H. Degenhardt, A. Klein, ”Filter design with optimized numbers of data streams for multi-pair two-way relaying under asymmetric rate requirements,” in *Proc. of the International ITG Workshop on Smart Antennas (WSA 2012), Dresden, Germany, March 2012.*

Filter design with optimized numbers of data streams for multi-pair two-way relaying under asymmetric rate requirements

Holger Degenhardt and Anja Klein
Institute of Telecommunications, Communications Engineering Lab,
Technische Universität Darmstadt, Merckstr. 25, 64283 Darmstadt, Germany
Email: {h.degenhardt, a.klein}@nt.tu-darmstadt.de

***Abstract*—A multi-pair two-way relaying scenario with multi-antenna nodes is considered. The bidirectional communications between the nodes are supported by an intermediate non-regenerative multi-antenna relay station. It is assumed that the nodes can subtract the back-propagated self-interference. In such a scenario, the required data rates for each direction of transmission are typically different which is considered by introducing asymmetric rate requirements. To maximize the achievable sum rate under asymmetric rate requirements, a strategy to optimize the number of simultaneously transmitted data streams of each node is presented. Furthermore, a weighted self-interference aware relay transceive filter as well as transmit and receive filters at the nodes are proposed. Additionally, the transmit powers of the nodes are optimized with respect to the transceive filter at the relay station and the asymmetric rate requirements. The aforementioned approach achieves higher sum rates compared to conventional approaches which do not optimize the number of simultaneously transmitted data streams of each node.**

I. Introduction

Relaying techniques can be used to expand the coverage of wireless networks and to increase the achievable throughput. To support multiple bidirectional communications via an intermediate half-duplex relay station RS, multi-antenna techniques can be used to spatially separate the communication pairs and to enable the simultaneous communication of all pairs [1]–[5]. Within each pair, the two-way relaying protocol of [6] can be applied to overcome the duplexing loss of conventional one-way relaying schemes. The achievable sum rates for two-way relaying depend on the overall channel and, therewith, on the transceive filter design at RS. Furthermore, the achievable sum rates depend on the available channel state information (CSI) and on the capability of the nodes to perform self-interference cancellation. If the nodes can subtract the back-propagated self-interference before recovering the desired signal, the achievable sum rates can be increased.

Non-regenerative multi-antenna two-way relaying in a single-pair scenario is investigated in [7]–[9]. A minimum mean square error (MMSE) transceive filter at RS exploiting self-interference cancellation is derived in [7]. In [8], a gradient based transceive filter approach for sum rate maximization is presented and in [9], joint source and relay precoding designs are investigated. Non-regenerative multi-pair two-way relaying with single-antenna nodes and a multi-antenna relay has been considered in [1]–[3] and different transceive filter designs based on zero-forcing block-diagonalization (ZFBD) are proposed to exploit self-interference cancellation. Considering multi-antenna nodes and exploiting the multiplexing gain increases the achievable sum rates. The authors of [4], [5] investigate a pairwise communication of multi-antenna nodes via an intermediate multi-antenna relay.

Typically, more or less symmetric data rates for the bidirectional communications are achieved in non-regenerative two-way relaying. However, many practical applications require asymmetric data rates, but this is not considered in [4], [5]. Asymmetric rate requirements for a single pair and a multi-user single cell scenario have been considered in [10] and [11], respectively.

In this paper, multi-pair two-way relaying with multi-antenna nodes under asymmetric rate requirements is considered. It is assumed that the nodes can perfectly cancel self-interference which is exploited for the transceive filter design at RS. In this scenario, maximizing the achievable sum rate under asymmetric rate requirements is a non-convex problem. Thus, we propose to decouple the overall optimization into different subproblems. First, the optimization of the transmit (Tx) and receive (Rx) filters at the nodes is considered based on the idea of ZFBD. Secondly, a weighted minimum mean square error self-interference aware relay transceive filter WMMSE-SI is proposed to tackle the asymmetric rate requirements. Afterwards, an approach for optimizing the number of simultaneously transmitted data streams of each node is introduced. Finally, the optimization of the transmit powers of the nodes is investigated and an alternating optimization with the WMMSE-SI transceive filter at RS is proposed to fulfill the asymmetric rate requirements and to achieve high sum rates.

The paper is organized as follows. In Section II, the system model is given and in Section III, the different considered subproblems are described. The Tx and Rx filter design at the nodes is presented in Section IV-A. The WMMSE-SI transceive filter at RS is introduced in Section IV-B. In Section V, an approach for optimizing the number of simultaneously transmitted data streams is presented. In Section VI, the optimization of the transmit powers of the nodes is investigated and an alternating optimization is introduced. Performance results in Section VII confirm the analytical investigations and Section VIII concludes the paper.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^{*}$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operators $\mathrm{tr}(\cdot)$, $\otimes$ denote the sum of the main diagonal elements of a matrix and the Kronecker product of matrices, respectively. The operators $\Re[\cdot]$ and $||\cdot||_2$ denote the real part of a scalar and the Frobenius norm of a matrix, respectively. The matrix vectorization operator $\mathrm{vec}(Z)$ stacks the columns of matrix Z into a vector. The operator $\mathrm{vec}^{-1}_{M,N}(\cdot)$ is the revision of the operator $\mathrm{vec}(\cdot)$, i.e., a vector of length MN is sequentially divided into N vectors of length M which are combined to a matrix with M rows and N columns. The operator $\mathrm{mod}_y x$ returns the modulus of x after division by y and $\mathbf{I}_M$ denotes an identity matrix of size $M \times M$.

