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Combining Interference Alignment and Two-Way Relaying in Partially Connected Networks with only Local CSI

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***Abstract*—In this paper, a network consisting of several partially connected subnetworks where each subnetwork includes one relay is considered. "Partially connected" means that not all nodes are connected to all relays. Some nodes may be connected to multiple relays. The entire bidirectional pairwise communication between the nodes takes place via the intermediate half-duplex amplify-and-forward relays, considering two-way relaying. The algorithm proposed in this paper is a closed-form solution which requires only local channel state information (CSI) to achieve interference alignment. The properness condition for the proposed algorithm is derived. The process of interference alignment is decomposed into signal alignment, channel alignment and transceive zero forcing. It is shown that all subnetworks can be investigated separately. This means the whole problem can be divided into sub-problems. The simulation results show that the degrees of freedom increase for the considered partially connected network in comparison with the fully connected version of the considered network.**

I. Introduction

In nowadays wireless communication systems, interferences are the major factor limiting performance when several links share a common communication medium [1]. The interference signal can be treated as noise if the interference signals are sufficiently weak [2], [3]. If the interference is too strong to be treated as noise, the conventional approaches to handle these interferences avoid the interference signal at the receivers by orthogonalizing the communication links, e.g., in time by applying TDMA (time division multiple access) or in frequency by applying FDMA (frequency division multiple access) [4]. By applying these methods in an interference channel with K source-destination node pairs, each source node gets at most a fraction of $1/K$ of the total channel resources. In sum, these methods can achieve at most 1 degree of freedom [5]. The degrees of freedom (DoF) of a wireless interfering network are the interference free signal dimensions.

Another promising technique to manage this interference, especially in the high SNR region, has been developed in the past few years and is called interference alignment (IA) [1]. IA can be achieved by dividing the whole receive space into two subspaces, the useful subspace (US) and the interference subspace (IS). At each receiver, all interference signals should be aligned in the IS and the US contains only the useful signal. This means that each communication pair can use half of the total resources interference free, so that in total $K/2$ DoF can be achieved [1]. By applying a zero-forcing-filter (ZF-filter) at the receivers, each node can suppress all interference signals [6]. The impact of relays in fully connected interference networks has been investigated in the literature, e.g., in [7]. It has been shown that the use of relays does not increase the DoF, however relays can help to achieve an IA solution which maximizes the DoF.

One common relaying protocol is the so called two-way relaying protocol. In this protocol, the pairs exchange their data in two phases, the multiple access (MAC) phase and the broadcast (BC) phase [8]. Relay aided IA considering the two-way relaying protocol in a fully connected network was investigated in [9]–[11]. Fully connected means that all channel coefficients are non-zero. Each of the $2K$ nodes in [9]–[11] wants to transmit d data streams and is equipped with N antennas. Each of the Q relays is equipped with R antennas. To perform IA with a single relay, the number of antennas at the relay needs to be larger than or equal to the number of communication pairs times the number of transmitted data streams, i.e., $R \geq Kd$ [9]. The case $R = Kd$ together with the concept of signal alignment (SA), channel alignment (CA), and zero-forcing (ZF) was introduced in [9]. As each node is able to suppress self-interference, it is not necessary to separate the aligned signals at the relay. After SA and CA, all Kd effective data streams are separated. This means that a relay with $R = Kd$ antennas can perform transceive zero forcing (TRxZF) [9]. In [10], the pair-aware interference alignment (PAIA) method was introduced for the case $R \geq Kd$. A multi-pair multi-relay fully connected network was presented in [12], which proposes an IA-algorithm and a minimum mean square error (MMSE)-algorithm.

In real-world scenarios, the assumption that all nodes are connected to all relays does not hold. The reasons for this are physical phenomena, e.g., high path losses or shadowing, leading to links of considerably different strengths. Hence, the received signal at each node comprises three signal types: the desired signal, strong interference signals, and weak interference signals. Sufficiently weak links can be approximated with

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zero which results in networks with partial connectivity [4], [13]. In [4], [13], it was shown that the number of antennas at the nodes can be reduced in the partially connected case compared to the fully connected case. Both works investigate the DoF of networks without relays. Relays which assist the communication cannot improve the DoF in a fully connected network [7], but it is conjectured in [7] that relays can improve the DoF if a network is partially connected. Relay aided IA in a partially connected network which considers the one-way relaying protocol is investigated in [14], where it was shown that partial connectivity can help to reduce the relay requirements in a two subnetwork case.

