Two-Way Relaying for Multiple Applications in Wireless Sensor Networks

Zhiliang Chen*, Alexander Kuehne*, Anja Klein*, Adrian Loch†, Matthias Hollick† and Joerg Widmer‡
*Communications Engineering Lab, Technische Universität Darmstadt, Germany
Email: {z.chen,a.kuehne,a.klein}@nt.tu-darmstadt.de
†Secure Mobile Networking Lab, Technische Universität Darmstadt, Germany
Email: {adrian.loch,matthias.hollick}@seemoo.tu-darmstadt.de
‡Institute IMDEA Networks, Spain
Email: joerg.widmer@imdea.org

***Abstract*—Recent work in wireless sensor networks implies possibilities of concurrent support of multiple applications. In this paper, we discuss a novel scheme called hybrid computation in two-way relaying, which introduces cooperation of three sensor nodes to support bi-directional communications of two applications. Applications in wireless sensor networks require different computations and forms of aggregation. In the proposed scheme, different computations at the intermediate node are integrated in a two-way relaying scheme. For computations and transmissions in the proposed scheme, data from all three nodes are considered. We propose a superposition coding protocol and a time division protocol to handle the transmission of the messages from the intermediate node. The problem of maximizing the sum rate is discussed. The results show that the superposition coding protocol outperforms the time division protocol.**

I. Introduction

Wireless Sensor Networks (WSNs) are networks with densely deployed sensor nodes. Sensor nodes observe information from the physical world and transmit it to the desired sinks [1]. In WSNs, computation (data aggregation) and transmission are always paired with each other, which highlights a property of WSNs that the data itself, rather than its identity, is important [2].

An efficient design of a WSN is a cross-layer problem, because joint optimization in cross-layer design is capable of considering data computation, routing, power consumption and throughput requirement [3]. Most works which jointly consider communications between sensor nodes and computations usually assume only a single application supported in a WSN [4], [5]. In this paper, we discuss a scheme where multiple applications are running in a WSN.

Recent works such as [6] and [7] start to consider sensor networks with concurrent support of multiple applications. In [6], a WSN is partitioned with a weighted balanced two-slice problem in order to support two applications running in a WSN concurrently. However, sensor nodes can only process a single application at a time in this work. This solution may compromise the coverage for each application since it reduces the number of sensor nodes activated for each application and lacks the possibility of introducing cooperation between sensor nodes. The work in [7] proposes a network layer protocol that forwards data packets of different applications one after another towards multiple gateways. This idea does not develop the feature of combining computation and transmission in a WSN, and there is no cooperation among sensor nodes. In this paper, sensor nodes cooperate to exchange messages.

In the case that data from two applications are forwarded in opposite directions in WSNs, bi-directional transmission has to be involved. A way to handle bi-directional transmission is to use two-way relaying [8]. In two-way relaying, two partner nodes exchange messages with the cooperation of a relay node within two transmission phases. At the relay node, messages are superimposed and broadcast. The partner nodes can perform an interference-free decoding of their partner's message by cancelling their own message in the received signal.

When two-way relaying is used in WSNs to support bi-directional communications, extra problems need to be considered in comparison to the two-way relaying introduced in [8]. Firstly, the relay node itself shall also be a node exchanging own messages of two applications with the partner nodes. Secondly, the relay node (or the intermediate node) can apply computation to the signal received in the multiple-access phase (MAC-phase). Therefore, Self-Interference Cancellation (SIC) may not be possible at the partner nodes in the broadcast phase (BC-phase). Works in [9] and [10] extend the usual two-way relaying in the first point.

In [9], the authors work on a bi-directional traffic requirement in a three-node network. In addition to the two-way relaying where two partner nodes S1 and S3 exchange information x_1 and x_3 via an intermediate node S2, this intermediate node piggybacks own extra information x_2 to the partner nodes, as shown in Figure 1. The extra information introduces interference at the partner nodes after SIC. Furthermore, two decoding orders at nodes S1 and S3 are possible to decode their partner's message and the extra message from the intermediate node. The authors of [9] proved that the maximum sum capacity of transmitting x_1, x_3 and x_2 in the BC-phase is achieved when both partner nodes firstly decode the information from the intermediate node having cancelled

This work was performed within the LOEWE Priority Program Cocoon (www.cocoon.tu-darmstadt.de), supported by the LOEWE research initiative of the state of Hesse/Germany.

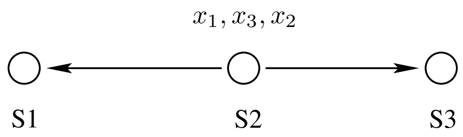

Fig. 1. Piggyback in two-way relaying

their own message from the received signal. The maximization of the sum rate is constrained by the total transmit power of the intermediate node S2 and by the requirement that the capacities of transmitting x_1 and x_3 in the BC-phase shall be no less than those in the MAC-phase. Combinatorial analysis and simulation results show that such piggyback solution gives larger sum rate than the solutions where messages are transmitted in time division in the BC-phase.

In [10], the authors analyse a two-way relaying scenario where each partner node has an individual private message for the intermediate node. Two MAC-phase solutions are proposed in this work. In the first solution, the private messages for the intermediate node are extracted as a computation over MAC in addition to the messages for exchange. The second solution applies a time sharing of two sub-schemes. In the first sub-scheme, the partner nodes transmit only the private messages for the intermediate node. In the second sub-scheme, the partner nodes transmit messages for exchange onto which the node with larger transmit power superimposes the private message for the intermediate node. A merge of the two MAC-phase solutions is shown by the simulation results in [10].

Works in [9] and [10] both extend the usual two-way relaying with the consideration of extra message exchange among the three-node network. However, the works did not cover the problem in WSNs when computation shall be introduced at the intermediate node in order to perform data aggregation. In this paper, we consider the scenario of multiple applications running simultaneously in a WSN where both of the problems are covered by jointly considering the computations and the communications. All three sensor nodes generate data of two applications requiring different aggregation functions and forward them to the other two nodes. Two-way relaying is employed for the bi-directional transmission while taking into account that the intermediate node transmits and receives extra messages along with the messages exchanged between the partner nodes.

