10th International ITG Conference on Systems, Communications and Coding (SCC), 2015, Hamburg, Germany

Power Control in Wireless Broadcast Networks using Game Theory

Alexander Kuehne*, Hong Quy Le*, Mahdi Mousavi*, Matthias Wichtlhuber†, David Hausheer†, and Anja Klein*
*Communications Engineering Lab, TU Darmstadt, Germany, {a.kuehne, m.mousavi, a.klein}@nt.tu-darmstadt.de
†Peer-to-Peer Systems Engineering Lab, TU Darmstadt, Germany, {mwichtlh,hausheer}@ps.tu-darmstadt.de

***Abstract*—Broadcast is an important application in wireless networks, e.g. for video streaming, file distribution, or event notification. Wireless nodes are battery powered, their network lifetime depends on the energy consumption. Thus, energy efficiency is an important metric when designing broadcast protocols in infrastructureless and all-wireless networks. In this paper, we apply game theory to solve the minimum energy broadcast tree construction problem using power control by formulating the problem into a non-cooperative game. Our game-based broadcast protocols are decentralized in nature. We demonstrate that our game-based algorithms provide better performance than other known decentralized algorithms and the performance loss of our approach is small compared to a centralized solution.**

I. Introduction

In wireless networks, broadcast is the process of sending data from one node to all other nodes. In case the signal from the source node cannot reach all other nodes, the network must change its operation to an ad hoc-like network where some nodes forward the data to other nodes. One of the key challenges when designing broadcast protocols in wireless networks is energy efficiency since the lifetime of wireless networks is limited by the battery energy in wireless nodes. Therefore, energy efficiency is a crucial when designing broadcast protocols in wireless networks. Furthermore, decentralized algorithms are needed since centralized schemes require a lot of signaling to the central node causing congestion and increasing delay in the network. Moreover, centralized approaches are less robust against topology changes in the network.

The problem of finding the minimum energy broadcast tree has already been studied in the literature. In [1], a Broadcast Link-based Minimum Spanning Tree (BLiMST) algorithm is proposed based on the use of the standard Minimum Spanning Tree (MST) as in wired networks in which a link cost is associated with each pair of nodes. This algorithm assumed unicast transmissions only. Thus, the wireless multicast advantage is ignored during the the construction of MST. When the multicast advantage is utilized, the problem of finding the minimum energy broadcast tree is NP-complete [1]. The authors of [1] proposed the Broadcast Incremental Power (BIP) algorithm which is a widely used heuristic approach. BIP constructs the broadcast tree by starting with the source node and adds one node at a time to the tree choosing the node with the minimum additional cost. This process is continued until all the nodes in the network are added to the tree. Implementation of BIP which require only local information were considered in [2], [3]. In [4] the authors proposed a decentralized algorithm for BIP-called dynamic incremental power (DynaBIP). DynaBIP constructs a broadcast tree in a similar manner to BIP. At each iteration, DynaBIP selects the link which requires the minimum incremental cost but in a decentralized manner. The corresponding node and link are then added to the tree.

Recently, the application of game theory in modeling and analysis of wireless communication networks has received considerable attention and has led to numerous works, e.g., [5], [6], [7]. Game theory can be applied for scenarios having the following features: there is a set of users, each user takes some actions based on certain objective, and the achievement of the objective depends on the actions taken by every user.

In [8], game theory has been applied to the problem of finding a minimum transmission broadcast tree, however, a fixed transmit power for each node is assumed, i.e., power control is not incorporated in the game leaving room for further improvements. In a broadcast scenario, each node connected to the network may decide to change its transmit power to achieve a desired trade-off between its power consumption and the coverage of its transmission. However, to avoid redundant transmissions, the transmit power of each node depends on the transmit power of other nodes. Therefore, the best action for a node depends on the actions adopted by the others, and it is not trivial to foresee the outcome of this interaction. Thus, game-theoretic tools have to be exploited.

In this paper, we apply game theory to design a distributed algorithm for finding the minimum energy broadcast tree taking into account *power control* by formulating the problem of assigning forwarding nodes and receiving nodes into a non-cooperative game. The paper is organized as follows. In Section II, we give the system model. In Section III, we formulate the minimum energy broadcast tree problem as a non-cooperative game and give the solution of the game. The performance of the distributed algorithm based on game theory is presented in Section IV.

