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Closed form solution and useful signal power maximization for interference alignment in multi-pair two-way relay networks

Rakash SivaSiva Ganesan*, Hong Quy Le*, Hussein Al-Shatri†, Tobias Weber† and Anja Klein*
*Communication Engineering Lab, Technische Universität Darmstadt, Merckstrasse 25, 64283 Darmstadt, Germany,
†Institute of Communications Engineering, University of Rostock, Richard-Wagner-Strasse 31, 18119 Rostock, Germany,
{r.ganesan, a.klein}@nt.tu-darmstadt.de, {hussein.al-shatri, tobias.weber}@uni-rostock.de

***Abstract*—In this paper, bidirectional pair-wise communication between $2K$ nodes is considered. Each node has N antennas and wants to transmit d data streams to its communication partner. A single non-regenerative half-duplex relay with R antennas supports the communication. In this scenario, the process of interference alignment can be decomposed into *partial signal alignment* (PSA), *partial channel alignment* (PCA) and zero forcing (ZF). PSA and PCA are dual problems and we focus on PSA in this paper. PSA is a bilinear problem. A closed form solution is possible only when there is a sufficient number of variables in the system. In this paper, a closed form solution is proposed and the condition for the feasibility of the closed form solution is derived in terms of K, N, R, and d. Besides this, in order to improve the performance at low and medium signal to noise ratios (SNRs), a gradient based algorithm to maximize the useful signal power is also proposed. It is shown through simulations that in some cases, it is better to serve less node pairs and utilize the additional degrees of freedom in the system to maximize the useful signal power.**

I. Introduction

Interference alignment (IA) has been introduced for a K-user interference channel in [1]. Using the concept of IA, in [1], it is shown that the capacity of wireless networks can be increased proportional to the number of node pairs in the system. Theoretically, each of the node pairs can achieve half of the degrees of freedom which is achievable in the absence of interference. IA can be performed in the dimensions of time, frequency, space or signal level. In this paper, we focus on IA in spatial dimension.

Recently, relay aided IA has been considered in [1]–[9] and the references therein, to manipulate the effective channel coefficients between the nodes in order to achieve IA at the receivers. In [1]–[7], IA is achieved based on one-way relaying. In [8] and [9], bidirectional communication between K communication pairs is considered and IA is achieved based on two-way relaying. In this paper, we focus on IA based on two-way relaying. Each of the $2K$ nodes has N antennas and wants to transmit d data streams to its communication partner. In [8], it is shown that in order to achieve interference-free communication, it is necessary that the number R of antennas at the relay should be greater than or equal to the number of data streams transmitted in either direction i.e., $R \geq Kd$.

The case $R = Kd$ is considered in [8]. The process of interference alignment is decomposed into *signal alignment* (SA), *channel alignment* (CA) and *zero forcing* (ZF) [8]. In the first time slot called multiple access (MAC) phase, all the nodes transmit their signals to the relay such that at the relay, the signal subspace of each node aligns with the signal subspace of its partner node. This is called signal alignment (SA) [8]. It is assumed that the self interference is known and can be cancelled at the receivers. Hence, the useful signal and self interference do not need to be separated at the relay. In the second time slot called broadcast (BC) phase, the receive filters of each node are designed such that the effective channel consisting of the channel between the relay and the node and the receive filter spans the same subspace as the effective channel of its partner node. This is called channel alignment (CA) [8]. After SA and CA, there are only Kd effective data streams and Kd effective channels. The relay with $R = Kd$ antennas can perform transceive zero forcing [8]. It is also shown in [8] that SA and CA are dual problems and hence, any algorithm solving SA conditions will also solve CA conditions. A closed form solution to achieve SA is proposed in [8].

