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# Perfect Versus Imperfect Interference Alignment Using Multiple MIMO Relays

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***Abstract*—Interference alignment is a promising approach to handle the interference in mobile radio networks especially at high SNR. In the present paper, the interference alignment is achieved using a number of MIMO relays in a scenario with $K$ single antenna node pairs. A two-phase transmission scheme is considered assuming the channel to be constant during the transmission phases. Firstly, the source nodes transmit to both the relays and the destination nodes. Secondly, the source nodes and the relays retransmit to the destination nodes. By adapting the relays' processing matrices to the channel while fixing the nodes' filters, a linear system of equations is formulated which is solved for the interference alignment. The interference signals can be aligned perfectly at the destination nodes if there are enough relays. A closed-form solution with minimum retransmit energy at the relays is selected if the linear system of equations is underdetermined. If the system of linear equations has no solution, i.e., if there are not enough relays, least squares based solutions are employed which minimize the total received interference. The results show that $K/2$ degrees of freedom are achieved if the interference signals are aligned perfectly. Furthermore, aligning the interference signals imperfectly achieves a capacity-gain over perfect interference alignment at low and moderate SNRs while requiring a lower number of relays.**

***Index Terms*—interference alignment, relaying, MIMO.**

## I. Introduction

Recently it was shown that the maximum number of degrees of freedom (DoF) in various interference channels is half of the one achieved in the absence of the interference [1]. Furthermore, the maximum DoF in time-varying interference channels are achieved by a technique named interference alignment (IA) [1]. IA is achieved by splitting the signal space at the destination nodes into two subspaces. The first subspace contains the interference signals and the second subspace contains only the desired signals. References [1] and [2] propose IA schemes which achieve asymptotically the DoF upper bound by aligning in time and frequency domains, respectively. However, these schemes require either infinite time extensions in time varying channels or infinite frequency extensions in frequency selective channels.

In [3] and [4], IA is realized in the spatial domain for constant MIMO interference channels. In [3], the transmit filters at the source nodes are designed jointly by solving an eigenvalue problem. Although this scheme successfully aligns the interference signals at every destination node, it achieves only $1/(K-1)$ DoF per user. Moreover, a closed-form solution for IA assuming different numbers of antennas at the nodes and different numbers of data symbols transmitted by each user is proposed in [4]. Besides this scheme requires a large number of antennas, it generally does not free half of the receive signal space for the useful signal.

The concept of relaying is commonly used in wireless communication for range extension. However, we use the relays to aid the interference alignment rather than to extend the range. It is shown in [5] that the maximum DoF of a time-varying interference channel can not be increased by adding relays if the network is fully connected. Nevertheless, the relays can help to achieve IA in time-invariant interference channels. A relay-aided IA scheme for a two-hop three node pairs scenario is proposed in [6]. The authors show that $1/2$ DoF per user are achievable without the need for time extensions. Furthermore, a $K$ single antenna node pair scenario with a MIMO relay is considered in [7]. It is shown that the $K/2$ DoF are achievable if the number of the antennas at the relay is at least $\sqrt{(K-1)(K-2)}$. However, no closed-form solution is proposed. A $K$ single antenna node pairs scenario with multiple single antenna relays is considered in [8]. For both fixed and adapted transmit and receive filters, a linear system of equations is required to be solved for IA.

A multiple antenna nodes scenario with a MIMO relay is considered in [9] and [10]. In [9], a scenario consisting of a MIMO relay and $K$ node pairs where each node is equipped with $N$ antennas is considered. The authors propose a scheme which adapts the relay's processing matrix so that the interference links through the relays are linearly depended on the direct interference links. $NK/2$ DoF are achieved requiring $N(K-1)$ antennas at the relay. Moreover, the IA scheme proposed in [10] adapts the transmit and receive filters partially to the channel as well to achieve $NK/2$ DoF with at least $N\sqrt{K(K-2)}$ antennas at the relay. Finally, a scenario consisting of two user pairs with two relays is considered in [11]. The authors propose a technique based on interference neutralization introduced in [12]. In this technique, the relays do beamforming to align the interference signals at the first hop. In the second hop, they perform transmit zero forcing to get rid of the interference at the destination nodes.

