R. S. Ganesan, H. Al-Shatri, T. Weber and A. Klein, ”Iterative MMSE filter design for multi-pair two-way multi-relay networks,” in *Proc. IEEE International Conference on Communications*, Jun. 2013.

©2008 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this works must be obtained from the IEEE.

Iterative MMSE filter design for multi-pair two-way multi-relay networks

Rakash SivaSiva Ganesan*, Hussein Al-Shatri†, Tobias Weber† and Anja Klein*
*Communication Engineering Lab, Technische Universität Darmstadt, Merckstrasse 25, 64283 Darmstadt, Germany,
†Institute of Communications Engineering, University of Rostock, Richard-Wagner-Strasse 31, 18119 Rostock, Germany,
{r.ganesan, a.klein}@nt.tu-darmstadt.de, {hussein.al-shatri, tobias.weber}@uni-rostock.de

***Abstract*—In this paper, a bi-directional communication between K node pairs is considered. Each of the $2K$ nodes has multiple antennas. There is no direct link between the nodes. Q half-duplex relays each with R antennas support the communication. Two-way relaying is assumed. The linear transmit filters, relay filters and receive filters are designed iteratively to minimize the mean square error (MMSE) subject to power constraints at the nodes and at the relays. It is guaranteed that the proposed scheme achieves a local minimum of the objective function. The proposed iterative MMSE algorithm can also be applied to a uni-directional communication based on one-way relaying. Simulation results show that at low and moderate signal to noise ratios, the proposed iterative MMSE scheme performs better than other MMSE based and interference alignment based two-way and one-way relaying schemes known in the literature.**

I. Introduction

Relays are employed for range extension and capacity enhancement in wireless networks. In this paper, bi-directional communication between K node pairs is considered and we focus on two-way relaying. A single relay with multiple antennas can support communication between K node pairs [1]–[8]. In [1]–[4], single antenna nodes are considered. The relay filters are designed based on different objectives. In [1], [2] the relay filters are designed to zero force, i.e., to null interference at the receiver. In [3], signal to interference plus noise ratio (SINR) and minimum mean square error (MMSE) are taken as the design criteria. A non zero-forcing based beamforming scheme is described in [4]. This scheme considers the trade-off between maximizing the useful signal and suppressing the interference signal.

In [5]–[8], multiple antennas are considered at the nodes. In [5], zero forcing (ZF) is performed at the relay and the multiple antennas at the nodes are used to maximize the effective channel gain. ZF and MMSE based relay filters are used in [6]. The transmit and receive filters are designed to maximize the received signal power. In [7], the relay and receive filters are calculated iteratively, to minimize the mean square error (MSE). For the design of the MMSE filter at the receiver, inter-pair interference is neglected due to limited channel knowledge at the receiver [7]. In [8] and the references therein, the transmit, relay and receive filters are designed jointly to perform interference alignment at the receivers.

Rakash SivaSiva Ganesan and Anja Klein are involved in the LOEWE Priority Program Cocoon (www.cocoon.tu-darmstadt.de).

In [9]–[11], multiple single antenna relays supporting multiple single antenna node pairs are considered. In [9] and [10], the relay coefficients are designed to suppress the interferences at the receiver nodes. In case the number of relays is not sufficient to suppress the interferences, a least squares solution to minimize the interferences at the receivers is described in [10]. In [11], the relay coefficients are designed to satisfy given SINR contraints at the receivers and to minimize the transmit power at the relays.

In this paper, we consider the general case where multiple relays with multiple antennas support a bidirectional communication between K node pairs with multiple antennas. The transmit, relay and receive filters are derived to minimize the MSE taking into account that the self-interference can be cancelled at the nodes. The term MSE refers to the overall MSE at all the $2K$ receiver nodes. As the joint optimization of the transmit, relay and receive filters is a non-convex oproblem, in this paper, the filters are derived in three steps. First, for fixed transmit and relay filters, the optimum receive filters are derived in closed form. Secondly, for fixed transmit and receive filters, the relay filters that minimize the MSE are formulated as a semidefinite programming (SDP) problem. The optimum relay filters are obtained using convex optimization. Similar to the second step, in the third step the optimum transmit filters are obtained through convex optmization by fixing the receive and the relay filters. These three steps are repeated iteratively. It is shown that the proposed iterative MMSE scheme converges. The special case of $K = 1$ is considered in [12], where the relay and receive filters are jointly optimized. Our proposed scheme in the current paper is a generalization of the iterative MMSE scheme given in [13] for one-way relaying. In addition to this, at the receiver the correlation of the interference signals forwarded through multiple relays, which is neglected in [13], is taken into account in the proposed scheme. We show how our proposed iterative MMSE algorithm can also be applied to a uni-directional communication based on one-way relaying.

The organization of the paper is as follows. The system model is introduced in Section II. In Section III, the proposed iterative MMSE scheme is described. Section IV evaluates the performance of the proposed scheme in terms of the sum rate of the system. Section V concludes the paper. We use lower case letters for scalars and lower case bold letters and upper case bold letters to denote column vectors and matrices, respectively.

