IEEE Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM'13), Madrid, Spain, June 2013

Practical OFDMA in Wireless Networks with Multiple Transmitter-Receiver Pairs

Adrian Loch, Robin Klose and Matthias Hollick
Technische Universität Darmstadt
Secure Mobile Networking Lab
Mornewegstr. 32, 64293 Darmstadt, Germany
{*adrian.loch, robin.klose, matthias.hollick*}@*seemoo.tu-darmstadt.de*

Alexander Kuehne and Anja Klein
Technische Universität Darmstadt
Communications Engineering Lab
Merckstr. 25, 64283 Darmstadt, Germany
{*a.kuehne, a.klein*}@*nt.tu-darmstadt.de*

***Abstract*—State-of-the-art physical layers such as OFDMA are widely used in infrastructure-based networks to enhance efficiency in one-to-many transmissions. Application to wireless mesh networks is highly promising, as the diversity increases in many-to-many scenarios. While theoretical work on OFDMA for this scenario exists, it has not yet been implemented in practice. In this demonstration, we show practical measurements of OFDMA in a topology with multiple transmitters and receivers, which represents a fully-connected segment of a mesh network. Moreover, we model our system analytically and in simulation. Our measurements show the validity of these models. We demonstrate over 90% reduction of the symbol error rate and a 29% channel capacity increase.**

I. Introduction

In Wireless Mesh Networks (WMN), typically multiple nodes share the same interference domain. While such a scenario is challenging regarding medium access control, it also offers high spatial diversity since many links are available. There exist cross-layer approaches that exploit this [1], [2], [3], but they often do not interact with the physical layer, i.e., they adapt protocols on the network layer in order to cope better with the lower layers, but do not directly influence or change them.

Transmission is often based on Orthogonal Frequency Division Multiplexing (OFDM), which is a state-of-the-art multiplexing technique in digital broadband communication. While OFDM only supports transmissions by one user, Orthogonal Frequency Division Multiple Access (OFDMA) has emerged as a matching medium access control technique which allows to assign individual subchannels within its bandwidth to different users. Hence, subchannels can be assigned to links according to their channel quality, leading to an overall higher capacity. OFDMA is currently in use in cellular systems [4], but its performance in conjunction with WMNs has barely been studied.

Our goal is to identify performance gains of OFDMA in WMNs. We do not consider the complete network, but a fully-connected segment of it, since such a scenario already allows us to get insights into the strong synergy between OFDMA and WMNs. On the one hand, OFDMA features fine-granularity of resources. On the other hand, WMNs have a large number of nodes among which resources can be shared. Thus, the combination of both is highly promising.

As an intuitive example, in OFDMA cellular networks one base station serves m users and gains can be achieved by assigning each subchannel to the user which experiences best channel conditions on the corresponding frequency. Hence, the overall performance which can be achieved in terms of e.g. throughput or bit/symbol error rate is the maximum out of the m available channels. On the contrary, in a fully-connected WMN segment with m users, subchannels can be distributed among $\frac{m(m-1)}{2}$ links. Thus, the overall performance is higher since the probability of finding a good communication link for a certain subchannel increases with the number of available links among which to choose.

We demonstrate the practical feasibility of OFDMA for such a WMN segment. Our contributions are as follows:

- We derive an analytical model to determine the performance gains in terms of channel capacity.
- We build a simulative model to gain insights into the bit error and symbol error rate gains of our approach.
- We implement our system on software-defined radios to measure gains in practice and validate our models.

II. System Design

A. OFDMA for Wireless Mesh Networks

While in OFDM a channel is divided into N_c subcarriers which are all assigned to one transmitter, in OFDMA each subcarrier can be assigned to a different node. If each is assigned only once, nodes send at orthogonal frequencies and thus there is no interference. We assume that there are n senders which always have data to send to n receivers. All nodes are half-duplex and thus there are n^2 links available. Moreover, we assume that all nodes have complete Channel State Information (CSI). We focus on scenarios where CSI stays approximately constant for long periods of time and thus the overhead to measure the CSI is comparatively low.

The key technique for beneficially combining OFDMA with WMNs is assigning subcarriers to links which experience good channel conditions at the corresponding frequencies. Note that the subcarriers are not just distributed among the n senders, but among the n^2 available links, since each link of a transmitter may have a different quality. For example, while the link from s_1 to r_1 might be good, the link from the same transmitter to r_2 might be very poor.

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We use a "best-out-of-n^2" allocation mechanism which assigns each subcarrier to the best available link. While this is unfair, it allows us to get insights on the best possible performance of OFDMA in a fully-connected WMN. Algorithm 1 describes our mechanism. Essentially, subcarriers are assigned to the link with the smallest $\left|\frac{N}{H(f)}\right|$ coefficient, which is the noise added to a symbol sent over a subcarrier at frequency f after undoing the effect of the channel.

Once all subcarriers are allocated, all nodes send synchronously at the same time. There is no interference as no subcarrier is assigned more than once. The system is equivalent to one transmitter sending to one receiver over the combined best channel out of the n^2 available links. Thus, we expect the resulting SNR to be higher compared to the average SNR of all n^2 links. This should also directly translate into lower bit and symbol error rates. Furthermore, the achievable capacity should also be higher.