In this paper, a scenario consisting of several single antenna node pairs and MIMO relays is considered. If the transmit and receive filters are fixed, linear processing at the relays is performed to manipulate the effective channel between the source nodes and the destination nodes including the relays

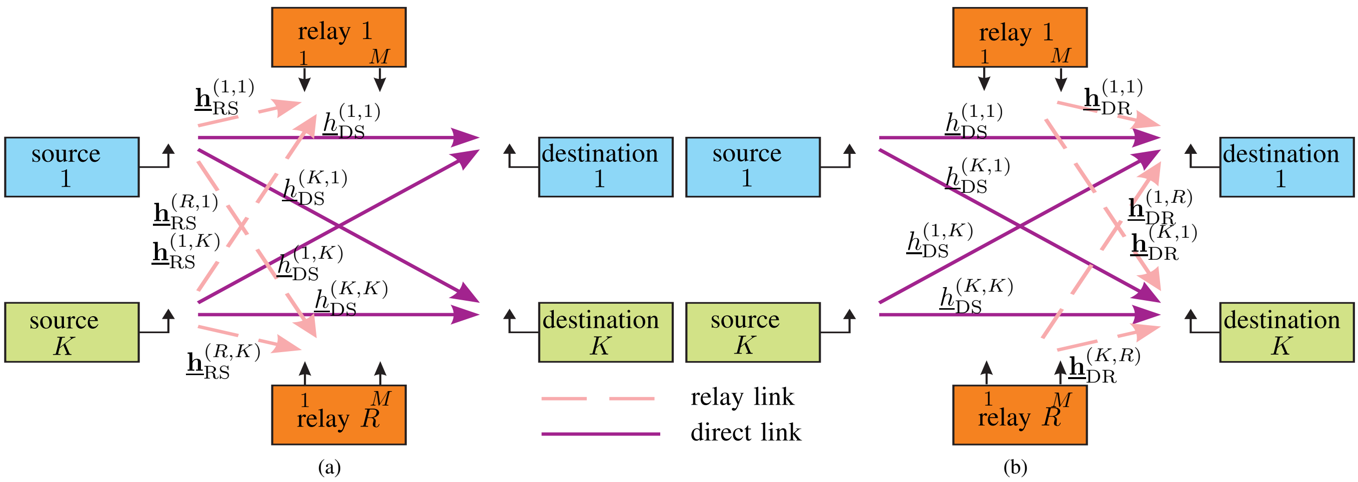


Fig. 1: Two transmission phases scenario: (a) the source nodes transmit to both the relays and the destination nodes, (b) both the source nodes and the relays retransmit to the destination nodes.

for achieving IA. To this end, a linear problem formulation for IA and the required number of relays and antennas per relay are derived. A closed-form solution is proposed if there are enough relays, i.e., for perfect IA. If the number of relays is not large enough, a pseudo solution based on the least squares method is obtained, i.e., imperfect IA is achieved.

The rest of the paper is organized as follows. The next section introduces the system model and the transmission scheme. In Section III, the sufficient and necessary conditions for IA are derived and the problem of IA is stated. A closed-form solution for perfect IA and a pseudo solution for imperfect IA are derived in Section IV and Section V, respectively. The performances of the proposed schemes are investigated in Section VI. Section VII concludes the paper.