The system model, the problem statement and the aggregation functions, are given in Section II. The hybrid computation in two-way relaying using superposition coding protocol is shown in detail in Section III. The time division protocol is presented in Section IV. In Section V, we give performance results. Section VI concludes this paper.

II. System Model, Problem Statement and Aggregation Functions

In this section, we firstly present the system model followed by the problem statement and the aggregation functions based on the concept of the divisible functions.

In this paper, a three-node network model as shown in Figure 2 is considered. Nodes S1, S2 and S3 observe data of two applications, denoted by x_1, y_1 at S1, x_2 , y_2 at S2 and x_3, y_3 at S3. We define the application by its function applied to the data. In such case, $x_i (i = 1, 2, 3)$ is the data for application f_X and $y_j (j = 1, 2, 3)$ is for application f_Y. Nodes S1, S2 and S3 are all half-duplex, i.e. they cannot transmit and receive at the same time. Furthermore, nodes S1 and S3 can only communicate with each other via the intermediate node S2.

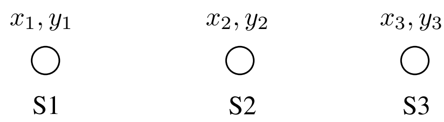

Fig. 2. Three sensor nodes and their data

We are interested in the case that two applications are running in a WSN concurrently. In Table I, it is shown that sensor nodes S1, S2 and S3 generate messages of two applications. We consider the case that the function output $f_X(x_1, x_2, x_3)$ is required at node S1 while the function output $f_Y(y_1, y_2, y_3)$ is required at node S3.

TABLE I
Data and function requirement

	S1	S2	S3
Message	x_1, y_1	x_2, y_2	x_3, y_3
Requirement	$f_X(x_1, x_2, x_3)$	-	$f_Y(y_1, y_2, y_3)$

In this paper, we assume f_X and f_Y are the download function and the mean function, respectively. These two functions are typical examples for WSNs. The download function usually refers to an application that a sensor network has to report its sensing status (GPS, object sensing, etc.) or its own status (battery status, geographical locations) to gateways. The mean function is useful when the application requires the average value of the sensed data. Temperature measurement, distributed voting, and other consensus problems [11], [12] belong to such an application. With the download function, node S1 requires the result of $f_X(x_1, x_2, x_3) = \{x_1, x_2, x_3\}$, with the requirement of the output of the mean function, node S3 requires the result of $f_Y(y_1, y_2, y_3) = \frac{1}{3}(y_1 + y_2 + y_3)$.

Since node S1 does not have messages of x_2 and x_3, communications are needed so that S1 can successfully determine $f_X(x_1, x_2, x_3)$ by receiving messages from node S2 and from node S3. Similarly, node S3 requires communications to receive messages from node S2 and node S1 in order to determine $f_Y(y_1, y_2, y_3)$. With the assumption that node S1 and S3 cannot directly communicate with each other, they have to forward their messages via the intermediate node S2.

In sensor networks, messages that are exchanged among sensor nodes are the computed data (function outputs) rather than the original data itself [2]. We use symbol ϕ to denote the function input variables which are not available at a sensor node [5]. In our requirement, nodes S1, S2 and S3 firstly calculate the functions of the two applications with their own data, the outputs are shown in Table II.

TABLE II
FUNCTION OUTPUTS AT S1, S2 AND S3

	S1	S2	S3
Application f_X	$f_X(x_1,\phi,\phi)$	$f_X(\phi,x_2,\phi)$	$f_X(\phi,\phi,x_3)$
Application f_Y	$f_Y(y_1,\phi,\phi)$	$f_Y(\phi,y_2,\phi)$	$f_Y(\phi,\phi,y_3)$

In Table II, $f_X(x_1,\phi,\phi)=\{x_1\}$, $f_X(\phi,x_2,\phi)=\{x_2\}$ and $f_X(\phi,\phi,x_3)=\{x_3\}$, $f_Y(y_1,\phi,\phi)=\frac{1}{3}y_1$, $f_Y(\phi,y_2,\phi)=\frac{1}{3}y_2$ and $f_Y(\phi,\phi,y_3)=\frac{1}{3}y_3$. To meet the requirement in Table I, node S1 transmits $f_Y(y_1,\phi,\phi)$ to node S2, node S2 calculates $f_Y(y_1,y_2,\phi)=\frac{1}{3}(y_1+y_2)$ and transmits it to node S3 where the final output $f_Y(y_1,y_2,y_3)$ can be calculated. Similarly, node S3 transmits $f_X(\phi,\phi,x_3)=\{x_3\}$ to node S2, node S2 determines $f_X(\phi,x_2,x_3)=\{x_2,x_3\}$ and transmits it to node S1. Node S1 yields the output $f_X(x_1,x_2,x_3)$. To simplify the notation, in the remainder of this paper, we omit the symbol ϕ when the variables are not available at the sensor nodes, e.g. $f_X(x_1,\phi,\phi)=f_X(x_1)$, etc..

Such as the download function and the mean function, the aggregation functions supported in WSNs which follow the "calculate-forward-calculate" way to calculate the function outputs in a distributed way are so-called *divisible functions*. Other examples of divisible functions are the max/min function, histogram, etc. [5].

The communication protocol for the requirement in Table I is still unknown. In sections III and IV, we propose a solution scheme called hybrid computation in two-way relaying which includes two realization protocols, namely superposition coding and time division. The term 'hybrid' indicates that multiple applications with different aggregation functions are running concurrently.

III. HYBRID COMPUTATION IN TWO-WAY RELAYING USING SUPERPOSITION CODING PROTOCOL

In this section, the hybrid computation in two-way relaying using superposition coding protocol for the requirement in Table I is briefly introduced. The details of the MAC-phase and the BC-phase are presented subsequently.