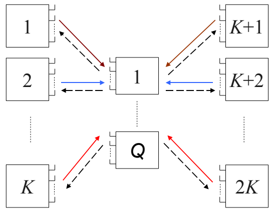

Fig. 1. K-pair two-way relay network

$(.)^*$, $(.)^{\mathrm{T}}$ and $(.)^{\mathrm{H}}$ denote the complex conjugate, transpose and complex conjugate transpose of the element within the brackets, respectively. The square root of a matrix $\mathbf{A}$ is denoted by $\mathbf{A}^{\frac{1}{2}}$ such that $\mathbf{A} = \mathbf{A}^{\frac{1}{2}\mathrm{H}}\mathbf{A}^{\frac{1}{2}}$. Tr$(.)$ and vec$(.)$ denote the trace and vectorization operations, respectively.

II. System Model

Figure 1 shows the K-pair two-way relay network with Q amplify and forward half-duplex relays each having R antennas. Each of the $2K$ nodes has N antennas and wants to transmit d data streams to its communication partner. Global channel knowledge is assumed at all the nodes and the relays. Let node j and node k be the communication partners for $j = 1, \ldots, 2K$ and $k = j + K$ if $j \leq K$ and $k = j - K$ if $j > K$. There is no direct link between the nodes and two-way relaying [9] is assumed. In the first time slot called multiple access (MAC) phase, the $2K$ nodes transmit the signal to the relays and in the second time slot called broadcast (BC) phase, the relays broadcast the signals to the $2K$ nodes. Let $\mathbf{d}_j$ and $\mathbf{V}_j$ denote the data symbols and the transmit filter matrix, respectively, of node j. Let $\mathbf{H}^{\mathrm{sr}}_{jq}$ denote the MIMO channel matrix between node j and relay q in the MAC phase. Let $\mathbf{n}_{1q}$ and $\mathbf{n}_{2k}$ denote the noise at relay q and node k, respectively. The components of the noise vectors of relay q and node k are i.i.d. complex Gaussian random variables which follow $\mathcal{CN}(0, \sigma^2_{1q})$ and $\mathcal{CN}(0, \sigma^2_{2k})$, respectively. The signal received at relay q is given by

$$\mathbf{r}_q = \sum_{i=1}^{2K} \mathbf{H}^{\mathrm{sr}}_{iq}\mathbf{V}_i\mathbf{d}_i + \mathbf{n}_{1q}. \tag{1}$$

Let $\mathbf{H}^{\mathrm{rd}}_{qk}$ denote the MIMO channel matrix between relay q and node k in the BC phase. Let $\mathbf{G}_q$ denote the matrix representing the linear signal processing performed at the relay. Each of the Q relays has a total transmit power P_{relay} available for transmission. The received signal $\mathbf{y}_k$ at node k is given by

$$\begin{aligned}\mathbf{y}_k = \; & \textstyle\sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{jq}\mathbf{V}_j\mathbf{d}_j + \sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{kq}\mathbf{V}_k\mathbf{d}_k \\ & + \textstyle\sum_{i=1, i\neq j,k}^{2K} \sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{iq}\mathbf{V}_i\mathbf{d}_i + \widetilde{\mathbf{n}}_k \end{aligned} \tag{2}$$

where $\widetilde{\mathbf{n}}_k = \sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{n}_{1q} + \mathbf{n}_{2k}$ is the effective noise at receiver k. In (2), the first term corresponds to the useful signal. The second and the third terms correspond to the self interference and unknown interferences, respectively. It is assumed that the self interference can be perfectly cancelled. Let $\mathbf{U}^{\mathrm{H}}_k$ denote the receive filter at node k. Further, let

$$\mathbf{A}_{jk} = \sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{jq}\mathbf{V}_j, \tag{3}$$

$$\mathbf{e}_k = \sum_{\substack{i=1 \\ i\neq j,k}}^{2K} \sum_{q=1}^{Q} \mathbf{H}^{\mathrm{rd}}_{qk}\mathbf{G}_q\mathbf{H}^{\mathrm{sr}}_{iq}\mathbf{V}_i\mathbf{d}_i. \tag{4}$$

Then the estimated data symbols are given by

$$\hat{\mathbf{d}}_j = \mathbf{U}^{\mathrm{H}}_k\left(\mathbf{A}_{jk}\mathbf{d}_j + \mathbf{e}_k + \widetilde{\mathbf{n}}_k\right). \tag{5}$$

The MSE of $\hat{\mathbf{d}}_j$ at receiver k is given by

$$MSE_k = \mathbb{E}\left\{\|\hat{\mathbf{d}}_j - \mathbf{d}_j\|^2\right\}. \tag{6}$$