The case $R \geq Kd$ is considered in [9]. In [9], the scheme *pair-aware interference alignment* (PAIA) is introduced. In PAIA, interference alignment is decomposed into *partial signal alignment* (PSA), *partial channel alignment* and zero forcing. It is partial in the sense that SA and CA are performed in a Kd dimensional subspace of the R dimensional relay space. The key idea is that out of all possible Kd dimensional subspaces in R dimensional relay space, one Kd dimensional subspace is chosen such that SA and CA are possible with a smaller number of antennas at the nodes as compared to that in [8]. Similar to SA and CA, PSA and PCA are also dual problems. Hence, only PSA is considered. In order to design the transmit filters satisfying PSA conditions, the PSA conditions are reformulated into a problem of finding a $(R - Kd)$ dimensional subspace which has at least a d dimensional intersection subspace with each of K given $2N$ dimensional subspaces, say, $S_1, S_2, \ldots, S_K$ [9]. This is a bilinear problem and an iterative algorithm is

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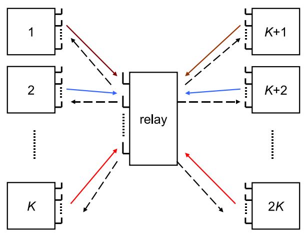

Fig. 1. K-pair two-way relay network

proposed in [9]. In addition to this, assuming that there is at least a d dimensional intersection subspace between arbitrary two subspaces S_m and S_n for $n \neq m$, a closed form solution is also proposed for some cases.

In this paper, we propose a closed form solution which is also applicable for the cases when there is no such d dimensional intersection subspace between two arbitrary subspaces S_m and S_n for $n \neq m$. The closed form solutions proposed in this paper and in [9] are feasible only if there are sufficient number of variables in the system. This means both the closed form solutions determine only a subset of all possible solutions. The condition for the feasibility of the proposed closed form solution is derived. In addition to this, a gradient based algorithm to maximize the useful signal power is proposed to improve the performance of interference alignment at low and medium signal to noise ratios (SNRs).

The paper is organized as follows. In Section II, the system model is introduced. The pair-aware interference alignment scheme [9] is briefly reviewed in Section III. In Section IV, the proposed closed form solution is given. In Section V, the gradient based method to maximize the useful signal power is described. The sum rate performance of the proposed algorithms is analyzed in Section VI. Section VII concludes the paper.

Throughout this paper, we use lower case letters, lower case bold letters and upper case bold letters to denote scalars, column vectors and matrices, respectively. $(.)^*$ and $(.)^{\mathrm{H}}$ denote the complex conjugate and complex conjugate transpose operations, respectively. We define two subspaces to be linearly independent if no vector of one subspace can be expressed as a linear combination of the basis vectors of the other subspace. Let A_1 and A_2 denote two N dimensional subspaces in an R dimensional vector space W. The intersection subspace of the two subspaces A_1 and A_2 is defined as

$$A_1 \cap A_2 \coloneqq \{\mathbf{q} \in W : \mathbf{q} \in A_1 \text{ and } \mathbf{q} \in A_2\}. \tag{1}$$

II. System Model

Figure 1 shows the K-pair two-way relay network with $2K$ nodes each having N antennas and one half-duplex non-regenerative relay having R antennas. Each node wants to transmit d data streams to its communication partner. It is assumed that $R \geq Kd$ [9] and $N \geq d$. The two-way relaying protocol [10] is considered. Without loss of generality it is assumed that nodes j and k are communication partners for $k = j + K$ when $j = 1, \ldots, K$ and $k = j - K$ when $j = K+1, \ldots, 2K$. The nodes and the relay have a maximum transmit power P_{n} and P_{r}, respectively. Let $\mathbf{d}_j$ and $\mathbf{V}_j$ denote the data vector and the transmit precoding matrix of node j, respectively. In the first time slot called multiple access (MAC) phase, all the nodes transmit to the relay. Let $\mathbf{H}_{\mathrm{r}j}$ denote the channel between node j and the relay. After linearly processing the signal received in the first time slot, the relay transmits the processed signal to all the nodes in the second time slot called broadcast (BC) phase. Let $\mathbf{H}_{k\mathrm{r}}$ denote the channel between the relay and node k. Let the vector $\mathbf{n}_{\mathrm{r}}$ denote the noise at the relay and the vector $\mathbf{n}_k$ denote the noise at node k. The components of the noise vectors $\mathbf{n}_{\mathrm{r}}$ and $\mathbf{n}_k$ are assumed to be i.i.d. complex Gaussian random variables which follow the complex normal distribution $\mathcal{CN}(0, \sigma_{\mathrm{r}}^2)$ and $\mathcal{CN}(0, \sigma_k^2)$, respectively. Let $\tilde{\mathbf{n}}_k = \mathbf{H}_{k\mathrm{r}}\mathbf{G}\mathbf{n}_{\mathrm{r}} + \mathbf{n}_k$ denote the effective noise at node k. The received signal at node k is given by

