Multi-Convex Optimization for Sum Rate Maximization in Multiuser Relay Networks

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***Abstract*—A scenario consisting of several single antenna source-destination node pairs communicating through multiple single antenna relays is considered. A two time-slot transmission scheme is considered. In the first time-slot, the source nodes transmit to both the relays and the destination nodes. Both the source nodes and the relays retransmit to the destination nodes in the second time-slot. As the relays cannot decode the received signals, an amplify and forward relaying strategy is assumed. In the present paper, the sum rate maximization problem is tackled. Due to the two transmissions of the source nodes and the two receptions of the destination nodes, there are temporal transmit and receive filters which can be optimized together with the relays' coefficients aiming at maximizing the sum rate. By partially adapting the filters and by introducing two sets of scaling factors, the sum rate maximization problem is reformulated as a tri-convex optimization problem. An iterative algorithm is proposed which maximizes the sum rate and guarantees a local optimum achievement. The results show that the proposed algorithm outperforms the previously proposed interference alignment scheme in all SNRs.**

***Index Terms*—multiuser relay network, sum rate maximization, multi-convex optimization.**

I. Introduction

For future multiuser wireless systems, maximizing the sum rate is the ultimate goal for enhancing the performance and making best use of the limited resources. These systems are in general interference-limited and maximizing the sum rate by optimizing the spatial filters at the source nodes and at the destination nodes is a well known non-convex problem [1], [2]. Even for scenarios with single antenna nodes, optimizing the power allocation for maximizing the sum rate is a non-convex problem [1], [2]. Nevertheless, many contributions in the last decades dealt with the sum rate maximization problem and many algorithms for different scenarios have been proposed. For some special scenarios, a global maximum is achievable using global optimization methods [3]–[5]. However, these methods suffer from a high computational complexity which limits their practicability. Therefore, several problem relaxations have been proposed and based on this, algorithms with a relatively low computational complexity have been found [6]. Due to the problem relaxations, these algorithms in general do not even guarantee a local maximum achievement of the original non-convex problem anymore.

Instead of solving the sum rate maximization problem, other alternative objectives like aligning the interferences or minimizing the sum mean square error (MSE) at the destination nodes have been studied. For multiuser relay networks, optimizing the spatial filters at the nodes and the relays' coefficients for getting rid of the interferences is a trilinear problem, i.e., the interference nulling problem is linear in either the transmit filters at the source nodes, the receive filters at the destination nodes or the relays' coefficients. Accordingly, iterative algorithms which alternatingly optimize the transmit filters, the receive filters and the relays' coefficients for interference nulling have been proposed [7], [8]. Furthermore, the sum MSE is a tri-convex function of the filters at the source nodes, the destination nodes and the relays. In other words, the sum MSE function is convex in either the transmit filters at the source nodes, the receive filters at the destination nodes or the relays' coefficients. This property is exploited to achieve a local minimum sum MSE in [9], [10]. The relation between the minimum mean square error (MMSE) and the signal to interference plus noise ratio (SINR) of an estimated symbol is exploited in [11]–[13]. The receive filters were designed as MMSE filters whereas the transmit filters and the relays' coefficients were optimized to maximize the sum rate by minimizing the weighted sum MSE. However, the authors of [11]–[13] did not optimize the transmit filters, the receive filters and relays' coefficients with respect to the sum rate.

In the present paper, a scenario consisting of several node pairs and multiple relays is considered. All nodes and relays are equipped with a single antenna each. By fixing certain filter coefficients at the nodes, the estimated data symbols at the destination nodes become linear in the other filter coefficients and the relays' coefficients. Unfortunately, the sum rate is a non-concave function of the filter coefficients to be adapted and the relays' coefficients. Fortunately, the sum rate maximization problem can be reformulated as a tri-convex optimization problem by adding two sets of scaling factors. In particular, a new term related to the received SINR at a destination node including a scaling factor is introduced. For the optimum scaling factor, this new term equals $1 + \mathrm{SINR}$.

