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Hybrid adaptive/non-adaptive multi-user OFDMA systems in the presence of user-specific imperfect channel knowledge

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Abstract

In this article, a hybrid multi-user OFDMA scheme which considers different user demands regarding channel access and user-specific imperfect channel quality information (CQI) is analytically described and evaluated. The considered hybrid scheme offers two possible modes to serve the user: firstly, via a non-adaptive mode which applies a discrete Fourier transform precoding to exploit frequency diversity and, thus, does not require any CQI at the transmitter. Secondly, via an adaptive mode which performs an adaptive resource allocation and link adaptation. Since this adaptation to the channel is done based on CQI, the adaptive mode is prone to imperfect CQI which results from estimation errors or time delays. Hence, in case of user-specific imperfect CQI, the question arises which user shall be served adaptively or non-adaptively and which resource shall be allocated to which user such that the system data rate is maximized while fulfilling a certain target bit error rate (BER) and minimum required user data rate. Analytical expressions of the system performance considering imperfect CQI and different user demands are derived. Based on these expressions, algorithms determining which user is served adaptively or non-adaptively subject to the BER and minimum data rate constraints are developed. Simulations show the superiority of the hybrid OFDMA scheme in terms of achievable data rate and user satisfaction compared to conventional pure adaptive and non-adaptive OFDMA schemes in the presence of user-specific imperfect CQI.

Keywords: OFDMA, Multi-user diversity, Imperfect channel knowledge

Introduction

Orthogonal frequency division multiple access (OFDMA) [1] is regarded as a promising candidate for future mobile radio systems. In case that reliable channel knowledge is available at the transmitter, good performances for transmissions can be accomplished by means of adaptation to the channel using, e.g., techniques like adaptive multi-user scheduling [2], and adaptive power loading, modulation and coding [3,4]. OFDMA schemes applying these techniques are referred to as adaptive OFDMA. For practial applications in OFDMA systems, fast heuristic algorithms such as in [5] and references within have been developed to solve the resource and power allocation problem. In [5], the solution of a simple single user case is employed

to design a heuristic for the general multiuser case. Furthermore, the resource and power allocation problem has been addressed in multi-cell scenarios, e.g. in [6]. Here, the authors analyze the problem of mitigating multicell interference while taking into acount heterogeneous user data rate requirements for multicell OFDMA systems with half-duplex Decode-and-Forward relaying. The resulting non convex and combinatorial problem is solved by dual decomposition and iterative resource allocation algorithms. The case of radio resource allocation in an OFDMA spectrum sharing environment has also been addressed, e.g. in [7]. Here, the goal is to maximize the weighted sum rate of the secondary users subject to the total transmit power of the secondary base station (BS) and the primary service collision probability for continuous and discrete rate strategies. The non convex resource allocation problem is solved applying the dual optimization method.

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In case that no reliable channel knowledge is available, the use of diversity-exploiting transmission schemes is the preferred strategy for provision of good performance [8]. OFDMA schemes applying this strategy are referred to as non-adaptive OFDMA. Adaptive OFDMA schemes require accurate channel knowledge at the transmitter and a considerable amount of signaling which limits the range of applications to scenarios with rather slowly changing channels, e.g., slowly moving MSs. In these scenarios, however, adaptive access scheme outperform non-adaptive access schemes [9]. Nevertheless, non-adaptive OFDMA schemes are more suitable in scenarios with fast changing channels due to the use of diversity combining techniques which do not need transmitter sided channel knowledge resulting in marginal overhead. As in a realistic scenario, both situations are present, i.e., static up to semi-static users and fast moving users exist, it is beneficial to combine both multiple access schemes in a hybrid OFDMA scheme to serve all users with respect to the given conditions.

Hybrid OFDMA systems which allow the co-existence or the switching between adaptive OFDMA transmission and non-adaptive OFDMA schemes, respectively, have already been introduced in the literature. In [9,10], the co-existence and adaptive selection of multiple access schemes in a hybrid OFDMA system is discussed. As adaptive scheme, adaptive chunk-based time division multiple access (TDMA)/OFDMA is applied. As non-adaptive scheme, block equidistant frequency division multiple access (B-EFMDA) is applied, where the subcarriers of a given user are blockwise equidistantly distributed over the bandwidth to exploit frequency diversity. Within a so called super-frame, chunks of subcarriers are pre-allocated for the two modes. Between super-frames, the allocation of the subcarriers can change. The preselection of the applied access scheme mode for the different users is amongst others based on the type of service, the channel quality of the downlink and the Signal-to-Interference-and-Noise-Ratio (SINR). During operation, the access schemes can dynamically be changed, i.e., a two step mechanism is applied.

Another OFDM-based hybrid multiple access scheme has been presented in [11]. Here, adaptive OFDMA is employed as adaptive transmission and frequency hopping (FH)-OFDMA is employed as non-adaptive scheme which exploits frequency diversity. To select the applied access scheme, three classes are defined, namely the mobility class, the service class and the environment class. The mobility class are (a) mobile users and (b) nomadic users with a rather low terminal velocity. Concerning service, real-time and non-real time services are considered. The environment classes are (a) low and (b) high intercell interference environments. According to the class affiliation of a given user, either adaptive OFDMA or FH-OFDMA is applied as multiple access scheme. In both works [9,11], the decision whether a user is served by an adaptive or non-adaptive access scheme is not done based on analytical calculations. For instance in [10], the decision whether the channel quality information (CQI) quality is good enough to apply the adaptive access scheme is based on simulative curves which are only valid for a certain set of simulation parameters. In [11], the expected throughput of either the adaptive or non-adaptive access scheme is used as criterion without considering the impact of imperfect CQI. Furthermore, concerning the mobility, only the coherence time of the channel of each user which has to be smaller than a given threshold to apply the adaptive scheme is used as criterion to select the access scheme. However, this approach totally disregards the impact of the number of users applying the adaptive access scheme on the multi-user diversity gain and, thus, the performance. Taking this into account, the decision whether a user is served adaptively or non-adaptively cannot be made user-wise but has to be done jointly considering all users. Moreover, the determination of the threshold value is rather heuristic since the actual achievable data rate is not calculated. From this, it follows that the hybrid multiple access schemes proposed in [9,11] cannot guarantee that certain quality of service requirements of each user are actually fulfilled as the multiple access scheme selection is not based on analytical calculations considering imperfect channel knowledge.

The consideration of imperfect channel state information (CSI) has been already discussed for OFDM-based schemes in the literature, for example in [12-25] and references therein. However, in all mentioned works, the problem of dealing with imperfect CSI is only addressed in pure adaptive OFDM-based systems but not in hybrid systems. In [26], a comparison of adaptive and non-adaptive multi-user OFDMA schemes in the presence of imperfect channel knowledge has been presented where the same degree of CQI imperfectness was assumed for each user. It is shown that at a certain level of CQI imperfectness, it is beneficial to switch from adaptive to non-adaptive transmission, i.e., depending on the quality of the channel knowledge, either all users apply the adaptive or non-adaptive transmission scheme. In a realistic scenario however, the level of CQI imperfectness differs from user to user.

Concerning the scheduling for adaptive OFDMA schemes, proportional fair scheduling (PFS) approaches provide a good trade-off between system throughput and fairness. PFS in combination with OFDMA is well discussed in the literature, e.g., [27,28]. If, furthermore, different user demands shall be considered, weighted proportional fair scheduling (WPFS) approaches can be

applied, which are discussed, e.g., in [29-31]. These WPFS algorithms favor high demand users to get channel access even if their channel gain is low which leads to a degradation of the system throughput compared to PFS approaches. Both PFS and WPFS require channel knowledge at the transmitter. However, in a realistic scenario with imperfect channel knowledge, the performance also degrades compared to the case of perfect channel knowledge. The joint impact of imperfect channel knowledge and different user demands on the performance of a hybrid OFDMA system appyling WPFS has not been mentioned in the literature so far.

To the author's knowledge, an analytical assessment of a hybrid OFDMA scheme with different user demands taking into account imperfect channel knowledge has not been provided so far. Moreover, the problem of selecting the multiple access schemes based on analytical performance calculations to fulfill certain quality of service requirements as the target bit error rate (BER) and minimum user data rates while maximizing the overall system performance has not been considered in the literature and will be addressed in this article.

The remainder of this article is organized as follows. In Section "System model and assumptions", the OFDMA system model together with the channel model and system assumptions is provided. Furthermore, the modelling of imperfect channel knowledge is presented assuming two sources of impairments: time delays and estimation errors. In Section "Adaptive and non-adaptive transmission", the two transmission modes of the hybrid scheme, namely adaptive and non-adaptive OFDMA, are introduced. In Section "Hybrid adaptive/non-adaptive OFDMA", the concept of a hybrid multi-user OFDMA system which is aware of imperfect user-specific CQI is proposed. Furthermore, the main problem formulation is introduced which aims at maximizing the system data rate while fulfilling a given BER and minimum data rate requirement for each user applying both the adaptive and non-adaptive OFDMA transmission modes. It is shown that this optimization problem can be split up into two subproblems, namely the signal-to-noise-ratio (SNR) threshold problem and the user serving problem. In Section "The SNR threshold problem", analytical derivations of the average data rate and BER taking into account imperfect CQI and different user demands are presented which are then used to solve the SNR threshold problem. Section "The user serving problem" provides solutions for the user serving problem and analyses the complexity of the algorithms. In Section "Simulation results", the achievable data rate of the hybrid OFDMA system is illustrated and compared to pure adaptive and pure non-adaptive OFDMA systems in the presence of imperfect user-specific CQI. Finally, conclusions are drawn in Section "Conclusions".

System model and assumptions

In this section, the considered system model is introduced. Furthermore, the concept of different user demands in terms of channel access is presented. Finally, the modeling of imperfect channel knowledge is discussed.