$$\mathbf{y}_k = \mathbf{H}_{k\mathrm{r}}\mathbf{G}\left[\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\mathbf{d}_j + \sum_{\substack{i=1,\\ i\neq j}}^{2K} \mathbf{H}_{\mathrm{r}i}\mathbf{V}_i\mathbf{d}_i\right] + \tilde{\mathbf{n}}_k \tag{2}$$

for $k = 1, \ldots, 2K$ [9]. It is assumed that self-interference can be perfectly cancelled at the receiver. Let $\mathbf{U}_k^{\mathrm{H}}$ denote the receive filter matrix at receiver k. Then, the estimated data stream at node k is given by

$$\hat{\mathbf{d}}_k = \mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k\mathrm{r}}\mathbf{G}\left[\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\mathbf{d}_j + \sum_{\substack{i=1,\\ i\neq j,k}}^{2K} \mathbf{H}_{\mathrm{r}i}\mathbf{V}_i\mathbf{d}_i\right] + \mathbf{U}_k^{\mathrm{H}}\tilde{\mathbf{n}}_k \tag{3}$$

[9].

III. Pair-Aware Interference Alignment

In this section, the pair-aware interference alignment (PAIA) scheme [9] is briefly explained. PAIA is introduced in [9] to achieve interference alignment in multi-pair two-way relay networks for the case $R \geq Kd$. In this paper, we assume that self-interference can be perfectly cancelled at the receivers. If self-interference cannot be cancelled, then the scheme proposed in [11] can be used. But for [11], $R \geq 2Kd$ antennas are required at the relay. In our proposed PAIA scheme, considering that self-interference can be perfectly cancelled, we need only $R \geq Kd$ antennas at the relay.

In PAIA, the process of interference alignment is decomposed into three steps, namely, partial signal alignment (PSA), partial channel alignment (PCA), and zero forcing (ZF). These three steps are explained below. In the MAC phase, each of the $2K$ nodes transmits its signal such that the subspace spanned by its d data streams aligns with that of its communication partner within a Kd dimensional subspace in the R dimensional relay space [9]. Let $\mathbf{T}^{\mathrm{H}}$ denote an orthonormal matrix that projects the received signal at the relay to a Kd dimensional subspace. Consider the communication pair (j, k). Then the condition for PSA is given by

$$\operatorname{span}\left\{\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\right\} = \operatorname{span}\left\{\mathbf{T}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}k}\mathbf{V}_k\right\} \tag{4}$$

for $k = j + K$ when $j = 1, \ldots, K$ and $k = j - K$ when $j = K+1, \ldots, 2K$ [9]. The intuition behind PSA is that the self interference can be cancelled at the receiver and hence, does not need to be separated from the useful signal. In the BC phase, PCA followed by ZF is performed to perform interference alignment at the receiver [9]. Let $\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{j\mathrm{r}}$ denote the effective channel of node j. With PCA, the subspace spanned by the effective channel of node j is made to align with the subspace spanned by the effective channel of its communication partner within a Kd dimensional subspace in the R dimensional relay space. Let $\mathbf{Q}$ denote the orthonormal matrix that maps the linearly processed signals from the Kd dimensional subspace to the R dimensional relay transmit signal space. Then the condition for PCA is given by

$$\operatorname{span}\left\{\left(\mathbf{U}_k^{\mathrm{H}}\mathbf{H}_{k\mathrm{r}}\mathbf{Q}\right)^{\mathrm{H}}\right\} = \operatorname{span}\left\{\left(\mathbf{U}_j^{\mathrm{H}}\mathbf{H}_{j\mathrm{r}}\mathbf{Q}\right)^{\mathrm{H}}\right\} \quad (5)$$