It is assumed that the data symbols are independent and zero mean complex Gaussian distributed with variance one. Hence, $\mathbb{E}\left\{\mathbf{d}_j\mathbf{d}^{\mathrm{H}}_j\right\} = \mathbf{R}_{\mathbf{d}_j}$ and $\mathbb{E}\left\{\mathbf{d}_j\mathbf{d}^{\mathrm{H}}_i\right\} = \mathbf{0}$ for $i \neq j$. Then (6) can be expressed as

$$\begin{aligned} MSE_k = & \operatorname{Tr}\left(\left(\mathbf{U}^{\mathrm{H}}_k\mathbf{A}_{jk} - \mathbf{I}\right)\mathbf{R}_{\mathbf{d}_j}\left(\mathbf{A}^{\mathrm{H}}_{jk}\mathbf{U}_k - \mathbf{I}\right)\right) + \\ & \operatorname{Tr}\left(\mathbf{U}^{\mathrm{H}}_k\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}^{\mathrm{H}}_k\right\}\mathbf{U}_k\right) + \operatorname{Tr}\left(\mathbf{U}^{\mathrm{H}}_k\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}^{\mathrm{H}}_k\right\}\mathbf{U}_k\right). \end{aligned} \tag{7}$$

In this paper, the objective is to design the transmit filters, receive filters and filters at the relays such that the MSE is minimized subject to the power constraint at the nodes and the relays. This is given by

$$\begin{aligned} \underset{\mathbf{V}_j, \mathbf{U}_k, \mathbf{G}_q}{\text{minimize}} \quad & \overline{MSE} = \sum_{k=1}^{2K} MSE_k \\ \text{subject to} \quad & \operatorname{Tr}\left(\mathbf{G}_q\mathbf{R}_q\mathbf{G}^{\mathrm{H}}_q\right) \leq P_{\text{relay}} \;\text{ for }\; q = 1, \ldots, Q \\ & \operatorname{Tr}\left(\mathbf{V}_j\mathbf{R}_{\mathbf{d}_j}\mathbf{V}^{\mathrm{H}}_j\right) \leq P_{\text{node}} \;\text{ for }\; j = 1, \ldots, 2K \end{aligned} \tag{8}$$

where

$$\mathbf{R}_q = \mathbb{E}\left\{\mathbf{r}_q\mathbf{r}^{\mathrm{H}}_q\right\} = \sum_{i=1}^{2K}\left(\mathbf{H}^{\mathrm{sr}}_{iq}\mathbf{V}_i\right)\mathbf{R}_{\mathbf{d}_i}\left(\mathbf{H}^{\mathrm{sr}}_{iq}\mathbf{V}_i\right)^{\mathrm{H}} + \mathbf{R}_{n1q}. \tag{9}$$

The optimization problem in (8) is non-convex [14]. In the following, we propose an iterative scheme to obtain a local minimum.

III. Proposed Iterative MMSE Algorithm

In this section, an iterative algorithm to find a local optimum for the minimization of the MSE is described. The optimization variables in (8) are the transmit, relay and receive filter coefficients. In the proposed scheme, one of the three types of variables are optimized while fixing the other two. In the following, first these three steps are explained in detail. Then it is shown how the proposed iterative algorithm can be applied to one-way relaying as special case.

A. Design of Receive Filters

In this subsection, for fixed transmit and relay filters, the optimum receive filters are derived in closed form. First we initialize the transmit and relay filters arbitrarily. As the MSE_k involves only the receive filter $\mathbf{U}_k$ at receiver k, the receive filters can be optimized independently. For the fixed transmit and relay filters, the optimization problem described in (8) is an unconstrained quadratic optimization problem. The optimum $\mathbf{U}_k$ is given by the condition

$$\frac{\partial MSE_k}{\partial \mathbf{U}^*_k} \overset{!}{=} \mathbf{0}. \tag{10}$$

Substituting (7) in (10) gives

$$\mathbf{A}_{jk}\mathbf{R}_{\mathbf{d}_j}\left(\mathbf{A}_{jk}^{\mathrm{H}}\mathbf{U}_k - \mathbf{I}\right) + \mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\}\mathbf{U}_k + \mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\mathbf{U}_k = \mathbf{0}. \tag{11}$$

From (11), the optimum $\mathbf{U}_k$ that minimizes MSE_k is given by

$$\mathbf{U}_k = \left[\mathbf{A}_{jk}\mathbf{R}_{\mathbf{d}_j}\mathbf{A}_{jk}^{\mathrm{H}} + \mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} + \mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\right]^{-1}\mathbf{A}_{jk}\mathbf{R}_{\mathbf{d}_j} \tag{12}$$

where

$$\mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\} = \sum_{\substack{i=1\\ i\neq j,k}}^{2K}\sum_{q=1}^{Q}\sum_{\overline{q}=1}^{Q}\left(\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\right)\mathbf{R}_{\mathbf{d}_i}\left(\mathbf{H}_{\overline{q}k}^{\mathrm{rd}}\mathbf{G}_{\overline{q}}\mathbf{H}_{i\overline{q}}^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{H}} \tag{13}$$

$$\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\} = \sum_{q=1}^{Q}\left(\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\right)\mathbf{R}_{n1q}\left(\mathbf{H}_{qk}^{\mathrm{rd}}\mathbf{G}_q\right)^{\mathrm{H}} + \mathbf{R}_{n2k}. \tag{14}$$