II. System model

We consider one source and N nodes, which are randomly distributed over a specified region. Let $\mathcal{N}$ denote the set of nodes. In the considered broadcast scenario, the same message shall be transmitted from the source to all N nodes in the network. The mobility of the nodes is not considered, i.e., the changes in the network topology are neglectable during a broadcast session. Each broadcast session is divided into two phases: a broadcast tree construction phase and a transmission phase. The duration of the broadcast tree construction phase is assumed to be short compared to the transmission phase such that the wireless channel between the nodes remains constant during the broadcast session. Reciprocity is assumed, i.e., the channel from node i to node j is equal to the channel from node j to node i. Each node measures the channel to other nodes with the help of a beacon signal. The noise level is equal at all nodes. Each node can choose its power level that does not exceed a given maximum value $P_{\max}$.

A. Power consumption model

In the following, the transmit power and the total power consumption of the nodes are discussed.

1) Transmit power in unicast transmission: A receiving node j is said to be in the transmission range of transmitter i, if the received power at node j is above a threshold P_{th}. Let $g_{i,j}$ be the channel gain between node i and node j, then the minimum required transmit power of node i is given by $P_{\mathrm{T,req}}^{(i,j)} = \frac{P_{\mathrm{th}}}{g_{i,j}}$.

2) Transmit power in multicast transmission: Due to the broadcast nature of the wireless medium, one transmitter can transmit the same signal to multiple receivers simultaneously. We consider a multicast transmission from a transmitter i to receivers $j = 1, 2, ..., K$. Let $P_{\mathrm{T,req}}^{(i,j)}$ be the minimum required transmit power of node i for a successful unicast transmission to node j, then the minimum required transmit power of node i for the multicast transmission is given by $P_{\mathrm{T,req}}^{(i)} = \max\{P_{\mathrm{T,req}}^{(i,1)}, ..., P_{\mathrm{T,req}}^{(i,K)}\}$.

3) Power consumption of communication module: Each wireless device is equipped with a communication module. We adopt the power consumption model proposed in [9]. The communication module consists of four main components: Baseband Signal Processing Unit, RF Unit, Power Amplifier and the antenna. The total power consumption P_{tot} is defined as $P_{\mathrm{tot}} = P_0 + P_{\mathrm{T,req}}$, where P_0 accounts for the total power consumption of the four communication module components.

B. Graph representation

We say that node j is a *neighbor* of node i if $P_{\mathrm{t,req}}^{(i,j)} \leq P_{\max}$. Under this definition of the neighborhood, the wireless network can be represented by a graph $G(V, E)$ where V is the set of vertices and E is the set of edges. A broadcast tree $\mathcal{T}$ is defined as a tree in the graph $G(V, E)$ which is rooted at the source node and each node in the tree has exactly one parent node. In Fig. 1, an example of a broadcast tree is given for a scenario with one source S and six nodes. In this example, source S is the parent of node 1, while node 1 is the parent of nodes 2 and 3. Finally, node 3 is the parent of nodes 4, 5 and 6.

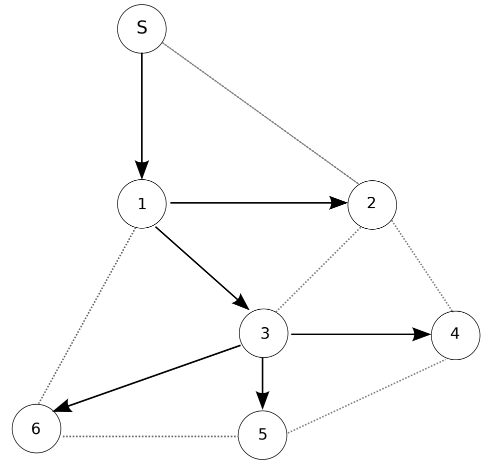

Fig. 1. Broadcast scenario consisting of one source and 6 nodes. The solid arrows represent the edges of the broadcast tree, the dashed lines indicate other possible links among the nodes.