A. Protocol Description

Using superposition coding in the scheme of the hybrid computation in two-way relaying, there are of two communication phases, the MAC-phase and the BC-phase. Before exchanging messages in the MAC-phase, nodes S1, S2 and S3 firstly apply functions f_X and f_Y to their own data of the two applications, i.e., $f_X(x_1)$ and $f_Y(y_1)$ at node S1, $f_X(x_2)$ and $f_Y(y_2)$ at node S2 and $f_X(x_3)$ and $f_Y(y_3)$ at node S3 are determined. In the MAC-phase, nodes S1 and S3 use Gaussian distributed codebooks to encode messages $f_Y(y_1)$ and $f_X(x_3)$ individually, yielding the output $f_Y^{\mathcal{C}}(y_1)$ and $f_X^{\mathcal{C}}(x_3)$, respectively, and transmit to the intermediate node S2. The super-script $\mathcal{C}$ is to denote that the message has been encoded with Gaussian distributed codebook. The intermediate node S2 decodes these two messages, calculates function outputs $f_Y(y_1,y_2)=\frac{1}{3}(y_1+y_2)$ and $f_X(x_2,x_3)=\{f_X(x_2),f_X(x_3)\}=\{x_2,x_3\}$ from the received message and its own one. In the BC-phase, node S2 encodes $f_Y(y_1,y_2)$, $f_X(x_2)$ and $f_X(x_3)$ with three independent Gaussian distributed codebooks, respectively, and allocates a portion of its total transmit power to each encoded message. The superimposed messages are broadcast to nodes S1 and S3. Similar to [9], we use τ_{MAC} and τ_{BC} to indicate the time allocated to the MAC-phase and the BC-phase ,respectively, and define their ratio as

$$\tau=\frac{\tau_{\mathrm{MAC}}}{\tau_{\mathrm{BC}}}. \tag{1}$$

In the following, details of the MAC-phase and the BC-phase are discussed.

B. MAC-Phase

Symbols P_1, P_2 and P_3 denote the transmit power of nodes S1, S2 and S3, respectively. Symbol h_{ij} denotes the channel between the transmit node i and the receive node j. The channels in the MAC-phase and the BC-phase are assumed to be memoryless, reciprocal and block static. The fading factors are hence $h_{12}=h_{21}=h_1$ and $h_{32}=h_{23}=h_3$. The receive noise at node S1, S2 and S3 is denoted as n_1, n_2 and n_3, respectively, where the noise power is assumed to be the same at all nodes, denoted as N. The transmit SNR is defined as $\gamma_i=P_i/N, i=1,2,3$. We denote by $W(f_Y^{\mathcal{C}}(y_1)), W(f_X^{\mathcal{C}}(x_3))$ the single letters of the codewords $f_Y^{\mathcal{C}}(y_1)$ and $f_X^{\mathcal{C}}(x_3)$ at nodes S1 and S3, respectively [9]. $W(f_Y^{\mathcal{C}}(y_1)), W(f_X^{\mathcal{C}}(x_3))$ are assumed to be complex normal distributed random variables with zero-mean and variance equal to one ($\mathcal{CN}(0,1)$). In the MAC-phase, nodes S1 and S3 allocate transmit power P_1 and P_3 to $W(f_Y^{\mathcal{C}}(y_1))$ and $W(f_X^{\mathcal{C}}(x_3))$, respectively, and transmit to the intermediate node S2. Node S2 receives signal

$$r_2=h_1\sqrt{P_1}W(f_Y^{\mathcal{C}}(y_1))+h_3\sqrt{P_3}W(f_X^{\mathcal{C}}(x_3))+n_2 \tag{2}$$

from nodes S1 and S3.

S2 decodes $f_Y(y_1)$ and $f_X(x_3)$ error-free with rates R_1 and R_3, respectively, which are bounded by the MAC capacity region [13]. The region is formulated as

$$\begin{aligned}\mathcal{R}^{\mathrm{MAC}}=\big\{[R_1,R_3]\in\mathbf{R}_+^2 :\\ R_1\le C_1^{\mathrm{MAC}},\\ R_3\le C_3^{\mathrm{MAC}},\\ R_1+R_3\le C_\Sigma^{\mathrm{MAC}}\big\}\ ,\end{aligned} \tag{3}$$

where

$$C_1^{\mathrm{MAC}} = \log\left(1+|h_1|^2\gamma_1\right) \tag{4}$$
$$C_3^{\mathrm{MAC}} = \log\left(1+|h_3|^2\gamma_3\right) \tag{5}$$
$$C_\Sigma^{\mathrm{MAC}} = \log\left(1+|h_1|^2\gamma_1+|h_3|^2\gamma_3\right)\ , \tag{6}$$

and $\mathbf{R}_+$ is the set of non-negative real numbers.

Node S2 computes the functions $f_X(x_1,x_2)=\{f_X(x_1),f_X(x_2)\}$ and $f_Y(y_1,y_2)=\frac{1}{3}(y_1+y_2)$ after successfully decoding $f_X(x_3)$ and $f_Y(y_1)$.

C. BC-Phase

In the BC-phase, node S2 encodes three messages $f_X(x_2)$, $f_X(x_3)$ and $f_Y(y_1, y_2)$ with three independent Gaussian distributed codebooks, where the output codewords are $f_X^{\mathcal{C}}(x_2)$, $f_X^{\mathcal{C}}(x_3)$ and $f_Y^{\mathcal{C}}(y_1, y_2)$, respectively. As done in the MAC-phase, $W(f_X^{\mathcal{C}}(x_2)), W(f_X^{\mathcal{C}}(x_3))$ and $W(f_Y^{\mathcal{C}}(y_1, y_2)) \sim \mathcal{CN}(0, 1)$ denote the single letters of the respective codeword.