[9]. After PSA and PCA, there are Kd effective data streams in the Kd dimensional subspace at the relay and there are Kd effective channels. Hence, the relay can perform transceive zero forcing in this Kd dimensional subspace [9]. Let $\mathbf{G}_{\mathrm{z}}$ denote the transceive zero forcing matrix. Then the linear signal processing performed at the relay can be written as

$$\mathbf{G} = \mathbf{Q}\mathbf{G}_{\mathrm{z}}\mathbf{T}^{\mathrm{H}}. \quad (6)$$

Now if we look at the receiver, after receive zero forcing only the useful signal and the self-interference signal will be present. All the inter-pair interferences are nullified after receive zero forcing. This means all the inter-pair interference signals are in a subspace orthogonal to the subspace spanned by the columns of the receive zero forcing matrix. Hence, all the interference signals are within an $N-d$ dimensional interference subspace and the useful and the self interference signals are within a d dimensional useful subspace. From (4) and (5) it can be seen that PSA and PCA are dual problems. An algorithm that solves PSA will also solve PCA and hence, only PSA is considered further in this paper.

IV. Proposed Closed Form Solution

In this section, first the problem of PSA is reformulated into a problem of finding a subspace intersecting multiple subspaces [9]. Then a closed form solution for the reformulated problem is proposed. The condition for the feasibility of the proposed closed form solution is derived.

A. Reformulation of Partial Signal Alignment

For PSA, the signals from the communication partners need to be pair-wise aligned within the Kd dimensional subspace at the relay. To this extent, we divide the R dimensional relay space into two orthogonal subspaces, namely, the Kd dimensional relay useful subspace $RUSS$ and the $R-Kd$ dimensional relay interference subspace $RISS$. Signals from the communication partners are pair-wise aligned within $RUSS$. Let $RUSS_j$ denote the d dimensional alignment subspace of communication partners j and k within $RUSS$. In $RISS$, the signals from communication partners do not necessarily align. This means that the signal transmitted from any node j can be either in $RUSS_j$ or in $RISS$ but not in the relay useful subspace $RUSS_i$ for $i \neq j, k$ corresponding to other node pairs. Note that $RUSS_j$ and $RUSS_k$ denote the same subspace.

Consider the communication partners j and k. Each of the nodes transmits d data streams. The signals from node j and k span at most a $2d$ dimensional subspace

$$S_{jk} = \operatorname{span}\left\{\left[\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j \quad \mathbf{H}_{\mathrm{r}k}\mathbf{V}_k\right]\right\}. \quad (7)$$

Let the dimension of S_{jk} be given by $2d-\delta$ where $0 \leq \delta \leq d$. In this paper, we consider $\delta = 0$. For $\delta > 0$, signal alignment [8] can be performed for δ data streams and PSA can be performed for $d-\delta$ data streams. The case $\delta > 0$ is a direct combination of the algorithm proposed in [8] and of the algorithm proposed in the current paper and hence, is not considered further. Therefore, the signals from nodes j and k are assumed to span a $2d$ dimensional subspace in the R dimensional relay space. However, $RUSS_j$ corresponding to this communication pair is of dimension d. Hence, to make sure that the received signals from the communication pair (j,k) do not interfere with other pairs' signals, d dimensions of the received signals should be within $RISS$ [9]. Therefore, $RISS$ should have a d dimensional intersection subspace with that of S_{jk} for $k = j + K$ when $j = 1, \ldots, K$ and $k = j - K$ when $j = K+1, \ldots, 2K$.

B. Closed Form Solution

In this section, a closed form solution to find a $RISS$ and hence, precoding matrices and projection matrices is introduced. The closed form solution is feasible only when there are sufficient variables in the system i.e., it finds only a subset of all solutions that are achievable through the iterative algorithm proposed in [9]. The condition for the feasibility of the closed form solution is also derived.