In this paper, we propose an algorithm to perform IA in a partially connected network, considering the two-way relaying protocol. Furthermore, we determine the required channel state information (CSI). The proposed algorithm is only feasible if there are sufficient antennas at the relays and at the nodes. The condition for the properness is derived. It is shown that the DoF increase for the considered scenario in comparison to the fully connected case. Furthermore, the sum rate performance of the proposed algorithm is investigated.

The present paper is organized as follows: Section II introduces the system model for a partially connected network. In Section III, the proposed IA algorithm is presented. The performance of the proposed algorithm is compared with different reference schemes in Section V with respect to the sum rate and the DoF. Section VI concludes this paper.

Notation: In the following, lower case letters represent scalars, lower case bold letters represent vectors, and upper case bold letters represent matrices. $(.)^*$, $(.)^T$, $(.)^H$, denote the complex conjugate, transpose, and complex conjugate transpose of the the element inside the brackets, respectively. The matrix $\mathbf{I}_N$ denotes an $N \times N$ identity matrix. $\|.\|_F^2$ and $\mathrm{tr}(.)$ denote the Frobenious norm and the trace, respectively. $|.|$ denotes the cardinality.

II. System Model

Consider a network consisting of K multi-antenna node pairs and Q amplify-and-forward half-duplex relays. Note that not all node-pairs are connected to all Q relays, i.e., the considered network is partially connected. The whole network is divided into Q subnetworks, so that each subnetwork includes one relay and all node pairs which are connected to this relay. An example for $K = 6$ and $Q = 3$ is shown in Figure 1. It is assumed that only pairs of nodes are connected to a relay, not single nodes.

Such a partially connected network contains two different types of nodes. The first type of nodes are connected to a single relay and therefore belongs to a single subnetwork. The second type are nodes which are connected to several relays, i.e., which belong to multiple subnetworks and are located in the so-called intersection area. For the communication between the nodes, the two-way relaying protocol is used [8]. The bidirectional communication is carried out in two steps, called the MAC phase and the BC phase. There is no direct link between the nodes. In the MAC phase, each relay receives signals from all nodes in its subnetwork. In the BC phase, it is important to distinguish between the two different types of nodes. Each node which belongs to a single subnetwork receives only interference from this subnetwork besides its useful signal and self-interference. The nodes inside the intersection area receive interference from several subnetworks, i.e., the union of these subnetworks. If a communication pair is located inside the intersection area, each node of this pair receives also the useful signal via several relays, besides the interference.

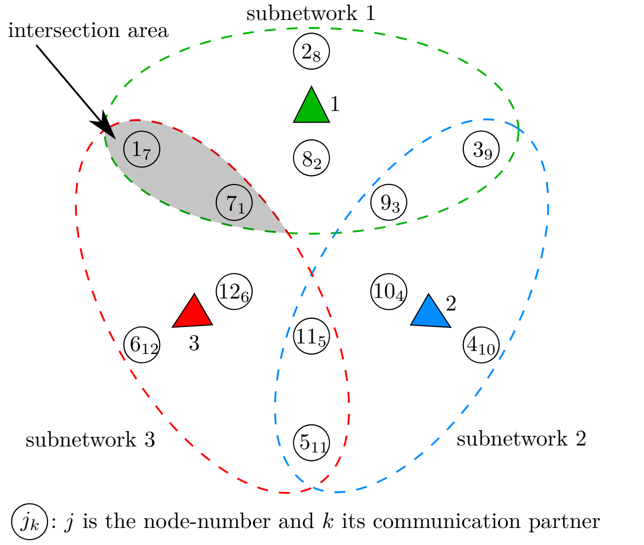

Fig. 1. Partially connected network consisting of $Q = 3$ subnetworks, where the subnetworks 1 and 2, 2 and 3, and 3 and 1 intersect. Only nodes inside the dashed area can communicate with the relay in this area, all other links are assumed to be zero.

