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Interference Alignment Using a MIMO Relay and Partially-Adapted Transmit/Receive Filters

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***Abstract*—Interference alignment is proposed for achieving the maximum degrees of freedom in interference channels. In the present paper, a scenario consisting of several pairs of multiple antenna nodes and a single MIMO relay helping the interference alignment is considered. The interference alignment is performed in two subsequent transmission phases assuming a time-invariant channel during the two transmission phases. Firstly, the relay and the destination nodes receive the signals from all the source nodes, i.e., also the direct links between the communicating node pairs are exploited. In the second transmission phase, both the source nodes and the relay retransmit the signals to the destination nodes. By adapting the relay's linear signal processing and partially adapting both the transmit and receive filters to the channel, a closed form solution for interference alignment is obtained. The performance of the proposed scheme is investigated by simulations. The results show that the proposed scheme achieves the maximum degrees of freedom and outperforms conventional relaying schemes at high SNRs.**

***Index Terms*—interference alignment, relaying, MIMO.**

I. Introduction

The performance of future wireless systems is limited by the interference among the users' transmissions. Recent contributions in multiuser information theory have shown that the maximum number of degrees of freedom (DoF) which can be achieved in an interference channel is half of the one which could be achieved if the interferences would not exist [1]. This upper bound of the DoF is achievable by transmission techniques named interference alignment (IA). IA aligns the received interferences at each destination node in a subspace of half of the dimension of the signal space while keeping the other half of the signal space interference-free for the useful signal. In early contributions on IA, see, e.g., [1] and [2], IA is performed in time and frequency dimensions by optimizing the transmit filters aiming at achieving IA and getting rid of the interferences using linear receive zero forcing. These proposals are difficult to be implemented in practice because they need infinite time extensions of the channel which has to be known at the source nodes in advance. In [3], every data symbol is transmitted twice where the interference links at the time instant of the first transmission are the inverse to the ones at the time instant of the second transmission. This means that by summing up the two receive signals at the destination nodes, the interferences can be cancelled out. Although the IA is successfully accomplished by this scheme, arbitrary long delays may result from waiting for the inverse interference channels. Furthermore, IA is realized in spatial domain for a constant MIMO interference channel in [4] and [5]. In [4], the number of antennas at the nodes is restricted to $K-1$ where K denotes the number of users. The optimum transmit filters for IA are obtained using an eigenvalue problem formulation. Although this scheme requires a large number of antennas at the nodes, it achieves only $1/\left(K-1\right)$ DoF per antenna for each user. An extension of this work for multiple data symbols per user is proposed in [5]. The authors propose a closed form solution for IA assuming different numbers of antennas at the nodes and different numbers of data symbols are transmitted by the users. The drawback of this scheme is that it requires a large number of antennas especially for a large number of users. Additionally, it does not achieve the maximum DoFs in general.

Wireless relaying networks are well studied in the literature and are already proposed for future cellular systems [6], [7], [8], [9]. The concept of relaying in wireless systems is commonly used for range extension. In particular, introducing a relay between a source node and a destination node can reduce the total transmit energy but the relay consumes additional resources for the retransmission. In this paper, relays are used for helping the interference alignment rather than for range extension. To this end, the effective channel between the source nodes and the destination nodes including the relays is manipulated by the choice of the relay processing matrix to achieve an effective channel with aligned interferences. It is shown in [10] that the maximum DoF of the channel cannot be increased by adding relays. However, the relays can help to achieve the maximum DoF especially in constant interference channels. A relay-aided IA is firstly proposed in [11]. The authors consider a two-hop transmission scheme for a three user single-antenna scenario with a single relay. They showed that $1/2$ DoF per user are achievable without the need for time extensions. Moreover, a K single-antenna user scenario with a multiple antenna relay is considered in [12]. It is shown that $1/2$ DoF per user are achievable if the number of antennas at the relay is at least $\sqrt{\left(K-1\right)\left(K-2\right)}$. However, the design of the relay processing matrix and the transmit and receiver filters is not unique and thus no closed-form solution is proposed. In [13], a K single antenna users scenario with multiple single antenna relays is considered. The IA problem is expressed as a linear system of equations for both fixed and optimized transmit and receive filters. In [14], the analysis of the maximum achievable DoF is extended

