Hybrid Rate and Power Adaptive Distributed Spectrum Optimization for DSL Systems

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***Abstract*— Besides rate-adaptive spectrum optimization, power minimization in multi-carrier Digital Subscriber Line (DSL) systems subject to far-end crosstalk (FEXT) is an important topic as it allows to reduce energy consumption and at the same time avoid excessive crosstalk on lines from other systems. However, an optimal solution for this class of problems requires centralized algorithms with high complexity which is not feasible for practical systems. In this work, a semi-distributed, low-complexity and near-optimal algorithm generalizing Distributed Spectrum Balancing (DSB) is proposed for a general class of spectrum management problems which allows the provider to trade off data rates against power consumption. Results from numerical simulations demonstrate near-optimal performance and fast convergence.**

I. Introduction

With constantly growing data rate and bandwidth demands, the problem of FEXT between copper wires in telephone binders has become increasingly important and is in fact the dominant impairmant in current DSL systems [1]. In recent years, extensive research has been carried out to develop multi-user transmission techniques commonly referred to as Dynamic Spectrum Management (DSM) which mitigate the performance loss due to FEXT [2].

DSM Level 1 and 2 attempt to minimize interference between loops (in the following also interchangeably referred to as *users*) by optimizing the transmit power spectra of modems employing Discrete Multitone (DMT) transmission. DSM Level 1 can be realized using Iterative Water-Filling (IWF) [3] where each modem repeatedly updates its own power allocation based on the current measured crosstalk plus noise profile as well as control parameters such as target data rate and signal-to-noise ratio (SNR) margin received from a Spectrum Management Center (SMC). It is well known that IWF converges to a Nash equilibrium point which can be quite suboptimal w.r.t. some social utility such as a weighted sum rate of the multi-user system. In contrast, DSL systems managed by DSM Level 2 can achieve a social optimum via joint power allocation at the SMC, such as Optimal Spectrum Balancing (OSB) [4], provided that knowledge about the complete interference channel is available. Unfortunately, the centralized nature and high complexity render OSB impractical for real-world DSL deployments. DSB [5] is an interesting approach to DSM Level 2 spectrum optimization which overcomes both of the aforementioned limitations of OSB with only minimal loss in performance.

While most literature mentioned above concentrates on rate-adaptive spectrum optimization where a weighted sum rate is to be maximized, interest has grown recently in considering other objectives such as power minimization which are of equally practical importance. In typical DSL deployments, data rates are assigned to subscribers according to a service level which may lie well below the theoretical rates achieved by OSB. This potential gap can be exploited by designing power-adaptive spectrum balancing algorithms which minimize the total utilized transmit power in order to reduce energy consumption and at the same time avoid excessive crosstalk on lines from other systems.

The authors of [6] present a centralized Spectrum Balancing algorithm for power minimization derived from OSB. In [7], the general equivalence of the sum power minimization and sum rate maximization problem in the dual domain is established. This theoretical consideration has been supplemented by [8] where a general framework for spectrum management problems is developed which allow to trade off data rate maximization against power minimization.

In this work, a semi-distributed, low-complexity and near-optimal algorithm is proposed based on DSB where each transmitter updates its own power spectral density (PSD) so that the multi-user system converges to an operating point where the trade-off between data rate and power optimization is determined by the operator. The iterative optimization is carried out in such a way that the transmit spectra of individual users quickly converge to a power allocation which permits a service with predefined data rate and SNR safety margin. In contrast to IWF, Pareto-optimal operating points can be approached by allowing exchange of messages between transmitters about the global system state.

The remainder of the paper is structured as follows: Section II defines the system model for the DSL binder. A generalized spectrum management problem for rate and power adaption is introduced in Section III. The proposed distributed Spectrum Balancing algorithm is presented in Section IV. Finally, Section V discusses performance results for the new scheme obtained from simulations.