II. System Model

As shown in Figure 1, K pairwise bidirectional communications via an intermediate multi-antenna relay station RS of $2K$ multi-antenna half-duplex nodes are considered. Nodes S_k and S_l form a bidirectional communication pair for $l = k - 1 + 2 \cdot \mathrm{mod}_2 k$, $k = 1, 2, ..., 2K$, i.e., S_1 and S_2, S_3 and S_4, ..., S_{2K-1} and S_{2K} form bidirectional communication pairs. It is assumed that all nodes are simultaneously transmitting to RS in the first time slot and within each pair, the two-way protocol of [6] is applied and the direct links are neglected. In the second time slot, RS retransmits a linear processed version of the received signals towards the nodes. This scheme is termed multi-pair two-way relaying. The transmit powers at each node and at RS are limited by $P_{\mathrm{MS,max}}$ and $P_{\mathrm{RS,max}}$, respectively. Each node is equipped with M antennas and it is assumed that the nodes can subtract the back-propagated self-interference. To enable the interference free transmission of at least one data stream per node, the number of antennas at RS is given by $L \geq (2K-1)$ considering perfect self-interference cancellation to be possible at the nodes. Furthermore, it is assumed that RS has perfect global channel state information (CSI) and the nodes have perfect knowledge of specific subchannels as described in Section IV-A to perform self-interference cancellation as well as Tx and Rx filter design.

The channel $\mathbf{H}_k \in \mathbb{C}^{L\times M}$ from S_k to RS is assumed to be constant during one transmission cycle of the multi-pair two-way scheme and channel reciprocity is assumed. All signals are assumed to be statistically independent and the noise at RS and at the nodes is assumed to be additive white Gaussian with variances $\sigma^2_{\mathrm{n,RS}}$ and σ^2_{n}, respectively. The system equations for multi-pair two-way relaying are presented in the following. The transmitted symbols of S_k are contained in the vector $\mathbf{x}_{\mathrm{S}_k}$ and the transmit filter at S_k is given by $\mathbf{Q}_k$. Thus, the

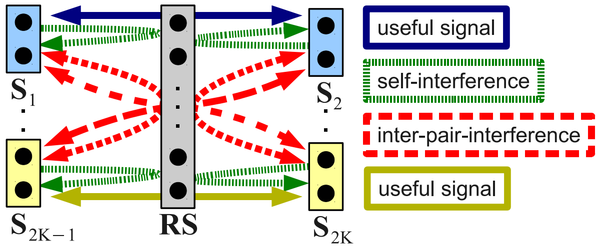

Fig. 1. Composition of useful signals and interferences in a bidirectional multi-pair two-way relaying scenario.

received baseband signal at RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \sum_{k=1}^{2K} \mathbf{H}_k \mathbf{Q}_k \mathbf{x}_{\mathrm{S}_k} + \mathbf{n}_{\mathrm{RS}}, \tag{1}$$

where $\mathbf{n}_{\mathrm{RS}}$ represents the complex white Gaussian noise vector at RS. RS linearly processes the received signal and the transceive filter at RS is given by

$$\mathbf{G} = \gamma \widetilde{\mathbf{G}}, \tag{2}$$

where $\widetilde{\mathbf{G}}$ is the transceive filter at RS which does not implicitly fulfill the power constraint and γ is a scalar value to satisfy the relay power constraint. It is given by

$$\gamma = \sqrt{\frac{P_{\mathrm{RS,max}}}{||\sum_{k=1}^{2K} ||\widetilde{\mathbf{G}}\mathbf{H}_k\mathbf{Q}_k||_2^2 + ||\widetilde{\mathbf{G}}||_2^2 \sigma^2_{\mathrm{n,RS}}}}. \tag{3}$$

The relay transmits the linearly processed version of $\mathbf{y}_{\mathrm{RS}}$ to all nodes. The received signal $\mathbf{y}_k$ using the receive filter $\mathbf{D}_k$ at S_k is given by

$$\mathbf{y}_{\mathrm{S}_k} = \mathbf{D}_k(\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{y}_{\mathrm{RS}} + \mathbf{n}_k), \tag{4}$$

where $\mathbf{n}_k$ represents the complex white Gaussian noise vector at S_k. The compositions of the receive signals are also illustrated in Figure 1. Each node receives its intended useful signals, receives interference from the signals intended for the other nodes termed inter-pair-interference, and receives back-propagated self-interference as well as noise. The inter-pair-interference has to be mitigated by the transceive filter at RS, but the back-propagated self-interference can be subtracted at each node [6] assuming that $\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_k$ is perfectly known at S_k. After self-interference cancellation, the received signal at S_k is given by

$$\hat{\mathbf{x}}_{\mathrm{S}_l} = \mathbf{D}_k\mathbf{H}_k^{\mathrm{T}}\mathbf{G} \sum_{j=1, j\neq k}^{2K} \mathbf{H}_j\mathbf{Q}_j\mathbf{x}_{\mathrm{S}_j} + \mathbf{D}_k(\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{n}_{\mathrm{RS}} + \mathbf{n}_k), \tag{5}$$

where $\hat{\mathbf{x}}_{\mathrm{S}_l}$ is the estimate of $\mathbf{x}_{\mathrm{S}_l}$ at node S_k which bidirectionally communicates with S_l. With $\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}$ the signal covariance matrix of $\mathbf{x}_{\mathrm{S}_k}$ and $\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}$ and $\mathbf{R}_{\mathbf{n}_{\mathrm{S}_l}}$ the noise covariance matrices at RS and S_l, respectively, the signal, interference and noise covariance matrices after self-interference cancellation

