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Interference Alignment Aided by Locally Connected Relays

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***Abstract*—In this paper, we consider a class of relay networks made up of two partially connected subnetworks. Every subnetwork consists of several relays along with the nearby source-destination node pairs being connected to the relays and, is assumed to be fully connected. The two subnetworks are coupled through the so called inter-subnetwork direct links. A linear interference alignment approach exploiting the one-way relaying protocol is applied. The feasibility conditions for interference alignment, i.e., the required numbers of relays in the whole network and in every subnetwork, are investigated. To this end, we develop a graph-based method to model the network topology and introduce the external constraints, from which an upper bound and a lower bound of the minimum required number of relays in a single subnetwork are obtained. Furthermore, we characterize the feasible region for achieving interference alignment in the considered networks using these bounds.**

I. Introduction

Recent research on interference management techniques in relay interference channel has focused on the required number of relays/relay antennas for achieving a certain number of degrees of freedom (DoF). Interference alignment (IA) aided by relays is a promising technique being able to achieve high per-user DoF with only few time extensions as well as few antennas at the source and at the destination nodes [1]–[4]. However, state of the art relay-aided IA algorithms assume fully connected relay networks, i.e., the networks with all communications links between the nodes and the relays having non-negligible channel gains. To achieve IA in fully connected networks, many relays/relay antennas are required [2]–[4], especially in large networks with lots of node pairs, which is one of the major difficulties for the implementation of relay-aided IA. In realistic scenarios, some of the communications links may be relatively weak as compared to the other links and, can be even neglected at reasonable signal-to-noise-ratios (SNRs), e.g., a relay may be only accessible by the nearby nodes. Such networks can be assumed to have partial connectivity. Partial connectivity has been exploited for IA without relays to increase the achievable DoF [5]–[7], but it has not yet been considered for relay-aided IA to reduce the required number of relays/relay antennas.

In this paper, we consider the relay networks consisting of several single-antenna source-destination node pairs and several single-antenna amplify-and-forward relays. Information shall be transmitted from the source nodes towards the destination nodes exploiting both the direct links and the relay links. Partial connectivity is introduced, i.e., some links with negligibly small channel gains are assumed to be absent. More specifically, the whole network are made up of two partially connected subnetworks. Each subnetwork includes a subset of the relays along with the nearby node pairs being connected to these relays and, is assumed to be fully connected. The two subnetworks have neither common relays nor common node pairs. However, the so called inter-subnetwork direct links between the source nodes in one subnetwork and the destination nodes in the other one may exist. For instance, Fig. 1 illustrates a network made up of two partially connected subnetworks. Due to the presence of the inter-subnetwork links, the IA problems in the two subnetworks are coupled, e.g., the required number of relays in a single subnetwork is influenced by the number of available relays in the other subnetwork. The influence will be interpreted as external constraints. We develop a graph-based method to investigate the external constraints. Furthermore, we also derive an upper bound and a lower bound of the minimum required number of relays in a single subnetwork and characterize the feasible region for IA using these bounds.

In Section II, the system model and the IA conditions are introduced. In Section III, we study the external constraints. These external constraints will then be exploited in Section IV for investigating the required numbers of relays in the considered networks. Finally, we compare the performances achieved by a few representative scenarios based on simulations and conclude our work.

II. System Model and Linear IA

Recall the relay network made up of two partially connected subnetworks as introduced in Section I. The whole network consists of a set of K single-antenna node pairs $\{(\mathrm{s}_1, \mathrm{d}_1), \ldots, (\mathrm{s}_K, \mathrm{d}_K)\}$ and a set of Q single-antenna relays $\{\mathrm{r}_1, \ldots, \mathrm{r}_Q\}$. Let K_n and Q_n denote the number of node pairs and the number of relays in the n-th subnetwork, respectively. Every subnetwork is assumed to have at least three node pairs. Each source node s_k transmits a single data symbol intended for the corresponding destination node d_k through a constant interference channel using two time slots. In the first time slot, each source node transmits to the connected destination nodes and all relays in the corresponding subnetwork. In the second time slot, the source nodes retransmit to the connected destination nodes while every relay forwards a scaled version