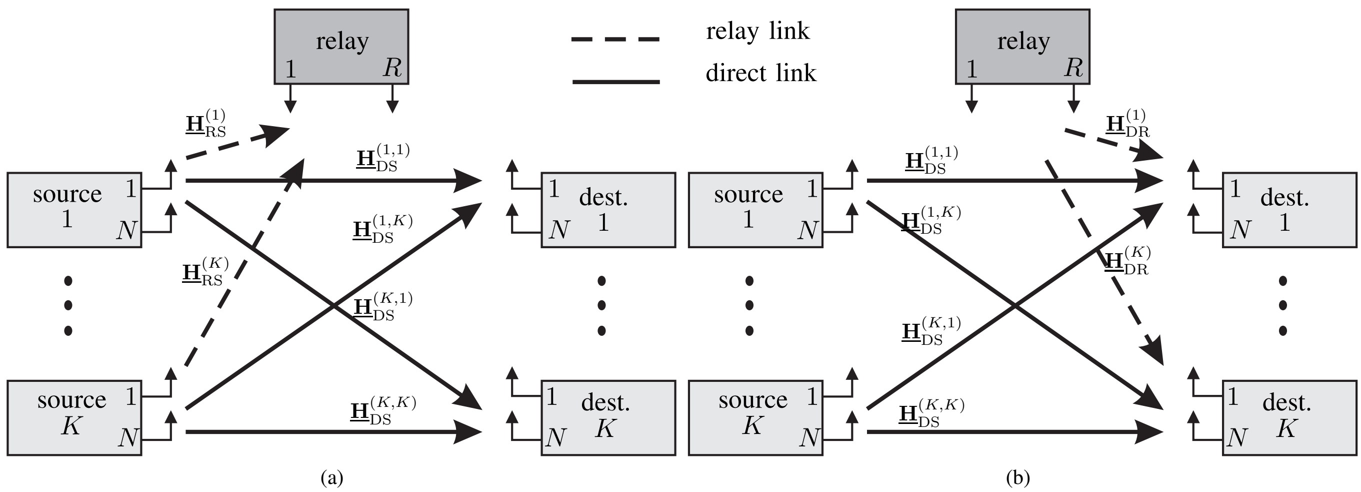

Fig. 1: Two transmission phases: (a) the source nodes transmit to both the relay and the destination nodes, (b) both the source nodes and the relay retransmit to the destination nodes.

to a scenario consisting of a multiple antenna relay and K node pairs where each node is equipped with N antennas. By adapting the relay processing matrix in such a way that the interference links through the relay become linearly dependent on the direct interference links, the interferences are aligned at the destination nodes and the maximum DoFs are achieved requiring $N(K-1)$ antennas at the relay. However, the direct link of the second time slot are not exploited as well as the transmit filters are not optimized in this scheme. In [15], the authors assign a multiple antenna relay to every destination node assuming that there are no cross links among the relays and the destination nodes. The feasibility conditions for IA are derived and an iterative algorithm for minimizing the interference leakage introduced in [11] is proposed.

In the present paper, a scenario consisting of several multiple antenna node pairs and a single multiple antenna relay is considered. The transmit and receive filters are exploited for IA by partially adapting them to the channel. To this end, a linear problem formulation for IA is derived and a closed form solution adjusting the relay's processing matrix, the transmit filters, and the receive filters, is obtained.

The rest of the paper is organized as follows. The following section describes the system model. In Section III, the sufficient and necessary conditions of IA are derived. A closed-form solution for IA is derived in Section IV. The relay's transmit energy is analyzed and discussed in Section V. The performance of the proposed scheme is investigated in Section VI. Section VII concludes the paper.

II. System Model and Transmission Phases

A scenario consisting of K source-destination node pairs and a single multiple antenna half duplex relay is considered. A two time slot transmission scheme is considered. In the first time slot, the source nodes transmit to both the destination nodes and the relay as depicted in Fig. 1a. Both the source nodes and the relay retransmit to the destination nodes at the second time-slot as shown in Fig. 1b. Every node is equipped with N antennas. The number R of antennas at the relay is small $R < KN$ so that the relay cannot separate and decode its received signals. Therefore, an amplify and forward relaying strategy is considered. Furthermore, full channel state information (CSI) is assumed to be available at all nodes and at the relay. Each source node transmits Q data symbols with $Q \leq N$ to its intended destination node in two transmission phases. Let $\underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)}$, $\underline{\mathbf{H}}_{\mathrm{RS}}^{(l)}$ and $\underline{\mathbf{H}}_{\mathrm{DR}}^{(k)}$ denote the $N \times N$ channel matrix between the l-th source node and the k-th destination node, the $R \times N$ channel matrix between the l-th source node and the relay and the $N \times R$ channel matrix between the relay and the k-th destination node, respectively. All channels are assumed to be constant throughout the transmission duration. Uncorrelated additive Gaussian noise with zero mean and the same variance σ^2 is assumed at all receive antennas of the relay and the destination nodes. The received noise vectors at the k-th destination node and at the relay are denoted by $\underline{\mathbf{n}}_{\mathrm{D}}^{(k)}$ and $\underline{\mathbf{n}}_{\mathrm{R}}$, respectively.