System model

In this work, a one cell downlink scenario is considered with one BS and U Mobile Stations (MSs) with user index $u = 1, \ldots, U$ located in the cell. The BS and the MSs are equipped with one antenna each, where the MSs are assumed to be uniformly distributed inside the cell. OFDMA is used and the bandwidth is subdivided into N orthogonal subcarriers with frequency spacing Δf. A block of Q adjacent subcarriers, also called chunk [32], is defined as a resource unit. Hence, a total number of $N_{\mathrm{ru}} = \lfloor N/Q \rfloor$ resource units is assumed with $\lfloor . \rfloor$ the nearest integer lower than or equal to the argument. It is assumed that Q is chosen in such a way that the channel does not vary significantly within a resource unit. Furthermore, the channels of adjacent resource units are assumed to be uncorrelated, i.e., the coherence bandwidth of the channel is smaller than the bandwidth of two adjacent resource units. The fast fading described by the transfer factor $H_u(n, k)$ of each user u on the resource unit with index $n = 1 \ldots, N_{\mathrm{ch}}$ in a time slot $k \in \mathbb{Z}$ is then modeled as a complex Gaussian distributed random process with variance one. It is assumed that the BS transmits with power P_{T} where the transmit power is equally shared among the N subcarriers. This assumption is justified by the fact that the achievable gains applying optimal power allocation are negligible compared to the increase in complexity especially as one has to keep in mind that optimal power allocation requires accurate channel knowledge which makes optimal power allocation also prone to imperfect channel knowledge.

With the noise power spectral density N_0, the average SNR $\bar{\gamma}_u$ at the MS of user u can be calculated by

$$\bar{\gamma}_u = \frac{P_{\mathrm{T}}}{N \cdot \Delta f \cdot N_0} \cdot \left(\frac{d_u}{d_0} \right)^{-\alpha}, \tag{1}$$

with d_u denoting the distance between the MS of user u and the BS, d_0 the minimum distance between any MS and the BS and α the pathloss coefficient. From this, it follows that the instantaneous SNR $\gamma_u(n, k)$ of user u on resource unit n in time slot k is given by

$$\gamma_u(n, k) = \bar{\gamma}_u \cdot |H_u(n, k)|^2, \tag{2}$$

i.e., $\gamma_u(n, k)$ follows an exponential distribution. In this work, these instantaneous SNR values are applied as CQI for the adaptive transmission scheme.

Different user channel access demands

In multi-user communication systems, not every user has the same requirement in terms of data rate or channel access, respectively, due to different applications and services like, e.g., video conferencing, online gaming or just voice transmission. Hence, it is reasonable to allocate the available resources according to the demands of the different users. For that purpose, the channel access demand vector **D** is introduced:

$$\mathbf{D} = [D_1, D_2, \dots, D_U] \tag{3}$$

with $D_u \in \mathbb{N}$ denoting the demand of user u in terms of number of resource units where

$$\sum_{u=1}^{U} D_u = N_{\mathrm{ru}}. \tag{4}$$

In the following, it is assumed that the user demands never exceed the available number of resource units, i.e., (4) is always fulfilled as the demand vector **D** is set by the BS. In case that the demand of the users is larger than the number of available resources, the BS dictates a feasible solution. In case that the demand is smaller than the number of available resources, the BS distributes the remaining resources among the users.

Users which have the same channel access demand D_u are arranged into demand groups $\mathcal{G}_i$ with $i = 1, \dots, G$ where G denotes the number of demand groups.

Note that with this definition of the user requirements, the user demand is independent of the channel conditions which allows a very simple verification of the feasibility of the user demand using (4). Furthermore, it is much easier to guarantee a certain amount of allocated resource units than to guarantee a certain data rate which for adaptive transmission schemes strongly depends on the quality of the channel and the quality of the channel knowledge as shown later on.

Nevertheless, for non-adaptive schemes which work independently from any instantaneous channel knowledge, it is possible to determine the achievable data rate a priori from D_u as shown later on. Since for the considered hybrid adaptive/non-adaptive system the minimum user data rate shall not be smaller than the achievable user data rate in a pure non-adaptive system as explained later on in details, the achievable minimum user data rates can be determined a priori from **D**, i.e., there is a correlation between the requested number of resources **D**.

Imperfect channel knowledge

In a realistic scenario, the CQI available at the BS cannot be assumed to be perfectly known. In the following, two sources of error together with the modeling and the parameters describing the CQI imperfectness are presented. It is assumed that the BS is able to measure these parameters, i.e., the impairment parameters are assumed to be perfectly known at the BS. Note that the resource unit index n is omitted for the sake of readability.

Noisy CQI

In a realistic scenario, the channel transfer function for the downlink has to be measured applying for example pilot assisted channel estimation (PACE) in a time division duplex (TDD) system, i.e., in the uplink each MSs transmits a sequence of M_P pilot symbols $\mathbf{d}_p = [d_{p,1}, \dots, d_{p,M_P}]^{\mathrm{T}}$ with $\mathbf{d}_p^H \mathbf{d}_p = M_P$ which are known to the BS. Since the adaptive resource allocation and modulation scheme selection at the BS require a certain amount of computation time it is reasonable to assume that the CQI is noisy, as the CQI has to be updated rapidly considering only a channel estimation based on few pilots. Furthermore, the overhead providing CQI at the BS should kept to minimum, i.e. one cannot spend any number of pilots for estimating the complete downlink channel for each MS, as the pilots of different MSs have to be transmitted orthogonal to each other in the uplink.

Applying a least squares (LS) criterion and assuming an average SNR $\bar{\gamma}_u$ during the training phase, the LS estimate $\hat{H}_u(k)$ of the channel $H_u(k)$ can be modeled by

$$\hat{H}_u(k) = H_u(k) + E_u, \tag{5}$$

where the estimation error E_u is a complex Gaussian distributed random variable with zero mean and variance $\sigma_{\mathrm{E},u}^2$ given by

$$\sigma_{\mathrm{E},u}^2 = \frac{1}{\bar{\gamma}_u \cdot M_P}. \tag{6}$$

Outdated CQI

Due to the time delay T between the time instant when measuring the SNR and the actual time of data transmissions, the CQI is outdated. In the following, the correlation coefficient ρ_u between the realization of the actual channel and the outdated channel is introduced as a figure of merit to determine the up-to-datedness of the CQI. From literature, e.g. [33], it is known when the angles of arrival for the different propagation paths can be assumed to be uniformly distributed, the distribution of the Doppler shifts corresponds to a Jake's spectrum. Then, the correlation coefficient ρ_u only depends on the time delay T and the maximum Doppler shift $f_{D,u}$ of user u given by $\rho_u = J_0(2\pi f_{D,u} T)$ with $J_0(x)$ denoting the 0th-order Bessel function of the first kind. With the carrier frequency f_0 and the speed of light c, $f_{D,u}$ is given by $f_{D,u} = \frac{f_0 \cdot |v_u|}{c}$ with v_u the radial component of the velocity of user u along a line from the user u to the BS. From this, it follows that the correlation coefficient ρ_u of the channel transfer factor of user u is given by

$$\rho_u = J_0 \left(2\pi f_0 T c^{-1} \cdot |v_u| \right). \tag{7}$$

In the following, it is assumed that each user has a different velocity $\mathbf{v}_u = [v_x, v_y]^T$, where the x- and y-components of $\mathbf{v}_u$ are independent from each other and normally distributed with zero mean and variance σ_v. From this, it follows that the velocity component v_ϕ in any direction with angle ϕ is normally distributed with zero mean and variance σ_v^2. Hence, the radial component of the velocity v_u of user u is also $\mathcal{N}(0, \sigma_v^2)$ distributed. The absolute value $|v_u|$ is then half-normally distributed with expectation value $\bar{v} = \sqrt{\frac{2}{\pi}} \cdot \sigma_v$. In the following, the dynamic of the user movements inside the cell is expressed by this average velocity $\bar{v}$.

Adaptive and non-adaptive transmission

In this section, the adaptive and non-adaptive transmission modes of the considered hybrid OFDMA system are introduced.

Adaptive transmission

In this section, the adaptive multi-user OFDMA transmission mode is introduced. Instead of combating the channel variations by applying some sort of averaging technique to exploit diversity, the variations in the channel of different users are capitalized to transmit data only on the strongest channels [34]. In order to take into account the current SNR conditions of the resource units and the different user demands, a WPFS approach is applied. WPFS requires information about the actual channel conditions, more precisely information about the SNR of the different resource units of different users. To incorporate different user demands, a user specific weighting factor p_u with $p_u \geq 1 \; \forall \; u \in \{1, \ldots, U\}$ is introduced. Based on that, the subcarriers of resource unit n in time frame k are allocated to the user $u^\star(n, k)$ with the highest ratio between the weighted instantaneous SNR and the average SNR $\bar{\gamma}_u$, leading to

$$u^\star(n, k) = \arg\max_u \left\{ \frac{p_u \cdot \gamma_u(n, k)}{\bar{\gamma}_u} \right\}. \tag{8}$$

By doing so, each resource unit is allocated to one user exclusively. The weighting can be interpreted as a virtual SNR boost, i.e., the higher the weighting factor, the higher the probability of getting access to the channel. In case that $p_u = 1 \; \forall \; u \in \{1, \ldots, U\}$, all users have the same channel access probability as with conventional PFS. Note that the calculation of the proper weighting factors is shown in Section "The SNR threshold problem" later on. After resource allocation, the modulation scheme is selected for each allocated resource unit based on the actual SNR values, i.e., for each subcarrier inside one resource unit the same modulation scheme is applied assuming the same transmit power per subcarrier. By doing so, the modulation is adapted to the pathloss and to the fast fading. In

this work, uncoded M-ary Quadrature Amplitude Modulation (M-QAM) and M-ary Phase Shift Keying (M-PSK) are considered.