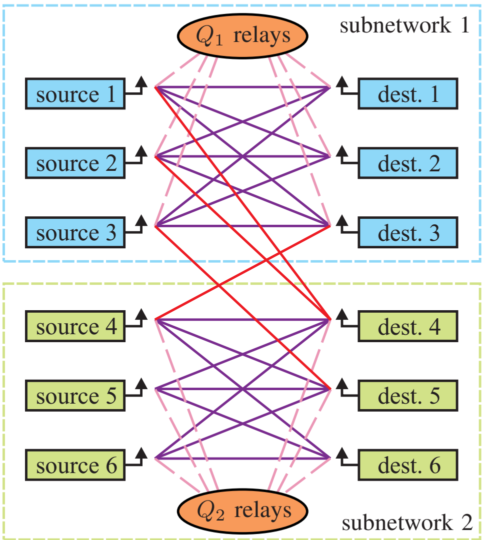

Fig. 1. A network made up of two partially connected subnetworks with Q_1 and Q_2 single-antenna relays, respectively

of its received signal to all the destination nodes in the corresponding subnetwork. Let $h_{\rm DS}^{(k,j)}$, $h_{\rm RS}^{(q,j)}$, and $h_{\rm DR}^{(k,q)}$ denote the channel coefficients of the links between s_j and d_k, between s_j and r_q, and between r_q and d_k, respectively. The channel coefficients of the present links are assumed to be continuously and independently distributed over the complex field. The other channel coefficients are set to zero. Global channel state information is assumed to be available at all nodes and relays. Furthermore, let $\mathbf{v}^{(j)} = (v_1^{(j)}, v_2^{(j)})^{\rm T}$ and $\mathbf{u}^{(k)} = (u_1^{(k)}, u_2^{(k)})^{\rm T}$ denote the temporal transmit and receive filters at s_j and d_k, respectively. Let $g^{(q)}$ denote the scaling factor at the relay r_q.

We exploit the linear algorithm proposed in [4] to achieve IA. On the one hand, IA requires that the interferences at all destination nodes shall be nulled:

$$\sum_{q=1}^{Q} h_{\rm DR}^{(k,q)} g^{(q)} h_{\rm RS}^{(q,j)} + h_{\rm DS}^{(k,j)} \left(\frac{v_2^{(j)}}{v_1^{(j)}} + \frac{u_1^{(k)*}}{u_2^{(k)*}} \right) = 0,\ \forall k, j,\ k \neq j, \tag{1}$$

where the relay scaling factors $g^{(q)}$ and the ratios of the filter coefficients $v_2^{(j)}/v_1^{(j)}$ and $u_1^{(k)*}/u_2^{(k)*}$ are chosen as variables. Let a solution to the interference-nulling conditions of (1) be written in the vector form $\mathbf{x} = (\mathbf{x}_1^{\rm T}, \mathbf{x}_2^{\rm T})^{\rm T}$, where $\mathbf{x}_n$ is a vector including the $R_n + 2K_n$ variables of the n-th subnetwork. We refer to the solution space W of (1) as the interference-nulling solution space. Note that if relay links are not available between two directly connected nodes s_j and d_k, the interference propagating through the direct link can only be suppressed by choosing the corresponding filters orthogonal, i.e., $v_2^{(j)}/v_1^{(j)} = -u_1^{(k)*}/u_2^{(k)*}$, almost surely. On the other hand, if an interference-nulling solution $\mathbf{x} \in W$ also fulfills the equality

$$\sum_{q=1}^{Q} h_{\rm DR}^{(k,q)} g^{(q)} h_{\rm RS}^{(q,k)} + h_{\rm DS}^{(k,k)} \left(\frac{v_2^{(k)}}{v_1^{(k)}} + \frac{u_1^{(k)*}}{u_2^{(k)*}} \right) = 0, \tag{2}$$

the solution $\mathbf{x}$ is called an invalid solution with respect to the k-th node pair since the useful signal at d_k will be nulled by the receive filter. Otherwise, we refer to $\mathbf{x}$ as a valid solution with respect to the k-th node pair. The invalid solution subspace with respect to the k-th node pair, which is implicitly defined by equality (2) in W, can be either a strict subspace of W having codimension one if (2) is linearly independent of the interference-nulling conditions, or be identical to W. IA is feasible if and only if a valid solution with respect to all node pairs exists. This requires sufficient relays to ensure that the invalid solution subspace with respect to every node pair is a strict subspace of W, e.g., in a fully connected network,

$$Q \geq K^2 - 3K + 2 \tag{3}$$

relays are required [3], [4]. For random channel coefficients, the required number of relays is usually derived in the almost sure sense.