B. Design of Relay Filters

In this subsection, the transmit and receive filters are fixed and the optimum relay filters are derived. For fixed transmit and receive filters, the optimization problem in (8) becomes a quadratically constrained quadratic minimization problem. The objective function can be expressed as

$$\overline{MSE} = \sum_{k=1}^{k=2K}\mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\left(\mathbf{A}_{jk}\mathbf{R}_{\mathbf{d}_j}\mathbf{A}_{jk}^{\mathrm{H}} + \mathbb{E}\left\{\mathbf{e}_k\mathbf{e}_k^{\mathrm{H}}\right\}\right)\mathbf{U}_k\right) - 2\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{A}_{jk}\mathbf{R}_{\mathbf{d}_j}\right)\right\} + \mathrm{Tr}\left(\mathbf{U}_k^{\mathrm{H}}\mathbb{E}\left\{\widetilde{\mathbf{n}}_k\widetilde{\mathbf{n}}_k^{\mathrm{H}}\right\}\mathbf{U}_k + \mathbf{R}_{\mathbf{d}_j}\right). \tag{15}$$

Substituting (13) and (14) in (15) and using the identity $\mathrm{Tr}\,(XYZ) = \mathrm{Tr}\,(YZX)$, we get

$$\begin{aligned}\overline{MSE} &= 2\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{G}_q\mathbf{B}_{1q}\right)\right\} - 2\mathrm{Re}\left\{\mathrm{Tr}\left(\mathbf{G}_q\mathbf{B}_{2q}\right)\right\} + \\ &\mathrm{Tr}\left(\left(\mathbf{D}_{2q}^{\frac{1}{2}}\mathbf{G}_q\mathbf{D}_{1q}^{\frac{1}{2}\mathrm{H}}\right)^{\mathrm{H}}\left(\mathbf{D}_{2q}^{\frac{1}{2}}\mathbf{G}_q\mathbf{D}_{1q}^{\frac{1}{2}\mathrm{H}}\right)\right) - \\ &\sum_{k=1}^{2K}\mathrm{Tr}\left(\left(\mathbf{D}_{4kq}^{\frac{1}{2}}\mathbf{G}_q\mathbf{D}_{3kq}^{\frac{1}{2}\mathrm{H}}\right)^{\mathrm{H}}\left(\mathbf{D}_{4kq}^{\frac{1}{2}}\mathbf{G}_q\mathbf{D}_{3kq}^{\frac{1}{2}\mathrm{H}}\right)\right) + \\ &\mathrm{Tr}\left(\left(\mathbf{D}_{2q}^{\frac{1}{2}}\mathbf{G}_q\mathbf{R}_{n1q}^{\frac{1}{2}\mathrm{H}}\right)^{\mathrm{H}}\left(\mathbf{D}_{2q}^{\frac{1}{2}}\mathbf{G}_q\mathbf{R}_{n1q}^{\frac{1}{2}\mathrm{H}}\right)\right) + \mathbf{Z}_{1q},\end{aligned} \tag{16}$$

where $\mathbf{Z}_{1q}$ represents terms independent of $\mathbf{G}_q$ and

$$\mathbf{B}_{1q} = \sum_{k=1}^{2K}\sum_{\substack{i=1,\\ i\neq k}}^{2K}\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\mathbf{R}_{\mathbf{d}_i}\sum_{\substack{\overline{q}=1,\\ \overline{q}\neq q}}^{Q}\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{\overline{q}k}^{\mathrm{rd}}\mathbf{G}_{\overline{q}}\mathbf{H}_{i\overline{q}}^{\mathrm{sr}}\mathbf{V}_i\right)^{\mathrm{H}}\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}, \tag{17}$$

$$\mathbf{B}_{2q} = \sum_{k=1}^{2K}\mathbf{H}_{jq}^{\mathrm{sr}}\mathbf{V}_j\mathbf{R}_{\mathbf{d}_j}\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}, \tag{18}$$

$$\mathbf{D}_{1q} = \sum_{i=1}^{2K}\mathbf{H}_{iq}^{\mathrm{sr}}\mathbf{V}_i\mathbf{R}_{\mathbf{d}_i}\mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{iq}^{\mathrm{srH}}, \quad \mathbf{D}_{2q} = \sum_{k=1}^{2K}\mathbf{H}_{qk}^{\mathrm{rdH}}\mathbf{U}_k\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}, \tag{19}$$

$$\mathbf{D}_{3kq} = \mathbf{H}_{kq}^{\mathrm{sr}}\mathbf{V}_k\mathbf{R}_{\mathbf{d}_k}\mathbf{V}_k^{\mathrm{H}}\mathbf{H}_{kq}^{\mathrm{srH}}, \quad \mathbf{D}_{4kq} = \mathbf{H}_{qk}^{\mathrm{rdH}}\mathbf{U}_k\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{qk}^{\mathrm{rd}}. \tag{20}$$