Our objective is to find a $RISS$ such that it has a d dimensional intersection subspace with each of the $2K$ subspaces S_{jk}. Note that S_{jk} and S_{kj} denote the same subspace. Hence, in this subsection, only S_{jk} for $j = 1, \ldots, K$ and $k = j+K$ is considered. $RISS$ can be of maximum dimension $R - Kd$. We define $\overline{RISS}$ as the union of d linearly independent subspaces $RISS_l$ for $l = 1, \ldots, d$ each of dimension $n = \lfloor \frac{R-Kd}{d} \rfloor$. This can be expressed as

$$\overline{RISS} = \bigcup_{\forall l} RISS_l. \quad (8)$$

Thus, $\overline{RISS} \subseteq RISS$. Replacing $RISS$ by a subspace $\overline{RISS}$ and decomposition of $\overline{RISS}$ as in (8) makes it possible to obtain a closed form solution. In this paper, we choose each $RISS_l$ for $l = 1, \ldots, d$ such that it has a one dimensional intersection subspace with each of the K subspaces S_{jk} for $j = 1, \ldots, K$ and $k = j + K$. Then the resulting $\overline{RISS}$ will have a d dimensional intersection subspace with each of the S_{jk}. The condition that each $RISS_l$ needs to have a one dimensional intersection subspace with each S_{jk} is more strict than the condition that $RISS$ needs to have d dimensional intersection with each S_{jk}. Hence, more variables are needed

$$\underbrace{\begin{pmatrix} \mathbf{H}_{\mathrm{r}1} & \mathbf{H}_{\mathrm{r}(K+1)} & \cdots & \mathbf{H}_{\mathrm{r}(n+1)} & \mathbf{H}_{\mathrm{r}(K+n+1)} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & & & & & & & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{H}_{\mathrm{r}(K-n)} & \mathbf{H}_{\mathrm{r}(2K-n)} & \cdots & \mathbf{H}_{\mathrm{r}K} & \mathbf{H}_{\mathrm{r}(2K)} \end{pmatrix}}_{\mathbf{H}} \begin{pmatrix} \mathbf{v}_1^l \\ \mathbf{v}_{K+1}^l \\ \vdots \\ \mathbf{v}_K^l \\ \mathbf{v}_{2K}^l \end{pmatrix} = \mathbf{0}. \tag{12}$$

to obtain a closed form solution than an iterative solution. In other words, the closed form solution finds only a subset of all possible solutions that are achievable through the iterative algorithm [9]. Let $\mathbf{s}_{jk}^l$ denote the basis vector of the one dimensional intersection subspace between S_{jk} and $RISS_l$. The probability of $\mathbf{s}_{jk}^l \in \text{span}\{\mathbf{H}_{\mathrm{r}j}\mathbf{V}_j\}$ or $\mathbf{s}_{jk}^l \in \text{span}\{\mathbf{H}_{\mathrm{r}k}\mathbf{V}_k\}$ is zero. Hence, without loss of generality, we assume that $\mathbf{s}_{jk}^l$ is within the subspace spanned by l^{th} data streams of the communication pair (j,k). This can be expressed as

$$\mathbf{s}_{jk}^l \in \text{span}\left\{\left[\mathbf{H}_{\mathrm{r}j}\mathbf{v}_j^l \quad \mathbf{H}_{\mathrm{r}k}\mathbf{v}_k^l\right]\right\} \tag{9}$$

where $\mathbf{v}_j^l$ is the l^{th} column vector of the precoding matrix $\mathbf{V}_j$. Since $\mathbf{v}_j^l$ and $\mathbf{v}_k^l$ are variables in the system, (9) can be written as

$$\mathbf{s}_{jk}^l = \mathbf{H}_{\mathrm{r}j}\mathbf{v}_j^l + \mathbf{H}_{\mathrm{r}k}\mathbf{v}_k^l. \tag{10}$$

Then $\chi_l = \{\mathbf{s}_{jk}^l \mid j = 1,2,\ldots,K,\ k = j+K\}$ denotes the set of one dimensional intersections of $RISS_l$ with S_{jk} for $j = 1,2,\ldots,K,\ k = j+K$. Now $RISS_l$ can be constructed as

$$RISS_l = \text{span}\{\chi_l\}. \tag{11}$$

However, $RISS_l$ can be of maximum dimension n. Hence, any $n+1$ vectors in χ_l should be within an n dimensional subspace. This means that any $n+1$ vectors in the set χ_l should be linearly dependent of each other. At least $Kd-n$ sets of $n+1$ vectors are necessary to make sure that all the vectors within χ are in a subspace of size n. This is given by (12). (12) is a system of homogenous linear equations with at least one non-trivial solution if the number of variables is greater than the number of equations given by

$$2KN \geq (K-n)R + 1. \tag{13}$$

We have such a constraint for each of the subspaces $RISS_l$ for $l = 1,\ldots,d$. Hence, the solution space of (12) should be of dimension of at least d. This results in the following condition:

$$2KN \geq (K-n)R + d. \tag{14}$$

Let

$$\left[\ \mathbf{A}_1\ \mathbf{A}_2\ \cdots\ \mathbf{A}_{2K}\ \right]^{\mathrm{H}} = \text{null}(\mathbf{H}). \tag{15}$$