III. External Constraints

In this section, we first define the external constraints. This involves the following vector spaces. Define orthogonal projections P_1 and P_2 in $\mathbb{C}^{(Q+2K)}$:

$$P_1 = \begin{pmatrix} \mathbf{I}_{Q_1+2K_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } P_2 = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{Q_2+2K_2} \end{pmatrix}, \tag{4}$$

where $\mathbf{I}$ is identity matrix. Performing P_n on the interference-nulling solution space W results in a space P_nW. In other words, for any vector $(\mathbf{x}_1^{\rm T}, \mathbf{x}_2^{\rm T})^{\rm T} \in W$, there are $(\mathbf{x}_1^{\rm T}, \mathbf{0})^{\rm T} \in P_1W$ and $(\mathbf{0}, \mathbf{x}_2^{\rm T})^{\rm T} \in P_2W$. Furthermore, let a subsystem of the interference-nulling conditions be formed by the equations corresponding to the intra-subnetwork interferences in the n-th subnetwork. Denote the solution space of this subsystem by W_n. Thus W_n consists of all the vectors that only null the intra-subnetwork interferences in the n-th subnetwork, leaving the remaining interferences unconsidered. Performing P_n on W_n yields a space P_nW_n. Obviously, P_nW is a subspace of P_nW_n, because W is a common subspace of W_1 and W_2. With the help of these vector spaces, a set of external constraints of the n-th subnetwork can be defined as a system of linear equations such that any vector $\mathbf{x} \in P_nW_n$ also belongs to P_nW if and only if $\mathbf{x}$ satisfies the system of linear equations.

Normally, the external constraints of a single subnetwork, e.g., the first one, depend on the channel realization. However, we will show that in special cases, e.g., if $Q_2 = 0$ and $Q_2 \geq K_2(K_2-1)$, the external constraints only depend on the network topology. In the following, we propose a graph-based method to identify a set of external constraints of the first subnetwork in each of the above cases. The same approach applies for the second subnetwork. Define graph G to be an undirected bipartite graph with the vertices being the source and the destination nodes of a given network and the edges being the direct interference links, i.e., the direct links except for the ones between the node pairs $(\mathrm{s}_k, \mathrm{d}_k)$. Particularly, we define an external path to be a path on the graph G with both ends belonging to the same subnetwork and all intermediate vertices belonging to the other subnetwork. We also introduce the following notations. If $v_2^{(j)}/v_1^{(j)} = -u_1^{(k)*}/u_2^{(k)*}$ holds, the filter $\mathbf{v}^{(j)}$ is orthogonal to $\mathbf{u}^{(k)}$. Let this be denoted by $\mathbf{v}^{(j)} \perp \mathbf{u}^{(k)}$. If $v_2^{(j)}/v_1^{(j)} = v_2^{(k)}/v_1^{(k)}$ holds, the filter $\mathbf{v}^{(j)}$ is aligned with $\mathbf{v}^{(k)}$. Let this be denoted by $\mathbf{v}^{(j)} \parallel \mathbf{v}^{(k)}$.