Non-adaptive transmission

In the following, the non-adaptive multi-user OFDMA transmission mode is introduced. The non-adaptive transmission mode is characterized by the fact that the transmitter does not require any instantaneous CQI. On that account, an optimal adaptation to the current channel condition is not possible. However, by exploiting diversity, the reliability of the transmission can be improved. In this work, a discrete Fourier transform (DFT) precoding of the data as done in interleaved frequency division multiple access (IFDMA) [35] or single carrier frequency division multiple access (SC-FDMA) is applied to exploit frequency diversity by averaging over the frequency variations of the channel. Since the transmitter does not have any instantaneous information about the channel conditions of different users, scheduling has to be done non-adaptively fulfilling the channel access demands **D** of the different users. To do so, the scheduler follows a round robin policy, i.e., the first D_1 available resource units are allocated to user 1, the next available D_2 resource units are allocated to user 2, and so on. Assuming that the average SNR $\bar{\gamma}_u$ of each user is known to the BS, one fixed modulation scheme is selected for all subcarriers of one user, i.e., the modulation is only adapted to the pathloss and not to the fast fading.

Hybrid adaptive/non-adaptive OFDMA

In this section, a hybrid adaptive/non-adaptive multi-user OFDMA system is proposed which is aware of the CQI impairments of the different users. In this hybrid scheme, both adaptive and non-adaptive transmission modes are supported where the adaptive and non-adaptive transmissions are multiplexed in frequency, i.e., different resource units in frequency direction are either used for non-adaptive or adaptive transmission over several time slots. In the following, the proposed hybrid OFDMA scheme is presented in more details. This then leads to the problem formulation. Finally, it is shown how to split up the problem in two subproblems which then can be solved successively.

Hybrid OFDMA scheme

Before the actual data transmission is done, the BS has to perform several operations. Figure 1 illustrates the operations which have to be done for each time frame k. In the following, these operations are described step-by-step.

Multiple access scheme selection

At first, the system has to select the applied multiple access scheme for each user u, i.e., it has to decide whether
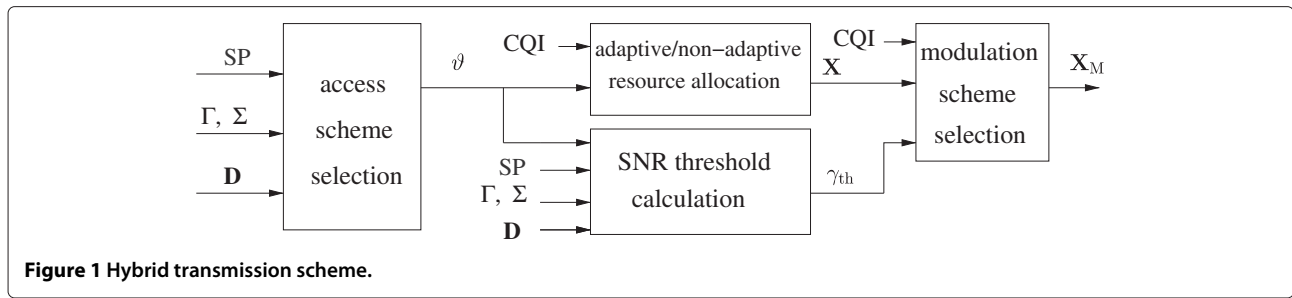
Figure 1 Hybrid transmission scheme.

a user is served adaptively or non-adaptively. This decision is based on the system parameters (SP) which are the number N_{ru} of available resource units, the number U of users to be served, the target BER BER_T and the user-specific average SNR $\bar{\gamma}_u$. Furthermore, the decision is based on the parameters describing the CQI imperfectness which are the correlation coefficients ρ_u stacked together in the vector

$$\mathbf{\Gamma} = [\rho_1, \ \rho_2, \ldots, \ \rho_U] \tag{9}$$

and the estimation error variances $\sigma_{E,u}^2$ given by

$$\mathbf{\Sigma} = [\sigma_{E,1}^2, \ \sigma_{E,2}^2, \ldots, \ \sigma_{E,U}^2]. \tag{10}$$

Note that it is assumed that these impairment parameters are perfectly known at the BS. Finally, the decision whether a user u is served adaptively or non-adaptively depends on the channel access demand vector $\mathbf{D}$ of (3) which is known at the BS. The outcome of the access scheme selection is the user serving vector

$$\vartheta = [\vartheta_1, \ldots, \vartheta_U]^{\mathrm{T}}, \tag{11}$$

where $\vartheta_u = 0$ if the user u is served non-adaptively and $\vartheta_u = 1$ if the user u is served adaptively.

Adaptive/non-adaptive resource allocation
In the following, the number of adaptively served users is denoted by $U_A = \vartheta^{\mathrm{T}}\vartheta$. Each user u demands access to D_u resource units on average resulting in $W_A = \sum_{u=1}^{U} \vartheta_u \cdot D_u$ resource units dedicated to the U_A adaptively served users and $W_{NA} = N_{ru} - W_A$ resource units dedicated to the $U - U_A$ non-adaptively served users. Together with the channel access demand vector $\mathbf{D}$ and the CQI values for each resource unit of each user, the user serving vector is used to perform the adaptive and non-adaptive resource allocation according to the following strategy: First, the resource units of the adaptively served users are allocated, i.e., WPFS is applied over all N_{ru} resource units taking into account only the U_A adaptive users. By doing so, the diversity of all N_{ru} resource units is exploited in the adaptive resource allocation process. However, the non-adaptively served users demand W_{NA} resource units which have to be re-allocated from the adaptive users. As for non-adaptive users it is not important which resource units

are allocated to them since the non-adaptive mode works independent from any CQI, W_{NA} out of the N_{ru} selected resource units with the lowest ratio between weighted instantaneous SNR and average SNR are re-allocated from the adaptive users to the non-adaptive users. By doing so, the best W_A out of N_{ru} resource units are selected for the adaptive users while the non-adaptive users still obtain their demanded number of resource units. Note that it is possible that an adaptive user u does not get access to exactly D_u resource units for a certain channel realization. However, on average, the amount of resource units allocated to user u is D_u. The outcome of the resource allocation is the $U \times N_{ru}$ allocation matrix $\mathbf{X}$. The elements $x_{u,n} \in \{0, 1\}$ of $\mathbf{X}$ denote whether the n-th resource unit is allocated to user u ($x_{u,n} = 1$) or not ($x_{u,n} = 0$).

SNR threshold calculation
Besides the resource allocation represented by matrix $\mathbf{X}$, the user serving vector ϑ and the SP, the impairment parameters Γ and Σ and the channel access demand vector $\mathbf{D}$ are used to determine the SNR threshold vectors

$$\gamma_{th}^{(u)} = [\gamma_{th,0}^{(u)}, \ldots, \gamma_{th,M}^{(u)}] \tag{12}$$

for each user u. Vector $\gamma_{th}^{(u)}$ denotes the SNR range in which a certain modulation scheme shall be applied for user u with $\gamma_{th,0}^{(u)} = 0$ and $\gamma_{th,M}^{(u)} = \infty$ assuming M available modulation schemes. Note that due to the user-specific estimation error variances $\sigma_{E,u}^2$ and correlation coefficients ρ_u, the SNR thresholds for different users are different. For example, a user with inaccurate CQI needs to adjust the SNR thresholds much more conservatively compared to a user with perfect CQI which will be explained in more details later on. Since the calculation of the SNR thresholds does not depend on the instantaneous CQI, the calculation can be performed in parallel to the resource allocation, i.e., both operations are independent from each other.

Modulation scheme selection
Finally, with the SNR threshold vector γ_{th}, the allocation matrix $\mathbf{X}$ and the CQI values for each resource unit of each user, the $U \times N_{ru}$ modulation scheme matrix $\mathbf{X}_M$ is computed where the elements $x_{M,u,n}$ denote which modulation

scheme is applied in the n-th resource unit allocated to user u in time frame k.

After the operations are completed, ϑ, $\mathbf{X}$ and $\mathbf{X}_{\mathrm{M}}$ are utilized for the actual data transmission applying the hybrid scheme. First, the binary data of user u is mapped on data symbols utilizing the u-th row $\mathbf{X}_{\mathrm{M}}^{(u)}$ of modulation scheme matrix $\mathbf{X}_{\mathrm{M}}$. The resulting data symbols are then either directly OFDM modulated according to the u-th row $\mathbf{X}^{(u)}$ of allocation matrix $\mathbf{X}$ or DFT precoded followed by the OFDM modulation depending on the user serving vector element ϑ_u. At the receiver, the channel, the OFDM modulation and the DFT precoding in case of a non-adaptively served user are inverted. Note that it is assumed that each user has perfect channel knowledge at the receiver and is informed about whether it is served adaptively or non-adaptively. This assumption can be justified by the fact that for channel estimation and equalization of the received data at the MSs, the duration of a whole downlink frame can be utilized leading to almost perfect channel knowledge.

Problem formulation

The goal of the considered hybrid system is to achieve a maximum average system data rate under the constraint of a minimum required user data rate and target BER. The two parameters which are adjustable by the system to accomplish this task are the user serving vector ϑ and the SNR threshold vector $\gamma_{\mathrm{th}}^{(u)}$ of each user u. In the following, the average user data rate is defined as

$$\bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) = \vartheta_u \bar{R}_{\mathrm{A}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) + (1 - \vartheta_u)\bar{R}_{\mathrm{N}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$$
$$(13)$$

with $\bar{R}_{\mathrm{A}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$ and $\bar{R}_{\mathrm{N}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$ denoting the achievable data rates applying the adaptive or non-adaptive transmission scheme, respectively. Consequently, the average user BER is defined as

$$\overline{\mathrm{BER}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) = \vartheta_u \overline{\mathrm{BER}}_{\mathrm{A}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$$
$$+ (1 - \vartheta_u)\overline{\mathrm{BER}}_{\mathrm{N}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$$
$$(14)$$

where $\overline{\mathrm{BER}}_{\mathrm{A}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$ and $\overline{\mathrm{BER}}_{\mathrm{N}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$ denote the BER of an adaptively served or non-adaptively served user, respectively. The system data rate $\bar{R}_{\mathrm{sys}}$ is then defined as the sum over the U different user data rates weighted by the factor $\left(\frac{D_u}{N_{\mathrm{ru}}}\right)$ which represents the probability of user u to get access to a given resource unit. This average system data rate shall be maximized over the vectors ϑ and $\gamma_{\mathrm{th}}^{(u)}$ subject to a minimum required user date rate $\bar{R}_{\mathrm{min}}^{(u)}$ and a target BER $\mathrm{BER}_{\mathrm{T}}$:

$$\bar{R}_{\mathrm{sys,opt}} = \max_{\vartheta, \gamma_{\mathrm{th}}^{(u)}} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \qquad (15)$$
$$\text{subject to}$$
$$\bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \geq \bar{R}_{\mathrm{min}}^{(u)}$$
$$\overline{\mathrm{BER}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \leq \mathrm{BER}_{\mathrm{T}}.$$

Note that the minimum user data rate $\bar{R}_{\mathrm{min}}^{(u)}$ each user u shall achieve is given by the average user data achievable when applying the pure non-adaptive transmission mode, i.e., no matter how bad the channel conditions are, each user shall achieve at least the data rate achievable when applying the robust non-adaptive transmission scheme, otherwise, any sophisticated adaptive transmission scheme would be pointless.