The rest of the paper is organized as follows. The next Section introduces the system model and the transmission scheme. Section III states the sum rate maximization problem. In Section IV, a new expression for the SINR is introduced. The optimization problem is reformulated and rewritten as a tri-convex optimization problem in Section V and Section VI, respectively. An iterative algorithm which maximizes the sum

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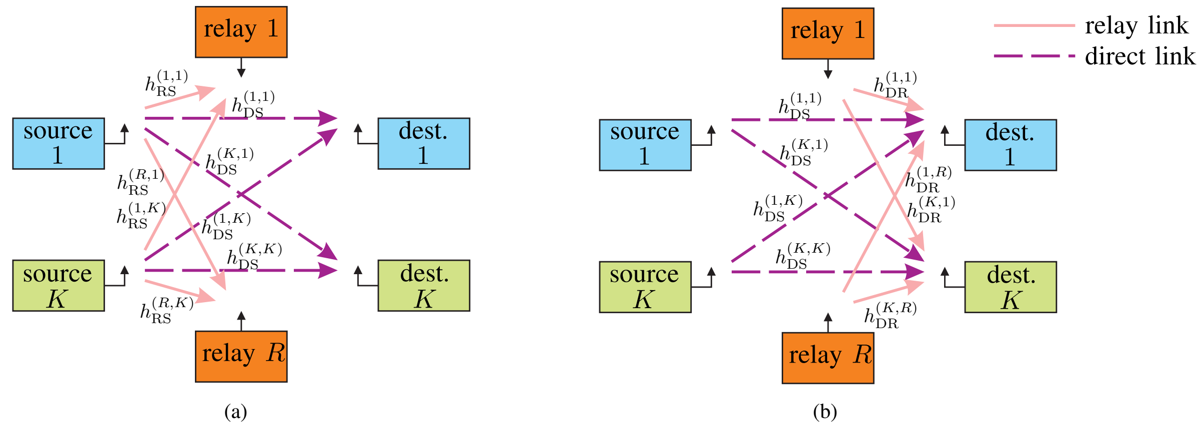

Fig. 1: Two time-slot transmission scheme: (a) the source nodes transmit to both the relays and the destination nodes in the first time-slot, (b) both the source nodes and the relays retransmit to the destination nodes in the second time-slot.

rate is proposed in Section VII. In Section VIII, numerical results which support the theoretical analysis are discussed. The conclusions are drawn in Section IX.

II. System Model and Transmission Scheme

A scenario consisting of K source-destination node pairs and R one-way relays is considered. A two time-slot transmission scheme is considered where τ denotes the time-slot. At the first time-slot $\tau = 1$, the source nodes transmit to both the relays and the destination nodes as shown in Fig. 1a. Both the source nodes and the relays retransmit to the destination nodes at the second time-slot $\tau = 2$ as illustrated in Fig. 1b. Every node and relay is equipped with a single antenna so that it is impossible for a relay to separate the received signals of the source nodes. Consequently, the amplify and forward relaying strategy is used. Frequency flat channels are assumed. The coefficients of the channels between the l-th source node and k-th destination node, the l-th source node and r-th relay and the r-th relay and the k-th destination node are denoted by $h_{\mathrm{DS}}^{(k,l)}$, $h_{\mathrm{RS}}^{(r,l)}$ and $h_{\mathrm{DR}}^{(k,r)}$, respectively. All channels are considered to be constant throughout the transmission duration and full CSI is assumed to be available at all the nodes and at the relays. Also, it is assumed that the noise signals $n_{\mathrm{D}}^{(k,\tau)}$, $k = 1, \ldots, K$, $\tau = 1, 2$, and $n_{\mathrm{R}}^{(r)}$, $r = 1, \ldots, R$, at the destination nodes and at the relays, respectively, are i.i.d. Gaussian noise with zero mean and the same variance σ^2.