In the case of $Q_2 = 0$, nulling the intra-subnetwork interference between two nodes in the second subnetwork requires the corresponding transmit and receive filters being orthogonal, almost surely. Consequently, if two nodes in the first subnetwork, e.g., s_j and d_k, are connected by an external path, the filters at any two neighboring nodes in the external path are almost surely orthogonal. Furthermore, the total number of edges in any external path connecting s_j and d_k must be odd because G is a partite graph. Therefore, if s_j and d_k are connected by at least one external path, $\mathbf{v}^{(j)} \perp \mathbf{u}^{(k)}$ almost surely follows. Similarly, two source nodes s_j and s_k or two destination nodes d_j and d_k in the first subnetwork being connected by at least one external path respectively results in the constraint $\mathbf{v}^{(j)} \parallel \mathbf{v}^{(k)}$ or $\mathbf{u}^{(j)} \parallel \mathbf{u}^{(k)}$, almost surely. We claim that in the case of $Q_2 = 0$, all the constraints resulting from the external paths of the first subnetwork as discussed above form a set of external constraints specifying the subspace P_1W in P_1W_1. To prove this, we need to show: (a) if $(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}})^{\mathrm{T}}$ is an interference-nulling solution, i.e., $(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}})^{\mathrm{T}}$ belongs to W, then $(\mathbf{x}_1^{\mathrm{T}}, \mathbf{0})^{\mathrm{T}}$ satisfies these constraints; (b) for any vector $(\mathbf{x}_1^{\mathrm{T}}, \mathbf{0})^{\mathrm{T}} \in W_1$ satisfying these constraints, there exists a vector $\mathbf{x}_2$ such that $(\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}})^{\mathrm{T}}$ belongs to W. The necessity, i.e., (a), is trivial since these constraints are deduced from the interference-nulling conditions and only involve the filters in the first subnetwork. To show the sufficiency, i.e., (b), we can first choose the filter at the end of an inter-subnetwork link in the second subnetwork to be orthogonal to the filter at the other end of the link. Afterwards, the remaining filters in the second subnetwork can be pairwise orthogonalized according to the previous one. This yields the required vector $\mathbf{x}_2$.

In the case of $Q_2 \geq K_2(K_2-1)$, a set of external constraints specifying the subspace P_1W in P_1W_1 can be formed by the constraints following from the external paths of the first subnetwork consisting only of the inter-subnetwork links. Similar to the case of $Q_2 = 0$, the necessity is trivial. To show the sufficiency, we first find all the external paths of the first subnetwork consisting only of the inter-subnetwork links. Each of these paths only has a single intermediate node in the second subnetwork, otherwise it includes at least one intra-subnetwork link. The filter at the intermediate node of any of these external paths can then be chosen orthogonal to the filters at the ends of the path. Finally, since the intra-subnetwork interference-nulling conditions in the second subnetwork form $K_2(K_2-1)$ linear equations, $K_2(K_2-1)$ relays are sufficient for solving these equations with arbitrarily fixed filters. Hence, the remaining filters and the relay scaling factors in the second subnetwork can be obtained. This yields the required vector $\mathbf{x}_2$.

However, the sets of external constraints derived above may be linearly dependent. To find a set of linearly independent external constraints in the case of $Q_2 = 0$, we define another

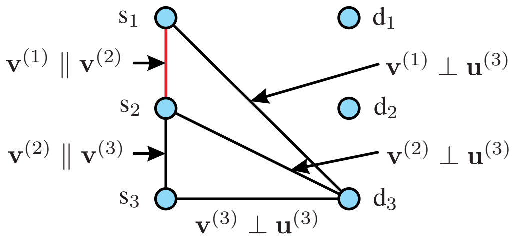

Fig. 2. Graph G_1 with respect to the network shown in Fig. 1

graph G_1. The vertices of G_1 are the source and the destination nodes in the first subnetwork. Every external constraint in the case of $Q_2 = 0$ corresponds to an edge of G_1. For instance, a diagram of the graph G_1 for the network shown in Fig. 1 is illustrated in Fig. 2. By graph theory, a set of linearly independent external constraints corresponds to the edges of a maximal forest of G_1, i.e., a maximal acyclic subgraph of G_1 including all vertices [8]. In Fig. 2, a maximal forest of G_1 has three edges corresponding to, e.g., $\mathbf{v}^{(1)} \parallel \mathbf{v}^{(2)}$, $\mathbf{v}^{(2)} \parallel \mathbf{v}^{(3)}$, and $\mathbf{v}^{(3)} \perp \mathbf{u}^{(3)}$, which are linearly independent. Similarly, define H_1 to be a graph with the same vertices as G_1 and the edges being the external constraints in the case of $Q_2 \geq K_2(K_2-1)$. For instance, the graph H_1 for the network shown in Fig. 1 has only one edge $\mathbf{v}^{(1)} \parallel \mathbf{v}^{(2)}$. The number of edges in a maximal forest of G_1 or H_1 can be denoted by $\mathrm{rank}(G_1)$ or $\mathrm{rank}(H_1)$, respectively.