The total transmit power P_2 of node S2 is distributed into three portions: β_1 for $W(f_X^{\mathcal{C}}(x_2))$, β_2 for $W(f_X^{\mathcal{C}}(x_3))$ and β_3 for $W(f_Y^{\mathcal{C}}(y_1, y_2))$ with $\beta_1 + \beta_2 + \beta_3 = 1$. Node S2 broadcasts the superimposed signal $\sqrt{\beta_1} W(f_X^{\mathcal{C}}(x_2)) + \sqrt{\beta_2} W(f_X^{\mathcal{C}}(x_3)) + \sqrt{\beta_3} W(f_Y^{\mathcal{C}}(y_1, y_2))$ to nodes S1 and S3. The receive signal at node S1 is

$$\begin{aligned} z_1 =& h_1 \sqrt{P_2} \Big(\sqrt{\beta_1} W(f_X^{\mathcal{C}}(x_2)) + \sqrt{\beta_2} W(f_X^{\mathcal{C}}(x_3)) \\ & + \sqrt{\beta_3} W(f_Y^{\mathcal{C}}(y_1, y_2)) \Big) + n_1. \end{aligned} \tag{7}$$

Similarly, node S3 receives signal

$$\begin{aligned} z_3 =& h_3 \sqrt{P_2} \Big(\sqrt{\beta_1} W(f_X^{\mathcal{C}}(x_2)) + \sqrt{\beta_2} W(f_X^{\mathcal{C}}(x_3)) \\ & + \sqrt{\beta_3} W(f_Y^{\mathcal{C}}(y_1, y_2)) \Big) + n_3. \end{aligned} \tag{8}$$

In order to calculate the function $f_X(x_1, x_2, x_3)$ at node S1, messages $f_X(x_2)$ and $f_X(x_3)$ shall be decoded from the receive signal z_1. However, the interference $f_Y(y_1, y_2)$ cannot be cancelled by SIC at node S1 because it does not contain the same message as $f_Y(y_1)$ which is transmitted from S1 in the MAC-phase. This highlights that in the hybrid computation in two-way relaying, a difference to the two-way relaying without computation is that SIC cannot always be performed at the partner nodes. At the other partner node S3, the message $f_X(x_3)$ of the download function remains the same as what has been transmitted from node S3 in the MAC-phase. Therefore, node S3 is able to perform SIC to cancel $f_X^{\mathcal{C}}(x_3)$ from its received signal z_3. This results in three unknown messages at node S1 and two unknown messages at node S3. Since superposition coding is employed in the BC-phase, different decoding orders are possible at nodes S1 and S3, shown in Table III. For simplicity of referring, we abbreviate the decoding orders listed in Table III using A1, A2, B1, ... B4, C1, C2. Decoding orders A1 and A2 at node S1 do not decode the unwanted message $f_Y(y_1, y_2)$, whereas decoding orders B1 - B4 decode message $f_Y(y_1, y_2)$ before decoding the wanted messages in order to eliminate $f_Y(y_1, y_2)$ as interference. According to Table III, 12 decoding order combinations at node S1 and node S3 are possible. In comparison to what has been discussed in [9], we cannot simply conclude which decoding order is always optimum. Therefore, all 12 decoding orders have to be discussed. In the further discussion, we use $\Lambda = \{$A1-C1, A1-C2, A2-C1, ... B4-C1, B4-C2$\}$ to denote all possible decoding order combinations at node S1 and node S3, where $\lambda \in \Lambda$ is the decoding order combination that the nodes S1 and S3 are using. For example, when $\lambda =$ B1-C2, node S1 firstly decodes message $f_X(x_2)$ with the interference caused by $f_Y(y_1, y_2)$ and $f_X(x_3)$. After cancelling $f_X(x_2)$, S1 decodes message $f_Y(y_1, y_2)$ with interference $f_X(x_3)$. Finally, S1 decodes $f_X(x_3)$ interference-free by cancelling $f_Y(y_1, y_2)$ from the receive signal. Similarly, node S3 in decoding order combination B1-C2 firstly decodes message $f_X(x_2)$ with the interference caused by $f_Y(y_1, y_2)$. Then it decodes $f_Y(y_1, y_2)$ interference-free after cancelling $f_X(x_2)$ from its received signal.

Besides the combinations of decoding orders at nodes S1 and S3, two rate constraints shall also be considered:

- In the BC-phase, the rate of transmitting $f_X(x_3)$ should be at least equal to the rate with which S3 transmits $f_X(x_3)$ in the MAC-phase. In the remainder of this work, we consider the equality in this constraint since it guarantees the error-free transmission of message $f_X(x_3)$ in the BC-phase.
- In the BC-phase, the rate of transmitting $f_Y(y_1, y_2)$ should be no less than the rate with which S1 transmits $f_Y(y_1)$ in the MAC-phase. We use 'no less than' here due to the fact that message $f_Y(y_1, y_2)$ includes data from node S1 as well as the data from node S2.

Taking into account the time ratio τ between the MAC and the BC-phase shown in (1), we can translate these two constraints to

$$C^{\mathrm{BC},x_3} = \tau R_3 \text{ and} \tag{9}$$

$$C^{\mathrm{BC},y} \geq \tau R_1 \ , \tag{10}$$

where C^{BC,x_3} denotes the maximum rate of transmitting $f_X(x_3)$ in the BC-phase and $C^{\mathrm{BC},y}$ is the maximum rate of transmitting $f_Y(y_1, y_2)$.

An optimization problem in the BC-phase is proposed to maximize the sum rate of node S2 broadcasting to nodes S1 and S3. In the BC-phase, message $f_X(x_3)$ requires only the same rate as that in the MAC-phase. Therefore, the optimization objective function considers only the sum rate of transmitting $f_X(x_2)$ and $f_Y(y_1, y_2)$ which are functions of the non-negative power portion factors β_1, β_2 and β_3 and the decoding orders λ at nodes S1 and S3. The maximization problem is given by

$$\begin{aligned} & \max_{\beta_1, \beta_2, \beta_3, \lambda} \left\{ C^{\mathrm{BC},x_2}(\beta_1, \beta_2, \beta_3, \lambda) + C^{\mathrm{BC},y}(\beta_1, \beta_2, \beta_3, \lambda) \right\} \\ & \text{s.t.} \\ & \beta_1 + \beta_2 + \beta_3 = 1, \\ & \beta_i \geq 0, \text{for } i = 1, 2, 3, \\ & \lambda \in \Lambda, \\ & C^{\mathrm{BC},x_3}(\beta_1, \beta_2, \beta_3, \lambda) = \tau R_3, \\ & C^{\mathrm{BC},y}(\beta_1, \beta_2, \beta_3, \lambda) \geq \tau R_1 \ , \end{aligned} \tag{11}$$

where C^{BC,x_2} denotes the maximum rate of transmitting $f_X(x_2)$ in the BC-phase.