Let $d^{(l)}$ denote the transmitted data symbol of the l-th source node. It is assumed that the data symbols are uncorrelated and that they have equal average energies

$$\mathrm{E}\left\{\left|d^{(l)}\right|^2\right\} = E_{\mathrm{d}}, \ \forall l. \tag{1}$$

The coefficient of the transmit filter at the l-th source node in the τ-th time-slot is denoted by $v_\tau^{(l)}$. In the first time-slot, the received signals at the k-th destination node

$$e_1^{(k)} = \sum_{l=1}^{K} h_{\mathrm{DS}}^{(k,l)} v_1^{(l)} d^{(l)} + n_{\mathrm{D}}^{(k,1)} \tag{2}$$

and the received signal at the r-th relay

$$e_{\mathrm{R}}^{(r)} = \sum_{l=1}^{K} h_{\mathrm{RS}}^{(r,l)} v_1^{(l)} d^{(l)} + n_{\mathrm{R}}^{(r)} \tag{3}$$

are obtained. In the second time-slot, the received signal at the k-th destination node reads

$$e_2^{(k)} = \sum_{l=1}^{K} h_{\mathrm{DS}}^{(k,l)} v_2^{(l)} d^{(l)} + \sum_{r=1}^{R} h_{\mathrm{DR}}^{(k,r)} g^{(r)} e_{\mathrm{R}}^{(r)} + n_{\mathrm{D}}^{(k)}, \tag{4}$$

where $g^{(r)}$ is the complex scaling factor of the r-th relay. Each destination node k receives twice and it combines the received signals with the weights $u_\tau^{(k)*}$, $\tau = 1, 2$. Consequently, the estimated data symbol at the k-th destination node reads

$$\hat{d}^{(k)} = u_1^{(k)*} e_1^{(k)} + u_2^{(k)*} e_2^{(k)}. \tag{5}$$

If $v_1^{(k)}$ and $u_2^{(k)}$ $\forall k$ are fixed, the received data symbol becomes a linear function of the variables $g^{(r)}$, $v_2^{(k)}$, and $u_1^{(k)}$, $\forall r, k$. With the vector

$$\mathbf{g} = \left(g^{(1)*}, \cdots, g^{(R)*} \middle| \, v_2^{(1)*}, \cdots, v_2^{(K)*} \middle| \, u_1^{(1)}, \cdots, u_1^{(K)} \right)^{\mathrm{T}} \tag{6}$$

of the variables and the vector

$$\begin{aligned}\mathbf{q}^{(k,l)} = \ & \big(u_2^{(k)*} v_1^{(l)} h_{\mathrm{DR}}^{(k,1)} h_{\mathrm{RS}}^{(1,l)}, \ldots, u_2^{(k)*} v_1^{(l)} h_{\mathrm{DR}}^{(k,R)} h_{\mathrm{RS}}^{(R,l)} \,| \\ & \underbrace{0, \ldots, 0}_{l-1}, u_2^{(k)*} h_{\mathrm{DS}}^{(k,l)}, \underbrace{0, \ldots, 0}_{K-l} \,|\, \underbrace{0, \ldots, 0}_{k-1}, v_1^{(l)} h_{\mathrm{DS}}^{(k,l)}, \underbrace{0, \ldots, 0}_{K-k} \big)^{\mathrm{T}}\end{aligned} \tag{7}$$

of the constants, the estimated data symbol at the k-th destination node can be written as

$$\begin{aligned}\hat{d}^{(k)} = & \sum_{l=1}^{K} \mathbf{g}^{*\mathrm{T}} \, \mathbf{q}^{(k,l)} d^{(l)} + u_2^{(k)*} n_{\mathrm{D}}^{(k,2)} \\ & + \mathbf{g}^{*\mathrm{T}} \big(u_2^{(k)*} h_{\mathrm{DR}}^{(k,1)} n_{\mathrm{R}}^{(1)}, \ldots, u_2^{(k)*} h_{\mathrm{DR}}^{(k,R)} n_{\mathrm{R}}^{(R)} \,| \\ & \underbrace{0, \ldots, 0}_{K} \,|\, \underbrace{0, \ldots, 0}_{k-1}, n_{\mathrm{D}}^{(k,1)}, \underbrace{0, \ldots, 0}_{K-k} \big)^{\mathrm{T}}.\end{aligned} \tag{8}$$

III. Problem Statement

Based on the system model introduced in the previous section, the received SINR at the k-th destination node is calculated as

$$\gamma^{(k)}(\mathbf{g}) = \frac{E_{\mathrm{d}}\mathbf{g}^{*\mathrm{T}}\mathbf{q}^{(k,k)}\mathbf{q}^{(k,k)*\mathrm{T}}\mathbf{g}}{\mathbf{g}^{*\mathrm{T}}\left(\sum\limits_{l\neq k} E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)\mathbf{g} + \sigma^2\left|u_2^{(k)}\right|^2}, \quad (9)$$

where $\mathbf{N}^{(k)}$ is a diagonal matrix with the first R diagonal elements being $\left|h_{\mathrm{DR}}^{(k,r)}\right|^2\left|u_2^{(k)}\right|^2$, the $R+K+k$-th diagonal element being one and the remaining diagonal elements being zero.