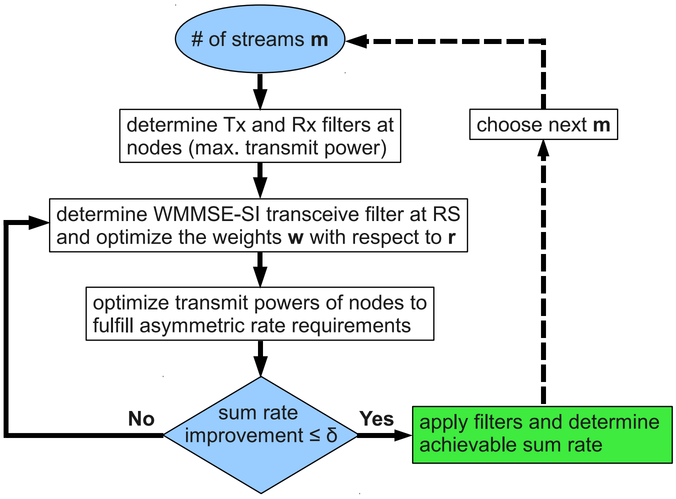

Fig. 2. Considered subproblems for maximizing the achievable sum rate under asymmetric rate requirements.

for the transmission from S_k to S_l are given by

$$\mathbf{A}^{\mathrm{S}_k} = \mathbf{H}_l^{\mathrm{T}}\mathbf{G}\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_l^*,$$
$$\mathbf{B}^{\mathrm{S}_k} = \mathbf{H}_l^{\mathrm{T}}\mathbf{G}\left(\sum_{j=1, j\neq k,l}^{2K}\mathbf{H}_j\mathbf{Q}_j\mathbf{R}_{\mathbf{x}_{\mathrm{S}_j}}\mathbf{Q}_j^{\mathrm{H}}\mathbf{H}_j^{\mathrm{H}}\right)\mathbf{G}^{\mathrm{H}}\mathbf{H}_l^*,$$
$$\mathbf{C}^{\mathrm{S}_k} = \mathbf{H}_l^{\mathrm{T}}\mathbf{G}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}\mathbf{G}^{\mathrm{H}}\mathbf{H}_l^* + \mathbf{R}_{\mathbf{n}_{\mathrm{BS}}}, \tag{6}$$

respectively.

Assuming that Gaussian codebooks are used for each data stream, the maximum achievable data rate from S_k to S_l considering self-interference cancellation is given by

$$C_{\mathrm{S}_k} = \frac{1}{2}\log_2|(\mathbf{I}_M + \mathbf{A}^{\mathrm{S}_k}(\mathbf{B}^{\mathrm{S}_k} + \mathbf{C}^{\mathrm{S}_k})^{-1})|. \tag{7}$$

Thus, the achievable sum rate is given by

$$C_{\mathrm{sum}} = \sum_{k=1}^{2K} C_{\mathrm{S}_k}. \tag{8}$$

In this paper, asymmetric rate requirements are considered. To consider asymmetric rate requirements, an auxiliary variable r_k, $k = 2, 3, ...2K$ is introduced which describes the ratio r_k between the data rate from Sk to Sl and the reference data rate from S1 to S2. It is assumed that specific asymmetric rate requirements have to be fulfilled. Thus, the constraints $r_k = C_{\mathrm{S}_k}/C_{\mathrm{S}_1}$, $k = 2, ..., 2K$ are considered for the sum rate maximization and the vector $\mathbf{r} = (r_2, r_3, ...r_{2K})$ contains the ratios for all links. The achievable sum rate under the asymmetric rate requirements is given by

$$C_{\mathrm{sum,constr.}} = 2K \cdot \min_k\left(C_{\mathrm{S}_k}/r_k\right), \ k = 1, .., 2K. \tag{9}$$

III. Subproblems for Sum Rate Maximization

The sum rate $C_{\mathrm{sum,constr.}}$ of (9) shall be maximized under the transmit power constraints $P_{\mathrm{MS,max}}$ and $P_{\mathrm{RS,max}}$ given in Section II. The sum rate maximization with respect to the optimization of the number of simultaneously transmitted data streams, the optimization of the Tx and Rx filters at the nodes, the optimization of the relay transceive filter and the optimization of the transmit powers of the nodes is a non-convex problem. Thus, we propose to decouple the overall optimization into different subproblems as shown in Figure 2. The proposed optimization of the subproblems is a suboptimal approach for maximizing $C_{\mathrm{sum,constr.}}$. The considered subproblems are:

- the optimization of the numbers m_k of simultaneously transmitted data streams of the nodes; these numbers are contained in the vector $\mathbf{m} = (m_1, m_2, ..., m_{2K})$ and the optimization is treated in Section V,
- the design of the Tx and Rx filters at the nodes treated in Section IV-A,
- the weighted self-interference aware relay transceive filter design WMMSE-SI treated in Section IV-B,
- the optimization of the transmit powers of the nodes to fulfill the asymmetric rate requirements treated in Section VI,
- and an alternating optimization between the relay transceive filter and the transmit powers of the nodes treated in Section VI.

IV. Filter Design

In this section, the designs of the Tx and Rx filters at the nodes based on a BD-approach and of the WMMSE-SI transceive filter at RS are considered. The vector $\mathbf{m}$ containing the numbers of simultaneously transmitted data streams of each node is assumed to be given.

A. Tx and Rx filter design at nodes

To maximize the sum rate $C_{\mathrm{sum,constr.}}$, the Tx and Rx filters at the nodes have to be optimized with respect to the available channel knowledge. To perform self-interference cancellation, it is assumed that node S_k has perfect knowledge of the self-interference channel $\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_k$. To design the Tx and Rx filters, additional channel knowledge is required and two cases are investigated in this paper:

- individual Tx and Rx filter design: node S_k performs a filter design based on its knowledge of its own individual channel H_k,
- joint Tx and Rx filter design: node S_k performs a filter design based on its knowledge of a receive and a transmit subchannel allocated to S_k and S_l by RS.