IV. Required Numbers of Relays

We then consider the required number of relays in a single subnetwork. Let a valid solution with respect to the n-th subnetwork be defined as an interference-nulling solution which is valid with respect to all node pairs belonging to the n-th subnetwork. Thus a valid solution with respect to a single subnetwork exists if every invalid solution subspace with respect to a node pair belonging to the subnetwork is a strict subspace of W, and vise versa. Hence, IA is feasible, i.e., all invalid solution subspaces are strict subspaces in W, if and only if there is a valid solution with respect to every single subnetwork. Thus, the number of relays in every subnetwork shall be sufficient for guaranteeing the existence of a valid solution with respect to the subnetwork. On the one hand, since each individual subnetwork is fully connected, the required number of relays in the n-th subnetwork without external constraints is $K_n^2 - 3K_n + 2$ by (3). On the other hand, the external constraints do not involve the relays. Thus from an engineering point of view, they are almost surely linearly independent of the intra-subnetwork interference-nulling conditions if there are sufficient relays. Therefore, satisfying each external constraint besides nulling the intra-subnetwork interferences requires one more relay. Hence, the required number of relays in the first subnetwork is at least $\overline{Q}_1 = K_1^2 - 3K_1 + 2 + \mathrm{rank}(G_1)$ if $Q_2 = 0$ holds, and $\underline{Q}_1 = K_1^2 - 3K_1 + 2 + \mathrm{rank}(H_1)$ if $Q_2 \geq K_2(K_2-1)$ holds. Furthermore, in the case of $Q_2 \geq K_2(K_2-1)$, the external constraints of the first subnetwork only depends on the inter-subnetwork connectivity. In other words, these constraints always needs to be satisfied, almost surely, regardless of the number of available relays in the second subnetwork. Consequently, $\underline{Q}_1$ is a lower bound of the minimum required number of relays in the first subnetwork. Accordingly, if the first subnetwork has at least $\overline{Q}_1$ relays, a valid solution with respect to the first subnetwork exists for any number of relays in the second subnetwork. Thus $\overline{Q}_1$ is an upper bound of the minimum required number of relays in the first subnetwork. Similarly, we can define $\overline{Q}_2$ and $\underline{Q}_2$, which are the upper and the lower bound of the minimum required number of relays in the second subnetwork, respectively.

Proposition 1: In a given network made up of two subnetworks with at least three node pairs each, $\overline{Q}_n$ and $\underline{Q}_n$ satisfy

$$\overline{Q}_1 - \underline{Q}_1 = \overline{Q}_2 - \underline{Q}_2. \tag{5}$$

Proof: First consider two disconnected subnetworks. Thus $\overline{Q}_n = \underline{Q}_n = K_n^2 - 3K_n + 2$ holds for both subnetworks in this case. Then we modify the network by adding inter-subnetwork links to it. Adding the first inter-subnetwork link does not produce any external path in the modified network. Thus equality (5) still holds. Without loss of generality, we assume that an inter-subnetwork link e_0 with the ends s_j in the first subnetwork and d_k in the second subnetwork is added to a network with at least one inter-subnetwork link, which corresponds to the graphs G, G_n and H_n. The resulting network corresponds to the graphs G', G'_n and H'_n. Then the following three cases shall be distinguished.

Case I. Neither s_j nor d_k is an end of the previously added inter-subnetwork links. If the second subnetwork has at least three node pairs, any two nodes in the second subnetwork are connected by a path in G. Therefore, adding e_0 results in at least one external path in G' between s_j and every end of the previously added links in the first subnetwork. Accordingly, new edges following from these paths shall be included in the modified graph G'_1. However, only one more edge ending at s_j is included in a maximal forest of G'_1. Thus, adding e_0 results in increasing $\overline{Q}_1$ by one. On the other hand, since d_k is not an end of any previously added link, all the new external paths resulting from adding e_0 involve intra-subnetwork links of the second subnetwork. Hence, the graph H'_1 is identical to H_1 and $\underline{Q}_1$ remains unchanged. Accordingly, $\overline{Q}_2$ is increased by one and $\underline{Q}_2$ remains unchanged.