Now we can separate the variable $\mathbf{G}_q$ in (16) by using the identities $\mathrm{vec}\left(\mathbf{YXZ}\right) = \left(\mathbf{Z}^{\mathrm{T}}\otimes\mathbf{Y}\right)\mathrm{vec}\left(\mathbf{X}\right)$ and $\mathrm{Tr}\left(\mathbf{XY}\right) = \mathrm{vec}^{\mathrm{H}}\left(\mathbf{X}^{\mathrm{H}}\right)\mathrm{vec}\left(\mathbf{Y}\right)$. This results in

$$\begin{aligned}\overline{MSE} =& 2\mathrm{Re}\left\{\mathrm{vec}^{\mathrm{H}}\left(\mathbf{G}_q^{\mathrm{H}}\right)\mathrm{vec}\left(\mathbf{B}_{1q} - \mathbf{B}_{2q}\right)\right\} \\ &+ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{G}_q\right)\mathbf{F}_q^{\frac{1}{2}\mathrm{H}}\mathbf{F}_q^{\frac{1}{2}}\mathrm{vec}\left(\mathbf{G}_q\right) + \mathbf{Z}_{1q}\end{aligned} \tag{21}$$

where

$$\begin{aligned}\mathbf{F}_q =& \left(\mathbf{D}_{1q}^{\frac{1}{2}*}\otimes\mathbf{D}_{2q}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\mathbf{D}_{1q}^{\frac{1}{2}*}\otimes\mathbf{D}_{2q}^{\frac{1}{2}}\right) - \sum_{k=1}^{2K}\left(\left(\mathbf{D}_{3kq}^{\frac{1}{2}*}\otimes\mathbf{D}_{4kq}^{\frac{1}{2}}\right)^{\mathrm{H}}\right. \\ &\left.\left(\mathbf{D}_{3kq}^{\frac{1}{2}*}\otimes\mathbf{D}_{4kq}^{\frac{1}{2}}\right)\right) + \left(\mathbf{R}_{n1q}^{\frac{1}{2}*}\otimes\mathbf{D}_{2q}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\mathbf{R}_{n1q}^{\frac{1}{2}*}\otimes\mathbf{D}_{2q}^{\frac{1}{2}}\right).\end{aligned} \tag{22}$$

Introducing an auxiliary variable t_1, the minimization of the sum MSE can be reformulated as

$$\begin{aligned}\underset{\mathbf{G}_q}{\text{minimize}}\quad & t_1 \\ \text{subject to}\quad & t_1 - 2\mathrm{Re}\left\{\mathrm{vec}^{\mathrm{H}}\left(\mathbf{G}_q^{\mathrm{H}}\right)\mathrm{vec}\left(\mathbf{B}_{1q} - \mathbf{B}_{2q}\right)\right\} \\ & - \mathrm{vec}^{\mathrm{H}}\left(\mathbf{G}_q\right)\mathbf{F}_q^{\frac{1}{2}\mathrm{H}}\mathbf{F}_q^{\frac{1}{2}}\mathrm{vec}\left(\mathbf{G}_q\right) \geq 0, \\ & \mathrm{Tr}\left(\mathbf{G}_q\mathbf{R}_q\mathbf{G}_q^{\mathrm{H}}\right) \leq P_{\text{relay}}\end{aligned} \tag{23}$$

for $q = 1, \ldots, Q$. The first constraint can be written as a positive semidefinite constraint using the Schur complement formula [14] as follows:

$$\begin{bmatrix}\mathbf{I} & \mathbf{F}_q^{\frac{1}{2}}\mathrm{vec}\left(\mathbf{G}_q\right) \\ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{G}_q\right)\mathbf{F}_q^{\frac{1}{2}\mathrm{H}} & t_1 - 2\mathrm{Re}\left\{\mathrm{vec}^{\mathrm{H}}\!\left(\mathbf{G}_q^{\mathrm{H}}\right)\mathrm{vec}\!\left(\mathbf{B}_{1q} - \mathbf{B}_{2q}\right)\right\}\end{bmatrix} \geq 0. \tag{24}$$

Similarly, the power constraint at the relay can be formulated as a positive semidefinite constraint

$$\begin{bmatrix}\mathbf{I} & \mathrm{vec}\left(\mathbf{R}_q^{\frac{1}{2}}\mathbf{G}_q^{\mathrm{H}}\right) \\ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{R}_q^{\frac{1}{2}}\mathbf{G}_q^{\mathrm{H}}\right) & P_{\text{relay}}\end{bmatrix} \geq 0. \tag{25}$$

Now, the optimization problem in (23) is a convex minimization with positive semidefinite constraints [14]. Convex optimization tools can be used to solve for the optimum relay filters $\mathbf{G}_q$. In this paper, we use CVX [14] to solve the problem.