Without loss of generality, we choose the decoding order combination $\lambda =$ B1-C2 at nodes S1 and S3 as an example to demonstrate the details of the maximization in (11). With the combination $\lambda =$ B1-C2, node S1 firstly decodes message

TABLE III
POSSIBLE DECODING ORDERS AT NODES S1 AND S3

S1	$f_X(x_2)$, $f_X(x_3)$: A1	$f_X(x_3)$, $f_X(x_2)$: A2	$f_Y(y_1, y_2)$, $f_X(x_2)$, $f_X(x_3)$: B1
	$f_X(x_2)$, $f_Y(y_1, y_2)$, $f_X(x_3)$: B2	$f_y(y_1, y_2)$, $f_X(x_3)$, $f_X(x_2)$: B3	$f_X(x_3)$, $f_Y(y_1, y_2)$, $f_X(x_1)$: B4
S3	$f_Y(y_1, y_2)$: C1	$f_X(x_2)$, $f_Y(y_1, y_2)$: C2	

$f_Y(y_1, y_2)$ treating $f_X(x_2)$ and $f_X(x_3)$ as interference with rate

$$C^{\mathrm{BC},y}_{\mathrm{S1,B1}} = \log\left(1 + \frac{|h_1|^2 P_2 \beta_3}{N + |h_1|^2 P_2 \beta_1 + |h_1|^2 P_2 \beta_3}\right). \quad (12)$$

We define symbol q_1 as $|h_1|^2\gamma_2 = 1/q_1$ and simplify (12) as

$$C^{\mathrm{BC},y}_{\mathrm{S1,B1}} = \log\left(1 + \frac{\beta_3}{q_1 + \beta_1 + \beta_3}\right) . \quad (13)$$

Afterwards, S1 subtracts $f_Y(y_1, y_2)$ and decodes $f_X(x_2)$ with the interference caused by $f_X(x_3)$. The rate of decoding $f_X(x_2)$ is

$$C^{\mathrm{BC},x_2}_{\mathrm{S1,B1}} = \log\left(1 + \frac{|h_1|^2 P_2 \beta_1}{N + |h_1|^2 P_2 \beta_2}\right) . \quad (14)$$

With the definition of q_1, (14) is simplified as

$$C^{\mathrm{BC},x_2}_{\mathrm{S1,B1}} = \log\left(1 + \frac{\beta_1}{q_1 + \beta_2}\right) . \quad (15)$$

The interference-free decoding of $f_X(x_3)$ is performed after cancelling $f_X(x_2)$ with the maximum rate

$$C^{\mathrm{BC},x_3}_{\mathrm{S1,B1}} = \log\left(1 + \frac{\beta_2}{q_1}\right) . \quad (16)$$

At node S3, message $f_X(x_3)$ is cancelled by SIC. With decoding order combination B1-C2, S3 decodes the unwanted message $f_X(x_2)$ at first treating $f_Y(y_1, y_2)$ as interference. By denoting $|h_3|^2\gamma_2 = 1/q_3$, the maximum rate of transmitting $f_X(x_2)$ is

$$C^{\mathrm{BC},x_2}_{\mathrm{S3,C2}} = \log\left(1 + \frac{\beta_1}{q_3 + \beta_3}\right) . \quad (17)$$

After eliminating $f_X(x_3)$, node S3 can decode $f_Y(y_1, y_2)$ interference-free with maximum rate

$$C^{\mathrm{BC},y}_{\mathrm{S3,C2}} = \log\left(1 + \frac{\beta_3}{q_3}\right) . \quad (18)$$

Because all the decoding has to be error-free, the rates of transmitting x_2 and transmitting y should be chosen as $C^{\mathrm{BC},x_2} = \min\{C^{\mathrm{BC},x_2}_{\mathrm{S1,B1}}, C^{\mathrm{BC},x_2}_{\mathrm{S3,C2}}\}$ and $C^{\mathrm{BC},y} = \min\{C^{\mathrm{BC},y}_{\mathrm{S1,B1}}, C^{\mathrm{BC},y}_{\mathrm{S3,C2}}\}$.

Using $\log(1+r_1) = \tau R_1$ and $\log(1+r_3) = \tau R_3$, we define two signal-to-interference-plus-noise ratio (SINR) parameters, $r_1 = 2^{\tau R_1} - 1$ and $r_3 = 2^{\tau R_3} - 1$. The rate constraint $C^{\mathrm{BC},x_3} = \tau R_3$ in (9) can be replaced by

$$\frac{\beta_2}{q_1} = r_3 , \quad (19)$$

and the constraint $C^{\mathrm{BC},y} \geq \tau R_1$ of (10) can be transformed to

$$\min\left\{\frac{\beta_3}{q_1 + \beta_1 + \beta_2}, \frac{\beta_3}{q_3}\right\} \geq r_1 . \quad (20)$$