The problem of optimizing $\mathbf{g}$ aiming at maximizing the sum rate with a total energy constraint can be stated as

$$\mathbf{g}_{\mathrm{opt}} = \underset{\mathbf{g}}{\operatorname{argmax}}\left\{\sum_{k=1}^{K}\log_2\left(1+\gamma^{(k)}(\mathbf{g})\right)\right\} \quad (10)$$

subject to

$$\mathbf{g}^{*\mathrm{T}}\mathbf{C}\mathbf{g} = E_2, \quad (11)$$

where $\mathbf{C}$ is a $R+2K \times R+2K$ diagonal matrix with the first R diagonal elements being the received energies at the relays, the next K diagonal elements being E_{d} and the last K diagonal elements being zero. The transmitted energy E_1 in the first time slot is fixed as $v_1^{(l)}$, $\forall l$ are fixed and E_2 denotes the energy transmitted in the second time-slot. For the optimization problem of (10)-(11), the energy constraint of (11) is a convex set but the sum rate $\sum\limits_k \log_2\left(1+\gamma^{(k)}(\mathbf{g})\right)$ is not a concave function of $\mathbf{g}$. As a result, the optimization problem of (10)-(11) is non-convex.

IV. Signal to Interference Plus Noise Ratio

The sum rate $\sum\limits_k \log_2\left(1+\gamma^{(k)}(\mathbf{g})\right)$ is a non-concave function of $\mathbf{g}$. The main difficulty on reformulating the sum rate as a multi-concave function is that both the nominator and the denominator of the SINR are functions of $\mathbf{g}$, see (9). To overcome this problem, a new term

$$\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right) = \frac{\mathrm{E}\left\{\left|w^{(k)}\ d^{(k)}\right|^2\right\}}{\mathrm{E}\left\{\left|\hat{d}^{(k)} - w^{(k)}\ d^{(k)}\right|^2\right\}} \quad (12)$$

which describes the received SINR at a destination node, is introduced. $w^{(k)}$ is a complex weighting factor. Using the function

$$\begin{aligned}\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right) &= \mathrm{E}\left\{\left|\hat{d}^{(k)} - w^{(k)}\ d^{(k)}\right|^2\right\}\\ &= \mathbf{g}^{*\mathrm{T}}\left(\sum_l E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)\mathbf{g} + \sigma^2\left|u_2^{(k)}\right|^2\\ &\quad - E_{\mathrm{d}}w^{(k)}\mathbf{q}^{(k,k)*\mathrm{T}}\mathbf{g} - E_{\mathrm{d}}w^{(k)*}\mathbf{g}^{*\mathrm{T}}\mathbf{q}^{(k,k)} + E_{\mathrm{d}}\left|w^{(k)}\right|^2,\end{aligned} \quad (13)$$

which is obtained from (8), $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ can be rewritten as

$$\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right) = \frac{E_{\mathrm{d}}\left|w^{(k)}\right|^2}{\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)}. \quad (14)$$

If $w^{(k)}$ is fixed, $\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ is a convex function of $\mathbf{g}$ as $\left(\sum\limits_l E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)$ is a positive semidefinite matrix. Taking the generalized derivative of $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ with respect to $w^{(k)}$ and equalizing it to zero yields

$$\frac{\partial\eta^{(k)}}{\partial w^{(k)}} \overset{!}{=} 0. \quad (15)$$

A single stationary point is found by solving (15) for $w^{(k)}$ and the optimum weighting factor is calculated as

$$w_{\mathrm{opt}}^{(k)} = \frac{\mathbf{g}^{*\mathrm{T}}\left(\sum\limits_l E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)\mathbf{g} + \sigma^2\left|u_2^{(k)}\right|^2}{E_{\mathrm{d}}\mathbf{q}^{(k,k)*\mathrm{T}}\mathbf{g}}. \quad (16)$$