C. Design of Transmit Filters

In this subsection, the transmit filters at each of the nodes are derived. Similar to the relay filter optimization, for fixed receive filters and relay filters, the optimization problem in (8) is a quadratically constrained quadratic minimization problem which is convex. The objective function and power constraints are formulated as positive semidefinite constraints and solved using convex optimization tools. The sum MSE can be expressed as

$$\begin{aligned}\overline{MSE} =& \mathrm{Tr}\left(\left(\mathbf{C}_{Qj}\mathbf{V}_j - \mathbf{I}\right)\mathbf{R}_{\mathbf{d}_j}\left(\mathbf{V}_j^{\mathrm{H}}\mathbf{C}_{Qj}^{\mathrm{H}} - \mathbf{I}\right)\right) \\ &+ \sum_{\substack{\overline{k}=1,\\ \overline{k}\neq k,j}}^{2K}\mathrm{Tr}\left(\mathbf{C}_{Q\overline{k}j}\mathbf{V}_j\mathbf{R}_{\mathbf{d}_j}\mathbf{V}_j^{\mathrm{H}}\mathbf{C}_{Q\overline{k}j}^{\mathrm{H}}\right) + \mathbf{Z}_{2j}\end{aligned} \tag{26}$$

where $\mathbf{Z}_{2j}$ is independent of $\mathbf{V}_j$ and

$$\mathbf{C}_{Qj} = \mathbf{U}_k^{\mathrm{H}} \sum_{q=1}^{Q} \mathbf{H}_{qk}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}}, \;\; \mathbf{C}_{Q\bar{k}j} = \mathbf{U}_{\bar{k}}^{\mathrm{H}} \sum_{q=1}^{Q} \mathbf{H}_{q\bar{k}}^{\mathrm{rd}} \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}}. \tag{27}$$

Similar to the previous subsection, the objective function can be formulated as minimization of an auxiliary variable t_2 subject to the following constraint:

$$\begin{bmatrix} \mathbf{I} & \mathbf{C}_{1j}^{\frac{1}{2}} \mathrm{vec}\left(\mathbf{V}_j\right) \\ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{V}_j\right) \mathbf{C}_{1j}^{\frac{1}{2}\mathrm{H}} & t_2 + 2\mathrm{Re}\left\{\mathrm{vec}^{\mathrm{H}}\!\left(\mathbf{V}_j^{\mathrm{H}}\right) \mathrm{vec}\!\left(\mathbf{R}_{\mathbf{d}_j} \mathbf{C}_{Qj}\right)\right\} \end{bmatrix} \geq 0 \tag{28}$$

where

$$\mathbf{C}_{1j} = \sum_{\substack{\bar{k}=1, \\ \bar{k}\neq j}}^{2K} \left(\mathbf{R}_{\mathbf{d}_j}^{\frac{1}{2}*} \otimes \mathbf{C}_{Q\bar{k}j}\right)^{\mathrm{H}} \left(\mathbf{R}_{\mathbf{d}_j}^{\frac{1}{2}*} \otimes \mathbf{C}_{Q\bar{k}j}\right) \tag{29}$$

for $j = 1, \ldots, 2K$. The power constraints are given by

$$\begin{bmatrix} \mathbf{I} & \mathrm{vec}\left(\mathbf{R}_{\mathbf{d}_j}^{\frac{1}{2}} \mathbf{V}_j^{\mathrm{H}}\right) \\ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{R}_{\mathbf{d}_j}^{\frac{1}{2}} \mathbf{V}_j^{\mathrm{H}}\right) & P_{\mathrm{node}} \end{bmatrix} \geq 0, \tag{30}$$

$$\begin{bmatrix} \mathbf{I} & \mathbf{C}_{3jq} \mathrm{vec}\left(\mathbf{V}_j\right) \\ \mathrm{vec}^{\mathrm{H}}\left(\mathbf{V}_j\right) \mathbf{C}_{3jq}^{\mathrm{H}} & P_{\mathrm{relay}} - \mathrm{Tr}\left(\mathbf{G}_q \mathbf{C}_{2jq} \mathbf{G}_q^{\mathrm{H}}\right) \end{bmatrix} \geq 0 \tag{31}$$

where

$$\mathbf{C}_{2jq} = \sum_{\substack{i=1, \\ i\neq j}}^{2K} \mathbf{H}_{iq}^{\mathrm{sr}} \mathbf{V}_i \mathbf{R}_{\mathbf{d}_i} \mathbf{V}_i^{\mathrm{H}} \mathbf{H}_{iq}^{\mathrm{srH}} + \mathbf{R}_{n1q}, \;\; \mathbf{C}_{3jq} = \mathbf{R}_{\mathbf{d}_j}^{\frac{1}{2}*} \otimes \mathbf{G}_q \mathbf{H}_{jq}^{\mathrm{sr}}. \tag{32}$$

for $j = 1, \ldots, 2K$ and $q = 1, \ldots, Q$. The optimum transmit filters minimizing the above SDP problem are obtained using convex optimization tools.

The receive, relay and transmit filters are optimized iteratively either till the MSE does not change significantly or till a specified number of iterations is reached. Since at each step the objective function (8) is minimized, the algorithm is guaranteed to converge to a minimum, though not necessarily to the global minimum.

D. One-Way Relaying as a Special Case

Consider a uni-directional communication of K node pairs i.e., K source nodes and K corresponding destination nodes. Modifying the above notation accordingly, each of the source nodes j, for $j = 1, \ldots, K$, transmits d data streams to its destination node k, for $k = j + K$. The proposed iterative MMSE scheme can be applied to the one-way relaying case by setting $\mathbf{d}_j = \mathbf{0}$ and $\mathbf{V}_j = \mathbf{0}$ for $j = k+1, \ldots, 2K$ and $\mathbf{U}_k^{\mathrm{H}} = \mathbf{0}$ for $k = 1, \ldots, K$ in the above derivation.