To each of the Q relays, an index $q \in \mathcal{Q} = \{1, ..., Q\}$ is assigned. Relay q, $q \in \mathcal{Q}$ is equipped with $R_q \geq 1$ antennas. Index q also represents the subnetwork index. The set of node indices of size $2K$ is given by $\mathcal{K} = \{1, ..., 2K\}$, where node j and node k are communication partners for $j \in \mathcal{K}$ and $k = j + K$ if $j \leq K$ and $k = j - K$ if $j > K$. Let (j, k) denote a communication pair. Both nodes of a communication pair (j, k) are in the same subnetwork. Each node in subnetwork q is equipped with N_q antennas and wants to transmit $d \leq N_{q,\min}$ data streams to its communication partner. For simplicity, we assume that all subnetworks are of the same structure and size, as shown in Figure 1. This means $N = N_q$ and $R = R_q$ hold.

Due to the partial connectivity, it is appropriate to introduce sets of nodes. Let $\mathcal{K}(q)$ denote the set of nodes which are connected to relay q and $\mathcal{R}(k)$ the set of relays which are connected to node k. Two different examplary sets in Figure 1 are the set $\mathcal{K}(1) = \{(1, 7), (2, 8), (3, 9)\}$ and the set $\mathcal{R}(1) = \{1, 3\}$. The whole set of nodes is given by

$$\mathcal{K} = \bigcup_{q \in \mathcal{Q}} \mathcal{K}(q). \tag{1}$$

Let $\mathbf{H}_{j,q}^{\mathrm{sr}} \in \mathbb{C}^{R \times N}$ and $\mathbf{H}_{q,j}^{\mathrm{rd}} \in \mathbb{C}^{N \times R}$ denote the channel matrices for the MAC phase and the BC phase, respectively. Without loss of generality it is assumed that channels of the pair (j,k) are linearly independent of those of all the other pairs. Further, $\mathbf{d}_j \in \mathbb{C}^{d\times 1}$ and $\mathbf{V}_j \in \mathbb{C}^{N\times d}$ denote the data vector and the transmit filter, respectively. Let $\mathbf{n}_{\mathrm{r}q} = \mathcal{CN}\left(0,\sigma^2_{\mathrm{r}q}\right) \in \mathbb{C}^{R\times 1}$ denote the noise at relay q and $\mathbf{n}_{\mathrm{n}k} = \mathcal{CN}\left(0,\sigma^2_{\mathrm{n}k}\right) \in \mathbb{C}^{N\times 1}$ denote the noise at node k, respectively. The components of the two noise vectors $\mathbf{n}_{\mathrm{r}q}$ and $\mathbf{n}_{\mathrm{n}k}$ are i.i.d. complex Gaussian random variables. The maximum transmit power of each node is denoted by $P_{\mathrm{n,max}}$. It is assumed that all nodes have the same maximum transmit power.

In the MAC phase, relay q receives a signal given by

$$\mathbf{y}_q = \sum_{k\in\mathcal{K}(q)} \mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\mathbf{d}_k + \mathbf{n}_{\mathrm{r}q}. \tag{2}$$

After linear signal processing the relay retransmits the received signal to all connected nodes. The processing matrix of relay q is denoted by $\mathbf{G}_q$ and relay q has a maximum transmit power $P_{\mathrm{r,max}}$, here it is assumed that all relays have the same maximum transmit power.

In the BC phase, node k receives the signal

$$\begin{aligned}\mathbf{y}_k = &\sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\mathbf{d}_j + \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\mathbf{d}_k \\ &+ \sum_{\substack{i\in\mathcal{K}(\mathcal{R}(k)),\\ i\neq k,j}} \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\ &+ \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{n}_{\mathrm{r}q} + \mathbf{n}_{\mathrm{n}k},\end{aligned} \tag{3}$$

where nodes j and k are communication partners.

In (3), the first and the second term are the useful signal and the self-interference signal, respectively. The third term represents the unknown interference and the last two terms represent the effective noise at node k.

It is assumed that the self-interference can be perfectly canceled. Let $\mathbf{U}^{\mathrm{H}}_k \in \mathbb{C}^{d\times N}$ denote the receive filter at node k. The estimated data vector at node k is denoted by $\hat{\mathbf{d}}_j$ and is given by

$$\begin{aligned}\hat{\mathbf{d}}_j = &\mathbf{U}^{\mathrm{H}}_k \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\mathbf{d}_j \\ &+ \mathbf{U}^{\mathrm{H}}_k \sum_{\substack{i\in\mathcal{K}(\mathcal{R}(k)),\\ i\neq k,j}} \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i\mathbf{d}_i \\ &+ \mathbf{U}^{\mathrm{H}}_k \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{n}_{\mathrm{r}q} + \mathbf{n}_{\mathrm{n}k}.\end{aligned} \tag{4}$$