C. Cost model

We define the cost of a unicast transmission from node r to node j as the total power required for a successful transmission, i.e., $C_r(\{j\}) = P_0^{(r)} + P_{\mathrm{t,req}}^{(r,j)}$. Under this definition, the cost of a multicast transmission from node r to its children nodes, denoted by the set $\mathcal{S}_r$, is $C_r(\mathcal{S}_r) = \max_{j \in \mathcal{S}_r} C_r(\{j\})$.

III. Game-based approach

The problem of finding the minimum cost broadcast tree can be interpreted as the problem of assigning a given child node to a given parent node such that every node can be reached from the source node by edges in the tree while the tree consumes minimum total cost. In a broadcast tree, each node must have exactly one parent node. Each internal node in the tree must spend a cost to forward the data to its children while only spending as much power as necessary to serve all its children, i.e., a power control is applied which is the main difference to the approach of [8] where all parent nodes always transmit with $P_{\max}$. Imagine that this cost should be compensated by the payment from the children nodes. Being a parent, each node has its cost-sharing rule which its children must follow. Being a child, each node has the incentive to find the best parent with whom its share is minimized or equivalently its utility is maximized. Under this setting, the problem of parent-selection can be formulated as a non-cooperative game of choosing a service provider. The players of the game are the nodes except for the source node. The providers are all the nodes.

A. Cost-sharing game and power control

We consider a non-cooperative cost-sharing game which consists of a set of players $\mathcal{P}$ and a set of providers $\mathcal{R}$. Each provider r has a cost function C_r and a cost-sharing rule f_r [10]. The game is defined as $G = (\mathcal{P}, \mathcal{R}, \{\mathcal{A}\}_{i \in \mathcal{N}}, \{f_r\}_{r \in \mathcal{R}}, \{C_r\}_{r \in \mathcal{R}})$ where the set of players $\mathcal{P}$ is the set of all nodes, except for the source node and $\mathcal{R}$

is the set of all nodes in the network. Set $\mathcal{A}_i$ is the set of all possible actions of player i, in our case, the set of neighboring nodes of node i. Let $\mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_N$ denote the set of joint action profiles of all players, $\mathbf{a} \in \mathcal{A}$ denotes the action of all nodes in the game and $\{\mathbf{a}\}_r$ denotes the set of nodes which node r chooses as provider in action profile $\mathbf{a}$, i.e., $\{\mathbf{a}\}_r = \{j \in \mathcal{N} \mid a_j = r\}$. According to the cost-sharing rule of node r, the share of node i if node i chooses r is $f_r(i, \{\mathbf{a}\}_r)$. Thus, for each node i, the utility function can be defined as

$$u_i(\mathbf{a}) = -f_r(i, \{\mathbf{a}\}_r)\ ,\ a_i = r. \tag{1}$$

Note that the election of a node to relay a message is not part of the game following our proposed game formulation, i.e., every potential parent node will always relay the data to its children nodes.

The broadcast tree construction game is a repeated game. In each iteration, each node considers the broadcast tree from the last iteration and chooses a neighbor to be its parent that maximizes its utility. Given $\mathbf{a}_{-i}$ which denotes the actions of all players except i in the last iteration, player i chooses the best parent node a_i^* such that $u_i(a_i^*, \mathbf{a}_{-i})$ is maximized known as best response. The repeated game continues until no player can increase its utility by changing only its own action, i.e., a Nash Equilibrium has been reached. Note that power control is considered in the game by incorporating the power adjustment of each parent node, as explained in Section II-A1 and II-A2, respectively, into the utility function of each children node. After finishing the parent-selection game, each parent node then correspondingly adjusts its transmit power to serve all its dedicated children nodes.

B. Action spaces

The action space of each node is a subset of the set of its neighboring nodes. However, if each node can freely choose its action from its neighboring nodes, then the resulting tree is not necessarily connected, i.e., it can happen that several nodes form a cycle resulting in a tree which is not connected. In order to prevent this, some constraints to the action spaces must be introduced. Based on the idea of [8], where the authors proposed a hop-distance as a rank attribute of each node, we propose a cost-distance rank attribute for each node. The cost-distance rank $\mathrm{rank}(i)$ of node i is defined as the minimum total cost on the path from node i to the source node. Using rank attributes, we can define the action spaces of the nodes. Node i can choose node a_i to be its parent if the rank of node i is higher than the rank of a_i, i.e.,

$$\mathcal{A}_i = \{r \mid r \text{ is neighbor of } i \text{ and } \mathrm{rank}(r) < \mathrm{rank}(i)\}\,. \tag{2}$$

C. Cost-sharing rules

We consider two well-known cost-sharing rules from the cooperative game theory literature: Marginal contribution and Shapley value [10].

