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Interference Alignment in Partially Connected Multi-User Two-Way Relay Networks

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***Abstract*—In this paper, a partially connected ad-hoc network with relays is considered. Partially connected means that not all nodes are connected to all relays, but each node may be connected to one or multiple relays. This leads to multiple partially connected subnetworks, where each subnetwork includes a single relay and all nodes connected to this relay. The most challenging part of such a partially connected network is the handling of the nodes which are connected to multiple relays. In this paper, a new closed-form solution to achieve interference alignment in partially connected networks with relays is proposed. It is shown that local channel state information (CSI) is sufficient to perform interference alignment in such a network. The properness condition for the proposed algorithm is derived using the method of counting the dimensions of signal spaces. The new algorithm to perform interference alignment is decomposed into what we call simultaneous signal alignment, simultaneous channel alignment and transceive zero forcing. The simulation results show that the degrees of freedom increase for the considered network in comparison with our reference scheme.**

I. Introduction

Interference is and remains a fundamental issue in wireless communication networks. The phenomenon of interference appears in all networks where multiple users are sharing the same resources [1]. A conventional method to avoid interference is the orthogonalization of the communication links [2], [3]. Another promising technique to mitigate interference and to approach the capacity limits of wireless networks was proposed in [1] and is well known as interference alignment (IA). The concept of IA is to align multiple interference signals at a receiver in a single subspace of a dimension being smaller than the number of interferers. IA can be achieved by dividing the whole receive space into two subspaces, the useful subspace (US) and the interference subspace (IS). At each receiver, all interference signals should be aligned in the IS and the US contains only the useful signal. This means that IA can maximize the achievable degrees of freedom (DoF) [1].

The impact of relays in interference networks has been investigated, e.g., in [4]. It has been shown that the use of relays does not increase the DoF, however relays can help to achieve an IA solution which maximizes the DoF. Relay aided IA considering the two-way relaying protocol in a fully connected network was investigated in [5]–[8]. Fully connected means that all nodes are connected to all relays. The two-way relaying protocol allows a bidirectional pair-wise exchange of data in two phases, the multiple access (MAC) phase and the broadcast (BC) phase [9]. To perform IA with a single relay, the concepts of signal alignment (SA), channel alignment (CA), and zero-forcing (ZF) were introduced in [5].

In practical communication systems, the assumption that all nodes are connected to all relays with similar signal strengths does not hold. Usually the received signals have quite different power levels due to physical phenomena, e.g., path loss or shadowing. Hence, the received signal at each node comprises three signal types: the desired signal, strong interference signals, and weak interference signals. Sufficiently weak links can be approximated by zero which results in networks with partial connectivity [2], [10]–[12]. Relays which assist the communication cannot increase the DoF in a fully connected network [4], but it is conjectured in [4] that relays can improve the DoF if a network is partially connected. This conjecture was confirmed in [11] by simulations.

In [11], a communication pair which is connected to multiple relays is only served by a single relay, the other relays treat these pair's signals as interference and suppress them, which is suboptimal. In this paper we propose a new algorithm: A communication pair which is connected to multiple relays will be served by these multiple relays exploiting that all connected relays can receive and retransmit the useful signal of this communication pair. To perform IA in this partially connected network, we propose an approach to decouple the process of IA into three linear problems: simultaneous signal alignment (SSA), simultaneous channel alignment (SCA) and transceive zero forcing, where simultaneous means that nodes which are connected to multiple relays have to fulfill the alignment conditions to all connected relays simultaneously.

The present paper is organized as follows: Section II introduces the system model for a partially connected network. In Section III, the proposed IA algorithm is presented. In Section IV the performance of the proposed algorithm is compared with a reference scheme with respect to the sum rate and the DoF. Section V concludes this paper.

Notation: In the following, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $\mathbb{C}$ represent the set of complex numbers. $(.)^*$, $(.)^{\mathrm{T}}$, $(.)^{\mathrm{H}}$, $(.)^{-1}$, denote the complex conjugate, transpose, the complex conjugate transpose and the inverse of the element inside the brackets, respectively. $\mathbf{I}_N$ denotes an $N \times N$ identity matrix. The Frobenious norm of $\mathbf{A}$ is denoted by $\|\mathbf{A}\|_{\mathrm{F}} = \sqrt{\mathrm{Tr}\left(\mathbf{A}^{\mathrm{H}}\mathbf{A}\right)}$. The trace of a matrix is denoted by $\mathrm{Tr}(.)$. $|\mathcal{K}|$ denotes the cardinality of