## II. System Model and Transmission Scheme

We consider a scenario consisting of $K$ single antenna node pairs and $R$ multiple antenna relays. The number $M$ of antennas at each relay is not large enough $M < K$ so that an individual relay can not decode its received signals. Therefore, an amplify and forward relaying strategy is considered. Full CSI is assumed to be available only at the relays. A two-phase transmission scheme is considered where $\tau$ denotes the index of the transmission phase. At the first phase $\tau = 1$, the source nodes transmit to both the relays and the destination nodes as shown in Fig. 1a. Both the source nodes and the relays retransmit to the destination nodes at the second transmission phase $\tau = 2$ as illustrated in Fig. 1b. The channel coefficient between the $l$th source node and the $k$th destination node is denoted by $\underline{h}_{\mathrm{DS}}^{(k,l)}$. Let $\underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)}$ and $\underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)}$ be the $M \times 1$ channel vector between the $l$th source node and the $r$th relay and the $1 \times M$ channel vector between the $r$th relay and the $k$th destination node, respectively. All channels are assumed to be constant throughout the transmission duration. It is also assumed that the antennas at the relays and at the destination nodes receive uncorrelated additive Gaussian noise with zero mean and the same variance $\sigma^2$ denoted as $\underline{\mathbf{n}}_{\mathrm{R}}^{(r)}$, $r = 1, ..., R$ and $\underline{n}_{\mathrm{D}}^{(k,\tau)}$, $k = 1, ..., K$, $\tau = 1, 2$, respectively.

Let $\underline{s}_{\tau}^{(l)}$ denote the transmit signal of the $l$th source node at the transmission phase $\tau$. At the first transmission phase $\tau = 1$, the received signal at the $k$th destination node and the $r$th relay are

$$\underline{e}_1^{(k)} = \sum_{l=1}^{K} \underline{h}_{\mathrm{DS}}^{(k,l)} \underline{s}_1^{(l)} + \underline{n}_{\mathrm{D}}^{(k,1)} \tag{1}$$

and

$$\underline{\mathbf{e}}_{\mathrm{R}}^{(r)} = \sum_{l=1}^{K} \underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)} \underline{s}_1^{(l)} + \underline{\mathbf{n}}_{\mathrm{R}}^{(r)}, \tag{2}$$

respectively. Each relay $r$ processes the received signal linearly with an $M \times M$ processing matrix $\underline{\mathbf{G}}^{(r)}$. At the second transmission phase $\tau = 2$, both the source nodes and the relays retransmit to the destination nodes. The retransmitted signal of the $r$th relay reads

$$\underline{\mathbf{s}}_{\mathrm{R}}^{(r)} = \underline{\mathbf{G}}^{(r)} \underline{\mathbf{e}}_{\mathrm{R}}^{(r)}. \tag{3}$$

Accordingly, the received signal at the $k$th destination node is

$$\underline{e}_2^{(k)} = \sum_{l=1}^{K} \underline{h}_{\mathrm{DS}}^{(k,l)} \underline{s}_2^{(l)} + \sum_{r=1}^{R} \underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)} \underline{\mathbf{s}}_{\mathrm{R}}^{(r)} + \underline{n}_{\mathrm{D}}^{(k,2)}. \tag{4}$$

The received signals of both transmission phases at the $k$th destination node can be combined as

$$\begin{pmatrix} \underline{e}_1^{(k)} \\ \underline{e}_2^{(k)} \end{pmatrix} = \sum_{l=1}^{K} \underline{\mathbf{H}}^{(k,l)} \begin{pmatrix} \underline{s}_1^{(l)} \\ \underline{s}_2^{(l)} \end{pmatrix} + \tilde{\underline{\mathbf{n}}}_{\mathrm{D}}^{(k)}, \tag{5}$$

where

$$\tilde{\underline{\mathbf{n}}}_{\mathrm{D}}^{(k)} = \begin{pmatrix} \underline{n}_{\mathrm{D}}^{(k,1)} \\ \sum\limits_{r=1}^{R} \underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)} \underline{\mathbf{G}}^{(r)} \underline{\mathbf{n}}_{\mathrm{R}}^{(r)} + \underline{n}_{\mathrm{D}}^{(k,2)} \end{pmatrix} \tag{6}$$