II. System Model

An N-user DSL binder is considered where users $n \in \mathcal{N}$ employ DMT transmission over K subcarriers (*tones*). Assuming perfect synchronization of the transmitters and a sufficiently long cyclic prefix, the tones $k \in \mathcal{K}$ can be modeled as K parallel interference channels where crosstalk from other loops is treated as noise. Let $g_k^{n,n} = |h_k^{n,n}|^2$ denote the channel gain of user n and $g_k^{n,m} = |h_k^{n,m}|^2$ ($n \neq m$) the FEXT coupling gain from disturber m to victim loop n on tone k. Let s_k^n denote the allocated transmit power of user n on tone k and σ_k^n the sum power of received noise and interference not managed by the DSM system. Using the Shannon-gap approximation [1], the number of bits per symbol that can be loaded on tone k of user n is given by

$$b_k^n = \log_2 \left(1 + \frac{1}{\Gamma} \frac{g_k^{n,n} s_k^n}{\sum_{m \neq n} g_k^{n,m} s_k^m + \sigma_k^n} \right) \tag{1}$$

where $\Gamma > 1$ denotes the SNR gap to capacity [1] which is a function of the code and target bit error rate (BER). In order to comply with DSL standards, the value of Γ can be chosen to include an SNR safety margin. The total utilized power P^n and data rate R^n of user n are given by

$$P^n = \sum_{k \in \mathcal{K}} s_k^n \quad \text{and} \quad R^n = f_s \sum_{k \in \mathcal{K}} b_k^n, \tag{2}$$

respectively, where f_s is the symbol rate of the DMT system.

III. The Generalized Spectrum Management Problem

This paper considers a generalized spectrum management problem similar to the one studied in [8]:

$$\begin{aligned} \min_{\boldsymbol{s}} \; & \sum_{n \in \mathcal{N}} (\alpha^n P^n - \beta^n R^n) && (3) \\ \text{subject to } & R^n \geq R^n_{\text{target}} && \forall n \in \mathcal{N} \\ & P^n \leq P^n_{\max} && \forall n \in \mathcal{N} \\ & 0 \leq s_k^n \leq s^n_{k,\text{mask}} && \forall n \in \mathcal{N}, k \in \mathcal{K} \end{aligned}$$

where the objective to be minimized w.r.t. the set $\boldsymbol{s} = \{s_k^n | n \in \mathcal{N}, k \in \mathcal{K}\}$ of power variables is the weighted sum of user transmit powers and data rates and (α^n, β^n) are the weights assigned to user n. Note that this choice of the objective can be reduced to either a purely rate-adaptive problem by setting $\alpha^n = 0$ for all $n \in \mathcal{N}$ or a power-adaptive problem where $\beta^n = 0$ for all $n \in \mathcal{N}$. The first constraint allows an operator to enforce a target data rate R^n_{target} for user n while the other two constraints impose aggregate power and spectral mask limits per user in order to account for the power limitation of the modem's analog front-end and comply with DSL standards. In the further study of problem (3) we assume that a feasible solution exists.

Due to the non-convexity of the bitloading (1) in the power variables s_k^n to be optimized, the target data rate constraint turns problem (3) into a global optimization problem which is known to be generally difficult to solve. Its dual w.r.t. power and data rate constraints is given by

$$\begin{aligned} \max_{\boldsymbol{\omega}, \boldsymbol{\lambda}} \; & g(\boldsymbol{\omega}, \boldsymbol{\lambda}) && (4) \\ \text{subject to } & \omega_n \geq 0 && \forall n \in \mathcal{N} \\ & \lambda_n \geq 0 && \forall n \in \mathcal{N} \end{aligned}$$

with dual objective

$$g(\boldsymbol{\omega}, \boldsymbol{\lambda}) = \inf_{\substack{\boldsymbol{s} \\ 0 \leq s_k^n \leq s^n_{k,\text{mask}}}} L(\boldsymbol{s}, \boldsymbol{\omega}, \boldsymbol{\lambda}) \tag{5}$$

and Lagrangian

$$\begin{aligned} L(\boldsymbol{s}, \boldsymbol{\omega}, \boldsymbol{\lambda}) &= \sum_{n \in \mathcal{N}} (\alpha^n P^n - \beta^n R^n) \\ &+ \sum_{n \in \mathcal{N}} \lambda^n (P^n - P^n_{\max}) + \sum_{n \in \mathcal{N}} \omega^n (R^n_{\text{target}} - R^n) \quad (6) \end{aligned}$$

where $\boldsymbol{\omega} = \{\omega^n | n \in \mathcal{N}\}$ is the set of dual variables ω^n associated with the per-user target rate constraints and $\boldsymbol{\lambda} = \{\lambda^n | n \in \mathcal{N}\}$ are the dual variables corresponding to the per-user power constraints of the primal (3). Note that the spectral mask constraints have not been replaced by dual variables since they can be easily taken into account directly. From basic duality theory it is known that (4) is convex although the primal is nonconvex. The slave problem of determining the value of $g(\boldsymbol{\omega}, \boldsymbol{\lambda})$ for fixed $(\boldsymbol{\omega}, \boldsymbol{\lambda})$ can be decoupled into per-tone subproblems [4] but remains difficult to solve due to the coupling across users.