Then,

$$\mathbf{v}_j^l = \mathbf{A}_j^{\mathrm{H}}\mathbf{w}^l. \tag{16}$$

The vector $\mathbf{w}^l$ selects a solution within the solution space. Let $\mathbf{W} = \left[\mathbf{w}^1 \quad \ldots \quad \mathbf{w}^d\right]$. Then the precoding matrix can be expressed as

$$\mathbf{V}_j = \mathbf{A}_j^{\mathrm{H}}\mathbf{W}. \tag{17}$$

In order to guarantee that $\mathbf{V}_j$ is of full rank d, the matrix $\mathbf{W}$ need to be chosen such that it is of full rank d. Once the precoding matrices are calculated, $RISS_l$ and, hence, $RISS$ can be obtained from (11) and (8), respectively.

Remark 1: In the closed form solution described above, $RISS$ is obtained as a union of many subspaces $RISS_l$ for $l = 1,\ldots,d$ and $RISS_l$ is obtained as the span of χ_l which is constrained to be in an n dimensional subspace. In general, $RISS$ can be obtained as the subspace spanned by all the Kd vectors $\mathbf{s}_{jk}^l$ for $j = 1,\ldots,K$ and $k = j+K$ and $l = 1,\ldots,d$ which is constrained to be in an $R-Kd$ dimensional subspace. However, in this case, one additionally needs to make sure that the resulting precoding matrices are of rank d.

Remark 2: It should be noted that if $R-Kd$ is not an integer multiple of d, then in the above closed form solution, $R-Kd-nd$ dimensions of $RISS$ are not utilized. Hence, for some cases it might be better to scale down d by a factor f and increase K so that $RISS$ is fully utilized. In this case, more degrees of freedom can be achieved compared to the case where $R-Kd$ is not an integer multiple of d.

V. Maximization of Useful Signal Power

In this section, we focus on the case when (14) is satisfied with strict inequality sign. In this case, there are infinitely many possible solutions and we choose the solution which maximizes the useful signal power (USP) at the relay. Interference alignment aims at maximizing the number of transmitted data streams and, hence, has good performance in the high SNR regime. The algorithms introduced in Section IV look for a common interference subspace and pair-wise useful subspace. The signal power in the common interference subspace $RISS$ is nullified before forwarding the signal to the receivers. Hence, the signal power in the $RISS$ is lost. Especially at low and moderate SNR regime, it might be useful to maximize the signal power as much as possible and transmit a smaller number of data streams than possible. In this section, we propose a method for maximizing the useful signal power in the $RUSS$. In other words, the projection matrix $\mathbf{T}$ is chosen such that the total useful signal power within $RUSS$ is maximized. The term useful signal power implies the power of the signal after receive