Due to (20), it can be seen that the problem in (11) is a combinatorial problem. We can re-formulate the maximization with decoding order combination $\lambda = $ B1-C2 as

$$\begin{aligned}
&\max\Bigg\{\min\left\{\log\left(1 + \frac{\beta_1}{q_1 + \beta_2}\right), \log\left(1 + \frac{\beta_1}{q_3 + \beta_3}\right)\right\} \\
&\quad + \min\left\{\log\left(1 + \frac{\beta_3}{q_1 + \beta_1 + \beta_2}\right), \log\left(1 + \frac{\beta_3}{q_3}\right)\right\}\Bigg\} \\
&\text{s.t.} \\
&\text{if } q_1 + \beta_1 + \beta_2 > q_3, \text{then} \\
&\beta_1 \leq \frac{1 - r_3 q_1 - r_1 q_1 - r_1 r_3 q_1}{1 + r_1}, \\
&\beta_2 = r_3 q_1, \\
&\beta_3 = 1 - \beta_2 - \beta_1; \\
&\text{else} \\
&\beta_1 \leq 1 - q_1 r_3 - r_1 q_3, \\
&\beta_2 = r_3 q_1, \\
&\beta_3 = 1 - \beta_2 - \beta_1 .
\end{aligned} \quad (21)$$

The total transmit power constraint at node S2 gives the condition $\beta_1 + \beta_2 + \beta_3 = 1$. Hence it is always possible to express one power portion factor with the other two. In the optimization problem in (21) with the given decoding order combination, we use a two dimensional numerical search to find the optimum solution of the power portion factors, which optimizes maximum sum rate in (11).

For the other decoding order combinations $\lambda \in \Lambda$, the objective functions and the two rate constraints of (9) and (10) are listed in Table IV.

IV. HYBRID COMPUTATION IN TWO-WAY RELAYING USING TIME DIVISION PROTOCOL

An alternative protocol which provides a performance benchmark for the superposition coding protocol is using time division (TD) and is introduced in the following.

The MAC-phase in the TD protocol is the same as for the superposition coding protocol. The difference is that in the BC-phase, node S2 transmits $f_X(x_2)$, $f_X(x_3)$ and $f_Y(y_1, y_2)$ one after another instead of superimposing them and transmitting them simultaneously. The time duration of the BC-phase will be split into three non-negative time portions α_1, α_2 and α_3, with $\alpha_1 + \alpha_2 + \alpha_3 = 1$, for transmitting $f_X(x_2)$, $f_X(x_3)$ and $f_Y(y_1, y_2)$, respectively.

In the TD protocol, S2 always allocates its total transmit power to the transmission in the BC-phase so that a fair comparison can be made with the superposition coding protocol. The maximum sum rate in the BC-phase transmitting three

TABLE IV
OBJECTIVE FUNCTIONS AND RATE CONSTRAINTS FOR DIFFERENT DECODING ORDER COMBINATIONS

λ	Objective function	
	Rate Constraint (9)	Rate Constraint (10)
A1-C1	$C_{\mathrm{S1,A1}}^{\mathrm{BC},x2} + C_{\mathrm{S3,C1}}^{\mathrm{BC},y}$	
	$\frac{\beta_1}{q_1+\beta_3} = r_3$	$\frac{\beta_3}{q_3+\beta_1} \geq r_1$
A2-C1	$C_{\mathrm{S1,A2}}^{\mathrm{BC},x2} + C_{\mathrm{S3,C1}}^{\mathrm{BC},y}$	
	$\frac{\beta_2}{q_1+\beta_1+\beta_3} = r_3$	$\frac{\beta_3}{q_3+\beta_1} \geq r_1$
A1-C2	$\max\{\min\{C_{\mathrm{S1,A1}}^{\mathrm{BC},x2}, C_{\mathrm{S3,C2}}^{\mathrm{BC},x2}\} + C_{\mathrm{S3,C2}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_3} = r_3$	$\frac{\beta_3}{q_3} \geq r_1$
A2-C2	$\max\{\min\{C_{\mathrm{S1,A2}}^{\mathrm{BC},x2}, C_{\mathrm{S3,C2}}^{\mathrm{BC},x2}\} + C_{\mathrm{S3,C2}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_1+\beta_3} = r_3$	$\frac{\beta_3}{q_3} \geq r_1$
B1-C1	$\max\{C_{\mathrm{S1,B1}}^{\mathrm{BC},x2} + \min\{C_{\mathrm{S1,B1}}^{\mathrm{BC},y}, C_{\mathrm{S3,C1}}^{\mathrm{BC},y}\}\}$	
	$\frac{\beta_2}{q_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_1+\beta_2}, \frac{\beta_3}{q_3+\beta_1}\} \geq r_1$
B1-C2	$\min\{C_{\mathrm{S1,B1}}^{\mathrm{BC},x2}, C_{\mathrm{S3,C2}}^{\mathrm{BC},x2}\} + \min\{C_{\mathrm{S1,B1}}^{\mathrm{BC},y}, C_{\mathrm{S3,C2}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_2}, \frac{\beta_3}{q_3+\beta_1}\} \geq r_1$
B2-C1	$C_{\mathrm{S1,B2}}^{\mathrm{BC},x2} + \min\{C_{\mathrm{S1,B2}}^{\mathrm{BC},y}, C_{\mathrm{S3,C1}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_2}, \frac{\beta_3}{q_3+\beta_1}\} \geq r_1$
B2-C2	$\min\{C_{\mathrm{S1,B2}}^{\mathrm{BC},x2}, C_{\mathrm{S3,C2}}^{\mathrm{BC},x2}\} + \min\{C_{\mathrm{S1,B2}}^{\mathrm{BC},y}, C_{\mathrm{S3,C2}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_2}, \frac{\beta_3}{q_3}\} \geq r_1$
B3-C1	$C_{\mathrm{S1,B3}}^{\mathrm{BC},x2} + \min\{C_{\mathrm{S1,B3}}^{\mathrm{BC},y}, C_{\mathrm{S3,C1}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_1+\beta_2}, \frac{\beta_3}{q_3+\beta_1}\} \geq r_1$
B3-C2	$\min\{C_{\mathrm{S1,B3}}^{\mathrm{BC},x2}, C_{\mathrm{S3,C2}}^{\mathrm{BC},x2}\} + \min\{C_{\mathrm{S1,B3}}^{\mathrm{BC},y}, C_{\mathrm{S3,C2}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_1} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_1+\beta_2}, \frac{\beta_3}{q_3}\} \geq r_1$
B4-C1	$C_{\mathrm{S1,B4}}^{\mathrm{BC},x2} + \min\{C_{\mathrm{S1,B4}}^{\mathrm{BC},y}, C_{\mathrm{S3,C1}}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_1+\beta_3} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_1}, \frac{\beta_3}{q_3+\beta_1}\} \geq r_1$
B4-C2	$\min\{C_{\mathrm{S1},3y2}^{\mathrm{BC},x2}, C_{\mathrm{S3},2y}^{\mathrm{BC},x2}\} + \min\{C_{\mathrm{S1},3y2}^{\mathrm{BC},y}, C_{\mathrm{S3},2y}^{\mathrm{BC},y}\}$	
	$\frac{\beta_2}{q_1+\beta_1+\beta_3} = r_3$	$\min\{\frac{\beta_3}{q_1+\beta_1}, \frac{\beta_3}{q_3}\} \geq r_1$