Moreover, it is assumed throughout this work that the required target BER is equal for all users. However, the problem can easily be extended to different target BERs.

Splitting up the problem into two smaller problems

In the following section, it is shown that it is possible to split up the problems into two smaller problems without changing or simplifying the original problem. Problem (15) can be rewritten as

$$\bar{R}_{\mathrm{sys,opt}} = \max_{\vartheta} \qquad\qquad\qquad (16)$$

$$\left\{ \begin{array}{c} \max_{\gamma_{\mathrm{th}}^{(u)}} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \\ \text{subject to} \\ \bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \geq \bar{R}_{\mathrm{min}}^{(u)} \\ \overline{\mathrm{BER}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \leq \mathrm{BER}_{\mathrm{T}} \end{array} \right\}$$

meaning that for the inner optimization problem, ϑ is kept fixed, i.e., for each given ϑ, the optimal SNR thresholds $\gamma_{\mathrm{th}}^{(u)}(\vartheta)$ as functions of ϑ are determined which optimize $\sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)})$ subject to the two constraints. The outer optimization problem is then solved by searching for the optimal ϑ. In a next step, the minimum user data rate constraint is not considered in the inner optimization problem but in the outer optimization problem:

$$\bar{R}_{\mathrm{sys,opt}} = \max_{\vartheta} \qquad\qquad\qquad (17)$$

$$\left\{ \begin{array}{c} \max_{\gamma_{\mathrm{th}}^{(u)}} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\mathrm{ru}}}\right) \bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \\ \text{subject to} \\ \overline{\mathrm{BER}}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}) \leq \mathrm{BER}_{\mathrm{T}} \end{array} \right\}$$
$$\text{subject to}$$
$$\bar{R}^{(u)}(\vartheta, \gamma_{\mathrm{th}}^{(u)}(\vartheta)) \geq \bar{R}_{\mathrm{min}}^{(u)}$$

From this it follows that the solution space of $\gamma_{\text{th}}^{(u)}(\vartheta)$ of the inner problem in (17) is larger than in (16). However, when searching for the optimal ϑ in the outer optimization problem, only the solutions of $\gamma_{\text{th}}^{(u)}(\vartheta)$ which fulfill the minimum user data rate constraint are taken into account, i.e., (16) and (17) are equivalent. The inner optimization problem of (17) can now be reformulated to

$$\left\{ \begin{array}{c} \max_{\gamma_{\text{th}}^{(u)}} \bar{R}_{\text{A/N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)}) \\ \text{subject to} \\ \overline{\text{BER}}_{\text{A/N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)}) \leq \text{BER}_{\text{T}} \end{array} \right\} \qquad (18)$$

since the user data rates are independent from each other, i.e., it is enough to optimize the individual user data rates in order to optimize the sum of the user data rates. Note that notation A/N means either applying the adaptive scheme (A) or applying the non-adaptive scheme (N).

Let $\bar{R}_{\text{A/N,opt}}^{(u)}(\vartheta)$ denote the optimized user data rates with respect to $\gamma_{\text{th}}^{(u)}$ as a function of ϑ:

$$\bar{R}_{\text{A/N,opt}}^{(u)}(\vartheta) = \max_{\gamma_{\text{th}}^{(u)}} \left(\bar{R}_{\text{A/N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)}) \right) \qquad (19)$$

$$\text{subject to}$$

$$\overline{\text{BER}}_{\text{A/N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)}) \leq \text{BER}_{\text{T}}.$$

In the following, this optimization problem is referred to as SNR threshold problem. Inserting (13) in (17), the outer optimization problem of (17) can be written as

$$\bar{R}_{\text{sys,opt}} = \max_{\vartheta} \sum_{u=1}^{U} \left(\frac{D_u}{N_{\text{ru}}} \right) \Big[\vartheta_u \bar{R}_{\text{A,opt}}^{(u)}(\vartheta)$$

$$+ (1 - \vartheta_u) \cdot \bar{R}_{\text{N,opt}}^{(u)}(\vartheta) \Big] \qquad (20)$$

$$\text{subject to}$$

$$\vartheta_u \bar{R}_{\text{A,opt}}^{(u)}(\vartheta) + (1 - \vartheta_u) \cdot \bar{R}_{\text{N,opt}}^{(u)}(\vartheta) \geq \bar{R}_{\text{min}}^{(u)}$$

In the following, this problem is referred to as user serving problem.

For the SNR threshold problem, an existing solution from the literature [25] can be applied while for the combinatorial user serving problem, it can be shown that the searching space can be significantly reduced. In the following two sections, solutions for the two problems (19) and (20) are presented. In Section "The SNR threshold problem", it is assumed that there exists a given user serving vector ϑ. For this ϑ, the optimal SNR thresholds are then determined solving (19). In Section "The user serving problem", it is then shown how to solve (20), i.e., how to find the optimal user serving vector ϑ.

The SNR threshold problem

In order to solve subproblem (19), analytical expressions for the user data rate $\bar{R}_{\text{A/N}}^{(u)}$ and BER $\overline{\text{BER}}_{\text{A/N}}^{(u)}$ for both the adaptive and non-adaptive transmission scheme have to be derived taking into account imperfect CQI and different user demands assuming that the user serving vector ϑ is known. In case of adaptive transmission, the weighting factors p_u of all adaptively served users have to be determined in advance to fulfill the desired user demands. Finally, the average user data rate is optimized subject to the target BER.

Non-adaptively served users

In this section, the average data rate and BER are derived analytically for the non-adaptively served users for a given realisation of the user serving vector ϑ.

SNR distribution

In case of non-adaptively served users, there are no adaptive scheduling decisions or adaptive modulation scheme selections to be made. Hence, only the probability density function (PDF) of the resulting SNR at the receiver of an allocated user is of interest to determine the performance of non-adaptively served users.

Each non-adaptive user gets randomly access to D_u resource units. Applying the non-adaptive transmission scheme as described in Section "Adaptive and non-adaptive transmission" leads to an averaging over the D_u different SNR conditions of the resource units allocated to a given user. In the following, the averaging over the D_u resource units is approximated by the arithmetic mean of the SNR of the different resource units. By assuming that the channels of these resource units are uncorrelated, it can be shown that the PDF $p_{\gamma}^{(u)}(\gamma)$ of the resulting SNR γ of user u is a chi-square distribution with $2D_U$ degrees of freedom [36] and given by

$$p_{\gamma}^{(u)}(\gamma) = \left(\frac{D_u}{\bar{\gamma}_u} \right)^{D_u} \cdot \frac{\gamma^{D_u-1}}{(D_u - 1)!} \cdot e^{-\frac{D_u\gamma}{\bar{\gamma}_u}} \qquad . \qquad (21)$$

Average user data rate

One fixed modulation scheme with index $m = 1, \ldots, M$ is used for all subcarriers of the allocated resource units. Hence, the data rate $\bar{R}_{\text{N}}^{(u)}$ of user u does not depend on the user serving vector ϑ and is given by

$$\bar{R}_{\text{N}}^{(u)} = b_m, \qquad (22)$$

where b_m denotes the number of bits per symbol corresponding to the applied modulation scheme. From this, it follows that $\bar{R}_{\text{N}}^{(u)}(\vartheta, \gamma_{\text{th}}^{(u)})$ as introduced in Section "Problem Formulation" is not a function of the user serving

vector ϑ and the SNR threshold vector $\gamma_{\text{th}}^{(u)}$ but only depends on the applied modulation scheme with index m.

Average user BER

The average BER using the modulation scheme with index m is then determined by

$$\overline{\text{BER}}_{\text{N}}^{(u)} = \int_0^\infty \text{BER}_m(\gamma) \cdot p_\gamma^{(u)}(\gamma) \, d\gamma, \tag{23}$$

where BER_m determines the BER of the applied modulation scheme with index m. In the following, the approximation for the BER for M-QAM and M-PSK modulation introduced in [37] is used given by

$$\text{BER}_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \tag{24}$$

with $\beta_m = \frac{1.6}{2^{b_m} - 1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9 b_m} + 1}$ using M-PSK modulation, respectively. Inserting (21) and (24) in (23), the average BER for non-adaptive users can be calculated according to

$$\overline{\text{BER}}_{\text{N}}^{(u)} = 0.2 \cdot \left(\frac{D_u}{D_u + \beta_m \bar{\gamma}_u} \right)^{D_u}. \tag{25}$$

Adaptively served users

In this section, the average data rate and BER for the adaptively served users are derived.

Calculation of channel access probability

Applying the adaptive transmission using WPFS, the weighting factors p_u for each adaptive user $u = 1, \ldots, U_A$ represented by the weighting vector $\mathbf{p} = [p_1, \ldots, p_{U_A}]$ have to be determined such that the user demands $\mathbf{D}$ are fulfilled. In this section, it is shown that there exists no linear relation between p_u and D_u. In fact, p_u does not only depend D_u but on the entire demand vector $\mathbf{D}$. Afterwards, it is shown how to calculate the user data rate and BER of adaptive users analytically.