By substituting (16) into (14), $\eta_{\mathrm{opt}}^{(k)}$ with the optimum weighting factor is calculated as

$$\begin{aligned}\eta_{\mathrm{opt}}^{(k)}(\mathbf{g}) &= \frac{\mathbf{g}^{*\mathrm{T}}\left(\sum\limits_{l=1}^{K} E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)\mathbf{g} + \sigma^2\left|u_2^{(k)}\right|^2}{\mathbf{g}^{*\mathrm{T}}\left(\sum\limits_{l\neq k} E_{\mathrm{d}}\mathbf{q}^{(k,l)}\mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2\mathbf{N}^{(k)}\right)\mathbf{g} + \sigma^2\left|u_2^{(k)}\right|^2}\\ &= 1 + \gamma^{(k)}(\mathbf{g}).\end{aligned} \quad (17)$$

From (14), the extreme values of $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ ranging from zero if $w^{(k)} = 0$ and 1 if $w^{(k)} = \infty$. Also, because there is a single stationary point and it is greater than or equal to one see (17), $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ is a concave function with respect to $w^{(k)}$. We observe that just the denominator of $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ is a function of $\mathbf{g}$ whereas both the nominator and the denominator of $\gamma^{(k)}(\mathbf{g})$ are functions of $\mathbf{g}$.

V. Problem Reformulation

Based on the result of (17), the vector of the unknowns $\mathbf{g}$ as well as the weighting vector

$$\mathbf{w} = \left(w^{(1)}, \ldots, w^{(K)}\right)^{\mathrm{T}} \quad (18)$$

can be jointly optimized for maximizing the sum rate. Based on this idea, using

$$\mathrm{f}_2(\mathbf{g}, \mathbf{w}) = \sum_{k=1}^{K}\log_2\left(\eta^{(k)}(\mathbf{g}, \mathbf{w})\right), \quad (19)$$

the optimization problem of (10)-(11) can be reformulated as

$$\left(\mathbf{g}_{\mathrm{opt}}, \mathbf{w}_{\mathrm{opt}}\right) = \underset{\mathbf{g},\mathbf{w}}{\operatorname{argmax}}\left\{\mathrm{f}_2(\mathbf{g}, \mathbf{w})\right\} \quad (20)$$

subject to

$$\mathbf{g}^{*\mathrm{T}}\mathbf{C}\mathbf{g} = E_2. \quad (21)$$

Clearly, the objective function $\mathrm{f}_2(\mathbf{g}, \mathbf{w})$ is concave with respect to $\mathbf{w}$ as $\eta^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ is a concave function of $w^{(k)}$ and the logarithm is a concave monotonic increasing function [14]. The objective function $\mathrm{f}_2(\mathbf{g}, \mathbf{w})$ can be rewritten as

$$\mathrm{f}_2(\mathbf{g}, \mathbf{w}) = \sum_{k=1}^{K} \log_2\left(E_\mathrm{d}\left|w^{(k)}\right|^2\right) - \sum_{k=1}^{K} \log_2\left(\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)\right). \quad (22)$$

In (22), only the second term depends on $\mathbf{g}$. Although $\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)$ with fixed $w^{(k)}$ is a convex function of $\mathbf{g}$, the function $\log_2\left(\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)\right)$ with fixed $w^{(k)}$ is not necessarily a convex function of $\mathbf{g}$ [15]. Hence, a problem reformulation which will be described in the next section is needed.

VI. Sum Rate as a Multi-Concave Function

In this section, K additional scaling factors are introduced and the optimization problem of (20)-(21) is reformulated as a tri-convex optimization problem. Consider the function

$$\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m}) = \ln(2) \sum_{k=1}^{K} \log_2\left(m^{(k)}\right) + \sum_{k=1}^{K} \log_2\left(E_\mathrm{d}\left|w^{(k)}\right|^2\right) - \sum_{k=1}^{K} m^{(k)} \mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right), \quad (23)$$

where

$$\mathbf{m} = \left(m^{(1)}, \ldots, m^{(K)}\right)^\mathrm{T} \quad (24)$$

is a vector of positive real variables

$$m^{(k)} > 0, \ \forall k. \quad (25)$$

To show the equivalence between $\mathrm{f}_2(\mathbf{g}, \mathbf{w})$ and $\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m})$, the first order optimality condition