Case II. Either s_j or d_k is an end of a previously added inter-subnetwork link. Firstly, assume that d_k is an end of a previously added link e_1 which has the other end s_i in the first subnetwork. For the same reason as in Case I, adding e_0 results in increasing $\overline{Q}_1$ by one. However, e_0 and e_1 form a new external path in G' consisting only of inter-subnetwork links. Therefore, the graph H'_1 includes a new edge between s_i and s_j. Although several previously added links may have d_k as a common end, only one more edge ending at d_k shall be included in a maximal forest of graph H'_1. Therefore, $\underline{Q}_1$ is increased by one. However, adding e_0 does not affect $\overline{Q}_2$ and $\underline{Q}_2$ because e_0 has the common end d_k with e_1 and, therefore, does not produce new external constraints for the second subnetwork. Secondly, if s_j instead of d_k is a common end with e_1, $\overline{Q}_1$ and $\underline{Q}_1$ remain unchanged whereas $\overline{Q}_2$ and $\underline{Q}_2$ are both increased by one.

Case III. Both s_j and d_k are ends of previously added inter-subnetwork links. Assume that d_k is an end of e_1 which has the other end s_i in the first subnetwork, and s_j is an end of e_2 which has the other end d_l in the second subnetwork. Then $\overline{Q}_1$ and $\overline{Q}_2$ remain unchanged, because G'_n is identical to G_n for both subnetworks. We only consider the influence of adding e_0 on $\underline{Q}_1$ and $\underline{Q}_2$. (a) If s_i and d_l are already connected by a previously added link e_3, then e_1, e_2 and e_3 form a path between s_j and d_k in G, which results in $\mathbf{v}^{(j)} \perp \mathbf{u}^{(k)}$. Thus, adding e_0 does not introduce a linearly independent interference-nulling condition to the network. Consequently, $\underline{Q}_1$ and $\underline{Q}_2$ remain unchanged. (b) If for any choice of e_1 and e_2, the ends s_i and d_l are not connected by a previously added link, then adding e_0 result in a new edge ending at s_j in a maximal forest of H'_1. Therefore, $\underline{Q}_1$ is increased by one. Accordingly, $\underline{Q}_2$ is increased by one as well.

To conclude, the equality of (5) will not be affected by adding an arbitrary number of inter-subnetwork links between two disconnected subnetwork, in arbitrary order. ∎

We will then characterize the feasible region for IA, i.e., the pairs of the required relay numbers (Q_1, Q_2) such that IA is feasible, in the considered networks.

Proposition 2: In a network consisting of two subnetworks with at least three node pairs each, the feasible region for IA is given by

$$\begin{cases} Q_1 + Q_2 > \overline{Q}_1 + \underline{Q}_2 - 2 & \text{(6a)} \\ Q_n \geq \underline{Q}_n, \quad n = 1, 2 & \text{(6b)} \end{cases}$$

in the almost sure sense.

Proof: If the two subnetworks are disconnected, then $\underline{Q}_n = \overline{Q}_n$ holds for both subnetworks. Consequently, the inequality of (6b) implies (6a). Besides, if (6b) holds, $Q_n \geq \overline{Q}_n$ holds as well in this case. Therefore, IA is almost surely feasible. Otherwise, IA is almost surely infeasible.

If there is at least one inter-subnetwork link, we will obtain (6a) by counting the number of variables N_V and the number of constraints N_C while taking the invalid solution subspaces into account. In other words, $N_\mathrm{V} > N_\mathrm{C} + 1$ shall hold so that the equality of (2) with respect to each node pair is linearly independent of the interference-nulling conditions. The total number of free variables N_V is simply $Q + 2K$. The total number of constraints consists of two parts. Firstly, the intra-subnetwork interference-nulling conditions correspond to $K_1(K_1-1)+K_2(K_2-1)$ constraints. Secondly, adding every inter-subnetwork link as we did in the proof of Proposition 1 produces an additional constraint that the filters at its ends need to be orthogonal, except for Case III(a), because it does not introduce a linearly independent constraint. Let N_1, N_2 and N_3 denote the numbers of times that Case I, II and III(b) occur when adding the inter-subnetwork links, respectively. Since the first link added between two disconnected subnetworks shall be counted additionally, the inter-subnetwork links introduce