and

$$\underline{\mathbf{H}}^{(k,l)} = \begin{pmatrix} \underline{h}_{\mathrm{DS}}^{(k,l)} & 0 \\ \sum\limits_{r=1}^{R} \underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)} \underline{\mathbf{G}}^{(r)} \underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)} & \underline{h}_{\mathrm{DS}}^{(k,l)} \end{pmatrix} \tag{7}$$

are the effective received noise at the $k$th destination node and the effective channel including the relays between the $l$th source node and the $k$th destination node, respectively. The effective channel of (7) between a source node and a destination node forms a virtual $2 \times 2$ MIMO channel. This effective channel is described by a $2 \times 2$ lower triangular matrix with equal diagonal elements corresponding to the direct link and the off-diagonal element corresponding to the link through the relays.

Each source node transmits a data symbol $\underline{d}_l$ with equal average symbol energy $\mathrm{E}\left\{|\underline{d}_l|^2\right\} = E_\mathrm{d},\ l = 1,\ldots,K$. Because each source node $l$ transmit twice, it has a two dimensional transmit signal space described by the transmit filter $\underline{\mathbf{v}}_l = \left(\underline{v}_1^{(l)}, \underline{v}_2^{(l)}\right)^{\mathrm{T}}$. Similarly, each destination node $k$ receives in the two transmission phases and thus it has a two dimensional receive signal space. A receive filter $\underline{\mathbf{u}}_k = \left(\underline{u}_1^{(k)}, \underline{u}_2^{(k)}\right)^{\mathrm{T}}$ which performs a zero forcing filtering for the interference nulling is considered at the destination nodes. Accordingly, the transmitted signal of the $l$th source node and the detected data symbol at the $k$th destination node are

$$\begin{pmatrix} \underline{s}_1^{(l)} \\ \underline{s}_2^{(l)} \end{pmatrix} = \underline{\mathbf{v}}_l\ \underline{d}_l, \tag{8}$$

and

$$\hat{\underline{d}}_k = \underline{\mathbf{u}}_k^{*\mathrm{T}} \begin{pmatrix} \underline{e}_1^{(k)} \\ \underline{e}_2^{(k)} \end{pmatrix}, \tag{9}$$

respectively. Based on the above mentioned assumptions, the covariance matrix of the received signal at the $r$th relay is

$$\underline{\mathbf{C}}_{\mathrm{rr}}^{(r)} = \left( E_\mathrm{d} \sum_{l=1}^{K} \left|\underline{v}_1^{(l)}\right|^2 \underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)} \underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)*\mathrm{T}} + \sigma^2 \mathbf{I}_M \right), \tag{10}$$

where $\mathbf{I}_M$ is the identity matrix of dimensions $M \times M$. Hence, the total retransmitted energy of the relays is calculated as

$$E_{\mathrm{Rtot}} = \sum_{r=1}^{R} \mathrm{tr}\left(\underline{\mathbf{G}}^{(r)} \underline{\mathbf{C}}_{\mathrm{rr}}^{(r)} \underline{\mathbf{G}}^{(r)*\mathrm{T}}\right), \tag{11}$$

where $\mathrm{tr}(\cdot)$ yields the trace of a matrix.

## III. IA Conditions and Linear System of Equations

For the considered scenario, the sufficient and necessary condition for the IA are

$$\underline{\mathbf{u}}_k^{*\mathrm{T}} \underline{\mathbf{H}}^{(k,l)} \underline{\mathbf{v}}_l = 0,\ \forall k, l,\ l \neq k, \tag{12}$$

and

$$\underline{\mathbf{u}}_k^{*\mathrm{T}} \underline{\mathbf{H}}^{(k,k)} \underline{\mathbf{v}}_k \neq 0,\ \forall k. \tag{13}$$

The first condition ensures that all interference signals received by each destination node $k$ are forced to zero by its receive filter $\underline{\mathbf{u}}_k$. Furthermore, the desired signal received by each destination node $k$ must not be fully aligned to the interference subspace and thus is not nulled by the receive filter $\underline{\mathbf{u}}_k$.