$$\frac{\partial \mathrm{f}_3}{\partial m^{(k)}} \overset{!}{=} 0 \quad (26)$$

with respect to $m^{(k)}$ is investigated. A single stationary point

$$m_\mathrm{opt}^{(k)} = \frac{1}{\mathrm{f}_1^{(k)}\left(\mathbf{g}, w^{(k)}\right)} \quad (27)$$

is found by solving (26) for $m^{(k)}$. By substituting (27) into (23), one obtains

$$\mathrm{f}_3\big(\mathbf{g}, \mathbf{w}, \mathbf{m}_\mathrm{opt}\big) = \mathrm{f}_2(\mathbf{g}, \mathbf{w}) + c_\mathrm{const}, \quad (28)$$

where c_const is a constant value. Accordingly, the optimization problem of (20)-(21) is equivalently stated as

$$\left(\mathbf{g}_\mathrm{opt}, \mathbf{w}_\mathrm{opt}, \mathbf{m}_\mathrm{opt}\right) = \underset{\mathbf{g},\mathbf{w},\mathbf{m}}{\operatorname{argmax}} \left\{\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m})\right\} \quad (29)$$

subject to

$$\mathbf{g}^{*\mathrm{T}} \mathbf{C} \mathbf{g} = E_2. \quad (30)$$

The optimization problem of (29)-(30) is a non-convex problem in the vectors $\mathbf{g}$, $\mathbf{w}$ and $\mathbf{m}$ when they are jointly optimized. However, if $\mathbf{w}$ and $\mathbf{g}$ are fixed, the objective function $\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m})$ is concave in $\mathbf{m}$. As described in the previous section, the objective function $\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m})$ is concave in $\mathbf{w}$ if both $\mathbf{g}$ and $\mathbf{m}$ are fixed. Furthermore, if both $\mathbf{w}$ and $\mathbf{m}$ are fixed, just the last term of (23) is required to be considered as an objective function. Accordingly, the optimization problem of (29)-(30) can be reformulated as

$$\mathbf{g}_\mathrm{opt} = \underset{\mathbf{g}}{\operatorname{argmin}} \left\{\mathbf{g}^{*\mathrm{T}} \mathbf{A} \mathbf{g} - \mathbf{b}^{*\mathrm{T}} \mathbf{g} - \mathbf{g}^{*\mathrm{T}} \mathbf{b}\right\} \quad (31)$$

subject to

$$\mathbf{g}^{*\mathrm{T}} \mathbf{C} \mathbf{g} = E_2, \quad (32)$$

where

$$\mathbf{A} = \sum_{k=1}^{K} m^{(k)} \left(E_\mathrm{d} \sum_{l} \mathbf{q}^{(k,l)} \mathbf{q}^{(k,l)*\mathrm{T}} + \sigma^2 \mathbf{N}^{(k)}\right) \quad (33)$$

is a positive semidefinite matrix as the scaling factors $m^{(k)}$, $\forall k$ are positive, see (25), and

$$\mathbf{b} = E_\mathrm{d} \sum_{k=1}^{K} m^{(k)} w^{(k)*} \mathbf{q}^{(k,k)}. \quad (34)$$

The optimization problem of (31)-(32) is convex as the objective $\left(\mathbf{g}^{*\mathrm{T}} \mathbf{A} \mathbf{g} - \mathbf{b}^{*\mathrm{T}} \mathbf{g} - \mathbf{g}^{*\mathrm{T}} \mathbf{b}\right)$ is a convex quadratic function and the constraint of (32) is a convex set. The structure of the problem of (31)-(32) is similar to the one of the sum MSE minimization problem which is solved in [16].

Based on the above discussion, the optimization problem of (29)-(30) is a convex problem for either $\mathbf{g}$, $\mathbf{w}$ or $\mathbf{m}$ individually. This class of optimization problems is known as multi-convex optimization problems [17]. In multi-convex optimization, this structure of the problem is exploited and several tools and techniques are proposed to find a local optimum as well as the global optimum, for more information see [17], [18].