$N_1 + N_2 + N_3 + 1$ constraints. Furthermore, the equations

$$N_1 - N_3 = \overline{Q}_1 - \underline{Q}_1 \tag{7}$$

$$N_2 + 2N_3 = \underline{Q}_1 - (K_1^2 - 3K_1 + 2) + \underline{Q}_2 - (K_2^2 - 3K_2 + 2) \tag{8}$$

can be summarized from the proof of Proposition 1. Hence, the total number of constraints is

$$N_\text{C} = \overline{Q}_1 + \underline{Q}_2 + 2K - 3. \tag{9}$$

Comparing N_V and N_C obtained above yields condition (6a). Furthermore, inequality (6b) is a necessary condition. Hence, Proposition 2 follows. ■

Proposition 2 also implies that the bounds $\overline{Q}_n$ and $\underline{Q}_n$ are tight for two subnetworks. Recall the example shown in Fig. 1. It is already derived in Section III that $\text{rank}(G_1) = 3$ and $\text{rank}(H_1) = 1$ hold. This results in $\overline{Q}_1 = 5$ and $\underline{Q}_1 = 3$. We can also derive $\overline{Q}_2 = 4$ and $\underline{Q}_2 = 2$ using the same approach. By Proposition 2, the feasible region for IA as partly shown in Table I is obtained.

TABLE I
FEASIBLE REGION FOR IA IN THE NETWORK SHOWN IN FIG. 1

Q_2 \ Q_1	0	1	2	3	4	5	6
0	×	×	×	×	×	1st	1st
1	×	×	×	×	×	1st	1st
2	×	×	×	×	√	√	√
3	×	×	×	√	√	√	√
4	2nd	2nd	2nd	√	√	√	√
5	2nd	2nd	2nd	√	√	√	√

√: Valid solutions w.r.t. both subnetworks exist.
1st/2nd: Valid solutions w.r.t. only one subnetwork exist.
×: All solutions are invalid w.r.t. both subnetworks.

V. Simulation Results

In this section, we evaluate the performance achieved in the network illustrated in Fig. 1 based on simulations. The present links are assumed to be i.i.d. Rayleigh fading with unit average gain. Zero mean additive white Gaussian noise is assumed at the relays and at the destination nodes. Equal power allocation is assumed among the source nodes. The total transmit power at the relays is assumed to be equal to the total power at the source nodes. The filters and the relay scaling factors are obtained from a randomly picked interference-nulling solution of (1). The performance is measured by the average sum-rate per time-slot C as a function of the pseudo SNR γ_PSNR, which is defined to be the ratio of the total transmitted energy by both the source nodes and the relays to the noise variance [4]. Fig. 3 shows the achieved performances with different relay numbers. With $(3, 3)$ or $(4, 2)$ relays, IA is almost surely feasible. The total number of achieved DoF is 3. There is no qualitative sum-rate difference between these two cases. With $(5, 0)$ or $(0, 4)$ relays, valid solutions with respect to only one subnetwork can be obtained and IA is almost surely infeasible. The total number of DoF is 1.5. As a reference scenario, we also assume that each relay is accessible by all node pairs. Then 20 global relays are required to achieve 3 DoF in the network.

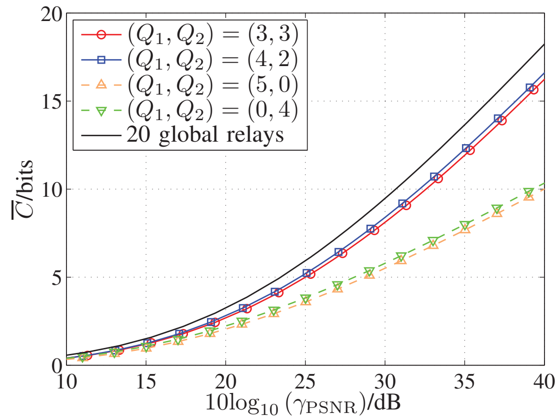

Fig. 3. Achieved performances in the network shown in Fig. 1

VI. Conclusion

In this paper, we apply a linear relay-aided IA algorithm to a class of networks made up of two partially connected subnetworks. We introduce the external constraints to investigate the coupling of the two subnetworks. A graph-based method is proposed to identify the sets of external constraints as well as the required numbers of relays. We show that using locally connected relays can help to achieve IA with less relays as compared to the one required in the fully connected networks.

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