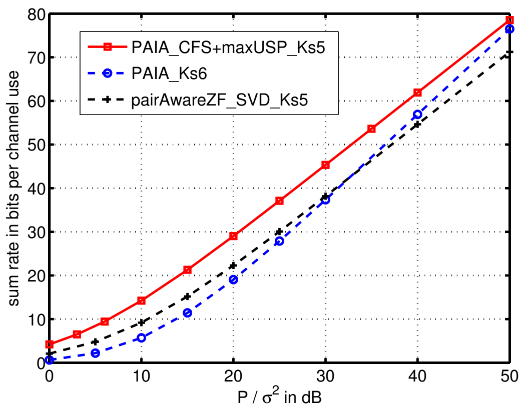

Fig. 2. Sum rate performance $K = 6,\ N = 2,\ R = 9$ and $d = 1$

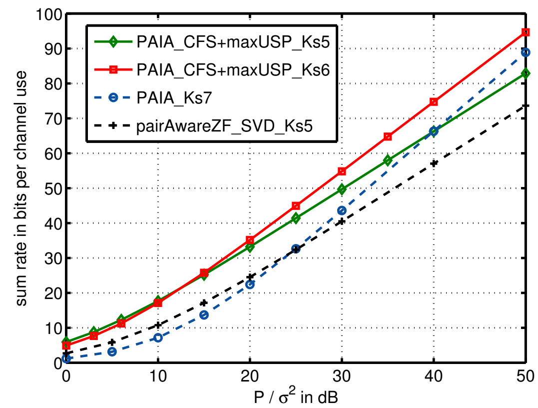

Fig. 3. Sum rate performance $K = 7,\ N = 3,\ R = 9$ and $d = 1$

zero forcing is performed at the relay to spatially separate Kd effective data streams in $RUSS$. In this paper, the sum of the SNRs of the signals from all the nodes in $RUSS$ is maximized. Similarly, in the BC phase $\mathbf{Q}$ can be chosen to maximize the sum of the SNRs of the useful signals at all the receivers. Consider the l^{th} data stream of node j for $l = 1, \ldots, d$ and $j = 1, \ldots, 2K$. The SNR of the l^{th} data stream from node j at the relay is given by

$$\text{SNR}_j^l = \frac{P_{\text{n}}}{\sigma_{\text{r}}^2} \frac{\mathbf{v}_j^{l\text{H}} \mathbf{H}_{\text{r}j}^{\text{H}} \mathbf{T} \mathbf{g}_j^l \mathbf{g}_j^{l\text{H}} \mathbf{T}^{\text{H}} \mathbf{H}_{\text{r}j} \mathbf{v}_j^l}{\mathbf{v}_j^{l\text{H}} \mathbf{v}_j^l} \tag{18}$$

where $\mathbf{g}_j^l$ denotes the receive zero forcing direction corresponding to the l^{th} data stream from node j. Due to the fact that the data streams from the communication partners j and k are aligned at the relay, we have $\mathbf{g}_j^l = \mathbf{g}_k^l$. The optimization problem is to maximize the sum of the SNRs subject to the partial signal alignment conditions. Let SSNR denote the sum of all the SNRs. The optimization problem becomes

$$\text{SSNR} = \max_{\mathbf{w}^1, \ldots, \mathbf{w}^d} \sum_{j=1}^{2K} \sum_{l=1}^{d} \frac{P}{\sigma_{\text{r}}^2} \frac{\mathbf{w}^{l\text{H}} \mathbf{A}_j \mathbf{H}_{\text{r}j}^{\text{H}} \mathbf{T} \mathbf{g}_j^l \mathbf{g}_j^{l\text{H}} \mathbf{T}^{\text{H}} \mathbf{H}_{\text{r}j} \mathbf{A}_j^{\text{H}} \mathbf{w}^l}{\mathbf{w}^{l\text{H}} \mathbf{A}_j \mathbf{A}_j^{\text{H}} \mathbf{w}^l}. \tag{19}$$

The above function is non-convex [12] in $\mathbf{w}^l$ for $l = 1, \ldots, d$ and a local solution is obtained iteratively using the gradient approach [12]. The initial value of $\mathbf{w}^l$ is chosen randomly. The gradient of the objective function is calculated and the variable $\mathbf{w}^l$ is updated iteratively using the following relation:

$$\mathbf{w}^l \rightarrow \mathbf{w}^l + \alpha \frac{\partial \text{SSNR}}{\partial \mathbf{w}^{l*}} \tag{20}$$

where the parameter α controls the step size. $\mathbf{w}^l$ and $\frac{\partial \text{SSNR}}{\partial \mathbf{w}^{l*}}$ are normalized after each iteration. The derivative can be obtained by calculating the derivative with respect to each element of $\mathbf{w}^{l*}$ [13], [14]. Note that receive zero forcing is performed at the relay to spatially separate all the Kd effective data streams. This ensures that the d data streams from any node j are linearly independent of each other and, hence, the matrix $\mathbf{W}$ and the precoding matrices $\mathbf{V}_j$ are of full rank d.