Without loss of generality, it is assumed that the users are sorted by their user demand in descending order, i.e. $D_{u-1} \geq D_u$. In order to determine how many resource units are allocated to an adaptive user u, it is essential to determine the probability that a resource unit is allocated to a given user u. Assuming there are N_{ru} resource units from which W_A are taken into account for scheduling the adaptive users, the probability $P^{(u)}(w, N_{\text{ru}}, \mathbf{p})$ that the w-th best resource unit out of N_{ru} resource units with $w = 1, \ldots, W_A$ is allocated to user u is given by

$$P^{(u)}(w, N_{\text{ru}}, \mathbf{p}) = N_{\text{ru}} \cdot \binom{N_{\text{ru}} - 1}{w - 1} \int_0^\infty \left(\frac{1}{p_u} \right)$$
$$\cdot e^{-\frac{\gamma}{p_u}} \cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_A} \left(1 - e^{-\frac{\gamma}{p_i}} \right) \right) \tag{26}$$
$$\cdot \left(1 - \prod_{i=1}^{U_A} \left(1 - e^{-\frac{\gamma}{p_i}} \right) \right)^{w-1}$$
$$\cdot \left(\prod_{i=1}^{U_A} \left(1 - e^{-\frac{\gamma}{p_i}} \right) \right)^{N_{\text{ru}} - w} d\gamma.$$

The first terms in the integral of (26) outside the bracket represents the probability that the weighted and normalized SNR value of user u has the value γ. The first bracket term represents the probability that the weighted and normalized SNR value of all other users in this resource unit is smaller than γ taking into account that γ is exponentially distributed resulting from (2). The second bracket term represents the probability that there are $w - 1$ resource units whose highest WPFS ratio is higher than the value γ. The third bracket term represents the probability that there are $N_{\text{ru}} - w$ resource units whose highest WPFS ratio is smaller than the value γ, i.e., user u has the highest WPFS ratio in the w-th best resource unit out of N_{ru} resource units. The factor N_{ru} in front of the integral takes into account the N_{ru} possible positions of the w-th best resource unit inside the total number N_{ru} of resource units. The factor $\binom{N_{\text{ru}} - 1}{w - 1}$ takes into account the possible positions of the $w - 1$ better resource units inside the remaining $N_{\text{ru}} - 1$ resource units.

Applying the binomial theorem, (26) can be written as

$$P^{(u)}(w, N_{\text{ru}}, \mathbf{p}) = N_{\text{ru}} \cdot \binom{N_{\text{ru}} - 1}{w - 1} \cdot \sum_{\varepsilon=0}^{w-1} \binom{w - 1}{\varepsilon} \cdot (-1)^\varepsilon$$
$$\cdot \sum_{v=1}^{U_A'} (-1)^{v-1} \cdot \frac{1}{p_u'} \cdot \sum_{|\eta|=v-1} \frac{1}{\frac{1}{p_u'} + \sum_{i=1}^{U_A'} \frac{\eta_i}{p_i'}} \tag{27}$$

with the extended weighting vector $\mathbf{p}'$ of length $U_A' = (\varepsilon + N_{\text{ru}} - w + 1) \cdot U_A$ given by

$$\mathbf{p}' = \underbrace{[\mathbf{p} \quad \mathbf{p} \quad \cdots \quad \mathbf{p}]}_{(\varepsilon + N_{\text{ru}} - w + 1) \text{ - times}} \tag{28}$$

and with the multi-index $\eta = [\eta_1, \eta_2, \ldots, \eta_{U_A'}]$ where $\eta_u = 0$ and $\eta_j \in \{0, 1\} \, \forall j = 1, \ldots, U_A'$ with $j \neq u$. From this it follows that the expression $|\eta| = v - 1$ represents all possible realizations of η where the sum over the elements of η equals $v - 1$.

As W_A resource units in total are allocated to the adaptive users, the average number of resource units allocated to user u is given by

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_A} P^{(u)}(i, N_{\mathrm{ru}}, \mathbf{p}). \tag{29}$$

Assuming that the weighting factor of the user u' with the lowest demand is set to $p_{u'} = 1$, $U_A - 1$ weighting factors have to be determined such that the user demands are fulfilled:

$$E\{N_{\mathrm{ru},u}\} = \sum_{i=1}^{W_A} P^{(u)}(i, N_{\mathrm{ru}}, \mathbf{p}) = D_u \tag{30}$$

for all u with $u = 1, \ldots, U_A - 1$. (30) defines a set of equations with $U_A - 1$ equations and $U_A - 1$ variables which has one unique solution for the weighting factors $\mathbf{p}$. Due to the complex structure of (27), it is not possible to derive a closed form solution for $\mathbf{p}$. One way to find a solution numerically is to apply the following constrained nonlinear optimization problem:

$$\mathbf{p} = \arg\min_{\mathbf{p}} \left\{ \sum_{u=1}^{U_A-1} \left| \sum_{i=1}^{W_A} P^{(u)}(i, N_{\mathrm{ru}}, \mathbf{p}) - D_u \right| \right\}$$
$$\text{s.t.} \tag{31}$$
$$p_u \geq 1$$

using for example the *fmincon* function in MATLAB™. However, problem (31) is non convex, i.e., it is not possible to guarantee that the unique solution is found in every case.

SNR distribution
To determine the PDF $p_{\hat{\gamma}}^{(u)}(\hat{\gamma})$ of the measured SNR values of the resource units allocated to user u, the PDF $p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma})$ of the measured SNR of the allocated resource units from the w-th best out of N_{ru} resource units has to be derived in a first step.

Since the estimated SNR values of different users and resource units are independent from each other and with the knowledge that the estimated SNR values $\hat{\gamma}_u$ are exponentially distributed with a PDF given by

$$p_{\hat{\gamma}_u}(\hat{\gamma}_u) = \frac{1}{\bar{\gamma}_{\mathrm{E},u}} \exp\left(-\frac{\hat{\gamma}_u}{\bar{\gamma}_{\mathrm{E},u}}\right) \tag{32}$$

with $\bar{\gamma}_{\mathrm{E},u} = \bar{\gamma}_u \cdot (1 + \sigma_{\mathrm{E},u}^2)$, the joint PDF of all $N_{\mathrm{ru}} \cdot U_A$ estimated SNR values $X_1, \ldots, X_{U_{N_{\mathrm{ru}} \cdot A}}$ is given by

$$p_{X_1,\ldots,X_{N_{\mathrm{ru}} \cdot U_A}}(x_1, \ldots, x_{N_{\mathrm{ru}} \cdot A}) = p_{\hat{\gamma}_u}(x_1) \cdot p_{\hat{\gamma}_u}(x_2)$$
$$\tag{33}$$
$$\cdots p_{\hat{\gamma}_u}(x_{N_{\mathrm{ru}} \cdot U_A}).$$

Hence, $p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma})$ is then given by the marginal PDF resulting from determining the integral over the joint PDF leading to

$$p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma}) = a_w(u)$$

$$\cdot \underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \cdots \int_0^{\frac{p_u}{p_{U_A}}\hat{\gamma}}}_{U_A-1 \text{ - times}} \underbrace{\int_0^{\frac{p_u}{p_1}\hat{\gamma}} \cdots \int_0^{\frac{p_u}{p_{U_A}}\hat{\gamma}}}_{(N_{\mathrm{ru}}-w)\cdot U_A \text{ - times}} \underbrace{\int_{\frac{p_u}{p_1}\hat{\gamma}}^{\infty} \cdots \int_{\frac{p_u}{p_{U_A}}\hat{\gamma}}^{\infty}}_{(w-1)\cdot U_A \text{ - times}}$$

$$p_{X_1,\ldots,X_{N_{\mathrm{ru}} \cdot U_A}}(x_1, \ldots, x_{N_{\mathrm{ru}} \cdot U_A}) \, dx_1 \ldots dx_{N_{\mathrm{ru}} \cdot U_A - 1}$$

$$= \sum_{\varepsilon=0}^{w-1} \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon} \cdot a_w(u) \left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}}\right) \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{\mathrm{E},u}}}$$

$$\cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_A \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)} \left(1 - e^{-\frac{p_u'\hat{\gamma}}{p_i'\bar{\gamma}_{\mathrm{E},u}}}\right) \right) \tag{34}$$

where the factor $a_w(u)$ ensures that $\int\limits_0^{\infty} p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma}) d\hat{\gamma} = 1$ leading to

$$a_w(u) = \frac{1}{P^{(u)}(w, N_{\mathrm{ru}}, \mathbf{p})}. \tag{35}$$

To finally determine the PDF $p_{\hat{\gamma}}^{(u)}(\hat{\gamma})$ of the measured SNR values of the resource units allocated to user u, the sum over the W_A PDFs $p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma})$ with $w = 1, \ldots, W_A$ weighted by the probability $P^{(u)}(w, N_{\mathrm{ru}}, \mathbf{p})$ has to be calculated leading to

$$p_{\hat{\gamma}}^{(u)}(\hat{\gamma}) = \sum_{w=1}^{W_A} \frac{P^{(u)}(w, N_{\mathrm{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_A} P^{(u)}(\xi, N_{\mathrm{ru}}, \mathbf{p})} \cdot p_{w,\hat{\gamma}}^{(u)}(\hat{\gamma}) \tag{36}$$

$$= \sum_{w=1}^{W_A} \frac{P^{(u)}(w, N_{\mathrm{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_A} P^{(u)}(\xi, N_{\mathrm{ru}}, \mathbf{p})} \sum_{\varepsilon=0}^{w-1} \cdot \binom{w-1}{\varepsilon} \cdot (-1)^{\varepsilon}$$

$$\cdot a_w(u) \cdot \left(\frac{1}{\bar{\gamma}_{\mathrm{E},u}}\right) \cdot e^{-\frac{\hat{\gamma}}{\bar{\gamma}_{\mathrm{E},u}}}$$