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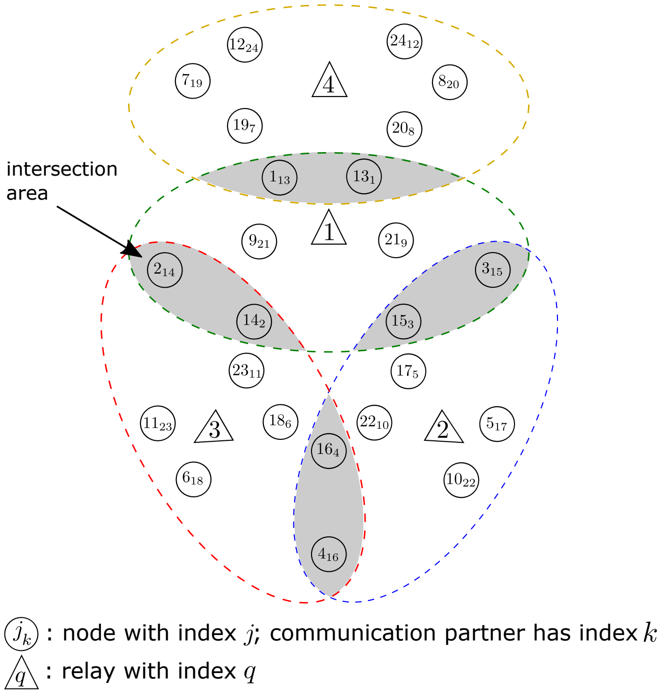

Fig. 1. Partially connected network with $K = 12$ communication pairs and $Q = 4$ subnetworks. The grey areas represent intersections of two subnetworks. Nodes inside the intersection areas are connected to two relays, all other nodes are connected to a single relay.

$\mathcal{K}$. $\mathbb{E}[.]$ denotes the expectation of the element inside the brackets. The null space of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is given by $\text{null}(\mathbf{A}) = \{\mathbf{x} \in \mathbb{C}^m : \mathbf{A}\mathbf{x} = \mathbf{0}\}$. The span of a matrix $\mathbf{A} \in \mathbb{C}^{n\times m}$ is denoted by $\text{span}(\mathbf{A}) = \{\mathbf{A}\mathbf{x} : \mathbf{x} \in \mathbb{C}^m\}$.

II. System Model

In this paper, a network consisting of multiple partially connected subnetworks is considered. The K multi-antenna node pairs communicate bidirectionally with the help of Q multi-antenna amplify-and-forward half-duplex relays. It is assumed that each subnetwork contains only a single relay, i.e., the number of relays is equal to the number of subnetworks. Due to the partial connectivity, only a subset of node pairs is connected to each relay. An example for $K = 12$ and $Q = 4$ is shown in Figure 1. It is assumed that always both nodes of a communication pair are connected to a single relay. In such partially connected networks, one can distinguish between two different types of nodes. Nodes of the first type are only connected to a single relay and therefore belong to a single subnetwork. Nodes of the second type are connected to more than one relay, thus belong to multiple subnetworks and are located in the so-called intersection area. The two-way relaying protocol [9] is exploited for the bidirectional communication between the communication partners. This bidirectional communication is carried out in two phases, called MAC phase and BC phase. Two-way relaying allows all nodes to simultaneously transmit to the relay in the MAC phase and the relays to retransmit back to all nodes in the BC phase. The direct links between the half-duplex nodes are irrelevant, because all nodes are transmitting or receiving simultaneously. In the MAC phase, each relay receives signals from all nodes in its subnetwork. In the BC phase, each node which belongs to a single subnetwork receives, beside its useful signal and self-interference, only interference from this subnetwork. Nodes inside the intersection area receive interference from several subnetworks, i.e., from all relays connected to these nodes, in the BC phase. Communication pairs inside the intersection area receive also the useful signal via several relays, beside the interference.

Each relay $q \in \mathcal{Q} = \{1, ..., Q\}$ in subnetwork q is equipped with $R_q \geq 1$ antennas. The set of the $2K$ nodes in the whole network is given by $\mathcal{K} = \{1, ..., 2K\}$. Let (j, k), $j, k \in \mathcal{K}$ denote a communication pair, where the communication partner index is given by

$$k = \begin{cases} j + K, & \forall j \leq K, \\ j - K, & \forall j > K. \end{cases}$$

We assume that both nodes of a communication pair (j, k) are always connected to the same relays. Each node of the K communication pairs is equipped with $N_k, \forall k \in \mathcal{K}$ antennas and wants to transmit $d_k \leq N_k$ data streams to its intended communication partner.

