Understanding Digital Subscriber Line Technology.* Prentice Hall PTRR, Upper Saddle River, NJ, USA, 1999.

[PS12] M. H. P. Silverman, "Achieving Quality IPTV over DSL," The Broadband Forum, Tech. Rep., March 2012.

[RWM+06] R.Cendrillon, W.Y, M.Moonen, J.Verlingen, and T.Bostoen, "Optimal multiuser spectrum balancing for digital subscriber lines," *IEEE Transactions on Communications*, pp. 922–933, 2006.

[SBB13] N. Skaljo, A. Begovic, and N. Behlilovic, "An example of improvement of access network triple-play ability by local loop shortening," in *Proc. 21st Telecommunications Forum (TELFOR)*, November 2013.

[SCGC02] K. B. Song, S. T. Chung, G. Ginis, and J. Cioffi, "Dynamic spectrum management for next-generation DSL systems," *IEEE Communications Magazine*, October 2002.

[SK13] W. Sarhan and A. Klein, "Optimization of the rate adaptation procedures in xDSL systems," in *Proc. IEEE 5th International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP)*, December 2013.

[SK14a] ——, "Combining virtual noise and seamless rate adaptation for user-specific requirements on quality of service in xDSL systems," in *Proc. IEEE 17th International OFDM Workshop 2012 (InOWo'14)*, 2014.

[SK14b] ——, "Tone grouping for improved rate adaptation procedures in xDSL systems," in *Proc. IEEE 6th International Symposium on International Symposium on Communications, Control, and Signal Processing (ISCCSP)*, 2014.

[SKK12] W. Sarhan, A. Klein, and M. Kuipers, "Jointly optimizing the virtual noise mask and the snr margin for improved service in xDSL systems," in *Proc. IEEE International Conference on Communications (ICC)*, June 2012.

[SKK13] W. Sarhan, M. Kuipers, and A. Klein, "Optimization of the per tone noise protection in xDSL systems employing virtual noise," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2013.

[SSMS03] T. Starr, M. Sobara, J. M.Cioffi, and P. J. Silverman, *DSL Advances.* Prentice Hall PTRR, Upper Saddle River, NJ, USA, 2003.

[ST00] A. Shalash and H. Takatori, "Theoretical and practical limits of next-generation high-speed digital subscriber loops," *IEEE Transactions on Communications*, vol. 48, September 2000.

[TTPH94] M. Tzannes, M. Tzannes, J. Proakis, and P. Heller, "DMT systems, dwmt systems and digital filter banks," in *Proc. IEEE International Conference on Communications (ICC)*, May 1994, pp. 311–315 vol.1.

[Van01] R. J. Vanderbei, *Linear Programming: Foundations and Extensions.* Springer-Verlag, 2001.

[VB05] J. Verlinden and D. Bruyssel, "Virtual noise mechanism," American National Standards Institute (ANSI), Tech. Rep. NIPP-NAI-2005-049, 2005.

[VVdBB04] J. Verlinden, E. Van den Bogaert, and T. Bostoen, "Protecting the robustness of ADSL and VDSL DMT modems when applying dsm," in *Proc. IEEE International Zurich Seminar on Communications (IZS)*, 2004.

[WAK+08] P. Whiting, A. Ashikhmin, G. Kramer, C. Nuzman, A. van Wijngaarden, M. Zivkovic, M. Peeters, M. Guenach, J. Maes, and J. Verlinden, "DSL crosstalk coefficient acquisition using SNR feedback," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, November 2008.

[WKHN10] A. Wakeel, D. Kronmueller, W. Henkel, and H. B. Neto, in *Leaking Interleavers for UEP Turbo Codes*. Brest France: 6th International Symposium on Turbo Codes and Iterative Information Processing, September 2010.

[WKK12] D. Wurtz, A. Klein, and M. Kuipers, "Distributed margin optimization using spectrum balancing in multi-user DSL systems," in *Proc. IEEE 21st 20th European Signal Processing Conference (EUSIPCO)*, August 2012.

[WLK05] A. M. Wyglinski, F. Labeau, and P. Kabal, "Bit loading with BER-constraint for multicarrier systems," *IEEE Transactions on Wireless Communications*, July 2005.

[WTN99] X. Wang, T. Tjhung, and C. Ng, "Modelling and analysis of error probability performance for DMT-based DSL system with impulse noise and crosstalk," *Journal of Communications and Networks*, December 1999.

[XSL08] T. Xue, H. Sorbara, and H. Lin, "Method and system for dynamic interleaver adaptation scheme in VDSL," United States Patent, Tech. Rep. US 7,466,758 B2, 2008.

[Yse04] G. Ysebaert, "Equlization and echo cancellation in DMT-based systems," Ph.D. dissertation, Katholieke Universiteit Leuven, 2004.

Lebenslauf

Name:	Wagih Sarhan
Anschrift:	Holzhofallee 32, 64295 Darmstadt
Geburtsdatum:	25.08.1983
Geburtsort:	Elgiza
Familienstand:	Verheiratet

Schulausbildung

09.90-06.93	Grundschule in Misr Language School
09.93-05.02	Gymnasium in Deutsche Evangelische Oberschule in Kairo
Schulabschluss	Abitur

Studium

10.02-03.07	Studium der Elektrotechnik an der Technischen Universität Darmstadt, Studienabschluß: Bachelor of Engineering
04.07-02.09	Studium der Elektrotechnik an der Technischen Universität Darmstadt, Studienabschluß: Master of Science

Berufstätigkeit

seit 10.09	Wissenschaftlicher Mitarbeiter am Fachgebiet Kommunikationstechnik, Institut für Nachrichtentechnik, Technische Universität Darmstadt

Erklärung laut §9 der Promotionsordnung

Ich versichere hiermit, dass ich die vorliegende Dissertation allein und nur unter Verwendung der angegebenen Literatur verfasst habe. Die Arbeit hat bisher noch nicht zu Prüfungszwecken gedient.

Darmstadt, 14. Januar 2008,