Let $\underline{\mathbf{s}}_{\tau}^{(l)}$ and $\underline{\mathbf{e}}_{\tau}^{(k)}$ with time slot index $\tau \in \{1,2\}$ denote the N dimensional transmitted vector of the l-th source node and the N dimensional received vector of the k-th destination node, respectively. The R dimensional received vector at the relay is denoted by $\underline{\mathbf{e}}_{\mathbf{R}}$. The received vectors in the first time slot at the k-th destination node and the relay are

$$\underline{\mathbf{e}}_1^{(k)} = \sum_{l=1}^{K} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} \underline{\mathbf{s}}_1^{(l)} + \underline{\mathbf{n}}_{\mathrm{D}}^{(k)} \tag{1}$$

and

$$\underline{\mathbf{e}}_{\mathrm{R}} = \sum_{l=1}^{K} \underline{\mathbf{H}}_{\mathrm{RS}}^{(l)} \underline{\mathbf{s}}_1^{(l)} + \underline{\mathbf{n}}_{\mathrm{R}}, \tag{2}$$

respectively. The relay linearly processes its received signal with an $R \times R$ processing matrix $\underline{\mathbf{G}}$. Then it retransmits the vector

$$\underline{\mathbf{s}}_{\mathrm{R}} = \underline{\mathbf{G}}\ \underline{\mathbf{e}}_{\mathrm{R}}. \tag{3}$$

As illustrated in Fig. 1b, both the source nodes and the relay retransmit to the destination nodes during the second time slot. The resulting received vector at the k-th destination node reads

$$\underline{\mathbf{e}}_2^{(k)} = \sum_{l=1}^{K} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} \underline{\mathbf{s}}_2^{(l)} + \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)} \underline{\mathbf{s}}_{\mathrm{R}} + \underline{\mathbf{n}}_{\mathrm{D}}^{(k)}. \tag{4}$$

The received vectors at the k-th destination node of both time slots can be combined as

$$\begin{pmatrix} \underline{\mathbf{e}}_1^{(k)} \\ \underline{\mathbf{e}}_2^{(k)} \end{pmatrix} = \sum_{l=1}^{K} \underline{\mathbf{H}}^{(k,l)} \begin{pmatrix} \underline{\mathbf{s}}_1^{(l)} \\ \underline{\mathbf{s}}_2^{(l)} \end{pmatrix} + \underline{\tilde{\mathbf{n}}}_{\mathrm{D}}^{(k)}, \tag{5}$$

where

$$\underline{\tilde{\mathbf{n}}}_{\mathrm{D}}^{(k)} = \begin{pmatrix} \underline{\mathbf{n}}_{\mathrm{D}}^{(k)} \\ \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)} \underline{\mathbf{G}}\ \underline{\mathbf{n}}_{\mathrm{R}} + \underline{\mathbf{n}}_{\mathrm{D}}^{(k)} \end{pmatrix}, \tag{6}$$

and

$$\underline{\mathbf{H}}^{(k,l)} = \begin{pmatrix} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} & \mathbf{0} \\ \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)} \underline{\mathbf{G}}\ \underline{\mathbf{H}}_{\mathrm{RS}}^{(l)} & \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} \end{pmatrix} \tag{7}$$

are the effective received noise at the k-th destination node and the effective channel matrix between the l-th source node and the k-th destination node, respectively. The effective channel matrix of (7) is a lower block triangular matrix with equal diagonal blocks describing the direct link and the lower off-diagonal block describing the link through the relay. Because of the two transmissions, the destination node's signal space dimension is doubled, see equation (5). Half of this signal space should be reserved for the received interferences.

In total Q data symbols are transmitted in both time slots by every user. The transmitted vector of the l-th source node as a function of the Q dimensional transmitted data vector $\underline{\mathbf{d}}_l = \left(\underline{d}_l^{(1)}, \ldots, \underline{d}_l^{(Q)}\right)^{\mathrm{T}}$ reads

$$\begin{pmatrix} \underline{\mathbf{s}}_1^{(l)} \\ \underline{\mathbf{s}}_2^{(l)} \end{pmatrix} = \begin{pmatrix} \underline{\mathbf{V}}_1^{(l)} \\ \underline{\mathbf{V}}_2^{(l)} \end{pmatrix} \underline{\mathbf{d}}_l, \tag{8}$$

where $\underline{\mathbf{V}}_\tau^{(l)}$ is the $N \times Q$ transmit filter matrix of the l-th source node at time slot τ. All data symbols are assumed to have the same average energy over the constellation $\mathrm{E}\left\{\left|\underline{d}_l^{(q)}\right|^2\right\} = E_{\mathrm{d}}$, $\forall l, q$. Denoting the $N \times Q$ receive filter matrix of the k-th destination node at time slot τ by $\underline{\mathbf{U}}_\tau^{(k)}$, the received data vector at the k-th destination node reads

$$\hat{\underline{\mathbf{d}}}_k = \begin{pmatrix} \underline{\mathbf{U}}_1^{(k)*\mathrm{T}} & \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \end{pmatrix} \begin{pmatrix} \underline{\mathbf{e}}_1^{(k)} \\ \underline{\mathbf{e}}_2^{(k)} \end{pmatrix}. \tag{9}$$