In the following, the designs of the Tx and Rx filters at the nodes for the two different cases are described. The proposed designs are based on using a relay transceive filter which minimizes the mean square error for given Tx and Rx filters at the nodes.

Individual Tx and Rx filter design: This case has also been investigated in previous publications, e.g., [4]. To obtain knowledge of channel H_k at node S_k for $k = 1, 2, ...2K$, pilot assisted channel estimation can be used, e.g., RS transmits successively one symbol per antenna element which is known at the nodes. For details on channel estimation see [12] and references therein. Based on this channel knowledge, the nodes select the strongest singular vectors for transmission and reception. Considering the singular-value decomposition (SVD) of the channel $\mathbf{H}_k = \mathbf{U}_k\mathbf{S}_k\mathbf{V}_k^{\mathrm{H}}$, the Tx and Rx filters are given by

$$\mathbf{Q}_k = P_{\mathrm{MS,max}}\mathbf{V}_{k,1:m_k} \tag{10}$$

$$\mathbf{D}_k = \mathbf{V}_{k,1:m_l}^{\mathrm{T}}, \tag{11}$$

respectively, where $\mathbf{V}_{k,1:m_k}$ contains the singular vectors which correspond to the m_k strongest singular values contained in $\mathbf{S}_k$.

Joint Tx and Rx filter design: In this case, the filter design at node S_k not only considers the own individual channel H_k, but rather considers the channels of all nodes by a design which is based on a receive and a transmit subchannel allocated to each pair by RS. In the beginning, an iterative optimization is performed at RS to obtain the transmit and receive subchannels for each pair. Afterwards, RS performs a pilot transmission through the subchannel filters and the nodes perform an SVD on the received channels to select the m_k strongest singular vectors for transmission and the m_l strongest singular vectors for reception. The iterative optimization to obtain the transmit and receive subchannels at RS can be separated into three steps. First, transmission and reception subchannels are determined for each pair based on the initial Tx and Rx filters at the nodes. Secondly, an update of the Tx and Rx filters of the nodes is computed at RS based on the obtained subchannels. Thirdly, the updated Tx and Rx filters are used to recalculate the transmit and receive subchannels for each pair and a repetitive optimization between the allocated subchannels and the Tx and Rx filters of the nodes is performed at RS. In the following, this will be explained in detail.

First, the transmit and receive subchannels are determined for each pair based on an MMSE extension of the ZFBD idea, which has been introduced for downlink spatial multiplexing in [13] and which is applied to the multi-user bidirectional two-way channels in [1]–[3] to determine a spatial subchannel for the bidirectional communication of each pair which is orthogonal to the subchannels of the other pairs. Using ZFBD, the interferences caused by the other pairs are forced to zero in each subchannel. Instead of completely suppressing inter-pair interference, we propose to allow some inter-pair interference in each spatial subchannel according to the MMSE principle to reduce the noise enhancement. To compute the transmit subchannels, let $\tilde{\mathbf{H}}_{\mathbf{j}}$ denote the transmit channel matrix of all nodes not belonging to the jth pair which is given by

$$\begin{aligned}\tilde{\mathbf{H}}_{\mathrm{TF},\mathbf{k}} = [&\mathbf{H}_1\mathbf{Q}_1, \mathbf{H}_2\mathbf{Q}_2, ..., \mathbf{H}_{2j-2}\mathbf{Q}_{2j-2},\\ &\mathbf{H}_{2j+1}\mathbf{Q}_{2j+1}, ..., \mathbf{H}_{2K}\mathbf{Q}_{2K}],\end{aligned} \tag{12}$$

where the Tx filters of (10) are used. The spatial transmit subchannel for Sk, $k = 2j-1, 2j$, which belongs to the jth pair is based on the SVD $\tilde{\mathbf{H}}_{\mathrm{TF},j} = \tilde{\mathbf{U}}_{\mathrm{TF},\mathbf{j}}\tilde{\mathbf{S}}_{\mathrm{TF},\mathbf{j}}\tilde{\mathbf{V}}_{\mathrm{TF},\mathbf{j}}^{\mathrm{H}}$. It is given by

$$\mathbf{H}_{\mathrm{TF},k} = \left(\tilde{\mathbf{U}}_{\mathrm{TF},j}\mathbf{D}_{\mathrm{TF},j}\right)^H \mathbf{H}_k, \tag{13a}$$

$$\text{with } \mathbf{D}_{\mathrm{TF},j} = \left(\tilde{\mathbf{S}}_{\mathrm{TF},j}\tilde{\mathbf{S}}_{\mathrm{TF},j}^{\mathrm{T}} + \frac{\sigma^2_{\mathrm{n,RS}}}{P_{\mathrm{MS,max}}}\mathbf{I}_L\right)^{-\frac{1}{2}}. \tag{13b}$$

The channel knowledge of $\mathbf{H}_{\mathrm{TF},k}$ can be obtained at Sk using a successive pilot transmission through the prefilter $\mathbf{G}_{\mathrm{TF},j} = \left(\tilde{\mathbf{U}}_{\mathrm{TF},j}\mathbf{D}_{\mathrm{TF},j}\right)^*$ at RS.