Because the CSI is not available at the nodes, the transmit filters and the receive filters are not adapted to the channel. As a result, the transmit subspaces at the source nodes and the interference subspaces at destination nodes are a priori known. Based on the first IA condition of (12), the relays' processing matrices are adapted to the channel so that the effective channel of the interference links, see (7), rotates the transmit directions to lay on the interference subspaces of the non-corresponding destination nodes. Because of the special structure of the effective channel of (7), the transmit directions are rotated if $\underline{u}_2^{(k)} \neq 0$ and $\underline{v}_1^{(l)} \neq 0$, $\forall k, l$. Because the direct useful links $\underline{h}_{\mathrm{DS}}^{(k,k)}$, $\forall k$ are not considered for IA, the useful signal almost surely will have a component orthogonal to the interference subspace.

For the effective interference link between the $l$th source node and the $k$th destination node, the interference alignment equation of (12) can be rewritten as

$$\begin{aligned} &\underline{v}_1^{(l)} \underline{u}_2^{*(k)} \sum_{r=1}^{R} \underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)} \underline{\mathbf{G}}^{(r)} \underline{\mathbf{h}}_{\mathrm{RS}}^{(r,l)} \\ &+ \underline{h}_{\mathrm{DS}}^{(k,l)} \left(\underline{v}_1^{(l)} \underline{u}_1^{(k)*} + \underline{v}_2^{(l)} \underline{u}_2^{(k)*}\right) = 0. \end{aligned} \tag{14}$$

If the transmit filters and the receive filters are fixed, equation (14) is linear in $\underline{\mathbf{G}}^{(r)}$, $r = 1, ..., R$. To simplify the analysis in the next section, we consider $\underline{\mathbf{G}}^{(r)*\mathrm{T}}$ as free variables. Considering the equation (14) for all interference links, a linear system of equations is formulated as

$$\underline{\mathbf{H}}\ \underline{\mathbf{x}} = -\underline{\mathbf{b}} \tag{15}$$

with

$$\underline{\mathbf{x}} = \left(\mathrm{vec}\left(\underline{\mathbf{G}}^{(1)*\mathrm{T}}\right)^{\mathrm{T}}, \ldots, \mathrm{vec}\left(\underline{\mathbf{G}}^{(R)*\mathrm{T}}\right)^{\mathrm{T}}\right)^{\mathrm{T}}, \tag{16}$$

where $\underline{\mathbf{H}}$ is a $K(K-1) \times RM^2$ full rank matrix and $\underline{\mathbf{b}}$ is $K(K-1)$ vector. In (16), $\mathrm{vec}(.)$ denotes the vectorization of a matrix. Each row of $\underline{\mathbf{H}}$ corresponds to a relays' interference link and each element of $\underline{\mathbf{b}}$ corresponds to a direct interference link. For instance, the structure of the row of $\underline{\mathbf{H}}$ and the element of $\underline{\mathbf{b}}$ corresponding to the effective interference link between the $l$th source node and the $k$th destination node are

$$\underline{v}_1^{(l)*} \underline{u}_2^{(k)} \left(\underline{\mathbf{h}}_{\mathrm{DR}}^{(k,1)*} \otimes \underline{\mathbf{h}}_{\mathrm{RS}}^{(1,l)*\mathrm{T}}, \ldots, \underline{\mathbf{h}}_{\mathrm{DR}}^{(k,R)*} \otimes \underline{\mathbf{h}}_{\mathrm{RS}}^{(R,l)*\mathrm{T}}\right) \tag{17}$$

and

$$\underline{h}_{\mathrm{DS}}^{(k,l)*} \left(\underline{v}_1^{(l)*} \underline{u}_1^{(k)} + \underline{v}_2^{(l)*} \underline{u}_2^{(k)}\right), \tag{18}$$

respectively where $\otimes$ denotes the Kronecker product. The linear system of equations of (15) has a unique solution if $RM^2 = K(K-1)$, but it has infinitely many solutions if $RM^2 > K(K-1)$. However, if $RM^2 < K(K-1)$ there is no solution to the linear system of equations of (15). For a given number $M$ of antennas at the relays and a given number $K$ of users, the equation $K(K-1)/M^2$ does not lead always to an integer number $R$ of relays. Therefore, the latter two inequalities are investigated in the following sections.