To reliably decode the desired signal, the unknown interference should be in the IS. The self-interference can be in the US or the IS and the US, which contains the useful signal, should be linearly independent of the IS. The US needs to have at least the dimension of the data vector. Then the zero-forcing filter $\mathbf{U}^{\mathrm{H}}_k$ can suppress all unknown interferences. This results in the following IA conditions:

$$\mathbf{\Lambda} = \mathbf{U}^{\mathrm{H}}_k \sum_{q\in\mathcal{R}(k)} \mathbf{H}^{\mathrm{rd}}_{q,k}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{i,q}\mathbf{V}_i \tag{5}$$

$$\mathbf{\Lambda} = \mathbf{0}, \quad \forall i \neq k,j, \tag{6}$$

$$\mathrm{rank}(\mathbf{\Lambda}) = d, \quad i = j. \tag{7}$$

III. Proposed algorithm

In this section, a closed-form solution to perform IA in a partially connected network is proposed. The concepts of signal alignment, channel alignment and transceive zero-forcing introduced in [9] for the case of fully connected networks are extended here to the case of partially connected networks.

It is assumed that the whole network consists of at least 3 subnetworks, i.e., $Q \geq 3$ and each subnetwork contains at least 3 communication pairs, i.e., $|\mathcal{K}(q)| \geq 3$. In the present paper, the number of communication pairs in an intersection area is restricted to one. The general case with several communication nodes inside an intersection area is left for future work. It is to note that in the proposed algorithm a communication pair inside an intersection area is only served by a single relay, the other connected relay treats these signals as interference. Hence, each relay serves all nodes which are only connected to this relay and the communication pair inside one of the intersection areas connected to this relay. This means that the served nodes $\mathcal{K}^{\mathrm{ser}}(q)$ are a proper subset of $\mathcal{K}(q)$, i.e. $\mathcal{K}^{\mathrm{ser}}(q) \subset \mathcal{K}(q)$. An example is shown in Figure 1, where relay 1 serves only the nodes $\{(1,7),(2,8)\}$, i.e., $\mathcal{K}^{\mathrm{ser}}(1) = \{(1,7),(2,8)\}$. Due to our assumption that all subnetworks are of the same structure, it is sufficient to investigate subnetwork q to find a solution for the whole network, i.e., the total number of subnetworks is irrelevant.

A. MAC-phase: Signal alignment

In this section, the method proposed in [9] to perform signal alignment is extended for partially connected networks. The total number of received data streams at relay q in the MAC phase is $|\mathcal{K}(q)|d$. To avoid inter-pair interference, the signals of all nodes which are served by relay $q \in \mathcal{Q}$ should be pair-wise aligned and linearly independent of each other at relay q. This means that each node designs its transmit filter such that the d-dimensional subspaces of the communication pair $(j,k) \quad \forall j \neq k \quad j,k \in \mathcal{K}(q)$ are pair-wisely aligned at relay q. This is called signal alignment. The signal alignment condition is given by

$$\mathrm{span}\left(\mathbf{H}^{\mathrm{sr}}_{j,q}\mathbf{V}_j\right) = \mathrm{span}\left(\mathbf{H}^{\mathrm{sr}}_{k,q}\mathbf{V}_k\right). \tag{8}$$

To apply signal alignment, it is necessary that the number N of antennas at each node and the number R of antennas at each relay are sufficiently large. The signals of the two nodes in the intersection area which are not aligned at relay q span a $2d$-dimensional subspace at relay q. All pair-wise aligned signals span a $|\mathcal{K}^{\mathrm{ser}}(q)|d/2$ dimensional space at relay q. The number of effective data streams at each relay results in a condition for the number of antennas at each relay, namely

$$R = 2d + \frac{1}{2}|\mathcal{K}^{\mathrm{ser}}(q)|d. \tag{9}$$

In order to satisfy (8), the subspaces spanned by $\mathbf{H}^{sr}_{jq}\mathbf{V}_j$ and $\mathbf{H}^{sr}_{kq}\mathbf{V}_k$ must intersect [9]. Hence, it is possible to determine the basis $\mathbf{H}^{\text{sr}}_{j,q}\mathbf{A}_j$ and the basis $\mathbf{H}^{\text{sr}}_{k,q}\mathbf{A}_k$ of the intersection subspace. The two matrices $\mathbf{A}_j$ and $\mathbf{A}_k$ can be chosen such that