To compute the receive subchannels, let $\tilde{\mathbf{H}}_{\mathrm{RF},\mathbf{j}}$ denote the receive channel matrix of all nodes not belonging to the jth pair which is given by

$$\begin{aligned}\tilde{\mathbf{H}}_{\mathrm{RF},\mathbf{j}} = [&\mathbf{D}_1\mathbf{H}_1^{\mathrm{T}}, \mathbf{D}_2\mathbf{H}_2^{\mathrm{T}}, ..., \mathbf{D}_{2j-2}\mathbf{H}_{2j-2}^{\mathrm{T}},\\ &\mathbf{D}_{2j+1}\mathbf{H}_{2j+1}^{\mathrm{T}}, ..., \mathbf{D}_{2K}\mathbf{H}_{2K}^{\mathrm{T}}],\end{aligned} \tag{14}$$

where the Rx filters of (11) are used. The spatial receive subchannel for Sk, $k = 2j-1, 2j$ is based on the SVD $\tilde{\mathbf{H}}_{\mathrm{RF},j} = \tilde{\mathbf{U}}_{\mathrm{RF},j}\tilde{\mathbf{S}}_{\mathrm{RF},j}\tilde{\mathbf{V}}_{\mathrm{RF},j}^{\mathrm{H}}$. It is given by

$$\mathbf{H}_{\mathrm{RF},k} = \mathbf{H}_k^T\tilde{\mathbf{V}}_{\mathrm{RF},j}\mathbf{D}_{\mathrm{RF},j}, \tag{15a}$$

$$\text{with } \mathbf{D}_{\mathrm{RF},j} = \left(\tilde{\mathbf{S}}_{\mathrm{RF},j}^{\mathrm{T}}\tilde{\mathbf{S}}_{\mathrm{RF},j} + \frac{\sigma^2_{\mathrm{n}}}{P_{\mathrm{RS}}}\mathbf{I}_L\right)^{-\frac{1}{2}}. \tag{15b}$$

The channel knowledge of $\mathbf{H}_{\mathrm{RF},k}$ can be obtained at Sk using a successive pilot transmission through the prefilter $\mathbf{G}_{\mathrm{RF},j} = \tilde{\mathbf{V}}_{\mathrm{RF},j}\mathbf{D}_{\mathrm{RF},j}$ at RS.

Secondly, the update of the Tx and Rx filters of each node is calculated based on an SVD of the spatial transmit and receive subchannels of each user. For node Sk, the m_k strongest singular vectors of the transmit subchannel are selected for transmission and the m_l strongest singular vectors of the receive subchannel are selected for reception. The transmit power is equally distributed between the selected singular vectors and water-filling is not considered.

Thirdly, the computation of the transmit and receive subchannels is repeated for a finite number of times using the updated Tx and Rx filters which are based on these subchannels as explained in the previous step instead of using the fixed filters of (10) and (11). The repetitive optimization can be solely performed at RS based on the available perfect CSI and investigations showed that, in general, five repetitions are sufficient.

Afterwards, pilot assisted channel estimation can be used to obtain channel knowledge of the final subchannels at the nodes. This channel knowledge is used to determine the Tx and Rx filters at the nodes by using SVD on the final transmit and receive subchannels, respectively. Node Sk selects the m_k strongest singular vectors of the transmit subchannel for transmission and the m_l strongest singular vectors of the receive subchannel for reception.

B. WMMSE-SI transceive filter at RS

In the following, a weighted self-interference aware MMSE relay transceive filter termed WMMSE-SI is presented. This derivation is based on the MMSE-SI filter derived in [5]. The

MMSE-SI filter in [5] is extended by considering weighting parameters w_k for each direction of transmission to tackle the asymmetric rate requirements. For completeness, the whole derivation is given in this paper. The error caused by self-interference is removed in the equations for the mean square error (MSE) so that back-propagated self-interference is only considered in the power constraint at RS and is not intentionally suppressed by the transceive filter design. The general equation for the transceive filter design at RS is given by

$$\mathbf{G} = \arg\min_{\mathbf{G}} \mathrm{E}\left\{\sum_{k=1}^{2K} w_k \left\|\mathbf{x}_{\mathrm{S}_k} - \hat{\mathbf{x}}_{\mathrm{S}_k}\right\|_2^2\right\}. \quad (16)$$

The convex problem described in (16) can be solved by using Lagrangian optimization. Let matrices $\boldsymbol{\Upsilon}^{(k)}$ and $\boldsymbol{\Upsilon}$ be given by

$$\boldsymbol{\Upsilon}^{(k)} = \mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}} + \frac{1}{2K-1}\mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}, \quad (17a)$$

$$\boldsymbol{\Upsilon} = \sum_{k=1}^{2K}\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\mathbf{Q}_k^{\mathrm{H}}\mathbf{H}_k^{\mathrm{H}} + \mathbf{R}_{\mathbf{n}_{\mathrm{RS}}}. \quad (17b)$$

Using matrices $\boldsymbol{\Upsilon}^{(k)}$ and $\boldsymbol{\Upsilon}$ of (17) in (16) and considering the power constraint at RS, the Lagrangian function with the Lagrangian multiplier η results in

$$L(\mathbf{G},\eta) = \sum_{k=1}^{2K} w_k F(\mathbf{G},k) - \eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right), \quad (18)$$

with

$$\begin{aligned} F(\mathbf{G},k) = {} & \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right) - 2\Re\left[\mathrm{tr}\left(\mathbf{D}_l\mathbf{H}_l^{\mathrm{T}}\mathbf{G}\mathbf{H}_k\mathbf{Q}_k\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}\right)\right] \\ & + \mathrm{tr}\left(\sum_{j=1,j\neq l}^{2K}\mathbf{D}_l\mathbf{H}_l^{\mathrm{T}}\mathbf{G}\boldsymbol{\Upsilon}^{(j)}\mathbf{G}^{\mathrm{H}}\mathbf{H}_l^{*}\mathbf{D}_l^{\mathrm{H}}\right) \\ & + \mathrm{tr}\left(\mathbf{D}_l\mathbf{R}_{\mathbf{n}_{\mathrm{S}_l}}\mathbf{D}_l^{\mathrm{H}}\right). \end{aligned} \quad (19a)$$