The set of nodes which is connected to relay q is denoted by $\mathcal{K}(q)$, and $\mathcal{R}(k)$ is the set of relays which are connected to node k. For the scenario in Figure 1, two example sets are $\mathcal{K}(1) = \{1, 2, 3, 9, 13, 14, 15, 21\}$ and $\mathcal{R}(1) = \{1, 4\}$. From our assumption that both nodes of a communication pair (j, k) are connected to the same relays follows that $\mathcal{R}(k) = \mathcal{R}(j)$. The sets of all nodes and relays are given by

$$\mathcal{K} = \bigcup_{q\in\mathcal{Q}} \mathcal{K}(q),$$

$$\mathcal{Q} = \bigcup_{k\in\mathcal{K}} \mathcal{Q}(k),$$

respectively. For simplicity of the notation, we assume only an intersection of at maximum two subnetworks in this paper. However, the presented concepts can easily be extended to the general case. The sets $\mathcal{K}^{\wedge}(q)$ and $\mathcal{K}^{\cap}(q_1, q_2)$ of nodes denote the nodes which are only connected to relay q and the nodes inside the intersection area between q_1 and q_2, respectively.

Let $\mathbf{H}^{\text{sr}}_{j,q} \in \mathbb{C}^{R_q\times N_j}$ and $\mathbf{H}^{\text{rd}}_{q,j} \in \mathbb{C}^{N_j\times R_q}$ denote the quasi-static Rayleigh fading channel matrices for the MAC phase and the BC phase, respectively. It is assumed that channel matrices are of full rank and mutually independent. Further, $\mathbf{V}_j \in \mathbb{C}^{N_j\times d_j}$ denotes the linear precoder and $\mathbf{d}_j \in \mathbb{C}^{d_j\times 1}$ the data vector of node j. It is assumed that the transmit symbols are independent and identically distributed (i.i.d.), so that $\mathbb{E}[\mathbf{d}_j\mathbf{d}_j^{\text{H}}] = \mathbf{I}_{d_j}$ holds. The precoders are normalized such that the maximum transmit power constraint is fulfilled, i.e., $\|\mathbf{V}_j\|_F^2 \leq P_{\text{n,max}}$, where $P_{\text{n,max}}$ denote the maximum transmit power of each node. It is assumed that all nodes have the same maximum transmit power. The K nodes transmits independent data, i.e., $\mathbb{E}[\mathbf{d}_k\mathbf{d}_j^{\text{H}}] = \mathbf{0}, \forall k \neq j$. Let

$\mathbf{n}_{\mathrm{r},q} = \mathcal{CN}(0, \sigma^2_{\mathrm{r},q}) \in \mathbb{C}^{R_q \times 1}$ denote the noise at relay q and $\mathbf{n}_{\mathrm{n},k} = \mathcal{CN}(0, \sigma^2_{\mathrm{n},k}) \in \mathbb{C}^{N_k \times 1}$ denote the noise at node k, respectively. The components of the two noise vectors $\mathbf{n}_{\mathrm{r},q}$ and $\mathbf{n}_{\mathrm{n},k}$ are i.i.d. complex Gaussian random variables.

In the MAC phase, relay q receives a signal given by

$$\mathbf{r}_q = \sum_{k \in \mathcal{K}(q)} \mathbf{H}^{\mathrm{sr}}_{k,q} \mathbf{V}_k \mathbf{d}_k + \mathbf{n}_{\mathrm{r},q}. \tag{1}$$

After linear signal processing, the relay retransmits the received signal to all connected nodes. The processing matrix of relay q is denoted by $\mathbf{G}_q$ and is normalized such that the maximum transmit power constraint is fulfilled, i.e., $\|\mathbf{G}_q \mathbf{r}_q\|_F^2 \le P_{\text{r,max}}$. The maximum transmit power of relay q is denoted by $P_{\text{r,max}}$. It is assumed that all relays have the same maximum transmit power.

In the BC phase, node k receives the signal

$$\begin{aligned}\mathbf{y}_k = &\sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{j,q} \mathbf{V}_j \mathbf{d}_j + \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{k,q} \mathbf{V}_k \mathbf{d}_k \\ &+ \sum_{q \in \mathcal{R}(k)} \sum_{\substack{i \in \mathcal{K}(q), \\ i \neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i \mathbf{d}_i \\ &+ \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k},\end{aligned} \tag{2}$$

where nodes j and k are communication partners.

The first and the second term of (2) are the useful signal and the self-interference signal, respectively. The third term of (2) represents the unknown interference and the last two terms represent the effective noise at node k.