By substituting $\tilde{\lambda}^n = \lambda^n + \alpha^n$ and $\tilde{\omega}^n = \omega^n + \beta^n$ in (6) it can easily be recognized that the resulting Lagrangian with dual variables $\tilde{\boldsymbol{\omega}} = \{\tilde{\omega^n} | n \in \mathcal{N}\}$ and $\tilde{\boldsymbol{\lambda}} = \{\tilde{\lambda^n} | n \in \mathcal{N}\}$ coincides with the Lagrangian of the rate-adaptive spectrum management problem. This implies that any dual optimization technique to solve the traditional rate-adaptive problem is also feasible for the power-adaptive dual problem (4) [9].

If the tone spacing Δf of the multi-carrier system is small enough so that the channel gains $g_k^{n,m}$ of adjacent tones expose a high degree of correlation, as is true for typical DSL binders, it has been shown that the duality gap of the spectrum management problem approaches zero and a primal solution can be obtained by solving (4) [10].

IV. Distributed Algorithm

While Spectrum Balancing algorithms such as OSB based on dual optimization can find a globally optimal solution with significantly reduced complexity compared to a exhaustive search in primal domain, they still exhibit a complexity that is infeasible for systems with more than a few users and additionally, rely on centralized optimization that has to take place at the SMC.

In recent years, significant progress has been made to derive low-complexity decentralized spectrum optimization techniques such as SCALE [11] or DSB [5] which are able to well approach the performance of OSB. In this section, we present a near-optimal algorithm for solving the hybrid power and rate-adaptive problem (3) based on the concepts of DSB which has originally been designed as a purely rate-adaptive algorithm. For this, we review the basic concepts of DSB and successively adapt them to our hybrid spectrum management problem.

The general approach of DSB is to find an approximation for the solution of the user-coupled spectrum management by iteration over N convex per-user subproblems which are decoupled from each other and have a water-filling type solution. Each user repeatedly updates its transmit power by solving its subproblem based on the currently measured noise profile similarily to IWF. However, in contrast to IWF, messages between modems are exchanged and incorporated into the local optimization process to achieve a jointly optimal solution. The iterative scheme of DSB is designed so that the system converges to a solution satisfying the Karush-Kuhn-Tucker (KKT) conditions of Problem (3) which may correspond to a local or global optimum.

The KKT conditions of Problem (3) are given by

$$\frac{\partial L(\boldsymbol{s},\boldsymbol{\omega},\boldsymbol{\lambda})}{\partial s_k^n} = 0 \qquad \forall n \in \mathcal{N}, k \in \mathcal{K} \qquad (7)$$
$$R^n \geq R^n_{\text{target}} \qquad \forall n \in \mathcal{N} \qquad (8)$$
$$P^n \leq P^n_{\max} \qquad \forall n \in \mathcal{N} \qquad (9)$$
$$0 \leq s_k^n \leq s^n_{k,\text{mask}} \qquad \forall n \in \mathcal{N}, k \in \mathcal{K} \qquad (10)$$
$$\omega_n \geq 0 \qquad \forall n \in \mathcal{N} \qquad (11)$$
$$\lambda_n \geq 0 \qquad \forall n \in \mathcal{N} \qquad (12)$$
$$\omega^n (R^n_{\text{target}} - R^n) = 0 \qquad \forall n \in \mathcal{N} \qquad (13)$$
$$\lambda^n (P^n - P^n_{\max}) = 0 \qquad \forall n \in \mathcal{N}. \qquad (14)$$

Note that each row can be split into per-user KKT conditions which, in a distributed scheme, can be solved locally at each transmitter.