$$\cdot \left(\prod_{\substack{i=1 \\ i \neq u}}^{U_A \cdot (\varepsilon + N_{\mathrm{ru}} - w + 1)} \left(1 - e^{-\frac{p_u'\hat{\gamma}}{p_i'\bar{\gamma}_{\mathrm{E},u}}}\right) \right)$$

where the factor $\left(\sum_{\xi=1}^{W_A} P^{(u)}(\xi, N_{\mathrm{ru}}, \mathbf{p})\right)^{-1}$ ensures that $\int_0^{\infty} p_{\hat{\gamma}}^{(u)}(\hat{\gamma}) d\hat{\gamma} = 1$. The cumulative distribution function (CDF) of the measured SNR values of the resource units allocated to user u is determined by integrating (36) resulting in

$$F_{\hat{\gamma}}^{(u)}(\hat{\gamma}) = \sum_{w=1}^{W_A} \frac{P^{(u)}(w, N_{\text{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_A} P^{(u)}(\xi, N_{\text{ru}}, \mathbf{p})} \sum_{\varepsilon=0}^{w-1} (-1)^{\varepsilon} \cdot \binom{w-1}{\varepsilon}$$

$$\cdot \left(\frac{a_w(u)}{\bar{\gamma}_{E,u}} \right) \cdot \sum_{v=1}^{U_A'} \sum_{|\eta|=v-1} \frac{(-1)^{v-1}}{\frac{1}{p_u'} + \sum_{i=1}^{U_A'} \frac{\eta_i}{p_i'}} \quad (37)$$

$$\cdot \left(1 - \exp \left(-\frac{p_u'\hat{\gamma} \left(\frac{1}{p_u'} + \sum_{i=1}^{U_A'} \frac{\eta_i}{p_i'} \right)}{\bar{\gamma}_{E,u}} \right) \right)$$

with U_A', $\mathbf{p}'$ and η as defined in (27) and (28).

Average user data rate

In the following, the average user data rate $\bar{R}_A^{(u)}$ of user u is defined as the sum rate of the different modulation constellations weighted by their probability. With the SNR threshold vector $\gamma_{\text{th}}^{(u)}$ and the CDF of the measured SNR values of the resource units allocated to user u given by (37), the average data rate $\bar{R}_A^{(u)}$ of user u can be written as

$$\bar{R}_A^{(u)} = \sum_{m=1}^{M} b_m \cdot \left(F_{\hat{\gamma}}^{(u)}(\gamma_{\text{th},m}^{(u)}) - F_{\hat{\gamma}}^{(u)}(\gamma_{\text{th},m-1}^{(u)}) \right). \quad (38)$$

Average user BER

The average BER $\overline{\text{BER}}_A^{(u)}$ applying imperfect CQI is formulated as the sum of the number of errors of the different modulation constellations divided by the average bit rate [25]. Introducing $p_{\gamma|\hat{\gamma}}^{(u)}(\gamma|\hat{\gamma})$ as the conditional PDF of the actual SNR γ and the outdated noisy SNR $\hat{\gamma}$ of user u, it can be shown that the conditional PDF is given by

$$p_{\gamma|\hat{\gamma}}^{(u)}(\gamma|\hat{\gamma}) = \frac{1}{\bar{\gamma}_u \sigma_{r,u}^2} \cdot e^{-\frac{\mu_u^2 \cdot \hat{\gamma} + \gamma}{\bar{\gamma}_u \sigma_{r,u}^2}} \cdot I_0 \left(\frac{2\mu_u \sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}_u \sigma_{r,u}^2} \right) \quad (39)$$

with $\sigma_{r,u}^2 = \frac{1+\sigma_{E,u}^2 - \rho_u^2}{1+\sigma_{E,u}^2}$, $\mu_u = \frac{\rho_u}{1+\sigma_{E,u}^2}$ and $I_0(x)$ denoting the 0th-order modified Bessel function of the first kind. The average BER $\overline{\text{BER}}_A^{(u)}$ is then given by

$$\overline{\text{BER}}_A^{(u)} = \frac{1}{\bar{R}_A^{(u)}} \sum_{m=1}^{M} b_m \cdot \int_{\gamma_{\text{th},m-1}^{(u)}}^{\gamma_{\text{th},m}^{(u)}} p_{\hat{\gamma}}^{(u)}(\hat{\gamma}) \quad (40)$$

$$\cdot \left[\int_0^\infty \text{BER}_m(\gamma) \cdot p_{\gamma|\hat{\gamma}}^{(u)}(\gamma|\hat{\gamma}) \, d\gamma \right] d\hat{\gamma}.$$

Inserting (24), (36) and (39) in (40) and introducing the functions

$$\Psi(m) = 1 + \beta_m \bar{\gamma}_u \sigma_{r,u}^2 \quad (41)$$

and

$$\Upsilon(m, \eta) = \left(1 + \sum_{i=1}^{U_A'} \frac{p_u' \cdot \eta_i}{p_i'} \right) \cdot \Psi(m) + \bar{\gamma}_{E,u} \beta_m \mu_u^2 \quad (42)$$

with η, $\mathbf{p}'$ and U_A' as defined in (27), (40) can be written as closed form as

$$\overline{\text{BER}}_A^{(u)} = \sum_{w=1}^{W_A} \frac{P^{(u)}(w, N_{\text{ru}}, \mathbf{p})}{\sum_{\xi=1}^{W_A} P^{(u)}(\xi, N_{\text{ru}}, \mathbf{p})} \cdot \sum_{\varepsilon=0}^{w-1} (-1)^{\varepsilon}$$

$$\cdot \binom{w-1}{\varepsilon} \cdot \frac{a_w(u)}{5 \cdot \bar{R}_A^{(u)}} \quad (43)$$

$$\cdot \sum_{m=1}^{M} b_m \cdot \sum_{v=1}^{U_A'} (-1)^{v-1} \sum_{|\eta|=v-1} \left(\frac{1}{\Upsilon(m, \eta)} \right)$$

$$\cdot \left[\exp \left(\frac{-\gamma_{\text{th},m-1}^{(u)} \Upsilon(m, \eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right) \right.$$

$$\left. - \exp \left(\frac{-\gamma_{\text{th},m}^{(u)} \Upsilon(m, \eta)}{\bar{\gamma}_{E,u} \Psi(m)} \right) \right].$$

Optimizing user data rate

In the following, the optimal SNR threshold vector $\gamma_{\text{th}}^{(u)}$ to solve (19) is determined.

Non-adaptively served users

As shown in Section "Average user data rate", the user data rate only depends one the applied modulation scheme with index m. Hence, optimizing the data rate of non-adaptive users, subproblem (19) can be simplified to

$$\bar{R}_{\text{N,opt}}^{(u)} = \max_m \left(\bar{R}_N^{(u)} \right) \quad (44)$$

$$\text{subject to}$$

$$\overline{\text{BER}}_N^{(u)} \le \text{BER}_T.$$

resulting in

$$\bar{R}_{\text{N,opt}}^{(u)} = \left\lfloor \frac{1}{A} \cdot \log_2 \left(B + \frac{C \cdot \bar{\gamma}_u (5\text{BER}_T)^{1/D_u}}{D_u (1 - (5\text{BER}_T)^{1/D_u})} \right) \right\rfloor \quad (45)$$

with $A = 1$, $B = 1$ and $C = 1.6$ using M-QAM modulation and $A = 1.9$, $B = -1$ and $C = 7$ using M-PSK modulation, respectively. Note that using (45), it is possible to determine the minimum user data rate $\bar{R}_{\text{min}}^{(u)} = \bar{R}_{\text{N,opt}}^{(u)}$ from the number D_u of requested resources as stated in Section "Different user channel access demands".

Adaptive users

To solve (19) for adaptive users, a Lagrange multiplier approach can be performed. As shown in [25], this leads to $M - 1$ equations given by

$$\frac{(1 - \lambda \text{BER}_\text{T})}{\lambda} = \frac{1}{b_{m+1} - b_m}$$
$$\cdot \left(\zeta^{(u)}(m, \gamma_{\text{th},m}^{(u)}, \sigma_{\text{E},u}^2, \rho_u) \cdot b_m \right.$$
$$\left. - \zeta^{(u)}(m+1, \gamma_{\text{th},m}^{(u)}, \sigma_{\text{E},u}^2, \rho_u) \cdot b_{m+1} \right) \quad (46)$$

with λ denoting the Lagrange multiplier and

$$\zeta^{(u)}(m, \hat{\gamma}, \sigma_{\text{E},u}^2, \rho_u) = \frac{0.2}{1 + \beta_m \bar{\gamma}_u \sigma_{r,u}^2} \cdot \exp\left(-\frac{\hat{\gamma} \rho_u^2 \beta_m}{1 + \beta_m \bar{\gamma}_u \sigma_{r,u}^2} \right) \quad (47)$$

the solution of the inner integral of (40). From (46) it can be seen that each element $\gamma_m^{(u)}$ of the optimal SNR threshold vector $\gamma_{\text{th,opt}}^{(u)}$ can be calculated using an initial value $\gamma_{\text{th},1}^{(u)}$. Thus, each threshold vector $\gamma_{\text{th}}^{(u)}$ is a function of the initial value $\gamma_{\text{th},1}^{(u)}$, i.e., $\gamma_{\text{th}}^{(u)} = f(\gamma_{\text{th},1}^{(u)})$. Determining the maximum average data rate subject to the target BER, the optimal initial value $\gamma_{\text{th},1,\text{opt}}^{(u)}$ which fulfills $\overline{\text{BER}}_\text{A}^{(u)}(f(\gamma_{\text{th},1,\text{opt}}^{(u)})) \leq \text{BER}_\text{T}$ has to be found resulting in $\bar{R}_{\text{A,opt}}^{(u)} = \bar{R}_\text{A}^{(u)}\left(\gamma_{\text{th,opt}}^{(u)}\right)$, which can be done numerically using for example the *fzero* function in MATLAB™. Note that from (37) and (38) it can be seen that $\bar{R}_\text{A}^{(u)}$ is a non convex function. Hence, it is not possible to guarantee that the global optimum can be found in any case.