Throughout the paper it is assumed that the self-interference can be perfectly canceled. Let $\mathbf{U}_k^{\mathrm{H}} \in \mathbb{C}^{d_k \times N_k}$ denote the receive filter at node k. The estimated data vector at node k is given by

$$\begin{aligned}\hat{\mathbf{d}}_j = &\mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{j,q} \mathbf{V}_j \mathbf{d}_j \\ &+ \mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \sum_{\substack{i \in \mathcal{K}(q), \\ i \neq k,j}} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i \mathbf{d}_i \\ &+ \mathbf{U}_k^{\mathrm{H}} \left(\sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{n}_{\mathrm{r},q} + \mathbf{n}_{\mathrm{n},k} \right).\end{aligned} \tag{3}$$

To achieve an IA solution and to reliably decode the useful signal, it is necessary that the unknown interference signal and the useful signal are in linearly independent subspaces at the receivers, i.e., the US and the IS have to be linearly independent. The self-interference can be in the US or the IS. The US needs to have at least the dimension of the data vector. Then, the receive filter $\mathbf{U}_k^{\mathrm{H}}$ can be designed as a zero-forcing filter that suppresses all unknown interferences. This results in the following IA conditions,

$$\mathbf{\Lambda} = \mathbf{U}_k^{\mathrm{H}} \sum_{q \in \mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k} \mathbf{G}_q \mathbf{H}^{\mathrm{sr}}_{i,q} \mathbf{V}_i, \tag{4}$$

$$\mathbf{\Lambda} = \mathbf{0}, \quad \forall i \in \{i \in \mathcal{K}(q) : i \neq k, j\}, \tag{5}$$

$$\mathrm{rank}(\mathbf{\Lambda}) = d_j, \quad \forall i \in \{i \in \mathcal{K}(q) : i = j\}. \tag{6}$$

III. Proposed Algorithm

This section presents a closed-form solution to perform IA in partially connected networks. The most challenging part of such a partially connected network is the handling of the nodes inside the intersection area. Nodes which are only connected to a single relay perform SA and CA at this relay, as proposed in [5]. In [11], a communication pair inside an intersection area is only served by one relay, the other connected relays treat this pair's signals as interference and zero-force them. In [11], the nodes inside an intersection area perform SA and CA only in the subnetwork which contains the relay which serves this communication pair. The new algorithm proposed in this paper offers the possibility for the nodes inside the intersection area to perform SA and CA simultaneously in multiple subnetworks, this new technique is called SSA and SCA. In addition to [11], the number of communication pairs in the intersection area is not restricted to one. For simplicity of the notation, as mentioned in Section II, it is assumed that an intersection area consists of at most an intersection of two subnetworks, i.e, the nodes inside the intersection area will be served by two relays. To determine the properness conditions for the new algorithm, we propose a new method based on counting the dimensions of signal spaces in Section III-C.

A. MAC-Phase: Simultaneous Signal Alignment

In this section, the method proposed in [5] to perform SA is extended to SSA. Therefore, SA is explained first. During the MAC phase, relay q receives in total $\frac{1}{2}\sum_{i \in \mathcal{K}(q)} d_i$ data streams. To avoid inter-pair interference, the signals of all node pairs which are connected to relay $q \in \mathcal{Q}$ should be pair-wise aligned and linearly independent of each other pair's signals at relay q [5]. This means that each node designs its transmit filter such that the spanned d-dimensional subspaces of the communication pair $(j, k) \quad \forall j \neq k \quad j, k \in \mathcal{K}(q)$ are pairwise aligned in a subspace of the entire signal space at relay q. The SA condition to align the signals from communication pair (j, k) at relay q is given by

$$\mathrm{span}\left(\mathbf{H}^{\mathrm{sr}}_{j,q} \mathbf{V}_{j,q}\right) = \mathrm{span}\left(\mathbf{H}^{\mathrm{sr}}_{k,q} \mathbf{V}_{k,q}\right). \tag{7}$$

In order to satisfy (7), the subspaces spanned by $\mathbf{H}^{\mathrm{sr}}_{jq} \mathbf{V}_{j,q}$ and $\mathbf{H}^{\mathrm{sr}}_{kq} \mathbf{V}_{k,q}$ must intersect at relay q [5]. The SA condition requires that both communication pairs are equipped with the same number of antennas, i.e., $N_k = N_j$. Further, it is assumed that both nodes of a communication pair transmit the same number of data streams i.e., $d_k = d_j$. To determine the entire solution space of valid precoding matrices $\mathbf{V}_{j,q}, \forall j \in \mathcal{K}$, (7) has to be rewritten as a homogeneous linear system of equations given by

$$\underbrace{\begin{bmatrix} \mathbf{H}^{\mathrm{sr}}_{j,q} & -\mathbf{H}^{\mathrm{sr}}_{k,q} \end{bmatrix}}_{\mathbf{H}^{\mathrm{ss}}_{j,k,q}} \cdot \underbrace{\begin{bmatrix} \mathbf{A}_{j,q} \\ \mathbf{A}_{k,q} \end{bmatrix}}_{\mathbf{A}_q} = \mathbf{0}. \tag{8}$$

The solution space $\mathbf{A}_q$ is determined by taking the null space of $\mathbf{H}^{\text{ss}}_{j,k,q} \in \mathbb{C}^{R_q \times 2N_k}$, given by

$$\underbrace{\begin{bmatrix}\mathbf{A}_{j,q}\\ \mathbf{A}_{k,q}\end{bmatrix}}_{\mathbf{A}_q} = \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,q}\right). \tag{9}$$