1) Marginal contribution sharing rule: According to the definition of Marginal Contribution (MC) sharing rule [10], each node $i \in \{\mathbf{a}\}_r$ must share a cost of

$$\begin{aligned} f_r^{MC}(i, \{\mathbf{a}\}_r) &= C_r(\{\mathbf{a}\}_r) - C_r(\{\mathbf{a}\}_r - \{i\}) \\ &= \max_{j \in \{\mathbf{a}\}_r} C_r(\{j\}) - \max_{k \in \{\mathbf{a}\}_r - \{i\}} C_r(\{k\}). \end{aligned} \tag{3}$$

Obviously, only the node that has the highest link-cost must share a non-zero cost. If we arrange the nodes in increasing link-cost order, e.g., $C_r(\{1\}) \le \cdots \le C_r(\{n\})$, then nodes $1, \ldots, n-1$ must share no cost, i.e., $f_r^{MC}(i, \{\mathbf{a}\}_r) = 0$, $\forall\ 1 \le i \le n-1$, but node n must share an amount of $f_r^{MC}(n, \{\mathbf{a}\}_r) = C_r(\{n\}) - C_r(\{n-1\})$.

Let $H_r^{(1)}(\{\mathbf{a}\}_r)$ and $H_r^{(2)}(\{\mathbf{a}\}_r)$ denote the highest and second-highest link-cost in $\{\mathbf{a}\}_r$, respectively. Each node i can calculate its own share as $f_r^{MC}(i, \{\mathbf{a}\}_r) = \max\{C_r(\{i\}) - H_r^{(2)}(\{\mathbf{a}\}_r), 0\}$. A node j which is not in $\{\mathbf{a}\}_r$ must know its possible share if it chooses r to be its parent. In order to calculate its possible share, node j must know both $H_r^{(1)}(\{\mathbf{a}\}_r)$ and $H_r^{(2)}(\{\mathbf{a}\}_r)$. If its link-cost to node r satisfies $C_r(\{j\}) \ge H_r^{(1)}(\{\mathbf{a}\}_r)$ then his share is $C_r(\{j\})$ - $H_r^{(1)}(\{\mathbf{a}\}_r)$.

2) Shapley value sharing rule: According to the Shapley Value (SV) sharing rule [10], each node $i \in S = \{\mathbf{a}\}_r$ must share an amount of

$$f_r^{SV}(i, S) = \sum_{T \subseteq S - \{i\}} \frac{(|T|!)(|S| - |T| - 1)!)}{|S|!} (C_r(T \cup \{i\}) - C_r(T)). \tag{4}$$

In general, the computation of individual payment is intractable for large $\{\mathbf{a}\}_r$. Fortunately, in our application the SV rule is easy to compute. Specifically, the cost-function satisfies the rule $C_r(\{\mathbf{a}\}_r) = \max_{i \in \{\mathbf{a}\}_r} C_r(\{i\})$, so if we reorder the single-node cost in increasing order, i.e., $C_r(\{1\}) \le \cdots \le C_r(\{n\})$, then according to the result in [11], node i must share

$$f_r^{SV}(i, \{\mathbf{a}\}_r) = \sum_{j=1}^{i} \frac{C_r(\{j\}) - C_r(\{j-1\})}{n+1-j} \tag{5}$$

where $C_r(\{0\}) = 0$ is used for convenience.