III. IA Conditions

Because every destination node receives twice, it has a $2N$ dimensional receive signal space. If IA is applied, the interfering received signals from the non-corresponding source nodes should span at most an N dimensional subspace at every destination node. Additionally, both the received useful signal and the interferences should not have a common subspace, i.e., they should not interact. Based on this, the receive filters for removing the interferences are designed according to the zero forcing strategy. Therefore, the first condition for interference alignment is stated as

$$\begin{pmatrix} \underline{\mathbf{U}}_1^{(k)*\mathrm{T}} & \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \end{pmatrix} \underline{\mathbf{H}}^{(k,l)} \begin{pmatrix} \underline{\mathbf{V}}_1^{(l)} \\ \underline{\mathbf{V}}_2^{(l)} \end{pmatrix} = \mathbf{0},\ l \neq k, \tag{10}$$

for all destination nodes $k = 1, \ldots, K$. In addition to zeroing the interferences which are received from the non-corresponding source nodes, it is required to ensure that the useful links would not be nulled. Mathematically, this condition can be written as

$$\begin{pmatrix} \underline{\mathbf{U}}_1^{(k)*\mathrm{T}} & \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \end{pmatrix} \underline{\mathbf{H}}^{(k,k)} \begin{pmatrix} \underline{\mathbf{V}}_1^{(k)} \\ \underline{\mathbf{V}}_2^{(k)} \end{pmatrix} = \underline{\mathbf{\Sigma}}^{(k)}, \tag{11}$$

where $\underline{\mathbf{\Sigma}}^{(k)}$ is a $Q \times Q$ diagonal matrix with non-zero diagonal entries which represents the k-th user useful effective link. Equation (11) ensures that the useful links are nonzero and that there are no interferences among different data symbols of the same user.

If the conditions (10) and (11) are fulfilled, every user can successfully transmit Q data symbols in two time slots. This means that $Q/2$ DoF are achieved by every user. Moreover, if every source node fully exploits its transmit signal space at each time slot with $Q = N$, the maximum DoF of the channel is achieved.

IV. Closed-Form Solution For IA

A. Partially-Adapted Transmit/Receive Filters

In the following, a closed-form solution which satisfies the IA conditions is derived. Considering the interference link between the l-th source node and the k-th destination node, the IA equations of (10) are rewritten as

$$\begin{aligned} &\underline{\mathbf{U}}_1^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} \underline{\mathbf{V}}_1^{(l)} + \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)} \underline{\mathbf{G}}\ \underline{\mathbf{H}}_{\mathrm{RS}}^{(l)} \underline{\mathbf{V}}_1^{(l)} \\ &\qquad + \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)} \underline{\mathbf{V}}_2^{(l)} = \mathbf{0}. \end{aligned} \tag{12}$$

The first term of (12) represents the direct link at the first time slot whereas the second one represents the link through the relay. The direct link of the second time slot is represented by the third term of (12). If the receive filter $\underline{\mathbf{U}}_1^{(k)}$ of the first time slot at the destination node is adapted to the channel while the transmit filter $\underline{\mathbf{V}}_1^{(l)}$ at the source node is kept fixed and vice versa during the second time slot, the IA equations of (12) are linear in $\underline{\mathbf{G}}$, $\underline{\mathbf{U}}_1^{(k)}$ and $\underline{\mathbf{V}}_2^{(l)}$. Similarly, the IA equations of the useful link for the k-th user of (11) are rewritten as

$$\begin{aligned} &\underline{\mathbf{U}}_1^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,k)} \underline{\mathbf{V}}_1^{(k)} + \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)} \underline{\mathbf{G}}\ \underline{\mathbf{H}}_{\mathrm{RS}}^{(k)} \underline{\mathbf{V}}_1^{(k)} \\ &\qquad + \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,k)} \underline{\mathbf{V}}_2^{(k)} = \underline{\mathbf{\Sigma}}^{(k)}. \end{aligned} \tag{13}$$

Following the same idea of fixing $\underline{\mathbf{V}}_1^{(k)}$ and $\underline{\mathbf{U}}_2^{(k)}$, the equations of (13) become linear in $\underline{\mathbf{G}}$, $\underline{\mathbf{U}}_1^{(k)}$ and $\underline{\mathbf{V}}_2^{(k)}$. Basically, the transmit filters in the first time slot are required to be full rank so that all data symbols are forwarded to the relay. For $Q = N$, the signal spaces of the source nodes at $\tau = 1$

are fully used. The selection of the transmit filters at $\tau = 1$ and the receive filters at $\tau = 2$ would not significantly affect the temporal relation between the interference subspace and the useful signal subspace of a destination node's signal space. In contrast, the distribution of the transmit power in the signal space will be influenced by the specific choice of the transmit filter. Similarly, the receive filter matrix should have full rank and only the spatial distribution of the receive signals of the individual data symbols will be influenced by the specific choice of the receive filter. If less data symbols $Q < N$ are transmitted, a beamforming towards the useful link direction reducing the gains of the interference links may be a reasonable choice for the fixed transmit and receive filter matrices.