## IV. PERFECT IA (PIA)

If $RM^2 > K(K-1)$, there are infinitely many solutions to the linear system of equations of (15). Among the infinitely many solutions of the linear system of equations, the one with the minimum total retransmit energy of the relays is selected. To this end, the total relays' transmitted energy of (11) is required to be written as a function of the vector $\underline{\mathbf{x}}$. Define $\underline{\mathbf{A}}$ as a $RM^2 \times RM^2$ block diagonal matrix where the $r$th $M$ blocks correspond to the $r$th relay and each block of these $M$ blocks equals $\underline{\mathbf{C}}_{\mathrm{rr}}^{(r)}$. Then the total retransmitted energy of the relays can be rewritten as $E_{\mathrm{Rtot}} = \underline{\mathbf{x}}^{*\mathrm{T}}\underline{\mathbf{A}}\ \underline{\mathbf{x}}$. As a result, the solution of (15) with the minimum total retransmitted energy of the relays is found using the optimization problem

$$\underline{\mathbf{x}}_{\mathrm{PIA}} = \underset{\underline{\mathbf{x}}}{\operatorname{argmin}}\left\{\underline{\mathbf{x}}^{*\mathrm{T}}\underline{\mathbf{A}}\ \underline{\mathbf{x}}\right\} \tag{19}$$

subject to

$$\underline{\mathbf{H}}\ \underline{\mathbf{x}} = -\underline{\mathbf{b}}. \tag{20}$$

The Lagrangian function of the optimization problem of (19) and (20) is

$$L(\underline{\mathbf{x}}, \underline{\lambda}) = \underline{\mathbf{x}}^{*\mathrm{T}}\underline{\mathbf{A}}\ \underline{\mathbf{x}} + \underline{\lambda}^{\mathrm{T}}\left(\underline{\mathbf{b}} + \underline{\mathbf{H}}\ \underline{\mathbf{x}}\right), \tag{21}$$

where $\underline{\lambda}$ is the vector of the Lagrangian multipliers each of which corresponds to a constraint of (20). Taking the derivatives of (21) with respect to $\underline{\mathbf{x}}$ and $\underline{\lambda}$ yields

$$\underline{\mathbf{A}}^{*\mathrm{T}}\underline{\mathbf{x}} + \underline{\mathbf{H}}^{*\mathrm{T}}\underline{\lambda}^{*} = 0, \tag{22}$$

and

$$\underline{\mathbf{b}} + \underline{\mathbf{H}}\ \underline{\mathbf{x}} = 0, \tag{23}$$

respectively. By substituting (22) in (23), the optimum $\underline{\lambda}^{*}$ reads

$$\underline{\lambda}^{*} = \left(\underline{\mathbf{H}}\ \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{H}}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{b}}. \tag{24}$$

The optimum $\underline{\mathbf{x}}_{\mathrm{PIA}}$ is obtained by substituting (24) into (22)

$$\underline{\mathbf{x}}_{\mathrm{PIA}} = -\left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{H}}^{*\mathrm{T}}\left(\underline{\mathbf{H}}\ \left(\underline{\mathbf{A}}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{H}}^{*\mathrm{T}}\right)^{-1}\underline{\mathbf{b}}. \tag{25}$$