where $l = k - 1 + 2\cdot\mathrm{mod}_2 k$. From the Lagrangian function, the Karush-Kuhn-Tucker (KKT) conditions can be derived:

$$\frac{\partial L}{\partial \mathbf{G}} = \sum_{k=1}^{2K} w_k f(\mathbf{G},k) - \eta\,\mathbf{G}^{*}\boldsymbol{\Upsilon}^{\mathrm{T}} = \mathbf{0}, \quad (20a)$$

$$\eta\left(\mathrm{tr}\left(\mathbf{G}\boldsymbol{\Upsilon}\mathbf{G}^{\mathrm{H}}\right) - P_{\mathrm{RS,max}}\right) = 0, \quad (20b)$$

$$\text{with}\quad \begin{aligned} f(\mathbf{G},k) = {} & -\mathbf{H}_l\mathbf{D}_l^{\mathrm{T}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}_k}}^{\mathrm{T}}\mathbf{Q}_k^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}} \\ & + \sum_{j=1,j\neq l}^{2K}\mathbf{H}_l\mathbf{D}_l^{\mathrm{T}}\mathbf{D}_l^{*}\mathbf{H}_l^{\mathrm{H}}\mathbf{G}^{*}\boldsymbol{\Upsilon}^{(j)^{\mathrm{T}}}. \end{aligned} \quad (21)$$

The KKT conditions can be used to determine the optimal transceive filter according to (16), because the optimization problem is convex for fixed transmit and receive filters at the nodes. In the following, matrix $\mathbf{K}$ is defined as

$$\begin{aligned} \mathbf{K} = {} & \sum_{k=1}^{2K}\sum_{j=1,j\neq k}^{2K} w_l\left[\boldsymbol{\Upsilon}^{(j)^{\mathrm{T}}} \otimes \left(\mathbf{H}_k^{*}\mathbf{D}_k^{\mathrm{H}}\mathbf{D}_k\mathbf{H}_k^{\mathrm{T}}\right)\right] \\ & + \left[\boldsymbol{\Upsilon}^{\mathrm{T}} \otimes \frac{1}{P_{\mathrm{RS,max}}}\mathrm{tr}\left(\sum_{k=1}^{2K}\mathbf{R}_{\mathbf{n}_{\mathrm{S}_k}}\right)\mathbf{I}_L\right], \end{aligned} \quad (22)$$

where $l = k - 1 + 2\cdot\mathrm{mod}_2 k$. Using (2), (3) and (22), the MMSE-SI filter at RS which solves problem (16) is given by

$$\mathbf{G} = \gamma\cdot\mathrm{vec}_{L,L}^{-1}\left(\mathbf{K}^{-1}\,\mathrm{vec}\left(\sum_{k=1}^{2K} w_l\mathbf{H}_k^{*}\mathbf{D}_k^{\mathrm{H}}\mathbf{R}_{\mathbf{x}_{\mathrm{S}_l}}\mathbf{Q}_l^{\mathrm{H}}\mathbf{H}_l^{\mathrm{H}}\right)\right),$$

$$\text{with } l = k - 1 + 2\cdot\mathrm{mod}_2 k. \quad (23)$$

The derived WMMSE-SI transceive filter at RS minimizes the weighted MSE assuming perfect self-interference cancellation and given Tx and Rx filters at the nodes. The weights w_k of the relay transceive filter are optimized with respect to the asymmetric rate requirements. The weights w_k which achieve the highest sum rate $C_{\mathrm{sum,constr.}}$ according to (9) are determined by numerical optimization.

V. Optimization of the number of simultaneously transmitted data streams

Up to now, the Tx and Rx filter optimization at the nodes as well as the relay transceive filter optimization have been investigated. Next, to maximize the sum rate $C_{\mathrm{sum,constr.}}$, the numbers $\mathbf{m} = \{m_1, m_2, ..., m_{2K}\}$ of simultaneously transmitted data streams of the nodes have to be jointly optimized. Considering one pair, the numbers of simultaneously transmitted data streams of the nodes S_k and S_l given by m_k and m_l, respectively, determine the selected singular vectors for transmission and reception and, therewith, determine the transmission and reception subspace of this pair. Thus, the values of m_k and m_l influence the achievable data rates of the other pairs, because the transmissions of the remaining pairs are performed in subspaces which only cause low interferences to the transmission subspace of pair Sk and Sl according to the MMSE principle of the applied WMMSE-SI relay transceive filter.

Every node is equipped with M antennas and can simultaneously transmit one up to M data streams. Thus, M^{2K} combinations for the numbers of simultaneously transmitted data streams exist in the considered scenario. Some of these combinations are not considered, because the number of antennas at RS has to be $L \geq \sum_{j=1,j\neq l}^{2K} m_j$ for $k = 1, 2, ..., 2K$, to enable the pairwise transmissions in orthogonal subspaces. All of the remaining combinations for the numbers of simultaneously transmitted data streams are used as an input vector $\mathbf{m}$ for the optimizations shown in Figure 2 which are described in the previous sections. The sum rate $C_{\mathrm{sum,constr.}}$ is determined for each combination and the combination which achieves the highest sum rate is used for transmission. The computations are performed at RS and the numbers of simultaneously transmitted data streams contained in $\mathbf{m}$ which achieve the highest sum rate $C_{\mathrm{sum,constr.}}$ are signaled to the nodes.