The transmit filters of the communication pair are a subset of $\mathbf{A}_q$, given by

$$\begin{bmatrix}\mathbf{V}_{j,q}\\ \mathbf{V}_{k,q}\end{bmatrix} \subseteq \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,q}\right). \tag{10}$$

In the following the extension to SSA will be explained. If a communication pair (j, k) inside the intersection area shall additionally perform signal alignment at a second relay $\tilde{q} \in \mathcal{Q}\backslash\{q\}$, this pair has to fulfill a second signal aliment condition simultaneously to (7). This second condition is given by

$$\text{span}\left(\mathbf{H}^{\text{sr}}_{j,\tilde{q}}\mathbf{V}_{j,\tilde{q}}\right) = \text{span}\left(\mathbf{H}^{\text{sr}}_{k,\tilde{q}}\mathbf{V}_{k,\tilde{q}}\right). \tag{11}$$

The corresponding homogeneous linear system of equations is given by

$$\underbrace{\begin{bmatrix}\mathbf{H}^{\text{sr}}_{j,\tilde{q}} & -\mathbf{H}^{\text{sr}}_{k,\tilde{q}}\end{bmatrix}}_{\mathbf{H}^{\text{ss}}_{j,k,\tilde{q}}} \cdot \underbrace{\begin{bmatrix}\mathbf{A}_{j,\tilde{q}}\\ \mathbf{A}_{k,\tilde{q}}\end{bmatrix}}_{\mathbf{A}_{\tilde{q}}} = \mathbf{0}. \tag{12}$$

If the two solution spaces $\mathbf{A}_q$ and $\mathbf{A}_{\tilde{q}}$ have an intersection, i.e, $\mathbf{A}_q \cap \mathbf{A}_{\tilde{q}} \neq \emptyset$, it is possible to achieve signal alignment simultaneously at two different relays q and $\tilde{q}$. The condition under which a common solution space exists will be introduced in Section III-C. The common solution space $\mathbf{A}_q \cap \mathbf{A}_{\tilde{q}} = \mathbf{A}$ is given by

$$\underbrace{\begin{bmatrix}\mathbf{A}_{j}\\ \mathbf{A}_{k}\end{bmatrix}}_{\mathbf{A}} = \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,q}\right) \cap \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k,\tilde{q}}\right). \tag{13}$$

Equation (13) can be rewritten as

$$\underbrace{\begin{bmatrix}\mathbf{A}_{j}\\ \mathbf{A}_{k}\end{bmatrix}}_{\mathbf{A}} = \text{null}\left(\begin{bmatrix}\mathbf{H}^{\text{ss}}_{j,k,q}\\ \mathbf{H}^{\text{ss}}_{j,k,\tilde{q}}\end{bmatrix}\right), \tag{14}$$

taking into account the properties mentioned in [13]. The columns of $\mathbf{A}$ span a $2N_k - R_q - R_{\tilde{q}}$ dimensional solution space which fulfills (7) and (11). If a node is only connected to relay q, the spanned solution space is of dimension $2N_k - R_q$. The precoding filters $\mathbf{V}_j$ and $\mathbf{V}_k$ are chosen from the solution space as

$$\begin{bmatrix}\mathbf{V}_{j}\\ \mathbf{V}_{k}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_{j}\\ \mathbf{A}_{k}\end{bmatrix} \cdot \mathbf{\Phi}_{\text{MAC}}, \tag{15}$$

where $\mathbf{\Phi}_{\text{MAC}}$ is a matrix with d_j columns and rank d_j selecting one possible solution of the whole solution space.

B. BC-Phase: Simultaneous Channel Alignment

Channel alignment is performed in the BC phase and was proposed in [5]. Each node served by relay q designs its receive filter such that the effective channels of the communication pair (j, k) span the same subspace at relay q. This condition is extended to SCA in this section. Like SA and CA are dual problems [5], SSA and SCA are also dual problems. The channel alignment condition for the nodes which are in the set $\mathcal{K}^{\wedge}(q)$ is given by

$$\text{span}\left(\mathbf{H}^{\text{rdH}}_{q,j}\mathbf{U}_{j,q}\right) = \text{span}\left(\mathbf{H}^{\text{rdH}}_{q,k}\mathbf{U}_{k,q}\right). \tag{16}$$