## V. IMPERFECT IA (IIA)

If there are not enough relays to solve the linear system of equations of (15), i.e., $RM^2 < K(K-1)$, a pseudo solution which minimizes the total interference in the system can be obtained as follows. The sum of the effective channel gains of the interference links can be written as

$$\sum_{k=1}^{K}\sum_{l\neq k}\left|\underline{\mathbf{u}}_k^{*\mathrm{T}}\underline{\mathbf{H}}^{(k,l)}\underline{\mathbf{v}}_l\right|^2 = \left\|\underline{\mathbf{b}} + \underline{\mathbf{H}}\ \underline{\mathbf{x}}\right\|^2, \tag{26}$$

where the elements of $\underline{\mathbf{b}}$ and the corresponding rows of $\underline{\mathbf{H}}\ \underline{\mathbf{x}}$ correspond to the direct interference links and the relays' interference links, respectively. Moreover, the solution to the linear system of equations would be the one where the interference signals received through the relays' links compensate the interference signals received through the direct links. An approximation to the solution is found using the unconstraint problem

$$\underline{\mathbf{x}}_{\mathrm{IIA}} = \underset{\underline{\mathbf{x}}}{\operatorname{argmin}}\left\{\left\|\underline{\mathbf{b}} + \underline{\mathbf{H}}\ \underline{\mathbf{x}}\right\|^2\right\}. \tag{27}$$

This problem is solved using the least squares method and the solution is

$$\underline{\mathbf{x}}_{\mathrm{IIA}} = -\left(\underline{\mathbf{H}}^{*\mathrm{T}}\underline{\mathbf{H}}\right)^{-1}\underline{\mathbf{H}}^{*\mathrm{T}}\underline{\mathbf{b}}. \tag{28}$$

This is an approximate solution of the interference alignment problem and the destination nodes will treat the unaligned interference signals like noise.

## VI. NUMERICAL RESULTS

In this section, the achieved sum-rate per transmission phase is taken as a measure of the system performance. The performances of the proposed schemes are investigated as a function of the pseudo signal to noise ratio $\gamma_{\mathrm{pSNR}} = \left(KE_{\mathrm{d}} + E_{\mathrm{R_{tot}}}\right)/\left(\sigma^2 K\right)$. The sum-rate is calculated as

$$C = \frac{1}{2}\sum_{k=1}^{K}\log_2\left(1+\gamma_k\right), \tag{29}$$

where

$$\gamma_k = \frac{E_{\mathrm{d}}}{\sigma^2}\frac{\left|\underline{\mathbf{u}}_k^{*\mathrm{T}}\underline{\mathbf{H}}^{(k,k)}\underline{\mathbf{v}}_k\right|^2}{\left|\underline{u}_1^{(k)}\right|^2 + \left|\underline{u}_2^{(k)}\right|^2\left(\sum_{r=1}^{R}\left|\underline{\mathbf{h}}_{\mathrm{DR}}^{(k,r)}\underline{\mathbf{G}}^{(r)}\right|^2 + 1\right) + I_k} \tag{30}$$

with

$$I_k = \sum_{l\neq k}\frac{E_{\mathrm{d}}}{\sigma^2}\left|\underline{\mathbf{u}}_k^{*\mathrm{T}}\underline{\mathbf{H}}^{(k,l)}\underline{\mathbf{v}}_l\right|^2 \tag{31}$$

are the received SINR and the unaligned interference signals normalized to the noise variance at the $k$th destination node, respectively. A scenario consisting of $K = 3$ node pairs and $R = 6$ single antenna relays is considered. To implement proper benchmark schemes, it is assumed that the relays are equipped with a single antenna $M = 1$. The node pairs and the relays are split into 3 groups each of which contains a node pair and 2 relays.

A Rayleigh fading channel is employed. The average channel gain for the channels within the same group is 16 times greater than the average channel gain for the channels between groups. The channels are normalized over the effective channel of a reference scenario consisting of a single node pair and all relays. For the reference scenario, the optimum transmission scheme which maximizes the received SNR with a given transmit energy is calculated. Because both the nodes and the relays are equipped with a single antenna, the reference scheme just solves a power allocation optimization problem. It finds the optimum the power allocation at the source node and the relays which achieves the maximum SNR at the destination node with a total power constraint. The normalization is done so that at high pseudo SNR, the pseudo SNR asymptotically equals the received SNR for the reference scheme.