IV. Performance Analysis

In this section, the sum rate performance of the proposed iterative MMSE scheme is investigated. Both bi-directional and uni-directional communication scenarios are considered. First bi-directional communication with $N = 2, R = 5, Q = 1$, $d = 1$ and $K = 4$ is considered. The proposed iterative MMSE (IterativeMMSE) scheme is compared with three reference schemes. The first two reference schemes are based on MMSE criteria [6] and self-interference aware MMSE criteria (SIA_MMSE) [7], respectively. In addition, for a fair comparison, we design the transmit and receive filters to maximize the received signal power. The third reference method is the pair aware interference alignment (PAIA) scheme proposed in [8]. For the proposed scheme, the transmit and relay filters are arbitrarily initialized.

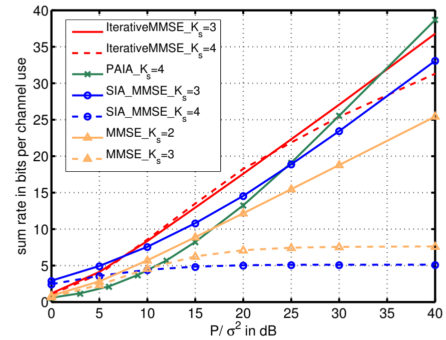

Fig. 2. Sum rate performance for a two-way relaying scenario with $K = 4, N = 2, \; R = 5, \; Q = 1$ and $d = 1$

Figure 2 shows the sum rate performance of each method as a function of P/σ^2. P is the transmit power available at each node. The relay has a transmit power $KP/Q = 4P$. σ^2 is the receive noise power at each of the relays and each of the receive nodes in the MAC and BC phases, respectively. The MIMO channel matrices are normalized such that, on average, the transmitted signal power is the same as the received signal power. The sum rate is calculated as an average value of 1000 channel realizations generated using the i.i.d. frequency-flat Rayleigh fading MIMO channel model [15].

From Figure 2 it can be seen that the proposed iterativeMMSE scheme has higher sum rates at moderate SNR values than all the considered reference schemes. As the relay has only 5 antennas, the MMSE and the SIA_MMSE can simultaneously support only $K_{\mathrm{s}} = 2$ and $K_{\mathrm{s}} = 3$ communication pairs, respectively. If the number of pairs is increased, the performance degrades significantly due to the additional interference in the system. The remaining communication pairs are served by assuming time division multiple access (TDMA) between different sets of pairs. The PAIA can simultaneously serve $K_{\mathrm{s}} = 4$ communication pairs. The proposed IterativeMMSE scheme can support $K_{\mathrm{s}} = 4$. However, the solution obtained is a local minimum solution and hence, there exist residual interferences at the receivers. At high SNR, the noise is negligible and the influence of the residual interference on the sum rate is significant. Hence, the interference alignment based scheme (PAIA) performs better than the iterativeMMSE at high SNR. In the proposed iterativeMMSE, if the number of simultaneously served users is reduced to $K_{\mathrm{s}} = 3$, then Figure 2 shows that performance improvement in comparison to the case $K_{\mathrm{s}} = 4$ can be achieved at high SNR. This gain in performance is due to the fact that the number of interferers is reduced and

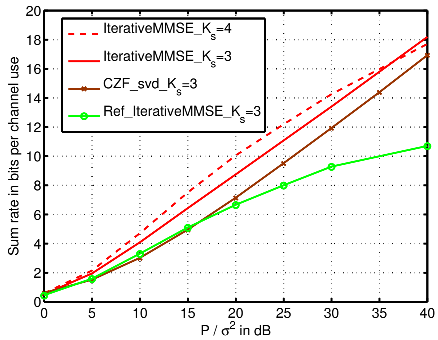

Fig. 3. Sum rate performance for a one-way relaying scenario with $K = 4, N = 2,\ R = 2,\ Q = 2$ and $d = 1$

the residual interferences at the receivers are almost zero. At very low SNR values, the SIA_MMSE is better than the other schemes. This is because at very low SNR, the influence of noise is significant and hence, it is better to improve the signal to noise ratio at the receivers than reducing the interferences.

Secondly, we consider a one-way relaying scenario. The proposed scheme can be directly applied to one-way relaying as shown in Section III-D. Here, $Q = 2$ relays each with $R = 2$ antennas and $K = 4$ transmitter-receiver node pairs each with $N = 2$ antennas each are considered. Two reference schemes are considered. The first scheme (CZF_svd) is based on the cooperative zero forcing scheme proposed in [16]. In [16], the transmitters and relays cooperate to nullify the interference signal at the receiver nodes. In [16], the receiver nodes have only a single antenna each. Hence, for a fair comparison in the simulations shown in Figure 3, we utilize the multiple antennas available at the receiver nodes to maximize the received signal power by performing singular value decomposition of the channel between the relays and the receiver. CZF_svd can support only $K_s = 3$. The second reference scheme (Ref_IterativeMMSE) is the iterative MMSE scheme proposed in [13]. Figure 3 shows that the proposed iterative MMSE scheme is better than both the reference schemes. In Ref_IterativeMMSE, the effect of the correlation of the interference signals forwarded through multiple relays is neglected. But in our proposed scheme this is taken into account and hence, has better performance. Figure 3 shows that for the case $K_s = 4$, the sum rate of the proposed iterative MMSE scheme is further increased at low and medium SNR. But now due to the additional data stream in the network, in comparison to the case $K_s = 3$, the influence of the residual MMSE is increased. Hence, at high SNR, the sum rate decreases.