If a communication pair is located inside the set $\mathcal{K}^{\cap}(q, \tilde{q})$, a second condition, given by

$$\text{span}\left(\mathbf{H}^{\text{rdH}}_{\tilde{q},j}\mathbf{U}_{j,\tilde{q}}\right) = \text{span}\left(\mathbf{H}^{\text{rdH}}_{\tilde{q},k}\mathbf{U}_{k,\tilde{q}}\right) \tag{17}$$

has to be fulfilled to perform SCA. Since SSA and SCA are dual problems, determining the solution space for SCA is dual to determining the SSA solution space. The channel alignment solution space is given by

$$\underbrace{\begin{bmatrix}\mathbf{B}_{j}\\ \mathbf{B}_{k}\end{bmatrix}}_{\mathbf{B}} = \text{null}\left(\begin{bmatrix}\mathbf{H}^{\text{ss}'}_{j,k,q}\\ \mathbf{H}^{\text{ss}'}_{j,k,\tilde{q}}\end{bmatrix}\right), \tag{18}$$

where $\mathbf{H}^{\text{ss}'}_{j,k,q} = \begin{bmatrix}\mathbf{H}^{\text{rdH}}_{q,j} & -\mathbf{H}^{\text{rdH}}_{q,k}\end{bmatrix} \in \mathbb{C}^{R_q \times 2N_k}$ and $\mathbf{H}^{\text{ss}'}_{j,k,\tilde{q}} = \begin{bmatrix}\mathbf{H}^{\text{rdH}}_{\tilde{q},j} & -\mathbf{H}^{\text{rdH}}_{\tilde{q},k}\end{bmatrix} \in \mathbb{C}^{R_{\tilde{q}} \times 2N_k}$. The columns of $\mathbf{B}$ span a $2N_k - R_q - R_{\tilde{q}}$ dimensional solution space which fulfills (16) and (17). If a node is only connected to relay q, the spanned solution space is of dimension $2N_k - R_q$. The two receive filters $\mathbf{U}^{\text{H}}_j$ and $\mathbf{U}^{\text{H}}_k$ are chosen from the solution space as

$$\begin{bmatrix}\mathbf{U}_{j}\\ \mathbf{U}_{k}\end{bmatrix} = \begin{bmatrix}\mathbf{B}_{j}\\ \mathbf{B}_{k}\end{bmatrix} \cdot \mathbf{\Phi}_{\text{BC}}, \tag{19}$$

where $\mathbf{\Phi}_{\text{BC}}$ is a matrix with d_j columns and rank d_j selecting one possible solution of the whole solution space.

The matrices $\mathbf{\Phi}_{\text{MAC}}$ of (15) and $\mathbf{\Phi}_{\text{BC}}$ of (19) can be optimized to maximize a given objective, e.g., the sum rate. Any selection of the matrices $\mathbf{\Phi}_{\text{MAC}}$ and $\mathbf{\Phi}_{\text{BC}}$ with d_j columns and rank d_j will lead to an IA solution. This optimization of the sum rate is left for future work.

C. Properness Condition

In this section the properness condition which has to be fulfilled to perform SSA and SCA is derived. We start with the derivation of the required number of antennas at each relay. The number of effective data streams at each relay results in the condition

$$R_q = \frac{1}{2}\sum_{i\in\mathcal{K}(q)} d_i, \tag{20}$$

for the number of required antennas at each relay.

The signal space at a node, has to be large enough such that the communication pair (j, k) can select a common subspace in the desired relay signal spaces q, if the node is inside the set $\mathcal{K}^{\wedge}(q)$, or at q and $\tilde{q}$, if the node is inside the set $\mathcal{K}^{\cap}(q, \tilde{q})$. This selection of a common subspace is necessary to perform SA and CA for the nodes inside $\mathcal{K}^{\wedge}(q)$ and to perform SSA and SCA for the nodes inside $\mathcal{K}^{\cap}(q, \tilde{q})$, respectively. The columns of $\mathbf{A}$ in Section III-A span a $2N_k - R_q$ dimensional solution space if a communication pair is inside the set $\mathcal{K}^{\wedge}(q)$ and a $2N_k - R_q - R_{\tilde{q}}$ dimensional solution space if a communication pair is inside the set $\mathcal{K}^{\cap}(q, \tilde{q})$. This results in the condition $2N_k - R_q - R_{\tilde{q}} \geq d_k$ to perform SSA and in the condition

$2N_k - R_q \geq d_k$ to perform SA. Hence, the required number of antennas at each node inside an intersection area is given by

$$N_k \geq \frac{R_q + R_{\tilde{q}} + d_k}{2}, \quad \forall k \in \mathcal{K}^{\cap}(q, \tilde{q}). \tag{21}$$

For nodes which are only connected to a single relay, the number of required antennas is given by

$$N_k \geq \frac{R_q + d_k}{2}, \quad \forall k \in \mathcal{K}^{\wedge}(q). \tag{22}$$