D. Best response and convergence

In each iteration of the repeated game, one of the nodes considers the state of the game from the last iteration and chooses its best action that maximizes its utility. More precisely, given the action profile of other players $\mathbf{a}_{-i}$ from the last iteration, player i determines the set $\mathcal{A}_i$ of possible actions, such that the rank-constraint (2) is satisfied. Now for each $r \in \mathcal{A}_i$, player i chooses the one that maximizes its utility compared to its current utility:

$$\begin{aligned} r_{\max} = \arg\max_{r \in \mathcal{A}_i} \quad & -f_r(i, \{r, \mathbf{a}_{-i}\}_r) \\ \text{s.t. } & -f_r(i, \{r, \mathbf{a}_{-i}\}_r) > -f_{a_i}(i, \{\mathbf{a}\}_{a_i}). \end{aligned} \tag{6}$$

Furthermore, the game we are considering with both MC and SV sharing rule is an exact potential game [10], i.e., the best-response dynamic converges to a Nash Equilibrium [10]. Thus, the convergence of the game is guaranteed and due to the rank constraints, the resulting broadcast tree is connected.

E. Implementation of the game-based algorithm

For the implementation of our algorithm, so called Hello messages are essential. The Hello message of node i includes the following information: its identity ID_i, its parent ID a_i, its rank $\mathrm{rank}(i)$. In case of the MC rule, it further includes the current highest and second highest link-costs $H_i^{(1)}$ and $H_i^{(2)}$ between itself and its current children. In case of the SV rule, the message includes all the current link-cost $C_i(\{j\})$ between node i and its current children. With this information, each node k can compute the expected utility if node k chooses i as parent.

With the designed Hello message, we can describe the algorithm for computing the rank attribute. At the beginning, the rank of the source node is set to zero. Each node puts his own rank value into a Hello message and disseminates the Hello message one-hop away. Each node i overhears the Hello messages from its neighbors and updates its rank as following

$$\mathrm{rank}(i) = \min_{j \in N(i)} \{\mathrm{rank}(j) + C_i(\{j\})\}. \tag{7}$$

This can be done, for example, whenever node i overhears the Hello message from its neighbor j, node i compares its current rank $\mathrm{rank}(i)$ with the value $\mathrm{rank}(j) + C_i(\{j\})$. If $\mathrm{rank}(j) + C_i(\{j\}) < \mathrm{rank}(i)$ then node i updates its rank: $\mathrm{rank}(i) \leftarrow \mathrm{rank}(j) + C_i(\{j\})$ and set its parent $a_i = j$ by sending a Leave message to its current parent and a Join message to node j. The rank of each node decreases after every update. Thus, the computation of the rank attribute must converge to an equilibrium. After computing the rank attribute, we obtain not only the rank attributes, but also a broadcast tree. This tree is then used as the initial broadcast tree for the game. After the rank computing process, each node's rank ends up being equal to its parent's rank plus the link-cost between them. The game can now be played as previously described. If a player wants to change its strategy, it needs to send a message to inform both the old and new parent. It sends a Leave message to the old parent and sends a Join message to the new parent and then a Hello message to inform other nodes about the change. The old and new parent, after receiving the Leave and Join message, need to broadcast the Hello message immediately to inform their neighbors about the change.

IV. Performance evaluation

In the following, we evaluate the performance of the game-based algorithms assuming a network with a specified number of nodes between 20 and 50 which are uniformly distributed in a square region of 1000m $\times$ 1000m. One of the nodes is randomly chosen to be the source node. The pathloss coefficient is considered to be $\alpha = 2$. The channel gains are assumed to be exponentially distributed given by $g_{i,j} = \left(\frac{d_0}{d_{i,j}}\right)^{\alpha} |h_{i,j}|^2 g_0$, where $d_{i,j}$ is the distance between node i and node j, $h_{i,j}$ is the complex Gaussian distributed channel coefficient and $g_0 = -60$dB is the reference channel gain at distance $d_0 = 100$m. The maximum transmit power $P_{\max}$ is set to 10dBm. It is assumed that there is no interference in the network and the noise power is set to -70dBm at receiving nodes. The threshold received power is $P_{\mathrm{th}} = -60$dBm. We consider different values for the power consumption of the hardware module, e.g., $P_0 = 0, P_0 = 0.05 \cdot P_{\max}, P_0 = 0.1 \cdot P_{\max}$. As reference schemes, we apply the Broadcast Link-based Minimum Spanning Tree (BLiMST) algorithm [1], the Broadcast Incremental Power (BIP) algorithm [1] and the dynamic incremental power (DynaBIP) [4].