VI. Transmit power optimization of the nodes and alternating optimization

Up to now, the Tx and Rx filter optimization at the nodes as well as the relay transceive filter optimization have been investigated. Furthermore, the optimization of the number of simultaneously transmitted data streams has been considered. Next, the transmit powers of the nodes have to be optimized to increase the sum rate $C_{\mathrm{sum,constr.}}$. In this paper, a low-complexity suboptimal approach is presented and the transmit power optimization of all nodes is separated into pairwise transmit power optimizations. As presented in Section II, each node has a transmit power limitation of $P_{\mathrm{MS,max}}$. Thus, the power optimization is performed by transmit power reduction of the nodes. Each node Sk which achieves a rate $C_{\mathrm{S}k} > \frac{r_k}{r_l} C_{\mathrm{S}l}$ reduces its transmit power $P_{\mathrm{S}k}$ to achieve $C_{\mathrm{S}k} = \frac{r_k}{r_l} C_{\mathrm{S}l}$, $l = k - 1 + 2 \cdot \mathrm{mod}_2 k$, $k = 1, 2, ..., 2K$. Thus, one node of each pair transmits with maximum power $P_{\mathrm{S}k} = P_{\mathrm{MS,max}}$ and the other one with reduced power. The Tx filters considering the optimized powers are given by

$$\mathbf{Q}_k = \frac{P_{\mathrm{S}k}}{P_{\mathrm{MS,max}}} \mathbf{Q}_k, \qquad (24)$$

where $\mathbf{Q}_k$ are the Tx filters given in Section IV-A.

The WMMSE-SI relay transceive filter presented in Section IV-B is based on given Tx and Rx filters at the nodes. Due to reducing the transmit powers of the nodes, the Tx filters have changed and an alternating optimization between the power optimization of the nodes and the WMMSE-SI relay transceive filter as shown in Figure 2 is proposed. During every run of the alternating optimization, the weights w_k, $k = 1, 2, ..., K$ of the relay transceive filter described in Section IV-B are also optimized. For the optimization of the ratio between the weights w_k and w_l, the transmit powers of both nodes Sk and Sl are assumed to be $P_{\mathrm{S}k} = P_{\mathrm{S}l} = P_{\mathrm{MS,max}}$.

VII. Performance results

In this section, numerical results on the achievable sum rates for the introduced optimization approaches are investigated. The channels between the nodes and RS are assumed to be i.i.d. Rayleigh fading channels with a channel gain of one. It is assumed that $P_{\mathrm{RS,max}} = 4P_{\mathrm{MS,max}}$, $\sigma^2_{\mathrm{n,RS}} = \sigma^2_{\mathrm{n}}$, $M = 2$ and $K = 2$. The ratio $P_{\mathrm{MS,max}}/\sigma^2_{\mathrm{n,RS}}$ between the maximum transmit power at the nodes and the noise power at RS is termed average receive signal to noise ratio (SNR) at RS.

The following approaches are compared: The approaches "opt. streams (joint)" and "opt. streams (individual)" perform the optimizations as shown in Figure 2 and optimize the number of simultaneously transmitted data streams as presented in Section V. The approach "opt. streams (joint)" uses the joint Tx and Rx filter design at the nodes given in Section IV-A and the approach "opt. streams (individual)" uses the individual Tx and Rx filter design. The approaches "max. streams (joint)" and "one stream (joint)" use the joint Tx and Rx filter design at the nodes given in Section IV-A and perform the optimizations shown in Figure 2 with a fixed number of data streams per node. The approach "max. streams (joint)" assumes that each node simultaneously transmits M data streams and the approach "one streams (joint)" assumes that each node simultaneously transmits one data stream. The performances of these approaches using the individual Tx and Rx filter design are not given, because the performances are slightly worse.

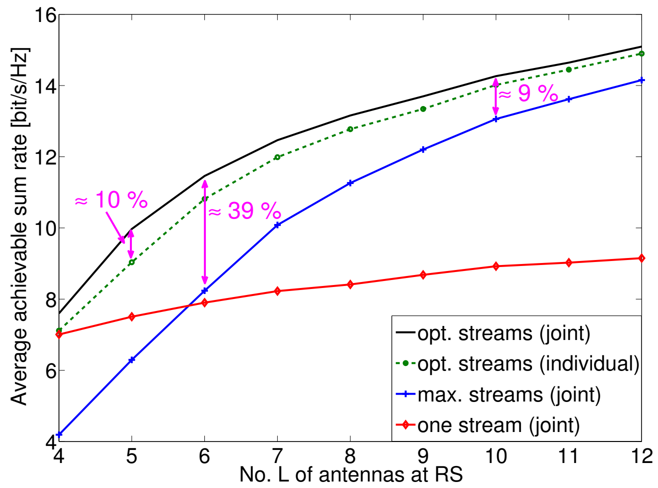

Fig. 3. Average achievable sum rates over number L of antennas at RS for $\mathbf{r} = (1, 0.5, 0.5, 0.25)$, $M = 2$, $K = 2$, average receive SNR at RS= 15dB.

The average achievable sum rates over different numbers L of antennas at RS for $\mathbf{r} = (1, 0.5, 0.5, 0.25)$ and an average receive SNR at RS of 15dB, are shown in Figure 3. The approach "opt. streams (joint)" achieves slightly higher sum rates than "opt. streams (individual)" due to optimizing the Tx and Rx filters with respect to the allocated subchannels. For $L = 5$ antennas at RS, the achievable sum rates can be increased by approximately 10%. The achievable sum rates of the approaches which optimize the number of simultaneously transmitted data streams "opt. streams (joint)" and "opt. streams (individual)" compared to achievable sum rates of the approaches which assume a fixed number of simultaneously transmitted data streams "max. streams (joint)" and "one stream (joint)" are approximately 39% higher for $L = 6$ antennas at RS and 9% higher for $L = 10$ antennas at RS. For a low number of antennas at RS, a low number of simultaneously transmitted data streams is optimal and the performance of "one stream (joint)" is better than the performance of "max. streams (joint)". For a high number of antennas at RS, it is advantageous to exploit the multiplexing gain and to transmit more data streams simultaneously. Thus, "max. streams (joint)" performs better than "one stream (joint)".