In the following, $\underline{\mathbf{v}}_k = \underline{\mathbf{u}}_k = 1/\sqrt{2}\,(1,1)^{\mathrm{T}}$, $\forall k$ is assumed for the transmit and receive filters in all IA schemes. Based on this setup, perfect IA with a unique solution can be achieved. For the same scenario, a scheme which ignores the channels between the groups and optimizes the transmission for each group individually is considered and named single-cell optimal relaying. Accordingly, the scheme finds the optimum transmission scheme for a single pair scenario with several relays as described above. Adding a relay to the scenario, the solution for perfect IA with a minimum total relays' retransmitted energy is found using the closed-form solution of (25). Finally, the imperfect IA scheme is employed when reducing the number of relays to 5 relays, 3 relays and 1 relay.

Fig. 2 shows the average achieved sum rate per transmission phase as a function of the pseudo SNR. At the low pseudo SNR regime, the noise is dominant and consequently the single cell optimal relaying scheme achieves significantly higher sum rates as compared to the other schemes. Furthermore, perfect IA outperforms the other schemes at high pseudo SNR, i.e., if the interference is dominant. It is also noted that by adding a relay less total relays' retransmit energy is consumed and thus the pseudo SNR is reduced by about 5 dB. Moreover, imperfect IA outperforms perfect IA at low and moderate pseudo SNR although it uses less relays. At the high pseudo SNR regime, the interference is severe so that imperfect IA schemes saturate at certain sum rates. At low pseudo SNR, it is noted that the smaller the number of relays, the higher the achieved sum rates using the imperfect IA scheme, i.e., less total relays' energy. At high pseudo SNR, as the number of relays is decreased, imperfect IA achieves lower sum rates, i.e., higher unaligned interference. Furthermore, it is interesting to note that the slope of the curves at high SNR corresponds to the total DoF. Therefore, total DoF of 1.5 is achieved by the perfect IA schemes. All other schemes achieve zero DoF because they allow some interference which is treated as noise at the destination nodes.

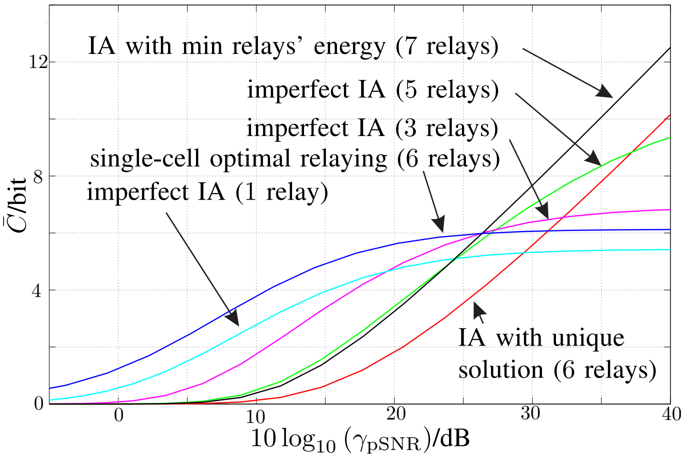

Fig. 2: Average sum rate as a function of the pseudo signal to noise ratio for $K = 3$ cells scenario.

## VII. Conclusion

In this paper, an IA scheme for a multiple single antenna node pairs scenario with several MIMO relays is considered. The problem of IA is formulated as a linear system of equations which is solved for the relays' processing matrices. The IA feasibility conditions are investigated. Particularly, the required number of relays is derived. If there are enough relays, perfect IA is achieved with the minimum total retransmitted energy at the relays. If there are not enough relays to perfectly solve the IA problem, the interference signals are aligned imperfectly by minimizing the total interference received at the destination nodes. The results for perfect IA show that as the number of relays increases, IA solutions with higher sum rates can be found. Furthermore, imperfect IA requires less relays and achieves significantly higher sum rates as compared to perfect IA at low and moderate SNRs.

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