$$\mathbf{H}^{\text{sr}}_{j,q}\mathbf{A}_j = \mathbf{H}^{\text{sr}}_{k,q}\mathbf{A}_k \tag{10}$$

is fulfilled, without loss of generality. This equation can also be written as

$$\underbrace{\begin{bmatrix}\mathbf{H}^{\text{sr}}_{j,q} & -\mathbf{H}^{\text{sr}}_{k,q}\end{bmatrix}}_{\mathbf{H}^{\text{ss}}_{j,k}} \cdot \underbrace{\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix}}_{\mathbf{A}} = \mathbf{0}, \tag{11}$$

so that the determination of $\mathbf{A}_j$ and $\mathbf{A}_k$ results in determining the null space of $\mathbf{H}^{\text{ss}}_{j,k} \in \mathbb{C}^{R\times 2N}$, given by

$$\underbrace{\begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix}}_{\mathbf{A}} = \text{null}\left(\mathbf{H}^{\text{ss}}_{j,k}\right). \tag{12}$$

The columns of $\mathbf{A}$ span a $2N-R$ dimensional solution space which fulfills (8). This results in the condition $2N-R \geq d$ to perform SA. Hence, the required number of antennas at each node is given by

$$N \geq \frac{R+d}{2}. \tag{13}$$

The two precoding filters $\mathbf{V}_j$ and $\mathbf{V}_k$ are chosen from the solution space as,

$$\begin{bmatrix}\mathbf{V}_j \\ \mathbf{V}_k\end{bmatrix} = \begin{bmatrix}\mathbf{A}_j \\ \mathbf{A}_k\end{bmatrix} \cdot \mathbf{\Phi}_{\text{MAC}}, \tag{14}$$

where $\mathbf{\Phi}_{\text{MAC}} \in \mathbb{C}^{(2N-R)\times d}$ is an arbitrary matrix to select one possible solution of the whole solution space.

B. BC-phase: Channel alignment

In this section, the method proposed in [9] to perform channel alignment is extended for partially connected network. Channel alignment is performed in the BC phase. Each node served by relay q designs its receive filter such that the effective channels of the communication pair (j,k) span the same subspace at relay q. The channel alignment condition is given by

$$\text{span}\left(\mathbf{H}^{\text{rdH}}_{q,j}\mathbf{U}_j\right) = \text{span}\left(\mathbf{H}^{\text{rdH}}_{q,k}\mathbf{U}_k\right). \tag{15}$$

Since signal alignment and channel alignment are dual problems [9], the determination of the solution space for channel alignment is similar to determining the signal alignment solution space, i.e., the channel alignment solution space is given by

$$\underbrace{\begin{bmatrix}\mathbf{B}_j \\ \mathbf{B}_k\end{bmatrix}}_{\mathbf{B}} = \text{null}\left(\mathbf{H}^{\text{ss}'}_{j,k}\right), \tag{16}$$

where $\mathbf{H}^{\text{ss}'}_{j,k} = \begin{bmatrix}\mathbf{H}^{\text{rdH}}_{q,j} & -\mathbf{H}^{\text{rdH}}_{q,k}\end{bmatrix} \in \mathbb{C}^{R\times 2N}$. The columns of $\mathbf{B}$ span a $2N-R$ dimensional solution space which fulfils (15). This results in the same condition as to perform signal alignment. The two receive-filters $\mathbf{U}^{\text{H}}_j$ and $\mathbf{U}^{\text{H}}_k$ are chosen from the solution space as,

$$\begin{bmatrix}\mathbf{U}_j \\ \mathbf{U}_k\end{bmatrix} = \begin{bmatrix}\mathbf{B}_j \\ \mathbf{B}_k\end{bmatrix} \cdot \mathbf{\Phi}_{\text{BC}}, \tag{17}$$

where $\mathbf{\Phi}_{\text{BC}} \in \mathbb{C}^{(2N-R)\times d}$ is an arbitrary matrix to select one possible solution of the whole solution space.