The average achievable sum rates over different average receive SNRs at RS for $\mathbf{r} = (1, 0.5, 0.5, 0.25)$, $L = 7$, are shown in Figure 4. For low SNRs, it is advantageous to transmit only a low number of data streams simultaneously to reduce the noise enhancement due to signal separation at RS and to benefit from higher SNRs of each stream. By increasing the SNR, the optimal number of simultaneously transmitted streams also increases. Thus, "max. streams (joint)" performs

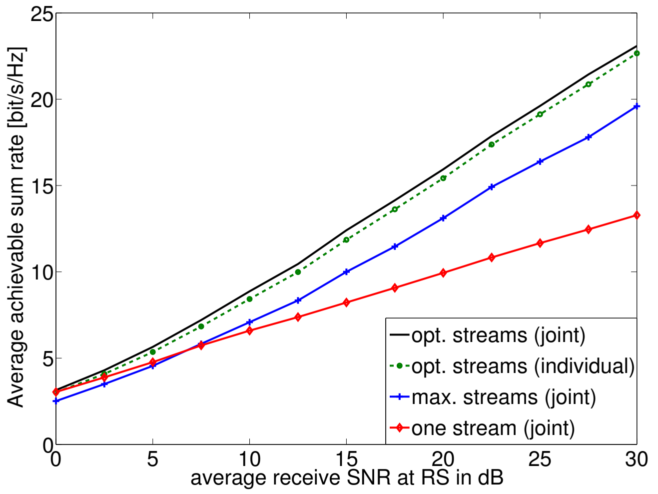

Fig. 4. Average achievable sum rates over different receive SNRs at RS for $\mathbf{r} = (1, 0.5, 0.5, 0.25)$, $M = 2$, $K = 2$, $L = 7$.

better than "one stream (joint)" for high SNR values and worse for low SNR values. Both approaches perform worse than the approaches "opt. streams (joint)" and "opt. streams (individual)" which optimize the number of simultaneously transmitted data streams.

VIII. Conclusions

Bi-directional communications in a multi-pair two-way relaying scenario under asymmetric rate requirements have been considered. The overall optimization problem of maximizing the achievable sum rate is non-convex. Thus, different subproblems for maximizing the achievable sum rate have been introduced and investigated, i.e., the optimization of the transmit and receive filters at the nodes, the design of a weighted self-interference aware relay transceive filter WMMSE-SI, the optimization of the number of simultaneously transmitted data streams of each node and the optimization of the transmit powers of the nodes. Performance results show that the achievable sum rate can be significantly improved under asymmetric rate requirements, if the number of simultaneously transmitted data streams is optimized.

Acknowledgment

The work of Holger Degenhardt is funded by the Deutsche Forschungsgesellschaft (DFG) under Grant No. Kl907/2-1 and is supported by the LOEWE Priority Program Cocoon and by the Graduate School of Computational Engineering at Technische Universität Darmstadt.

References

[1] E. Yilmaz, R. Zakhour, D. Gesbert, and R. Knopp, "Multi-Pair Two-Way Relay Channel with Multiple Antenna Relay Station," in *Proc. IEEE International Conference on Communications (ICC)*, 2010, pp. 1–5.

[2] A. Amah and A. Klein, "Pair-aware transceive beamforming for non-regenerative multi-user two-way relaying," in *Proc. IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, 2010, pp. 2506–2509.

[3] C. Y. Leow, Z. Ding, K. Leung, and D. Goeckel, "On the Study of Analogue Network Coding for Multi-Pair, Bidirectional Relay Channels," *IEEE Transactions on Wireless Communications*, vol. 10, no. 2, pp. 670–681, 2011.

[4] J. Joung and A. Sayed, "Multiuser Two-Way Amplify-and-Forward Relay Processing and Power Control Methods for Beamforming Systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1833–1846, 2010.

[5] H. Degenhardt and A. Klein, "Self-interference aware MIMO filter design for non-regenerative multi-pair two-way relaying," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2012.

[6] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 379–389, 2007.

[7] T. Unger, "Multi-antenna two-hop relaying for bi-directional transmission in wireless communication systems," Ph.D. dissertation, TU Darmstadt, June 2009.

[8] K.-J. Lee, K. W. Lee, H. Sung, and I. Lee, "Sum-Rate Maximization for Two-Way MIMO Amplify-and-Forward Relaying Systems," in *IEEE Vehicular Technology Conference (VTC Spring)*, 2009, pp. 1–5.

[9] R. Wang and M. Tao, "Joint Source and Relay Precoding Designs for MIMO Two-Way Relay Systems," in *IEEE International Conference on Communications (ICC)*, 2011, pp. 1–5.

[10] H. Degenhardt and A. Klein, "Non-regenerative multi-antenna two-hop relaying under an asymmetric rate constraint," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, 2011, pp. 1454–1459.

[11] H. Degenhardt, T. Unger, and A. Klein, "Self-interference aware MMSE filter design for a cellular multi-antenna two-way relaying scenario," in *Proc. 8th International Symposium on Wireless Communication Systems (ISWCS)*, 2011, pp. 261–265.

[12] B. Hassibi and B. Hochwald, "How much training is needed in multiple-antenna wireless links?" *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 951–963, 2003.

[13] Q. Spencer, A. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Transactions on Signal Processing*, vol. 52, no. 2, pp. 461–471, 2004.