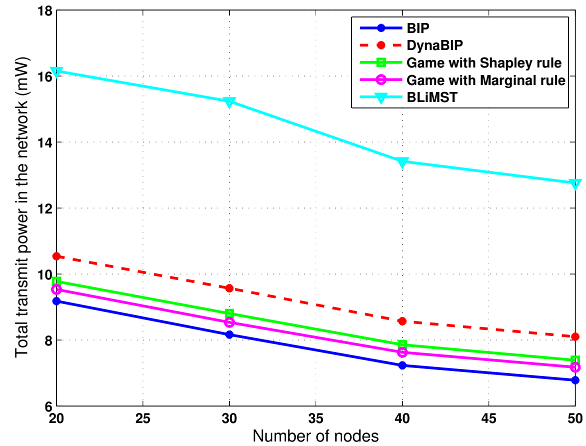

Fig. 2. Total transmit power vs. number N of nodes for $P_0 = 0$.

A. Case with $P_0 = 0$

Figure 2 shows the total required transmit power of the broadcast trees obtained by different algorithms assuming $P_0 = 0$. As the number of nodes increases, the total transmit power decreases since the distance between neighboring nodes decreases. According to Fig. 2, BLiMST performs worst because, in contrast to the others, it uses only unicast transmission and, thus, does not utilize the advantage of the broadcast nature of the wireless medium. The performance of BIP is best due to its centralized nature. Among the decentralized algorithms, the two proposed game-based algorithms with SV and MC rules perform approximately 8% and 10% better than DynaBIP, respectively. The reason why MC outperforms SV is due to the fact that in MC rule, among the children of a node, only the one which has the worst condition pays the cost and the others pay nothing. This leads to a broadcast tree with larger multicast group and therefore fewer parents compared to the SV rule in which each child node has to pay a part of the cost. In other words, in contrast to the SV rule, applying MC rule reduces the number of transmissions, the network can benefit from multicast transmission more and this reduces the final power consumption in the network.

B. Case with $P_0 > 0$

Figures 3 and 4 show the performance of the algorithms for the case when the power consumption of the hardware

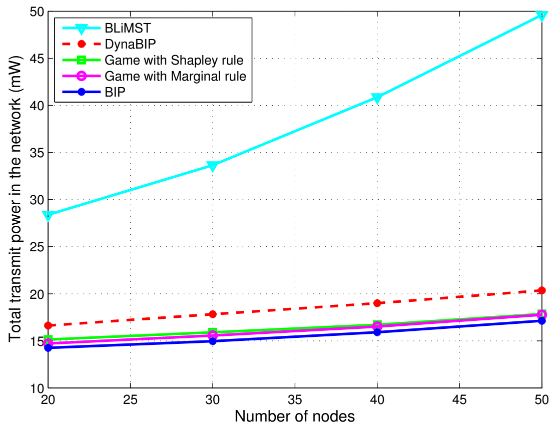

Fig. 3. Total used power vs. number N of nodes for $P_0 = 0.05 \cdot P_{\max}$

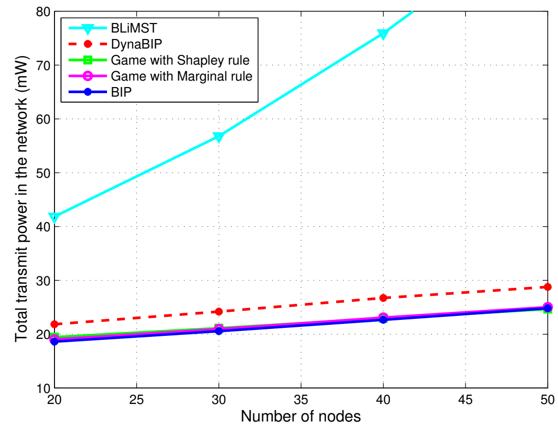

Fig. 4. Total used power vs. number N of nodes for $P_0 = 0.1 \cdot P_{\max}$

part is included in the cost assuming $P_0 = 0.05 \cdot P_{\max}$ and $P_0 = 0.1 \cdot P_{\max}$, respectively. In other words, the required power for each link is the transmission power plus a fixed value of internal power P_0. It can be seen that by considering the internal power, the game-based algorithms still perform approximately 10% better than DynaBIP and the performance is very close to the BIP algorithm. It also can be observed that by increasing the number of nodes in the network, the total power consumption increases. This is due to the fact that by increasing the number of nodes, the number of transmissions in the network increases and each transmission contains a fixed internal power regardless of the transmission power.