C. Transceive zero forcing

Transceive zero forcing is a combination of receive and transmit zero forcing [15]. This means that the relay matrix is given by

$$\mathbf{G}_q = \mathbf{G}^{\text{TX}}_q \cdot \mathbf{G}^{\text{RX}}_q, \tag{18}$$

where $\mathbf{G}^{\text{TX}}_q$ denotes the transmit zero forcing matrix and $\mathbf{G}^{\text{RX}}_q$ the receive zero forcing matrix. To satisfy the transceive zero forcing condition,

$$\mathbf{I}_R = \mathbf{H}^{\text{BC}}_{\text{eff}q} \cdot \mathbf{G}_q \cdot \mathbf{H}^{\text{MAC}}_{\text{eff}q} \tag{19}$$

must be fulfilled. The effective channels in the BC and MAC phase are given by (20) and (21), as shown on top of the next page, where $\mathbf{P}_{x,q} = \mathbf{H}^{\text{sr}}_{x,q}\mathbf{V}_x, \forall x \in \mathcal{K}(q)$ and $\mathbf{O}_{x,q} = \mathbf{U}^{\text{H}}_x\mathbf{H}^{\text{rd}}_{q,x}, \forall x \in \mathcal{K}(q)$, respectively. Equation (19) represents a system of linear equations, which results in the transceive zero forcing matrix at relay q, given by

$$\mathbf{G}_q = \left(\mathbf{H}^{\text{MAC}}_{\text{eff}q} \cdot \mathbf{H}^{\text{BC}}_{\text{eff}q}\right)^{-1}. \tag{22}$$

The relay processing matrix $\mathbf{G}_q$ is uniquely determined by (22). If the conditions (9) and (13) are fulfilled and the channels are random, the matrix $\left(\mathbf{H}^{\text{MAC}}_{\text{eff}q} \cdot \mathbf{H}^{\text{BC}}_{\text{eff}q}\right)$ is non-singular and thus invertible with a probability of one.

If the different subnetworks are of different size, but still only one communication pair is located in the intersection area, the adaptation of the proposed algorithm is straightforward and each subnetwork has to be investigated separately. To do so, it is necessary to introduce the variables N_q and R_q in the proposed algorithm and to prove that the conditions (13) and (9) are fulfilled in each of the q subnetworks.

D. Required CSI

In this section, the required CSI at each node and at each relay to perform IA and to estimate the transmitted data streams is determined. The transmit filter $\mathbf{V}_k$ and the receive filter $\mathbf{U}^{\text{H}}_k$ at node k only depend on pair-wise CSI, see (8) and (15). This means that a communication pair has to know its own channel to the relay and the channel of its communication partner to the relay. The relay processing matrix $\mathbf{G}_q$ depends on the effective channel of all nodes which are connected to relay q, given by (20) and (21). Subnetwork CSI means that a relay knows all MAC and BC channels of its own subnetwork and hence the transmit and receive filters of all served nodes. From (20) and (21) it is obvious that relay q needs also the information about the transmit and receive filter of the communication pairs which are not served by relay q, beside subnetwork CSI. These transmit filters and

$$\mathbf{H}_{\text{eff}q}^{\text{MAC}} = \begin{bmatrix} \mathbf{P}_{x,q} & \cdots & \mathbf{P}_{y,q} \end{bmatrix}, \quad (x, y \in \mathcal{K}(q); x \neq y | \text{span}(\mathbf{P}_{x,q}) \neq \text{span}(\mathbf{P}_{y,q})) \tag{20}$$

$$\mathbf{H}_{\text{eff}q}^{\text{BC}} = \begin{bmatrix} \mathbf{O}_{x,q}^{\text{H}} & \cdots & \mathbf{O}_{y,q}^{\text{H}} \end{bmatrix}^{\text{H}}, \quad (x, y \in \mathcal{K}(q); x \neq y | \text{span}(\mathbf{O}_{x,q}) \neq \text{span}(\mathbf{O}_{y,q})) \tag{21}$$

receive filters depend on the channels to a relay in another subnetwork. In the following, the required CSI at the relays to perform transceive zero forcing is referred to as extended subnetwork CSI. This extended subnetwork CSI is a proper subset of global CSI, where global CSI means that all MAC and BC are available at all relays.

After IA, all K communication pairs are separated and the filters $\mathbf{V}_k$, $\mathbf{U}_k^{\text{H}}$, and $\mathbf{G}_q$ are fixed. For data estimation the nodes have to use pilot sequences to estimate the resulting channel between the communication pairs because pair-wise CSI is not sufficient for data estimation; see (4).