B. Analysis

We derive theoretical capacity formulas for a baseline OFDM approach and our "best-out-of-n^2" mechanism. In our demonstration, practical measurements are compared to these theoretical upper boundaries. This (a) validates our implementation and (b) shows its efficiency.

For OFDM, the derivation of the capacity is well-known. The Probability Density Function (PDF) p_γ of the SNR γ assuming Rayleigh fading is given by $p_\gamma = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}$, i.e., γ is exponentially distributed with $\bar{\gamma}$ being the average SNR. The capacity can be derived as shown in Equation 1, where B is the bandwidth and $E_1(x)$ the exponential integral.

$$\begin{aligned} C_{\mathrm{OFDM}} &= B \int_0^\infty \log_2(1+\gamma) p_\gamma \,\mathrm{d}\gamma \\ &= \frac{B}{\ln 2} e^{\frac{1}{\bar{\gamma}}} E_1\left(\frac{1}{\bar{\gamma}}\right) \end{aligned} \tag{1}$$

For the "best-out-of-n^2" scheme, the Cumulative Distribution Function (CDF) of the best SNR $\gamma_{\max}$ out of n^2 channel realizations is given by Equation 2.

$$F_{\gamma_{\max}} = F_{\gamma_1} \cdot F_{\gamma_2} \cdot \ldots \cdot F_{\gamma_{n^2}} = (F_\gamma)^{n^2} = \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^{n^2} \tag{2}$$

Hence, the PDF is the derivative given in Equation 3.

$$p_{\gamma_{\max}} = \frac{dF_{\gamma_{\max}}}{d\gamma_{\max}} = n^2 \left(1 - e^{-\frac{\gamma_{\max}}{\bar{\gamma}}}\right)^{n^2-1} \frac{1}{\bar{\gamma}} e^{-\frac{\gamma_{\max}}{\bar{\gamma}}} \tag{3}$$

We can now obtain the capacity for OFDMA again by integrating the Shannon capacity formula multiplied by the PDF. For simplicity, we refer to $\gamma_{\max}$ as γ.

$$C = B \int_0^\infty \log_2(1+\gamma) \frac{n^2}{\bar{\gamma}} \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^{n^2-1} e^{-\frac{\gamma}{\bar{\gamma}}} \,\mathrm{d}\gamma \tag{4}$$

To solve this integral, we apply Equations 1.111 and 4.337 from [5]. We then obtain C_{OFDMA} as shown in Equation 5.

$$C = \frac{B \cdot n^2}{\ln 2} \sum_{k=0}^{n^2-1} \binom{n^2-1}{k} (-1)^k \left(\frac{1}{k+1}\right) e^{\frac{k+1}{\bar{\gamma}}} E_1\left(\frac{k+1}{\bar{\gamma}}\right) \tag{5}$$

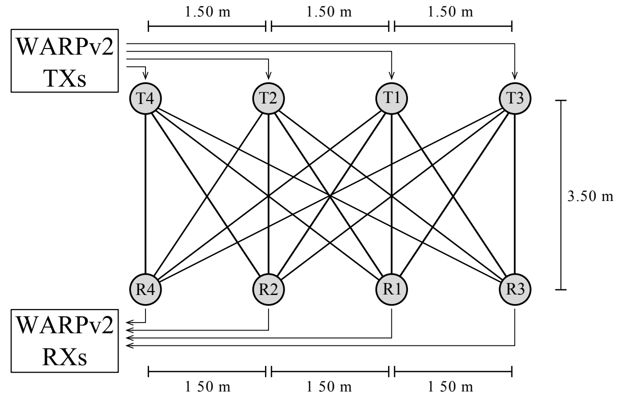

Figure 1: Experiment setup for practical measurements.

III. Demonstration

In our demonstration, we use the Wireless Open-Access Research Platform (WARP) Software-Defined Radio (SDR) [6]. Additionally, we use the WARPLab Reference Design, which is a framework that allows for rapid prototyping of physical layer algorithms in Matlab. Our setup includes four transmitters and four receivers, as shown in Figure 1. On each side nodes are connected to a single WARP board in order to guarantee synchronization among them. This is possible as the boards feature four radio interfaces. However, data sent and received by each radio is treated independently, as if each antenna was an independent node. Experiments are controlled by a computer connected to both boards. The parameters of our setup are given in Table I.

The demonstration allows to run experiments interactively and observe how subcarriers are allocated to the available links. Moreover, the measured capacity and bit/symbol error rates are displayed. For each transmission the experiment parameters can be adjusted in order to observe their impact. The measurements are compared to the theoretical bounds derived in Section II-B and to pre-computed simulative results, allowing to assess the efficiency of the system. In order to observe how channel conditions impact the allocation of subcarriers, two of the four transmitters can be configured

Algorithm 1 Best-out-of-n^2 subcarrier assignment.