VI. Performance Analysis

In this section, the sum rate performance of the proposed algorithm is compared with two reference schemes. The first reference scheme (pairAwareZF) is based on the idea of pair-wise transceive zero forcing [15]. The multiple antennas at the relay are used to perform pair-wise transceive zero forcing and multiple antennas at the nodes are utilized to transmit the data streams in the direction corresponding to the largest singular values. The second reference scheme is the PAIA scheme (PAIA) proposed in [9]. Here, interference alignment is achieved through partial signal alignment (PSA), partial channel alignment (PCA) and zero forcing.

Let $P_{\text{n}} = P$ denote the transmit power at each node. $P_{\text{r}} = KP$ is the transmit power available at the relay. The channel matrices corresponding to the channel between the nodes and the relay are generated randomly using the i.i.d. frequency flat Rayleigh MIMO channel model [16]. The channel matrices are normalized such that the average received power is the same as the average transmit power. For the simulations, the channels are assumed to be reciprocal.

We consider two scenarios. First $K = 6, N = 2, R = 9$ and $d = 1$. Let K_{s} denote the number of simulataneously served node pairs. For all the three schemes, whenever $K_s \leq K$, time sharing is assumed between different sets of pairs in order to serve all the K pairs. In pairAwareZF scheme with $R = 9$, at most $K_{\text{s}} = 5$ pairs can be served at the same time. In the PAIA scheme, according to [9], at most $K_{\text{s}} = 6$ pairs can be simultaneously served interference-free. The iterative method proposed in [9] is used to find the interference alignment solution. The closed form solution proposed in [9] cannot be applied in this scenario. However, the closed form solution proposed in the current paper can be applied to this scenario and the solution is feasible when $K_{\text{s}} \leq 5$. For $K_{\text{s}} = 5$, (14) is satisfied with inequality sign. Hence, a closed form solution is possible and there are infinitely many possible solutions. Using the proposed useful signal power maximization (maxUSP) algorithm, the solution which maximizes the useful signal power at the relay is chosen. In the result curves, the number of iterations is fixed to 50. Investigations have shown that typically, the algorithm is close to convergence already after 20 iterations. Figure 2 shows the sum rate performance of all the three schemes. The proposed closed form solution combined with useful signal power maximization PAIA_CFS+maxUSP performs better than both the reference schemes. In PAIA_CFS+maxUSP, only $K_{\mathrm{s}} = 5$ pairs of nodes are served simultaneously, whereas in PAIA, $K_{\mathrm{s}} = 6$ pairs of nodes are served. However, the performance of PAIA_CFS+maxUSP is better than that of PAIA. This is due to the fact that in PAIA, the nodes and the relay do not care about the power lost in the $RISS$. For $K_{\mathrm{s}} = 6$, the total power loss is very high so that even at an SNR of 50 dB, PAIA_CFS+maxUSP, which maximizes the useful signal power in the $RUSS$, performs better than PAIA.

In the second scenario $K = 7, N = 3, R = 9$ and $d = 1$. It can be seen from Figure 3 that PAIA_CFS+maxUSP for $K_s = 6$ performs better than both the reference schemes. Reducing the number of simulataneously served user to $K_{\mathrm{s}} = 5$ will provide more variables in the system to maximize the useful signal power and, hence, improve the performance at low SNR. However, at medium and high SNR regime, the sum rate is reduced as the total number of data streams transmitted is decreased. It is to be noted that in Figures 2 and 3, the slope of each of the curves is proportional to the degrees of freedom achieved by the corresponding scheme.

VII. Conclusion

In this paper, interference alignment in a multi-pair two-way relay network is considered. A single relay with R antennas assists K node pairs in achieving interference alignment. A closed form solution to achieve interference alignment is proposed for the cases when there is a sufficient number of variables in the system. This means, the solutions found are only a subset of all possible solutions that are achievable through different initializations in the iterative algorithm. The condition for the feasibility of the closed form solution is derived as $2KN \geq (K-n)R + d$, where $n = \lfloor \frac{R-Kd}{d} \rfloor$. When the feasibility condition is satisfied with strict inequality sign, infinitely many solutions are possible. A gradient based suboptimal algorithm to choose the solution that maximizes the useful signal power is proposed. Through simulation results it has been shown that at low and medium SNRs, the performance of the interference alignment scheme can be improved by serving less users than the maximum number possible and utilizing the available variables to improve the useful signal power.

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