VII. Iterative Algorithm

In this section, an iterative algorithm which alternately maximizes the objective function $\mathrm{f}_3(\mathbf{g}, \mathbf{w}, \mathbf{m})$ over either $\mathbf{g}$, $\mathbf{w}$ or $\mathbf{m}$ and achieves a local maximum is described. Let ϵ be an arbitrary small tolerance value. Then, the proposed algorithm is summarized as follows:

set initial values for $\mathbf{w}^{(0)}$ and $\mathbf{m}^{(0)}$
in every iteration i
  calculate $\mathbf{g}^{(i)}$ given $\mathbf{w}^{(i-1)}$ and $\mathbf{m}^{(i-1)}$ ▷ see [16] for solving (31)-(32)
  calculate $\mathbf{w}^{(i)}$ given $\mathbf{g}^{(i)}$ ▷ using (16)
  calculate $\mathbf{m}^{(i)}$ given $\mathbf{g}^{(i)}$ and $\mathbf{w}^{(i)}$ ▷ using (27)
stop if
$\left|\mathrm{f}_3\left(\mathbf{g}^{(i)}, \mathbf{w}^{(i)}, \mathbf{m}^{(i)}\right) - \mathrm{f}_3\left(\mathbf{g}^{(i-1)}, \mathbf{w}^{(i-1)}, \mathbf{m}^{(i-1)}\right)\right| \leq \epsilon$

This algorithm guarantees a local optimum achievement [17].

VIII. Numerical Results

In this section, the sum rate per time-slot

$$C = \frac{1}{2} \sum_{k=1}^{K} \log_2\left(1 + \gamma^{(k)}(\mathbf{g})\right) \quad (35)$$

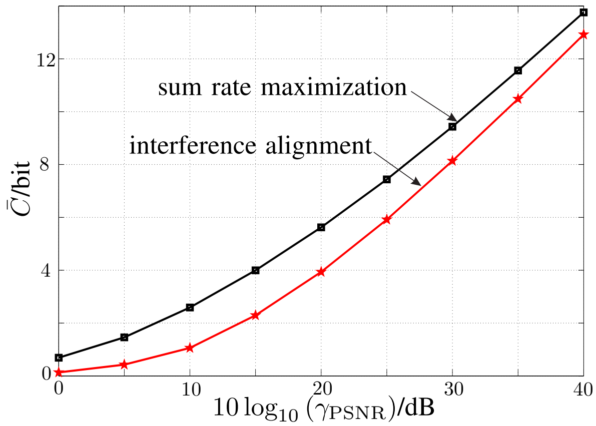

Fig. 2: Average sum rate as a function of the pseudo SNR.

is considered as a performance measure. The performance of the proposed algorithm is calculated as a function of the pseudo SNR

$$\gamma_{\mathrm{PSNR}} = \frac{E_1 + E_2}{\sigma^2}, \tag{36}$$

where the fixed coefficients of the transmit filters are chosen such that the half of the total energy is transmitted in the first time slot $E_1 = E_2$. Moreover, the energy is distributed equally over the source nodes in the first time-slot. In the following, a scenario with $K = 3$ node pairs and $R = 2$ relays is considered. This number $R = 2$ of relays is the minimum one required for interference alignment (IA) [19]. The channels between the node pairs and between the nodes and the relays are modeled as frequency flat i.i.d. Rayleigh fading channels with average gain one. The IA scheme proposed in [19] where an arbitrary IA solution is picked and scaled to satisfy the total energy constraint is considered as a reference scheme. In this reference scheme, the filters at the node are partially adapted together with the relays coefficients aiming at nulling the interferences at the destination nodes.

The average sum rate per time-slot $\bar{C}$ for many different channel realizations is depicted as a function of pseudo SNR in decibels in Fig. 2. As can be seen from Fig. 2, the sum rate maximization algorithm outperforms the IA scheme for all pseudo SNRs.

IX. Conclusion

The present paper considers the sum rate maximization problem in multiuser relay networks. By partially adapting the filters at the nodes and by adding two sets of scaling factors, the sum rate maximization problem is reformulated as a tri-convex optimization problem. Simple low complexity iterative algorithms achieving local maxima can be developed using this new formulation. The numerical results support the mathematical analysis and show that the proposed sum rate maximization scheme outperforms the state of the art at IA scheme.

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