messages is then scaled by the time portion in addition to the time scaling τ between the MAC-phase and the BC-phase as introduced in (1). Therefore we can directly give the rates of transmitting $f_X(x_2)$, $f_X(x_3)$ and $f_Y(y_1, y_2)$ as

$$
\begin{aligned}
C_{\mathrm{TD}}^{\mathrm{BC},x_2} &= \alpha_1 \log\left(1 + |h_1|^2\gamma_2\right) , && (22)\\
C_{\mathrm{TD}}^{\mathrm{BC},x_3} &= \alpha_2 \log\left(1 + |h_1|^2\gamma_2\right) \text{ and} && (23)\\
C_{\mathrm{TD}}^{\mathrm{BC},y} &= \alpha_3 \log\left(1 + |h_3|^2\gamma_2\right) , && (24)
\end{aligned}
$$

respectively.

The rate constraints in the TD protocol are identical to the constraints in the superposition coding protocol. The maximum rate of transmitting $f_X(x_3)$ at S1 is bounded by

$$C_{\mathrm{TD}}^{\mathrm{BC},x_3} = \tau R_3 . \tag{25}$$

At node S3, the maximum rate of transmitting message $f_Y(y_1, y_2)$ fulfils the rate constraint given by

$$C_{\mathrm{TD}}^{\mathrm{BC},y} \geq \tau R_1 . \tag{26}$$

In the following, a maximization problem with the same objective function as utilized in Section III is formulated. However, the constraints are changed according to the TD protocol in the BC-phase. This yields

$$
\begin{aligned}
&\max_{\alpha_1,\alpha_2,\alpha_3} \left\{C_{\mathrm{TD}}^{\mathrm{BC},x_2}(\alpha_1) + C_{\mathrm{TD}}^{\mathrm{BC},y}(\alpha_3)\right\}\\
&\text{s.t.}\\
&\alpha_1 + \alpha_2 + \alpha_3 = 1,\\
&\alpha_i \geq 0, \text{for } i = 1, 2, 3,\\
&C_{\mathrm{TD}}^{\mathrm{BC},x_3}(\alpha_2) = \tau R_3,\\
&C_{\mathrm{TD}}^{\mathrm{BC},y}(\alpha_3) \geq \tau R_1 .
\end{aligned} \tag{27}
$$

The two rate constraints in (27) give the solution of the parameter

$$\alpha_2 = \frac{\tau R_3}{\log\left(1 + |h_1|^2\gamma_2\right)} \tag{28}$$

and the range

$$\alpha_3 \geq \frac{\tau R_1}{\log\left(1 + |h_3|^2\gamma_2\right)} \tag{29}$$

by using equations (23) - (26). The objective function in (27) can be simplified by using the solution of α_2, α_3 and the relation $\alpha_1 + \alpha_2 + \alpha_3 = 1$. The monotonicity of the resulting objective function depends on the relation between the channel coefficients $|h_1|^2$ and $|h_3|^2$, which results in the following solution:

$$
\begin{aligned}
&\text{if} \quad |h_3|^2 \geq |h_1|^2\\
&\max_{\alpha_1,\alpha_2,\alpha_3} \left\{C_{\mathrm{TD}}^{\mathrm{BC},x_2} + C_{\mathrm{TD}}^{\mathrm{BC},y}\right\}\\
&= \left(1 - \frac{\tau R_3}{\log(1 + |h_1|^2\gamma_2)}\right) \log(1 + |h_3|^2\gamma_2) ,\\
&\text{if} \quad |h_3|^2 < |h_1|^2\\
&\max_{\alpha_1,\alpha_2,\alpha_3} \left\{C_{\mathrm{TD}}^{\mathrm{BC},x_2} + C_{\mathrm{TD}}^{\mathrm{BC},y}\right\}\\
&= \left(1 - \frac{\tau R_1}{\log(1 + |h_3|^2\gamma_2)}\right) \log(1 + |h_1|^2\gamma_2)\\
&\quad + \tau(R_1 - R_3) .
\end{aligned} \tag{30}
$$

V. PERFORMANCE RESULTS

In this section, we analyse the performance of the hybrid computation in two-way relaying using the superposition protocol and compare it with the time division protocol. We consider the channel parameters $|h_1|^2 = 1$ and $|h_3|^2 = 0.5$ together with the simulation parameters given in Table V as these parameters lead to feasible non-negative solutions of (11) and (30).

TABLE V
SIMULATION PARAMETERS

γ_1	1
γ_2	11
γ_3	1
τ	0.5

In Figure 3, the sum rate in the BC-phase is depicted as a function of the rates R_1 and R_3 in the MAC-phase in bit/s/Hz.