B. Linear System of Equations

Define the matrices $\underline{\mathbf{H}}_{\mathrm{RL}}^{(k,l)} = \underline{\mathbf{V}}_1^{(l)\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{RS}}^{(l)\mathrm{T}} \otimes \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DR}}^{(k)}$, $\underline{\mathbf{H}}_{\mathrm{TDL}}^{(k,j)} = \mathbf{I}_Q \otimes \underline{\mathbf{U}}_2^{(k)*\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)}$, and $\underline{\mathbf{H}}_{\mathrm{RDL}}^{(k,l)} = \underline{\mathbf{V}}_1^{(l)\mathrm{T}} \underline{\mathbf{H}}_{\mathrm{DS}}^{(k,l)\mathrm{T}} \otimes \mathbf{I}_Q$, where $\mathbf{I}_Q$ is the $Q \times Q$ identity matrix and $\otimes$ denotes the Kronecker product [16]. Then the IA equations of (12) and (13) can be represented as

$$\left(\underline{\mathbf{H}}_{\mathrm{RL}}^{(k,l)}, \underline{\mathbf{H}}_{\mathrm{TDL}}^{(k,l)}, \underline{\mathbf{H}}_{\mathrm{RDL}}^{(k,l)}\right) \begin{pmatrix} \mathrm{vec}\left(\underline{\mathbf{G}}\right) \\ \mathrm{vec}\left(\underline{\mathbf{V}}_2^{(l)}\right) \\ \mathrm{vec}\left(\underline{\mathbf{U}}_1^{(k)*\mathrm{T}}\right) \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \tag{14}$$

and

$$\left(\underline{\mathbf{H}}_{\mathrm{RL}}^{(k,k)}, \underline{\mathbf{H}}_{\mathrm{TDL}}^{(k,k)}, \underline{\mathbf{H}}_{\mathrm{RDL}}^{(k,k)}\right) \begin{pmatrix} \mathrm{vec}\left(\underline{\mathbf{G}}\right) \\ \mathrm{vec}\left(\underline{\mathbf{V}}_2^{(k)}\right) \\ \mathrm{vec}\left(\underline{\mathbf{U}}_1^{(k)*\mathrm{T}}\right) \end{pmatrix} = \mathrm{vec}\left(\underline{\mathbf{\Sigma}}^{(k)}\right), \tag{15}$$

respectively where $\mathrm{vec}\left(.\right)$ denotes the vectorization of a matrix. Stacking all interference and useful links together, we obtain the linear system of equations

$$\underline{\mathbf{H}}\ \underline{\mathbf{x}} = \underline{\mathbf{b}} \tag{16}$$

where

$$\underline{\mathbf{x}} = \begin{pmatrix} \mathrm{vec}\left(\underline{\mathbf{G}}\right) \\ \mathrm{vec}\left(\underline{\mathbf{V}}_2^{(1)}\right) \\ \vdots \\ \mathrm{vec}\left(\underline{\mathbf{V}}_2^{(K)}\right) \\ \mathrm{vec}\left(\underline{\mathbf{U}}_1^{(1)*\mathrm{T}}\right) \\ \vdots \\ \mathrm{vec}\left(\underline{\mathbf{U}}_1^{(K)*\mathrm{T}}\right) \end{pmatrix}, \underline{\mathbf{b}} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \mathrm{vec}\left(\underline{\mathbf{\Sigma}}^{(1)}\right) \\ \vdots \\ \mathrm{vec}\left(\underline{\mathbf{\Sigma}}^{(K)}\right) \end{pmatrix}, \tag{17}$$

and $\underline{\mathbf{H}}$ is defined in (18). The total number of links, i.e., the interference and useful links, in the system of equations of (16) is K^2 and for every link Q^2 constraints have to be satisfied. Therefore, the number of equations of (16) is $K^2 Q^2$. In addition to the R^2 variables at the relay, the transmit and receive filters at a single time slot are optimized resulting in KQN variables for the transmit filters and exactly the same number of variables for the receive filters. To solve the system of equations of (16), the number of variables has to be equal to or higher than the number of equations. This means that the number of antennas at the relay has to be

$$R \geq \sqrt{KQ\left(KQ - 2N\right)}. \tag{19}$$

Because the right hand term of the inequality (19) is a square root term, equality would not be reachable in general for any integer number of K, Q and N. In general, $R > \sqrt{KQ\left(KQ - 2N\right)}$ would hold and there will be infinitely many solutions to the linear system of equations of (16).