IV. Iterative Algorithm

In this section, an iterative reference algorithm for the proposed algorithm is presented. Equation (5) implies that at each receiver, all interference signals are suppressed. The algorithm presented in this section minimizes leakage interference [16] to approach an IA solution in a fully connected network. This reference algorithm allows the comparison of the DoF between fully and partially connected networks. The variables introduced in Section II are still valid, but in this section all channel coefficients are non-zero, i.e., all nodes are connected to all relays. The minimization problem is given by

$$\min_{\mathbf{V}_j, \mathbf{G}, \mathbf{U}_k^{\text{H}}} \mathbf{U}_k^{\text{H}} \mathbf{H}_k^{\text{rd}} \mathbf{G} \mathbf{H}_j^{\text{sr}} \mathbf{V}_j, \quad \forall j \neq k, \tag{23}$$

where $\mathbf{H}_k^{\text{rd}} = \begin{bmatrix} \mathbf{H}_{1,k}^{\text{rd}} & \mathbf{H}_{2,k}^{\text{rd}} & \cdots & \mathbf{H}_{Q,k}^{\text{rd}} \end{bmatrix}$ and $\mathbf{H}_j^{\text{sr}} = \begin{bmatrix} \mathbf{H}_{j,1}^{\text{srH}} & \mathbf{H}_{j,2}^{\text{srH}} & \cdots & \mathbf{H}_{j,Q}^{\text{srH}} \end{bmatrix}^{\text{H}}$. $\mathbf{G}$ is a concatenated block diagonal matrix, which contains the sub-matrices $\mathbf{G}_q, \quad \forall q \in \mathcal{Q}$.

The whole minimization takes place in three sequential steps, as shown in Figure 2. Each of these steps is a separate minimization problem, over only one of the three matrices $\mathbf{V}_j, \mathbf{G}, \mathbf{U}_k^{\text{H}}$. In the initializing step, the matrices $\mathbf{V}_j$ and $\mathbf{G}$ are arbitrarily chosen from the complex space $\mathbb{C}$. The final result of this algorithm strongly depends on these initialization values because the optimization problem is non-convex. It cannot be guaranteed that the algorithm converges to the global minimum. It is obvious from (23) that this algorithm requires global CSI in order to approach an IA solution.

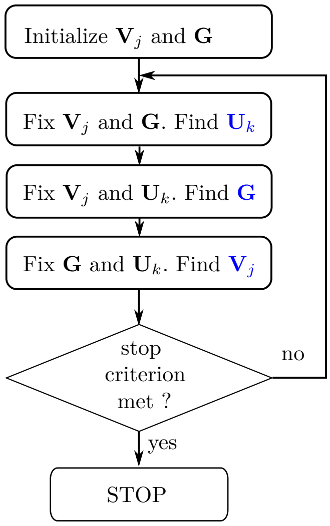

Fig. 2. Iterative IA algorithm to approach to an IA solution in a fully connected network

V. Performance Analysis

In this section, the sum rate performance together with the DoF of the proposed algorithm (closed_IA_partially) are analyzed. For the simulation of the partially connected network, we consider a scenario with $K = 6$ communication pairs as shown in Figure 1. For the simulation of the fully connected network, we consider scenarios with $K = 4$, $K = 5$, and $K = 6$ communication pairs. The simulation results for fully connected networks are required to investigate the DoF. Each of the $2K$ nodes is equipped with $N = 3$ antennas and wants to transmit d data stream to its communication partner. There is one relay equipped with $R = 4$ antennas in each of the $Q = 3$ subnetworks. From the conditions (9) and (13) it is known that the chosen scenario is feasible for our proposed IA algorithm.

The channels between the nodes and the relays are randomly generated i.i.d. Rayleigh fading channels [15]. Channel reciprocity is assumed and the channel coefficients are constant during the MAC and BC phase. Furthermore, it is assumed that the noise power at each node is the same as the relay's noise power $\sigma^2 = \sigma_k^2 = \sigma_q^2, \quad \forall k \in \mathcal{K}, \forall q \in \mathcal{Q}$. Figure 3 shows the sumrate performance of the different methods as a function of P/σ^2, where $P = P_{\text{n,max}}$ denotes the transmit power at each node. The transmit power $P_{\text{r,max}}$ at each relay is adjusted according to the different algorithms to guarantee a fair comparison.