Let int_k^n and rec_k^n denote the measured interference power and total received power of user n, respectively, which are both available in standard compliant DSL modems. By first rewriting $L(\boldsymbol{s},\boldsymbol{\omega},\boldsymbol{\lambda})$ as a difference of convex (d.c.) functions, the stationarity condition (7) can be transformed into a fixed-point equation w.r.t. s_k^n [5], yielding

$$s_k^n = \left[\frac{\frac{f_s}{\log(2)}\tilde{\omega}^n}{\tilde{\lambda}^n + W_k^n} - \frac{\text{int}_k^n}{\Gamma^{-1} g_k^{n,n}} \right]_0^{s^n_{k,\text{mask}}} \qquad (15)$$

where $[x]_0^{s^n_{k,\text{mask}}} = \min\left(\max(x,0), s^n_{k,\text{mask}}\right)$ and

$$W_k^n = \sum_{m \notin n} \frac{f_s g_k^{m,n}}{\log(2)} V_k^m \qquad (16)$$
$$V_k^n = \tilde{\omega}_n \left(\frac{1}{\text{int}_k^n} - \frac{1}{\text{rec}_k^n} \right) \qquad (17)$$
$$\text{int}_k^n = \sum_{m \neq n} g_k^{n,m} s_k^m + \sigma_k^n \qquad (18)$$
$$\text{rec}_k^n = \Gamma^{-1} g_k^{n,n} s_k^n + \text{int}_k^n. \qquad (19)$$

The term W_k^n, although dependent on s_k^n, is considered as constant in the fixed-point update rule (15).

The above equations (15)-(19) constitute the building blocks for the design of a distributed algorithm in the spirit of DSB. Given fixed int_k^n and W_k^n, the local subproblem for user n at each iteration is

$$\min_{\{s_k^n | k \in \mathcal{K}\}} \sum_{k \in \mathcal{K}} W_k^n s_k^n + \alpha^n P^n - \beta^n \tilde{R}^n \qquad (20)$$
$$\text{subject to } \tilde{R}^n \geq R^n_{\text{target}}$$
$$P^n \leq P^n_{\max}$$
$$0 \leq s_k^n \leq s^n_{k,\text{mask}} \qquad \forall k \in \mathcal{K}$$

where

$$\tilde{R}^n = f_s \sum_{k \in \mathcal{K}} \log_2 \left(1 + \frac{1}{\Gamma} \frac{g_k^{n,n} s_k^n}{\text{int}_k^n} \right) \qquad (21)$$

It can be verified that a KKT stationary solution for the convex problem (20) is found by (15). Thus, transmitter n can use (15) to find a stationary point for given dual variables (ω^n, λ^n) and updates (ω^n, λ^n) until conditions (8)-(14) are met. Once int_k^n and W_k^n have converged for all $n \in \mathcal{N}$ after repeatedly solving (20) und updating W_k^n via (16), the system reaches a KKT point satisfying conditions (7)-(14). For the distribution of messages W_k^n, assistance of the SMC according to Algorithm 1 is required which is why this scheme is not fully distributed. Note that a similar approach has been proposed in [12] for the power-adaptive problem based on the SCALE [11] algorithm.

Algorithm 1 Central steering algorithm of SMC

repeat
  Receive messages V_k^n $(k \in \mathcal{K})$ from users $n \in \mathcal{N}$
  Compute messages W_k^n $(k \in \mathcal{K})$ for users $n \in \mathcal{N}$ using (16)
  Transmit each W_k^n $(k \in \mathcal{K})$ to line $n \in \mathcal{N}$
until global convergence of spectra $\boldsymbol{s}$

We propose Algorithm 2 to solve the subproblem (20) of user n. It uses the subgradient rule

$$\omega^n \leftarrow \left[\omega^n + \mu(R^n_{\text{target}} - R^n)\right]^+ \qquad (22)$$

with step size μ and $[x]^+ = \max(0, x)$ to update ω^n and bisection search to find the optimal value of λ^n for a given value of ω^n. The maximum value $\lambda_{\max}$ can be chosen so that $\lambda_{\max} \gg \tilde{\omega}$. It has to be noted that while W_k^n have not fully converged, in contrast to the power constraint (9), the data rate constraint (8) cannot necessarily be satisfied. Thus the outer update loop of the associated dual variable ω^n has to stop when an increase of ω^n does not change the spectrum and/or data rate. Still, the outer loop should attempt to optimize ω^n so that a KKT point is approximated as closely as possible in order for the system to converge fast.