C. Relay Processing Matrix and Transmit/Receive Filters

In the proposed scheme, the relay processing matrix is adjusted so that the effective interference links including the transmit and receive filters through the relay are inverse to the direct interference links. It follows that the interferences are canceled out at the destination nodes. Moreover, reducing the direct links' interference energy leads to a lower relay's

$$\underline{\mathbf{H}} = \begin{pmatrix}
\underline{\mathbf{H}}_{\mathrm{RL}}^{(2,1)} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(2,1)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(2,1)} & \mathbf{0} & \cdots & \mathbf{0} \\
\vdots & \vdots & \vdots & & \vdots & & \ddots & & & \ddots \\
\underline{\mathbf{H}}_{\mathrm{RL}}^{(K,1)} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(K,1)} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(K,1)} \\
\vdots & \ddots & & \ddots & & \vdots & & & & \vdots \\
\vdots & & \ddots & & \ddots & \vdots & & & & \vdots \\
\underline{\mathbf{H}}_{\mathrm{RL}}^{(1,K)} & \mathbf{0} & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(1,K)} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(1,K)} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\
\vdots & \vdots & & \vdots & \vdots & \ddots & \ddots & & & \\
\underline{\mathbf{H}}_{\mathrm{RL}}^{(K-1,K)} & \mathbf{0} & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(K-1,K)} & \mathbf{0} & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(K-1,K)} & \mathbf{0} \\
\underline{\mathbf{H}}_{\mathrm{RL}}^{(1,1)} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(1,1)} & \mathbf{0} & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(1,1)} & \mathbf{0} & \cdots & & \mathbf{0} \\
\vdots & & \ddots & & & & \ddots & & & \\
\underline{\mathbf{H}}_{\mathrm{RL}}^{(K,K)} & \mathbf{0} & \cdots & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{TDL}}^{(K,K)} & \mathbf{0} & \cdots & & \mathbf{0} & \underline{\mathbf{H}}_{\mathrm{RDL}}^{(K,K)}
\end{pmatrix} \tag{18}$$

transmit energy required to compensate the interferences of the direct links. Therefore, reducing the transmit filters' gains results in less relay's transmit energy. Because the relay gain goes along with the relay's retransmit energy, reducing the relay gain is considered as an optimization goal in the following. Among the solutions of the linear system of equations of (16), the one which minimizes both the relay gain and the transmit filters' gains is selected. This solution can be found using the optimization problem

$$\underline{\mathbf{x}}_{\text{opt}} = \underset{\underline{\mathbf{x}}}{\operatorname{argmin}} \left\{ \underline{\mathbf{x}}^{*\text{T}} \mathbf{W} \underline{\mathbf{x}} \right\} \tag{20}$$

subject to

$$\underline{\mathbf{H}}\ \underline{\mathbf{x}} = \underline{\mathbf{b}}, \tag{21}$$

where $\mathbf{W}$ is a $\left(R^2 + 2KQN\right) \times \left(R^2 + 2KQN\right)$ diagonal matrix with ones at the first $R^2 + KQN$ entries of the main diagonal and zeros at the remaining entries. The first $R^2 + KQN$ entries correspond to the vectors $\operatorname{vec}\left(\underline{\mathbf{G}}\right)$ and $\operatorname{vec}\left(\underline{\mathbf{V}}_2^{(l)}\right)$, $\forall l$. The Lagrangian function of the optimization problem of (20)-(21) is

$$L\left(\underline{\mathbf{x}}, \underline{\lambda}\right) = \underline{\mathbf{x}}^{*\text{T}} \mathbf{W} \underline{\mathbf{x}} + \underline{\lambda}^{T} \left(\underline{\mathbf{b}} - \underline{\mathbf{H}}\ \underline{\mathbf{x}}\right), \tag{22}$$

where $\underline{\lambda}$ is a vector of Lagrangian multipliers each of which corresponds to a constraint of (21). Taking the derivatives of (22) with respect to $\underline{\mathbf{x}}$ and $\underline{\lambda}$ gives

$$\mathbf{W}\underline{\mathbf{x}} = \underline{\mathbf{H}}^{*T} \underline{\lambda}^{*}, \tag{23}$$

and

$$\underline{\mathbf{b}} - \underline{\mathbf{H}}\mathbf{W}\underline{\mathbf{x}} - \underline{\mathbf{H}}\left(\mathbf{I} - \mathbf{W}\right)\underline{\mathbf{x}} = 0, \tag{24}$$

respectively. By substituting (23) in (24), the optimum $\underline{\lambda}$ reads

$$\underline{\lambda}^{*} = \left(\underline{\mathbf{H}}\ \underline{\mathbf{H}}^{*\text{T}}\right)^{-1} \left(\underline{\mathbf{b}} - \underline{\mathbf{H}}\left(\mathbf{I} - \mathbf{W}\right)\underline{\mathbf{x}}\right). \tag{25}$$

The optimum $\underline{\mathbf{x}}_{\text{opt}}$ is obtained by substituting (25) into (23)

$$\underline{\mathbf{x}}_{\text{opt}} = \left(\mathbf{W} + \underline{\mathbf{H}}^{+}\underline{\mathbf{H}}\left(\mathbf{I} - \mathbf{W}\right)\right)^{-1} \underline{\mathbf{H}}^{+}\underline{\mathbf{b}}, \tag{26}$$

where $\underline{\mathbf{H}}^{+}$ is the right pseudoinverse of $\underline{\mathbf{H}}$.