D. Transceive Zero Forcing

Transceive zero forcing is a combination of receive and transmit zero forcing [14]. Let $\mathbf{G}_q^{\mathrm{RX\,H}}$ and $\mathbf{G}_q^{\mathrm{TX}}$ denote the receive and transmit zero forcing matrices. The effective channels in the MAC and BC phase are given by (23) and (24), as shown on top of the next page. $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}$ is of dimension $R_q \times \frac{1}{2}\sum_{i\in\mathcal{K}(q)} d_i$ and $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}$ is of dimension $\frac{1}{2}\sum_{i\in\mathcal{K}(q)} d_i \times R_q$. Taking (20) into account results in square matrices $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}$ and $\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}$, which are non-singular with probability one. Then $\mathbf{G}_q^{\mathrm{RX\,H}}$ and $\mathbf{G}_q^{\mathrm{TX}}$ are uniquely determined by

$$\mathbf{G}_q^{\mathrm{RX\,H}} = \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}}\right)^{-1}, \tag{25}$$

$$\mathbf{G}_q^{\mathrm{TX}} = \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}\right)^{-1}. \tag{26}$$

The entire relay processing matrix is given by

$$\mathbf{G}_q = \beta \cdot \mathbf{G}_q^{\mathrm{TX}} \cdot \mathbf{G}_q^{\mathrm{RX\,H}} = \beta \cdot \left(\mathbf{H}_{\mathrm{eff}q}^{\mathrm{MAC}} \cdot \mathbf{H}_{\mathrm{eff}q}^{\mathrm{BC}}\right)^{-1}, \tag{27}$$

where β is determined such that the relay transmit power constraint is fulfilled.

E. Required CSI to Perform IA

Typically, relay aided IA algorithms in fully connected networks require global CSI [5], where global CSI means that all MAC and BC channels are known at all nodes and relays. In partially connected networks, this condition can be relaxed by exploiting the fact that some interference links among the subnetworks are missing [11].

For the proposed algorithm the nodes which are just connected to a single relay q require only pair-wise CSI to determine the transmit filter $\mathbf{V}_k$ and the receive filter $\mathbf{U}_k^{\mathrm{H}}$, see (7) and (16). Pair-wise CSI means that a communication pair has to know its own channel to the connected relay and the channel of its communication partner to the connected relay. This is the same amount of required CSI at the nodes to perform SA and CA as in [11]. The nodes inside the intersection area which perform SSA and SCA require multiple pair-wise CSI. Multiple pair-wise CSI means that a node has to know its own channels to all connected relays and the channels of its communication partner to all connected relays.

Relay q has to know the channels of all nodes inside its own subnetwork, referred to as subnetwork CSI, as well as the transmit and receive filter of all nodes in is own subnetwork. These filters depends on multiple pair-wise CSI, if a node pair is inside the intersection area. Hence, relay q requires more than just subnetwork CSI to determine the relay processing matrix $\mathbf{G}_q$, but less than global CSI. In summary, it can be said that the required CSI to perform interference alignment in a partially connected network is less than global CSI and we call it local CSI.

IV. Performance analysis

In this section, the sum rate performance together with the DoF of the proposed algorithm are analyzed. We define the DoF as the total number of data streams received by all K nodes without interference. To investigate our proposed SSA and SCA closed form solution for partially connected networks (SSCP_closed) we consider a scenario with $K = 12$ communication pairs distributed over $Q = 4$ subnetworks, as shown in Figure 1. Every subnetwork contains one relay equipped with $R_q = R = 4, \forall q \in \mathcal{Q}$ antennas, so that (20) is fulfilled. Each of the $2K$ nodes is equipped with $N_k = N = 5, \forall k \in \mathcal{K}$ antennas and wants to transmit $d_k = d = 1$ data streams to its communication partner, which fulfills (21) and (22). For the simulations it is assumed that the channels between the nodes and the relays are random i.i.d. Rayleigh fading channels [14]. The channel matrices are normalized such that the average received signal power is the same as the average transmit signal power. Furthermore, we assume channel reciprocity and that the channel coefficients are constant during the MAC and BC phase. The noise power at each node and at each relay is assumed to be the same for the simulation, i.e., $\sigma^2 = \sigma_k^2 = \sigma_q^2, \quad \forall k \in \mathcal{K}, \forall q \in \mathcal{Q}$.

The chosen reference scheme which performs SA and CA in a partially connected network (SCP_closed) was proposed in [11]. In [11], a communication pair which is connected to multiple relays is only served by a single relay. The other relays treat these signals as interference and suppress them. For the simulations, both algorithms choose an arbitrary signal and channel alignment solution from the entire solution space, i.e., the matrices $\mathbf{\Phi}_{\mathrm{MAC}}$ and $\mathbf{\Phi}_{\mathrm{BC}}$ are arbitrarily selected complying with the conditions in Section III-B. This has no influence on the achievable DoF.