Moreover, by increasing the internal power, the performance of the SV rule becomes closer to the MC rule and the BIP algorithm. This behavior can be explained based on how these algorithms react to a fixed increment in the links' required power. For instance, in case of applying the MC rule, based on (3), P_0 will be canceled in the cost model and would not affect the nodes' decisions. In case of applying the SV rule, although there would be more transmissions in the network, P_0 will be considered in the cost sharing calculations and a better broadcast tree compared to the MC rule could be achieved. This also explains why the performance difference between the game-based algorithm with SV rule and the other two schemes, i.e., MC rule and BIP, becomes smaller for a higher value of P_0.

V. Conclusion

We have successfully applied the non-cooperative game theory framework in solving the problem of finding an energy-efficient broadcast tree in wireless broadcast networks using power control. We formulated the energy-efficient broadcast tree problem as a non-cooperative cost-sharing game between the nodes in the network applying two different cost-sharing rules: the Marginal contribution and the Shapley value sharing rule. The resulting cost-sharing game under these two cost-sharing mechanisms is a potential game and, thus, a Nash Equilibrium is guaranteed. We have provided a decentralized implementation of the game-based algorithms and have shown that our proposed algorithms outperform other decentralized solutions.

Acknowledgment

This work has been funded by the German Research Foundation (DFG) as part of project B03 within the Collaborative Research Center (CRC) 1053 – MAKI. The authors would like to thank M. Sc. Roland Kluge working in project A01 of MAKI for helpful comments on broadcast tree construction.

References

[1] J. E. Wieselthier and G. D. Nguyen, "Energy-efficient broadcast and multicast trees in wireless networks," *Mobile Networks and Applications*, vol. 7, pp. 481–492, December 2002.

[2] J. Cartigny, D. Simplot, and I. Stojmenovic, "Localized minimum-energy broadcasting in ad hoc networks," in *Proc. IEEE International Conference on Computer Communications (INFOCOM)*, April 2003.

[3] F. Ingelrest and D. Simplot-Ryl, "Localized broadcast incremental power protocol for wireless ad hoc networks," *ACM Wireless Networks*, vol. 14, pp. 309–319, June 2008.

[4] C. Miller and C. Poellabauer, "A decentralized approach to minimum-energy broadcasting in static ad hoc networks," in *Proc. International Conference on Ad Hoc Networks and Wireless (ADHOC-NOW)*, 2009.

[5] D. E. Charilas and A. D. Panagopoulos, "A survey on game game theory application in wireless networks," *Computer Networks: The International Journal of Computer and Telecommunications Networking*, vol. 54, pp. 3421–3430, 2010.

[6] S. Lasaule and H. Tembine, *Game theory and learning for wireless networks: fundamentals and applications*. Elsevier Publisher, 2011.

[7] Z. Han, D. Niyato, W. Saad, T. Basar, and A. Hjorunges, *Game theory in wireless and communication networks: theory, models and applications*. Cambridge University Press, 2012.

[8] F. Chen and J. Kao, "Game-based broadcast over reliable and unreliable wireless links in wireless multihop networks," *IEEE Transactions on Mobile Computing*, vol. 12, no. 8, August 2013.

[9] Q. Wang, M. Hempstead, and W. Yang, "A realistic power consumption model for wireless sensor network devices," in *Proc. IEEE Communications Society on in Sensor and Ad Hoc communications and Networks (SECON)*, 2006.

[10] R. Gopalakrishnan, J. R. Marden, and A. Wieman, "Characterizing distribution rules for cost sharing games," in *Proc. International Conference on Network Games, Control and Optimization (NetGCoop)*, 2011.

[11] S. C. Littlechild and G. Owen, "A simple expression for the shapley value in a special case," *Management Science, Theory Series*, vol. 20, no. 3, 1973.