The first reference method for a fully connected network is the iterative IA algorithm (iter_IA_fully) presented in Section IV. The transmit power at each relay is the same as that for the proposed algorithm and given by $P_{\text{r,max}}^{\text{IA}} = \frac{1}{3} K P$.

The second reference method for a partially connected network is a time division multiple access (TDMA) approach (SVD_TRxZF_TDMA) [15]. All nodes which are not located in the intersection area can transmit $d = 3$ data streams each in the first time slot. The 3 pairs in the intersection area require 3 additional time slots. This results in 8 time slots for a bidirectional communication considering two-way relaying.

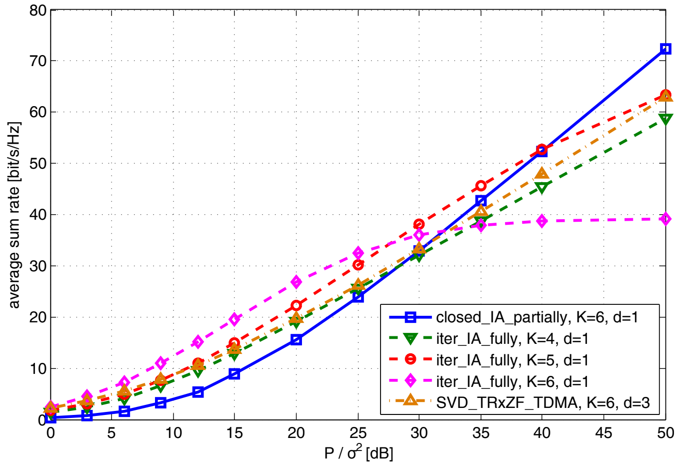

Fig. 3. Sum rate performance of the proposed algorithm in comparison with two reference methods for the scenario $Q = 3$, $R = 4$, $N = 3$

Each transmitting node designs its transmit filter according to the strongest singular vectors of the channel from the transmitting node to the relay. The relays spatially separate the data streams and perform transceive zero forcing. The transmit power at each relay is given by, $P_{\text{r,max}}^{\text{TDMA}} = \frac{1}{3}\frac{6}{4}KP$.

It can be seen in Figure 3 that the proposed algorithm "closed_IA_partially" outperforms the two reference schemes "iter_IA_fully" and "SVD_TRxZF_TDMA" in the high SNR region. Note that the reference methods are already optimized, i.e., there are no free variables for further optimization. The matrices $\mathbf{\Phi}_{\text{MAC}}$ of (14) and $\mathbf{\Phi}_{\text{BC}}$ of (17) are arbitrarily chosen from the complex space $\mathbb{C}$. Hence, the proposed algorithm has the potential for further optimization. The optimization of the two matrices $\mathbf{\Phi}_{\text{MAC}}$ and $\mathbf{\Phi}_{\text{BC}}$ to maximize the sum rate, is left for future work. It is to be expected that an optimization of these matrices results in a performance gain over the whole SNR region. The slope of the curves corresponds to the achieved DoF of the different schemes. The "iter_IA_fully" algorithm for $K = 6$ communication pairs converges to a finite sum rate in the high SNR region, due to interference leakage. Therefore, the "iter_IA_fully" algorithm is interference limited in the high SNR region. This implies that IA is infeasible for this scenario. The proposed algorithm reaches the highest DoF of all considered algorithms for the scenario in Figure 1.

VI. Conclusion

In this paper, a network consisting of Q partially connected subnetworks where each subnetwork includes one relay is considered. The entire bidirectional pair-wise communication between the nodes takes place via the intermediate relays, considering the two-way relaying protocol. In total, Q relays assists the K communication pairs in the whole network to achieve interference alignment. The feasibility conditions for the proposed closed form solution are derived as $R = 2d + \frac{1}{2}|\mathcal{K}^{\text{ser}}(q)|d$ and $N \geq \frac{R+d}{2}$. It is shown that pair-wise CSI at the nodes and extended subnetwork CSI at the relays is sufficient to perform interference alignment in a partially connected network. The so-called extended subnetwork CSI is always a proper subset of global CSI. Through simulation results it has been shown that the number of DoF increases if the network is partially connected and the proposed algorithm outperforms all reference schemes in the high SNR region.

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