V. Simulation results

In this section, performance of the proposed hybrid power and rate-adaptive DSB algorithm which we refer to

Algorithm 2 Local algorithm of user n

repeat
  Receive messages W_k^n ($k \in \mathcal{K}$) from SMC
  repeat
    $\lambda^n \leftarrow \lambda_{\max}/2$
    $\Delta\lambda \leftarrow \lambda^n/2$
    repeat
      Update $\{s_k^n\}$ using (15)
      Calculate aggregate power P^n and data rate R^n using (2)
      if $P^n > P_{\max}^n$ **then**
        $\lambda^n \leftarrow \lambda^n + \Delta\lambda$
      else
        $\lambda^n \leftarrow \lambda^n - \Delta\lambda$
      end if
      $\Delta\lambda \leftarrow \Delta\lambda/2$
    until λ^n converged for given ω^n
    Update ω^n using (22)
  until spectrum $\{s_k^n\}$ converged
  Calculate messages V_k^n ($k \in \mathcal{K}$) using (17) and send to SMC
until global convergence of spectra $\boldsymbol{s}$

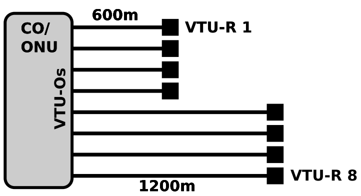


Fig. 1: 8-user VDSL2 upstream scenario

here as H-DSB is evaluated. For this, MATLAB simulations in an 8-user VDSL2 upstream scenario as depicted in Figure 1 have been carried out. Due to the near-far problem, this type of scenarios offers significant potential gains from spectrum optimization. Table I summarizes the relevant system parameters used for the simulations.

Figure 2 shows the rate region of $600\,\mathrm{m}$ loops versus $1200\,\mathrm{m}$ loops achieved by purely rate-adaptive DSB and IWF. The gap between both rate regions e.g. along the line of operating points where the $1200\,\mathrm{m}$ loops achieve a data rate of $1\,\mathrm{Mbps}$ indicates the potential savings in energy consumption by employing power-adaptive Spectrum Balancing rather than IWF. Since the rate-adaptive DSB and IWF algorithms only yield operating points on the boundary of the rate region, for a target rate of $1\,\mathrm{Mbps}$ for the $1200\,\mathrm{m}$ loops, IWF finds a power allocation corresponding to point A, while DSB finds a power allocation corresponding to point C. Unlike DSB however, besides point C, H-DSB can also find spectra so that the system operates at either point A, B, C, or any point of the rate region. Compared to IWF, H-DSB in power-adaptive mode can find a power allocation for point A with significantly reduced sum power. Indeed, simulation results reveal that the spectra obtained from IWF lead to a system sum power of $32.87\,\mathrm{mW}$ while the power allocation from H-DSB causes a system power consumption of only $9.03\,\mathrm{mW}$ which is a reduction of 72%.

TABLE I: Simulation parameters

Cable type	26-AWG [13]
FEXT model	ETSI 1% worst-case [13] without FSAN sum
Background noise level N_0	$-140\,\mathrm{dBm/Hz}$
Alien noise model	ETSI MD_EX [13]
VDSL2 band profile	998-ADE17M2x-B without US0 [14]
Symbol rate f_s	$4\,\mathrm{kHz}$
number of tones K	4096
Tone spacing Δf	$4.3125\,\mathrm{kHz}$
SNR gap Γ including $6\,\mathrm{dB}$ margin	$12.8\,\mathrm{dB}$

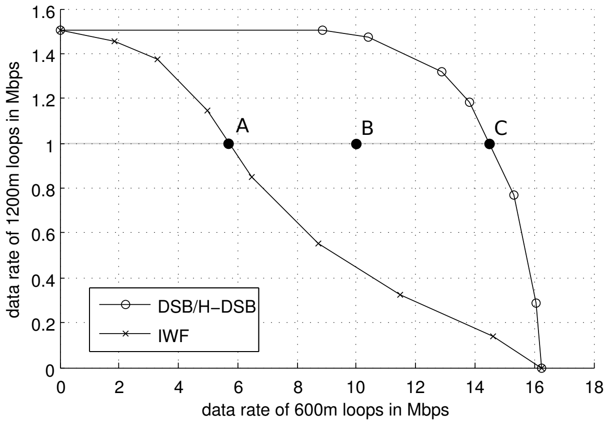
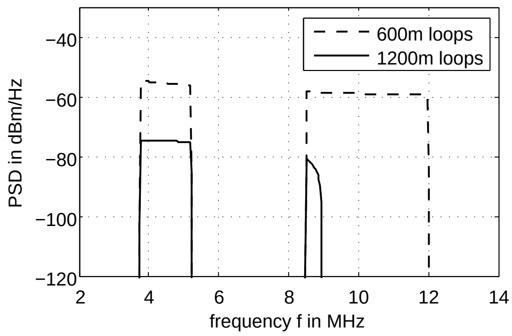