The user serving problem

For the analytical calculation of the user performance and the optimization of the SNR thresholds of the applied modulation schemes shown in Section "The SNR threshold problem", it was assumed that the user serving vector ϑ was already given. In the following, it is shown how to determine ϑ such that the average system data rate is maximized while all users fulfill the target BER and the minimum data rate requirements.

The problem to be solved is given by (20). As stated before, the minimum user data rate $\bar{R}_{\min}^{(u)}$ each user u shall achieve is given by the average user data $\bar{R}_{\text{N,opt}}^{(u)}$ achievable when applying the non-adaptive transmission mode, i.e., $\bar{R}_{\min}^{(u)} = \bar{R}_{\text{N,opt}}^{(u)}$. In the following, an exhaustive search algorithm and a reduced complexity algorithm are presented.

Exhaustive search algorithm

The most time-consuming way to solve (20) is an exhaustive search, i.e., all possible user serving vectors ϑ are tested to find the best vector according to (20), i.e., for each possible number U_A of adaptive users there exists $\binom{U}{U_\text{A}}$ possible realisations of ϑ. Hence, $\sum_{u=1}^{U} \binom{U}{u} = 2^U$ possible realizations of ϑ have to be tested, which can become prohibitively complex for large numbers U of users.

Reduced complexity algorithm

From the analytical expressions of the average user data derived in Section "The SNR threshold problem" it could be seen that besides the SNR thresholds, the data rate of user u depends on the weighting vector **p**. To be more precise, it depends on the number $|\mathcal{G}_i|$ of adaptive users in each demand group $\mathcal{G}_i$ with $i = 1, \ldots, G$, i.e., the number of users with a certain weighting factor p_i against which user u has to compete successfully in order to get access to a given resource unit. From this, it follows that for the calculation of $\bar{R}_{\text{A,opt}}^{(u)}(\vartheta)$ it is not decisive which of the users are served adaptively inside a certain demand group $\mathcal{G}_i$, but only how many users $|\mathcal{G}_i|$ are served inside this group. Exploiting this fact, an algorithm with lowered complexity referred to as RedCom algorithm can be found which optimally solves (20). Like in an exhaustive search, all possible numbers U_A of adaptive users are tested. Assuming there are G different demand groups, for each possible number U_A of adaptive users there exist a G-tuple

$$\Xi_{U_\text{A}} = \{\mu_{U_\text{A},1}, \mu_{U_\text{A},2}, \ldots, \mu_{U_\text{A},G}\} \quad (48)$$

with $\mu_{U_\text{A},i}$ denoting the number of adaptively served users inside demand group $\mathcal{G}_i$ with $\mu_{U_\text{A},i} \leq |\mathcal{G}_i|$ and $\sum_{i=1}^{G} \mu_{U_\text{A},i} = U_\text{A}$. Note that for G demand groups with $|\mathcal{G}_i|$ users in the i-th demand group $\mathcal{G}_i$, there exist

$$N_{\text{tup}} = \prod_{i=1}^{G} (|\mathcal{G}_i| + 1) \quad (49)$$

different G-tuples in total.

Since the data rate of each user u does not depend on the user serving vector, but only on the number of adaptive users inside each demand group, it is enough to determine for each G-tuple Ξ_{U_A} the $\mu_{U_\text{A},i}$ users in each demand group $\mathcal{G}_i$ which achieve the highest gain when served adaptively compared to the case when served non-adaptively, instead of testing all $\binom{U}{U_\text{A}}$ possible user serving vectors. In the end, the system data rates of the best user serving vectors for all possible numbers U_A of adaptive users have to be compared to find the optimal user serving vector. Note that for the extreme case of $G = U$ with $|\mathcal{G}_i| = 1$, i.e., each user has a different weighting factor p_u, the number of tuples to be checked equals $N_{\text{tup}} = \prod_{i=1}^{U}(1 + 1) = 2^U$ i.e., in this case the RedCom algorithm is equivalent to the ES algorithm. The pseudo code of the RedCom algorithm is outlined as follows:

(1) Determine $\bar{R}_{\text{N,opt}}^{(u)}$ for each user u.
(2) Determine $\bar{R}_{\text{A,opt}}^{(u)}(U_\text{A}, \rho_u, \sigma_{\text{E},u}^2)$ for each G-tuple Ξ_{U_A} for $U_\text{A} = 1, \ldots, U$ for each user u.
(3) Determine $\bar{R}_{\text{sys}}(0)$ for the case of no adaptive user ($U_\text{A} = 0$), i.e. $\vartheta_u = 0 \; \forall \; u$.
(4) Set the number of adaptive users to $U_\text{A} = 1$.

(5) Determine the difference
$\Delta_u(\Xi_{U_A}) = \bar{R}_{A,opt}^{(u)}(\Xi_{U_A}, \rho_u, \sigma_{E,u}^2) - \bar{R}_{N,opt}^{(u)}$ for each
G-tuple Ξ_{U_A} for each user u.

(6) For each demand group $\mathcal{G}_i$ find the $\mu_{U_A,i}$ users
with the highest non-negative $\Delta_u(\Xi_{U_A})$.

(7) If there exist no $\mu_{U_A,i}$ users with non-negative
$\Delta_u(\Xi_{U_A})$ for none of the G-tuples Ξ_{U_A}, store
$\bar{R}_{sys}(U_A) = 0$ and go to 10), else set $\vartheta_u(\Xi_{U_A}) = 1$
for these users.

(8) For each G-tuple Ξ_{U_A} compute $\bar{R}_{sys}(\Xi_{U_A})$ and
determine the G-tuple which achieves the highest
system data rate for U_A adaptive users.

(9) Store the user serving vector corresponding to the
best G-tuple as $\vartheta(U_A)$ and the corresponding
system data rate as $\bar{R}_{sys}(U_A)$.

(10) If $U_A = U$, go to 11), else increase $U_A \to U_A + 1$
and go back to 5).

(11) Find the optimal number of adaptive users $U_{A,opt}$
by determining the maximum system data rate
$\bar{R}_{sys}(U_{A,opt}) = \max_{U_A} \bar{R}_{sys}(U_A)$ with $U_A = 0, \ldots, U$.
The optimal user serving vector is then given by
$\vartheta(U_{A,opt})$.

Complexity analysis

As the user serving vector has to be updated at regular intervals, it is important for practical applications to know the complexity of the different algorithms. For the SNR threshold problem, the computational complexity is less critical, as the calculation of $\bar{R}_{N,opt}^{(u)}$ and $\bar{R}_{A,opt}^{(u)}(\Xi_{U_A}, \rho_u, \sigma_{E,u}^2)$ for all G-tuples Ξ_{U_A} with $U_A = 1, \ldots, U$ for all users can be performed offline for certain sets of system and CQI imperfectness parameters and stored in a look-up table, so this computational complexity is not considered.

ES algorithm

Applying the ES algorithm, U values have to be read out from the look-up table 2^U times. Further on, U values have to be added 2^U times. Finally, 2^U values have to be compared, resulting in a total number of

$$N_{O,ES} = 2^U \cdot (2U + 1) \qquad (50)$$

operations.

RedCom algorithm

Applying the RedCom algorithm, $N_{tup} \cdot U$ times a value from the look-up table has to be read out. Furthermore, $N_{tup} \cdot U$ substractions are performed. For each G-tuple, G sorting operations of $|\mathcal{G}_i|$ values with $i = 1, \ldots, G$ have to be done. Further on, for each G-tuple, U additions have to be performed. Finally, N_{tup} comparisons are made resulting in a total number of

$$N_{O,RedCom} = \left(\prod_{i=1}^{G} (|\mathcal{G}_i| + 1) \right) \cdot \left(2U + \sum_{i=1}^{G} |\mathcal{G}_i|^2 + 1 \right) \qquad (51)$$

operations. Note that for the complexity considering the sorting of U unsorted values, the worst-case complexity [38] is assumed, i.e., U^2 operations are assumed. In Figure 2, the number of required operations is depicted as a function of the number U of users for the different algorithms for different numbers G of demand groups where it is assumed that $|\mathcal{G}_i| = \lfloor \frac{U}{G} \rfloor \; \forall \, i = 1, \ldots, G$. It can be seen that the higher the number G of different demand groups, the higher the complexity. For the case $G = U$, the complexity of the RedCom algorithm is equivalent to the ES algorithm. However, for cases with $G < U$, the reduction of complexity of the RedCom-algorithm compared to the ES algorithm is tremendous, especially for large number U of users. It has to be noted that for practical applications, it is reasonable to assume that the CQI imperfectness parameters ρ_u and $\sigma_{E,u}^2$ do not change significantly over several OFDM symbols, i.e., the user serving vector can be kept constant for several OFDM symbols. From this it follows the user serving problem does not have to be solved within an OFDM symbol but in a larger time scale which makes it feasible for practical applications.

Simulation results

In the following, the hybrid transmission scheme is compared with conventional pure adaptive and the pure non-adaptive OFDMA schemes in the presence of imperfect user-dependent CQI. The two parameters describing the CQI impairment are the estimation error variance $\sigma_{E,u}^2$ and the correlation coefficient ρ_u. As $\sigma_{E,u}^2$ is directly linked with the average SNR of user u and, thus, determined by the scenario, only ρ_u is the remaining CQI impairment
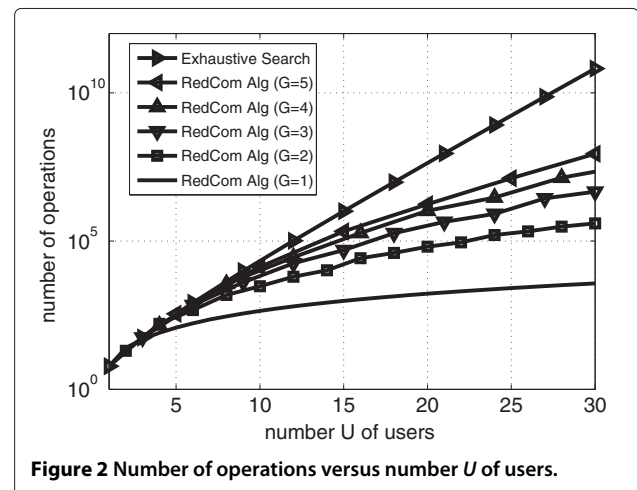

Figure 2 Number of operations versus number *U* of users.

parameter which is used as variable to analyze the system performance. As ρ_u is directly linked with the MS velocity of each user and each user has a different velocity as stated in Section "System model and assumptions", the average MS velocity $\bar{v}$ is the variable which indicates in the following how much outdated the CQI is in the cell.