Require: average noise power N and CSI $H(f)$ of all n^2 links
Ensure: allocation of N_c subcarriers to n^2 available links

set $\mathrm{subAlloc}[N_c]$ to zeros
for all subcarriers sc **do**
  set $\mathrm{linkPerf}[n^2]$ to zeros
  for all senders s **do**
    for all receivers r **do**
      set $\mathrm{currentLink} = n(s-1)+r$
      set $\mathrm{linkPerf}[\mathrm{currentLink}]$ to $\left|\frac{N}{H(sc)}\right|$
    end for
  end for
  set $\mathrm{subAlloc}[sc]$ to the index of smallest element in $\mathrm{linkPerf}$
end for

Table I: Experiment parameters.

Parameter	Value	Parameter	Value
# of subchannels	32	Symbol duration	6.8 μs
Pilot symbols	20	Guard space	400 ns
Data symbols	20	Subcarrier spacing	147,1 kHz
Bits per symbol	$[1, 2, 4, 6, 8]$	Bandwidth	18,8 MHz

Table II: Validation of theory, simulation and practice.

Metric	Theory	Simulation	Practice
Capacity OFDM [mbps]	175.29	175.35	172.00
Capacity OFDMA [mbps]	222.12	220.96	222.94
Capacity gain	26.71%	26.01%	29.61%

to generate narrow-band interferences. Also, simply placing objects between the antennas affects the allocation. The demonstration offers high degree of interaction by enabling attendees to set and tune all of the above parameters.

Our setup allows to ascertain in practice the simulative results shown in Figures 2 and 3. Our simulation is implemented in Matlab using a Rayleigh channel model. We average 1000 experiment repetitions. The 95% confidence intervals virtually match the average values and are not shown for clarity. Figure 2 shows the capacity gain, which for typical indoor SNRs of about 25 dB is close to 30%. The gain is not linear with each additional node, but becomes smaller, since from a certain n onwards, diversity only increases marginally, as good links for all subcarriers are already available. Figure 3 presents the SER improvement in percentage for two selected modulation schemes. For typical indoor SNRs, OFDMA can reduce the SER by about 90% even for 256-QAM, which is the highest modulation scheme envisioned by state-of-the-art technologies such as 802.11ac.

Tables II and III show capacity and SER results as measured in our demonstration setup. Note that in Table II the practical case features a larger gain than in theory and simulation, since the capacity of OFDM is lower. The reason is that practical issues arise when some of the subcarriers have poor quality. As OFDM cannot choose subcarriers, it cannot avoid these issues, as opposed to OFDMA.

Table III: Practical BER/SER for OFDM and OFDMA.

Metric	BPSK	4QAM	16QAM	64QAM	256QAM
SER_{OFDM}	0.017%	0.061%	0.692%	3.780%	17.435%
SER_{OFDMA}	0.000%	0.000%	0.000%	0.000%	1.136%

IV. Conclusion

This demonstration shows the benefits of OFDMA in fully-connected Wireless Mesh Networks (WMN) with multiple transmitter-receiver pairs. Essentially, gains can be achieved by allocating each subcarrier to the pair which experiences best quality on the corresponding frequency. Attendees can interactively carry out experiments and observe the impact of parameters as well as interference on the subcarrier allocation. Moreover, direct comparison to theoretical and simulative results is provided.

Acknowledgments

This work was supported by LOEWE Cocoon, LOEWE CASED and the DFG as part of the CRC 1053 MAKI.

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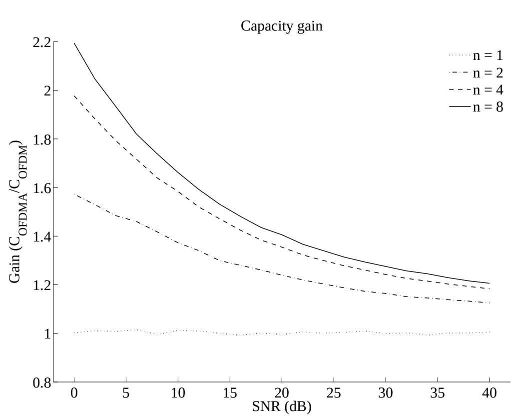

Figure 2: Capacity gain of OFDM compared to OFDMA.

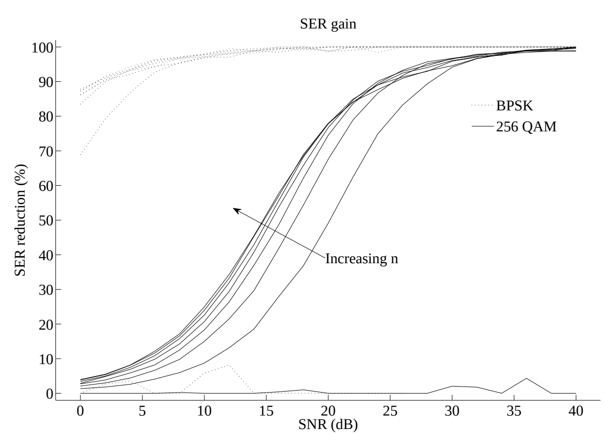

Figure 3: Symbol error rate improvement with OFDMA.