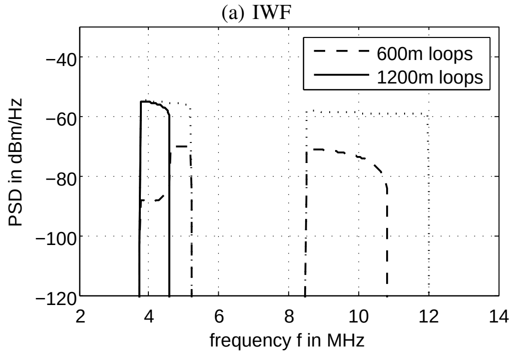
Fig. 2: Rate region of $600\,\mathrm{m}$ loops versus $1200\,\mathrm{m}$ loops

To better interpret the achievable power reduction from spectrum balancing, we study spectra for point A obtained from IWF and power-adaptive H-DSB shown in Figure 3. Clearly, IWF has to fully exploit the available power, in this scenario constrained by PSD masks, of at least one user in order to reach an operating point on the boundary of the rate region. In contrast, H-DSB induces politeness by imposing a power back-off on the lower tones of the $600\,\mathrm{m}$ users, thus actively combating the near-far problem. The obtained spectra make clear that for a given point in the rate region, multiple power allocations exist. In fact, by altering the weight factors α^n, an infinite number of feasible power allocations can be obtained from H-DSB for any tuple of target rates.

Finally, Figure 4 shows the convergence of user data rates and aggregate powers during the iterative update of power-adaptive H-DSB in the considered upstream scenario when starting with all-zero transmit powers and approaching operating point B with target rates $10\,\mathrm{Mbps}$ and $1\,\mathrm{Mbps}$ for the $600\,\mathrm{m}$ and $1200\,\mathrm{m}$ loops, respectively. It shows that after each user has updated its power allocation approximately four times, the desired target rates are achieved by all users and aggregate powers minimized.



(a) IWF



(b) H-DSB

Fig. 3: Transmit spectra for point A obtained from IWF and H-DSB.

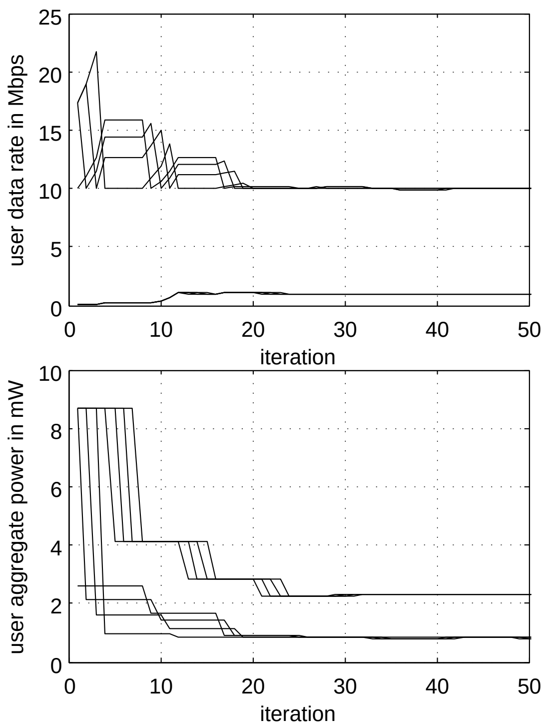


Fig. 4: Convergence of user data rate and aggregate powers during iteration of power-adaptive H-DSB

VI. Conclusion

In this work, a low-complexity semi-distributed Spectrum Balancing algorithm has been proposed which generalizes rate-adaptive DSB so that spectrum optimization can also be carried out power-adaptive. Simulation results showed that significant performance gains in terms of data rates or power reduction can be achieved compared to IWF. Also, very fast convergence of the scheme makes the scheme attractive for scenarios where the DSM system has to adapt to changing conditions in the binder.

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