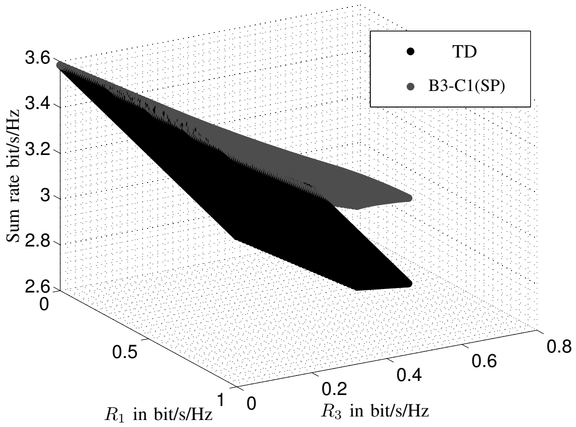

Fig. 3. Simulation Results (TD - Time Division Protocol, SP - Superposition Coding Protocol)

Therefore, the projection of all planes in the figure to the bottom R_1-R_3 plane is the MAC capacity region, where the corner value is $[C_\Sigma^{\mathrm{MAC}} - C_3^{\mathrm{MAC}}, C_3^{\mathrm{MAC}}] = [0.737, 0.585]$bit/s/Hz and $[C_1^{\mathrm{MAC}}, C_\Sigma^{\mathrm{MAC}} - C_1^{\mathrm{MAC}}] = [1, 0.3219]$bit/s/Hz. For comparison, the performances of the TD protocol and of the superposition coding protocol with decoding order combination B3-C1 are shown. To maintain the readability, we omit the performance of the superposition coding protocol with other decoding order combinations, because the performances are worse. When nodes S1 and S3 do not transmit anything in the MAC-phase, i.e., $R_1^\star = R_3^\star = 0$, only node S2 transmits messages to nodes S1 and S3 in the BC-phase, the performances of the TD protocol and the superposition coding protocol with decoding order combination B3-C1 are the same. For other rate pairs than $R_1^\star = R_3^\star = 0$, the superposition coding protocol with the decoding order combination B3-C1 outperforms the TD protocol for the chosen parameters. Approximately 14% gain is achieved by the superposition coding protocol compared to the TD protocol at the rate pair $[R_1^\star, R_3^\star] = [C_\Sigma^{\mathrm{MAC}} - C_3^{\mathrm{MAC}}, C_3^{\mathrm{MAC}}]$. The performance of the superposition coding protocol with other decoding order combinations can be worse than the performance of the TD protocol, i.e., the decoding order combination of the superposition coding protocol is crucial for its performance considering our proposed hybrid computation in two-way relaying scheme.

VI. Conclusion

In this paper, we consider a three-node wireless sensor network which runs two applications with the download function and the mean function, respectively, as the aggregation functions. In order to support the requirement that one partner node in the three-node network downloads all the sensor nodes' data of the first application and the other partner node calculates the mean value of all the sensor nodes' data of the second application, we propose a communication scheme named hybrid computation in two-way relaying with two protocols, the superposition coding protocol and the time division protocol. In the proposed scheme, there are two communication phases, the MAC-phase and the BC-phase. In the MAC-phase, the two partner nodes transmit messages of both applications to the intermediate node where hybrid computations are performed. In the BC-phase, the intermediate node broadcasts the computed data to the partner nodes where the required application functions are determined. In both protocols, we give optimization problems which maximize the sum rate.In the superposition coding protocol, we show that 12 different decoding order combinations at the partner nodes are possible since SIC cannot be performed at one of the partner nodes. The sum rate is maximized with respect to the power portion for each superimposed message, and the constraints guarantee the successful transmission of the messages from the partner nodes. In the time division protocol, the optimization is performed with respect to the time portion allocated to the message that is broadcast from the intermediate node to the partner nodes. Simulation results shows that the decoding order combination is crucial to the performance of the superposition coding protocol. Under the given system settings, we show that the superposition coding protocol with certain decoding order combination outperforms the time division protocol.

References

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "A survey on sensor networks," *IEEE Communications Magazine*, vol. 40, pp. 102–114, Aug. 2002.

[2] R. Rajagopalan and P. K. Varshney, "Data-aggregation techniques in sensor networks: A survey," *IEEE Communications Survey and Tutorials*, vol. 8, no. 4, pp. 48 – 63, 2006.

[3] I. F. Akyildiz, M. C. Vuran, and O. B. Akan, "A cross-layer protocol for wireless sensor networks," in *Proc. 40th Annual Conference on Information Sciences and Systems*, 2006.

[4] M. Goldenbaum, S. Stanczak, and M. Kaliszan, "On function computation via wireless sensor multiple-access channels," in *Proc. IEEE Wireless Communications and Networking Conference*, 2009.

[5] A. Giridhar and P. R. Kumar, "Computing and communicating functions over sensor networks," *IEEE Journal on Selected Areas in Communications*, vol. 23, pp. 755 – 764, Apr. 2005.

[6] M. Shazly, E. S. Elmallah, and J. Harms, "Balanced slices in wireless sensor networks," in *Proc. 2011 IEEE Wireless Communications and Networking Conference*, 2011.

[7] A. Alomar and A. Akbar, "CЯC: Carrefour routing for cluster-based wireless sensor network to support multiple applications in multiple gateway environments," in *Proc. International Conference on Mobile Ad-hoc and Sensor Networks*, 2009.

[8] B. Rankov and A. Wittneben, "Achievable rate regions for the two-way relay channel," in *Proc. 2006 IEEE International Symposium on Information Theory*, 2006.

[9] T. J. Oechtering and H. Boche, "Piggyback a common message on half-duplex bidirectional relaying," *IEEE Transactions on Wireless Communications*, vol. 7, no. 9, pp. 3397 – 3406, Sept. 2007.

[10] C. K. Ho, K. T. Gowda, and S. Sun, "A generalized two-way relay channel with private information for the relay," in *Proc. 2009 IEEE International Conference on Communications*, 2009.

[11] H. Ayad and M. Kamel, "Cumulative voting consensus method for partitions with a variable number of clusters," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 30, no. 1, pp. 160– 173, Jan. 2008.

[12] T. C. Aysal, M. E. Yildiz, A. D. Sarwate, and A. Scaglione, "Broadcast gossip algorithms for consensus," *IEEE Transactions on Signal Processing*, vol. 57, no. 7, pp. 2748 – 2761, Jul. 2009.

[13] T. Cover and J. Thomas, *Elements of Information Theory*. John Wiley and Sons, 1991.