V. Conclusion

In this paper, an iterative scheme to minimize the MSE is proposed for a multi-pair two-way relaying network with multiple relays. The transmit, relay and receive filters are optimized iteratively in three steps. In each step, one of the three different filters is optimized and the other two filters are fixed. First, the optimum receive filters that minimize the MMSE for fixed transmit and relay filters are derived in closed form. Secondly, for fixed transmit and receive filters, the objective function and the power constraints are reformulated into positive semidefinite constraints and the optimum relay filters are obtained using convex optimization tools. Similarly, in the third step the transmit filters are obtained using convex optimization tools. The algorithm is guaranteed to converge to a local minimum. One-way relaying is shown to be a special case of proposed iterative MMSE algorithm for two-way relaying. Simulation results show that the proposed iterative MMSE scheme perform better than the reference MMSE, zero-forcing and interference alignment based schemes.

Acknowledgment

This work is funded by the Deutsche Forschungsgesellschaft (DFG) under Grant No. Kl907/5-1 and WE2825/11-1.

References

[1] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, "Multi-Pair Two-Way Relay Channel with Multiple Antenna Relay Station," in *Proc. IEEE International Conference on Communications*, May 2010, pp. 1 –5.

[2] A. Amah and A. Klein, "Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying," in *Proc. IEEE International Conference on Acoustics Speech and Signal Processing*, Mar. 2010, pp. 2506 –2509.

[3] Y. Shen, Z. Fei, C. Xing, and J. Kuang, "Beamforming design for multi-pair two-way AF MIMO relaying networks using quadratic programming," in *Proc. IEEE International Conference on Signal Processing, Communications and Computing*, Sept. 2011, pp. 1 –5.

[4] C. C. Chai and C. Yuen, "On two-way communications for cooperative multiple source pairs through a multi-antenna relay," in *Proc. Conference Record of the Forty Fourth Asilomar Conference on Signals, Systems and Computers*, Nov. 2010, pp. 908 –912.

[5] H. Xin, Y. Peng, C. Wang, Y. Yang, and W. Wang, "Coordinated Eigen Beamforming for Multi-Pair MIMO Two-Way Relay Network," in *Proc. IEEE Global Telecommunications Conference*, Dec. 2011, pp. 1 –6.

[6] J. Joung and A. Sayed, "Multiuser Two-Way Amplify-and-Forward Relay Processing and Power Control Methods for Beamforming Systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1833 –1846, Mar. 2010.

[7] H. Degenhardt and A. Klein, "Self-interference aware MIMO filter design for non-regenerative multi-pair two-way relaying," in *Proc. IEEE Wireless Communications and Networking Conference*, Apr. 2012, pp. 272 –276.

[8] R. S. Ganesan, H. Al-Shatri, A. Kuehne, T. Weber, and A. Klein, "Pair-Aware Interference Alignment in Multi-user Two-Way Relay Networks," *submitted to IEEE Transactions on Wireless Communications - under third round review*.

[9] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379 –389, Feb. 2007.

[10] C. Wang, H. Chen, Q. Yin, A. Feng, and A. Molisch, "Multi-User Two-Way Relay Networks with Distributed Beamforming," *IEEE Transactions on Wireless Communications*, vol. 10, no. 10, pp. 3460 –3471, Oct. 2011.

[11] T. Wang, B. P. Ng, Y. Zhang, and M. H. Er, "Multiple peer-to-peer communications for two-way relay networks," in *Proc. International Conference on Information, Communications and Signal Processing*, Dec. 2011, pp. 1 –5.

[12] E. Park, K.-J. Lee, and I. Lee, "Joint MMSE Transceiver Design for MIMO Amplify-and-Forward Relay Systems with Multiple Relays," in *Proc. IEEE Vehicular Technology Conference*, Sep. 2009, pp. 1 –5.

[13] S. Ma, C. Xing, Y. Fan, Y.-C. Wu, T.-S. Ng, and H. Poor, "Iterative transceiver design for MIMO AF relay networks with multiple sources," in *Proc. Military Communications Conference*, Nov. 2010, pp. 369 –374.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge, 2004.

[15] A. Goldsmith, A. Paulraj, and H. V. Poor, *MIMO Wireless Communications*. Cambridge, UK: Cambridge, 2007.

[16] R. S. Ganesan, H. Al-Shatri, T. Weber, and A. Klein, "Cooperative Zero Forcing in Multi-Pair Multi-Relay Networks," in *Proc. International Symposium on Personal, Indoor and Mobile Communication Systems*, Sep. 2012.