V. Analysis of The Relay Retransmit Energy

When doing a performance assessment of IA exploiting a relay, it is important not only to consider the source nodes' transmitted energies but also the relay's retransmitted energy. The covariance matrix of the relay's transmitted vector $\underline{\mathbf{s}}_{\text{R}}$ is calculated as

$$\underline{\mathbf{C}}_{\text{rr}} = \underline{\mathbf{G}} \left(\sum_{l=1}^{K} \underline{\mathbf{H}}_{\text{RS}}^{(l)} \underline{\mathbf{C}}_{\text{ss}}^{(l)} \underline{\mathbf{H}}_{\text{RS}}^{(l)*\text{T}} + \underline{\mathbf{C}}_{\text{nn}} \right) \underline{\mathbf{G}}^{*\text{T}}, \tag{27}$$

where $\underline{\mathbf{C}}_{\text{ss}}^{(l)}$ and $\underline{\mathbf{C}}_{\text{nn}}$ are the covariance matrix of the l-th source node's transmitted vector at the first time slot and the covariance matrix of the relay's received noise vector $\underline{\mathbf{n}}_{\text{R}}$, respectively. The received noises at different relay antennas are i.i.d., and thus $\underline{\mathbf{C}}_{\text{nn}} = \sigma^2 \mathbf{I}_{\text{R}}$ holds. Furthermore, it is assumed that all data symbols are uncorrelated and the fixed matrices $\underline{\mathbf{V}}_1^{(l)}$, $\forall l$, are chosen in such a way that $\underline{\mathbf{V}}_1^{(l)} \underline{\mathbf{V}}_1^{(l)*\text{T}}$, $\forall l$ are diagonal matrices, i.e., $\underline{\mathbf{C}}_{\text{ss}}^{(l)} = E_{\text{d}} \underline{\mathbf{V}}_1^{(l)} \underline{\mathbf{V}}_1^{(l)*\text{T}}$. The total energy retransmitted by the relay is

$$\begin{aligned} E_{\text{R}_{\text{tot}}} = {} & E_{\text{d}} \operatorname{tr} \left(\sum_{l=1}^{K} \underline{\mathbf{G}}\ \underline{\mathbf{H}}_{\text{RS}}^{(l)} \underline{\mathbf{V}}_1^{(l)} \underline{\mathbf{V}}_1^{(l)*\text{T}} \underline{\mathbf{H}}_{\text{RS}}^{(l)*\text{T}} \underline{\mathbf{G}}^{*\text{T}} \right) \\ & + \sigma^2 \operatorname{tr} \left(\underline{\mathbf{G}}\ \underline{\mathbf{G}}^{*\text{T}} \right) \end{aligned} \tag{28}$$

where $\operatorname{tr}\left(.\right)$ denotes the trace of a matrix. It can be seen from (28) that the relay retransmit noise power grows linearly with the relay's gain.

VI. Numerical Results

In this section, the achieved average sum rate is taken as a measure of the performance. The performance of the proposed scheme is investigated as a function of the pseudo signal to noise ratio (PSNR) which is defined as the ratio of the total energy transmitted in two time slots by the source nodes and the relay to the noise power density at an antenna of a destination node

$$\gamma_{\text{PSNR}} = \left(KQE_{\text{d}} + E_{\text{R}_{\text{tot}}}\right) / \sigma^2. \tag{29}$$

The average sum rate is calculated as

$$C = \frac{1}{K} \frac{1}{N} \sum_{k=1}^{K} \sum_{q=1}^{Q} \operatorname{ld}\left(1 + \gamma_{k,q}\right), \tag{30}$$

where

$$\begin{aligned} \gamma_{k,q} = {} & \frac{E_{\text{d}}}{\sigma^2} \left| \left(\underline{\mathbf{u}}_{1,q}^{(k)*T}, \underline{\mathbf{u}}_{2,q}^{(k)*T} \right) \underline{\mathbf{H}}^{(k,k)} \begin{pmatrix} \underline{\mathbf{v}}_{1,q}^{(k)} \\ \underline{\mathbf{v}}_{2,q}^{(k)} \end{pmatrix} \right|^2 \\ & \Bigg/ \left(\left\| \underline{\mathbf{u}}_{1,q}^{(k)} \right\|^2 + \left\| \underline{\mathbf{u}}_{2,q}^{(k)} \right\|^2 + \underline{\mathbf{u}}_{2,q}^{(k)*\text{T}} \underline{\mathbf{H}}_{\text{DR}}^{(k)} \underline{\mathbf{G}}\ \underline{\mathbf{G}}^{*\text{T}} \underline{\mathbf{H}}_{\text{DR}}^{(k)*\text{T}} \underline{\mathbf{u}}_{2,q}^{(k)} \right) \end{aligned} \tag{31}$$

denotes the received SNR of the received q-th data symbol at the k-th destination node with $\underline{\mathbf{v}}_{\tau,q}^{(k)}$ and $\underline{\mathbf{u}}_{\tau,q}^{(k)}$ denoting the q-th columns of the $\underline{\mathbf{V}}_{\tau}^{(k)}$ and $\underline{\mathbf{U}}_{\tau}^{(k)}$ matrices, respectively.