For the pure adaptive system, two types of schemes are considered: Firstly, a naive approach where the BS always assumes perfect CQI, i.e., the SNR threshold vectors are calculated assuming perfect CQI for all users. Secondly, a pure adaptive scheme which is aware of the CQI imperfectness of each user and which adapts the SNR threshold vectors correspondingly, i.e., in case of imperfect CQI, the selection of the applied modulation schemes is performed more conservatively compared to the naive approach in order to fulfill the BER requirements. In case that the target BER is not fulfilled, the data rate of a user u is defined to be zero, i.e., $\bar{R}^{(u)} = 0$.

In the following, an OFDMA scenario with the parameters given in Table 1 is assumed.

The transmit power P_T is adjusted in such a way that a user at the cell border with no reliable CQI can achieve the target BER applying the non-adaptive transmission scheme. Furthermore, the time delay between the CQI updates is assumed to be $T = 4\,\mathrm{ms}$ and the CQI values are noisy estimates based on $M_P = 1$ pilot. Moreover, the applied modulation schemes range from QPSK for users at the cell edge up to 128-QAM for users near the BS.

Furthermore, only one user demand group is assumed, i.e., $G = 1$ and the user demand vector is set to $\mathbf{D} = [5, 5, \ldots, 5]$ meaning that each of the $U = 25$ users demands five out of the $N_\mathrm{ru} = 125$ resource units. To evaluate the performance, 10,000 independent user position realizations in the cell are generated assuming uniformly distributed users as stated in Section "System model and assumptions". Note that with each position of a user u in the cell and the corresponding average SNR $\bar{\gamma}_u$, the minimum user data rate $R_{\min}^{(u)}$ this user shall achieve is determined by calculating the achievable user data rate serving this user non-adaptively using (45). For each of these user

position realizations, different MS velocities v_u are generated where the radial components of the MS velocities are half-normally distributed with mean $\bar{v}$ as shown in Section "System model and assumptions". The average system data rate is then averaged over these 10,000 realizations.

In Figure 3, the average system data rate is depicted as a function of the average MS velocity $\bar{v}$ in the cell for the different transmission schemes. As one can see, the pure non-adaptive scheme achieves a constant system data rate, since it does not depend on the reliability of the CQI, neglecting the effect of intercarrier interference due to Doppler shifts. In case of $\bar{v} = 0\,\mathrm{km/h}$, the pure adaptive transmission scheme and the hybrid transmission scheme achieve the same system data rate and outperform the non-adaptive scheme. However, when increasing the average MS velocity in the cell and, thus, the unreliability of the CQI, the performances of the pure adaptive scheme dramatically decrease, especially for the naive approach since now, due to the imperfect CQI, wrong users and modulation schemes are selected for transmission. This results in a BER which no longer fulfills the target BER requirements. For the pure adaptive scheme which is aware of the imperfect CQI, the decrease is less dramatic. However, at some point the system performance is worse than for the pure non-adaptive transmission scheme. Applying the hybrid scheme for an increasing MS velocity in the cell, the system performance is always equal to or better than both the pure adaptive and pure non-adaptive scheme. For large velocities, the performance of the hybrid scheme converges to the one of the pure non-adaptive scheme, since now all the users in the hybrid scheme are served applying the non-adaptive scheme due to the totally outdated CQI. This effect is also shown in Figure 4, where the average number U_A of adaptively served users is depicted as a function of the MS velocity $\bar{v}$.

Table 1 System parameters

Bandwidth	10 MHz
Number N of subcarriers	500
Frequency block size Q	4
Number U of users	25
Carrier frequency f_0	2 GHz
Target BER BER$_\mathrm{T}$	10^{-3}
Cell radius R	300 m
Minimum distance BS-MS d_0	10 m
Pathloss coefficient α	2.6

Figure 3 System data rate versus average MS velocity $\bar{v}$.

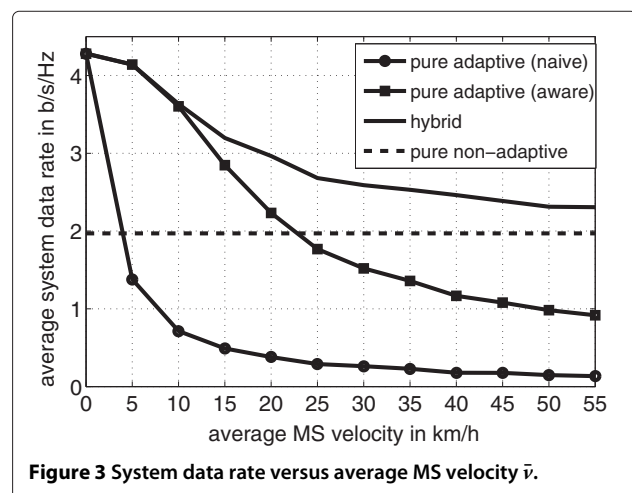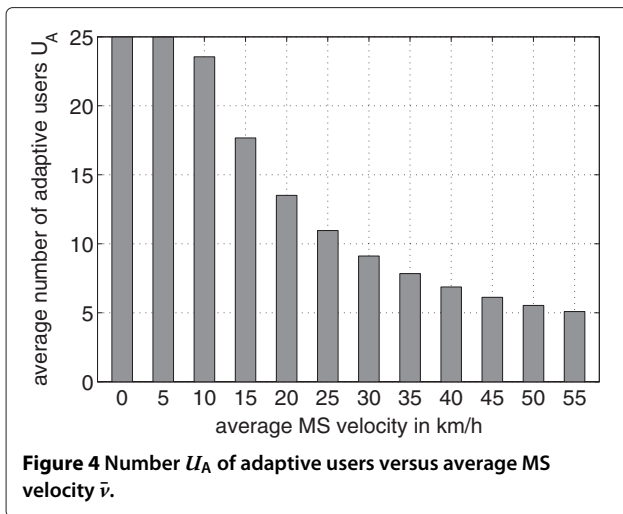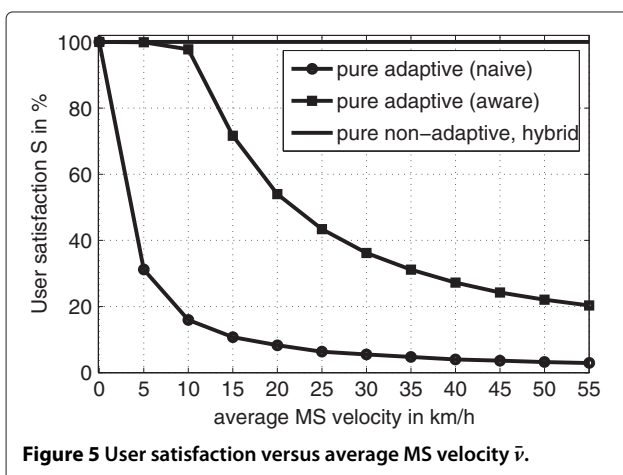**Figure 4 Number U_A of adaptive users versus average MS velocity $\bar{v}$.**

One can see that for low velocities, almost all of the $U = 25$ users are served adaptively. When increasing $\bar{v}$, more and more users are served non-adaptively.

To further compare the hybrid scheme with the conventional ones, another metric is introduced, namely the user satisfaction S which is defined as the percentage of users for which the minimum rate requirement is fulfilled. In Figure 5, the user satisfaction S is depicted as a function of the MS velocity $\bar{v}$. While applying the pure non-adaptive and the hybrid scheme, each user always achieves at least the minimum data rate, the user satisfaction decreases dramatically applying the pure adaptive schemes. Hence, the hybrid scheme outperforms the pure adaptive schemes also in terms of user satisfaction.

Conclusions

This article deals with the analytical description and evaluation of a hybrid multi-user OFDMA transmission scheme with different channel access user demands assuming user-specific imperfect CQI. The considered

hybrid transmission scheme offers two possible modes to serve the user: Firstly, via a non-adaptive OFDMA mode which applies a DFT precoding to exploit frequency diversity and, thus, does not require any channel knowledge at the transmitter. Secondly, via an adaptive OFDMA mode which performs an adaptive resource allocation and modulation scheme selection based on CQI to adjust to the current channel conditions. Assuming perfect CQI at the transmitter, the adaptive mode outperforms the non-adaptive mode due to a better adaptation to the channel. However, as the system performance of the adaptive mode suffers from CQI impairments such as estimation errors and time delays which could probably lead to a worse performance compared to the non-adaptive mode, the question arises which user shall be served adaptively or non-adaptively and which resource shall be allocated to which user such that the total system data rate is maximized while each user achieves a certain target BER and minimum user data rate. To answer this question, analytical expressions of the performances of the adaptive and non-adaptive transmission schemes as function of the parameters describing the CQI impairments and the user demands have been derived. Based on these expressions, algorithms which determine which user is served adaptively or non-adaptively subject to the BER and minimum data rate constraints have been developed. Simulations have shown that the hybrid OFDMA scheme outperforms pure adaptive and pure non-adaptive OFDMA transmission schemes in terms of achievable data rate and user satisfaction in the presence of user-specific imperfect CQI.

Methods

All simulations and calculations have been performed using MATLAB™.

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Figure 5 User satisfaction versus average MS velocity $\bar{v}$.

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