Figure 2 shows the sum rate performance of the proposed "SSCP_closed" algorithm in comparison to the "SCP_closed" reference scheme as a function of $\frac{P}{\sigma^2}$, where $P = P_{\mathrm{n,max}}$ denotes the transmit power of each node. σ^2 is the noise power per antenna at each relay and at each node. The transmit power at each relay is adjusted to $P_{\mathrm{r,max}} = \frac{1}{Q} KP$.

The proposed IA algorithm "SSCP_closed", see the solid line in Figure 2, can serve $K = 12$ communication pairs simultaneously whereas the reference algorithm "SCP_closed", see the dashed line with the circles in Figure 2, can only serve $K = 8$ communication pairs simultaneously. The remaining 4 communication pairs can be served using TDMA (time division multiple access). Hence, the proposed "SSCP_closed" IA algorithm achieves more DoF for a given number of antennas at each relay. The reason for this is that the nodes inside the intersection area perform SSA and SCA instead of SA and CA like in the reference scheme.

If one additional antenna is added to each of the Q relays, the reference algorithm "SCP_closed" for $R = 5$ antennas,

$$\mathbf{H}_{\text{eff}q}^{\text{MAC}} = \left[\mathbf{H}_{x,q}^{\text{sr}}\mathbf{V}_x \quad \cdots \quad \mathbf{H}_{y,q}^{\text{sr}}\mathbf{V}_y\right], \quad x, y \in \left\{x, y : x, y \in \mathcal{K}(q); x \neq y; \text{span}\left(\mathbf{H}_{x,q}^{\text{sr}}\mathbf{V}_x\right) \neq \text{span}\left(\mathbf{H}_{y,q}^{\text{sr}}\mathbf{V}_y\right)\right\} \tag{23}$$

$$\mathbf{H}_{\text{eff}q}^{\text{BC}} = \begin{bmatrix} \mathbf{U}_x^{\text{H}}\mathbf{H}_{q,x}^{\text{rd}} \\ \vdots \\ \mathbf{U}_y^{\text{H}}\mathbf{H}_{q,y}^{\text{rd}} \end{bmatrix}, \quad x, y \in \left\{x, y : x, y \in \mathcal{K}(q); x \neq y; \text{span}\left(\mathbf{U}_x^{\text{H}}\mathbf{H}_{q,x}^{\text{rd}}\right) \neq \text{span}\left(\mathbf{U}_y^{\text{H}}\mathbf{H}_{q,y}^{\text{rd}}\right)\right\} \tag{24}$$

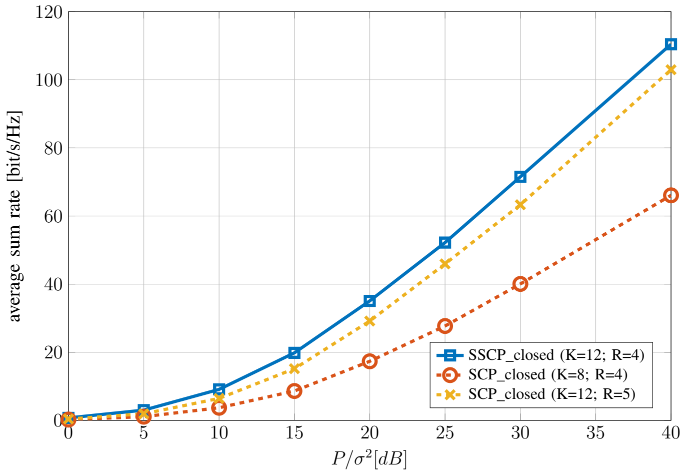

Fig. 2. Sum rate performance of the proposed algorithm in comparison with one reference methods for the scenario $Q = 4$, $N = 5$, $d = 1$

see the dashed line with the asterisks in Figure 2, achieves the same DoF as the proposed "SSCP_closed" algorithm. However, the proposed algorithm is still better than the reference algorithm in terms of the sum rate. One reason for this is that in the reference algorithm the nodes inside the intersection area will be only served by one relay, while the other relay treats this signal as interference and suppresses it, which results in a power loss.

V. Conclusion

In this paper, a network consisting of Q partially connected subnetworks where each subnetwork contains one relay is considered. The bidirectional pair-wise communication between the nodes takes place via the intermediate relays, using the two-way relaying protocol. In total, Q relays assist the K communication pairs to achieve interference alignment in the entire network. A new technique called simultaneous signal and channel alignment has been introduced to perform signal and channel alignment at multiple relays simultaneously. The properness conditions for the proposed closed form solution are derived using the introduced method of counting the required dimensions at every subnetwork. It was shown that local CSI is sufficient to perform interference alignment, while typically relay aided interference alignment algorithms in fully connected networks requires global CSI. It is shown that the proposed algorithm achieves more degrees of freedom than the reference algorithm and has a better sum rate performance.

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