In the following, a scenario with $K = 4$ node pairs, $N = 3$ antennas at the nodes and $Q = 3$ data symbols per user is considered. A relay with $R = 9$ antennas positioned in the center between the communication partners is used. A non-selective Rayleigh fading channel is employed with a unit average channel gain for the direct links. Assuming an attenuation exponent of $\alpha = 4$, the average channel gain of the relay's links, i.e., the channels between the source nodes and the relay and the channels between the relay and the destination nodes, is four because the relay is placed in between the communication partners. The sum rate averaged over many different channel realizations is investigated. For the proposed scheme, the transmit filter at $\tau = 1$ and the receive filter at $\tau = 2$ are chosen as identity matrices which are special cases of the required full rank matrices. For the numerical simulations, scalars are multiplied to the transmit filters so that equal energies per data symbol are transmitted by each user, i.e., the columns' norms of the normalized transmit filter matrices are one. Accordingly, the resulting normalized transmit filters are used as transmit filters.

For the same scenario two benchmark transmission schemes are evaluated. The first benchmark scheme employs transceive zero forcing at the relay. In the first transmission phase, the source nodes transmit $Q = N$ data symbols over their N antennas to the relay. The relay separates the KQ data symbols from the source nodes by linear zero forcing, i.e., 3 source nodes will be active simultaneously with $R = 9$ antennas at the relay. If there are more source nodes, 3 of them can be scheduled at a certain time instant, but for the per user capacity, this scheduling scheme is irrelevant. In the second transmission phase, the relay retransmits to the destination nodes using transmit zero forcing. The same transmit energy per data symbol is used by the source nodes and by the relay.

In the second benchmark scheme only a single user is served at a certain time instant. In the first transmission phase, Q data symbols are transmitted to the relay which again uses transceive zero forcing. The retransmission to the destination node takes place in the second transmission phase. Both the source node and the relay use the same transmit energy. As compared to the first benchmark scheme less users are served simultaneously but a beamforming gain is achieved at the rely.

The achieved average sum rates normalized to the number of required time slots are depicted as a function of the PSNR γ_{PSNR} in decibel in Fig. 2. At low and moderate PSNR, the other conventional relaying schemes outperform the proposed IA scheme. But at high PSNR, the proposed IA using a MIMO relay achieves higher sum rates as compared to the benchmark schemes. Furthermore, it is interesting to notice that the slope of the curves at high PSNR corresponds to the DoFs. Therefore, the maximum total DoFs of $\frac{4\times3}{2KN} = \frac{1}{2}$ is achieved by IA using a MIMO relay, $\frac{3\times3}{2KN} = \frac{3}{8}$ DoFs are achieved by the multiuser relaying scheme and $\frac{4\times3}{8KN} = \frac{1}{8}$ DoFs are achieved by the single user relaying scheme.

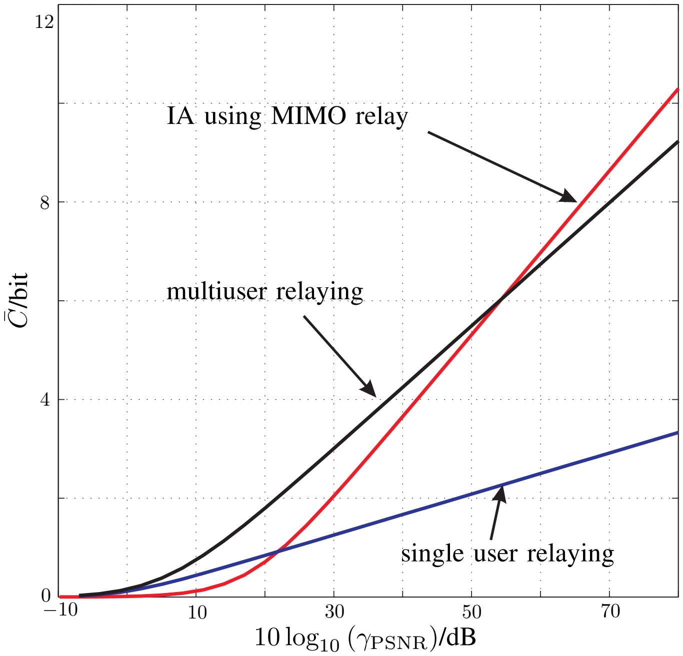

Fig. 2: Average sum-rate as a function of the PSNR.

VII. Conclusion

In this paper, an IA scheme exploiting a MIMO relay is investigated. By partially-adapting the transmit and the receive filters and fully-adapting the relay processing matrix to the channel, the IA equations become linear. A closed form solution with a reduced total transmit energy is proposed. It is shown that the proposed scheme achieves the maximum DoFs for the interference channel. In terms of the achieved sum rates, the proposed scheme outperforms conventional relaying schemes at high SNRs.

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