H. Degenhardt, A. Klein ”Non-regenerative multi-antenna two-hop relaying under an asymmetric rate constraint,” in *Proc. of the IEEE Wireless Communications and Networking Conference (WCNC 2011)*, Cancun, Mexiko, March 2011.

Non-regenerative multi-antenna two-hop relaying under an asymmetric rate constraint

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***Abstract*—In this work, the combination of one-way and two-way relaying is examined in a scenario where two multi-antenna nodes exchange information under an asymmetric rate constraint via a half-duplex non-regenerative multi-antenna relay. Two-way relaying overcomes the multiplexing loss of one-way relaying, but additional optimizations are required to fulfill the asymmetric rate constraint. To enable the consideration of suboptimal low-complexity approaches, a hybrid one-way / two-way scheme is suggested which ensures that the asymmetric rate constraint can always be fulfilled. The optimization problem of maximizing the overall sum rate for the considered scenario is formulated and an approach for transceive filter and power optimization considering the asymmetric rate constraint is derived. Simulation results for different link qualities and rate constraints confirm the theoretical considerations and show that the proposed algorithm performs close to the upper bound in case of high asymmetric rate constraints or highly asymmetric channels.**

I. Introduction

In scenarios where a direct communication between two nodes S1 and S2 is not possible, a relay station RS can be inserted in between the nodes and a two-hop relaying scheme can be applied. In this paper, non-regenerative two-hop relaying is considered, i.e., linear signal processing is applied at RS. RS is assumed to be half-duplex and time-division duplex is used. Conventional two-hop relaying, i.e., one-way relaying, requires four time slots to establish a bidirectional communication between the nodes, because each node requires separate orthogonal resources for the transmission and reception to and from RS, respectively. To overcome this drawback, the authors in [1] proposed a two-way relaying scheme requiring only two time slots to support the bidirectional communication. In two-way relaying, S1 and S2 transmit simultaneously to RS, which receives, linearly processes and retransmits the superimposed signals. The desired signal at each node can be recovered by subtracting the back propagated self-interference in the receive signal, which requires channel state information (CSI) at the nodes. Two-way relaying is in particular suitable for the bidirectional symmetric transmission between two nodes and recent transceive filter approaches maximize the overall sum rate or minimize the mean square error without considering the individual rates.

The authors of [2] compare different transceive filters and derive sum-rate upper bounds for a two-way relaying scenario with single-antenna nodes and a multi-antenna relay. In [3], multi-antenna one-way and two-way relaying is extensively studied. In most cases, it is assumed that RS is equipped with at least the total number of antennas of both nodes and different transceive filters are compared. The transceive filters are designed to maximize the sum rate or to minimize the mean square error without considering any asymmetric rate constraint. The authors of [4] present an overview of recent advances in non-regenerative two-hop relaying and state that in multi-antenna two-way relaying, $2N$ data streams can be simultaneously transmitted if the number of antennas at RS is larger or equal to $2N$. In [5] and [6], the optimal transceive filter at RS is derived for one-way relaying with multiple antennas. A relay transceive strategy for a multi-antenna scenario maximizing the weighted sum of the Frobenius norms of the effective single-user channels is introduced in [7] and extended in [8]. The authors of [9] characterize the capacity region of non-regenerative two-way relaying for single antenna nodes S1 and S2 and a multi-antenna relay RS. For this scenario, they present the optimal relay transceive filter structure to attain a boundary rate pair which is equivalent to the sum rate maximization under an asymmetric rate constraint. So far, a combination of one-way and two-way relaying as well as an asymmetric rate constraint in a multi-antenna scenario has not been examined.

In this paper, a scenario is considered where two multi-antenna nodes exchange information via a half-duplex non-regenerative multi-antenna relay under an asymmetric rate constraint, which means that the instantaneous data rate from S1 to S2 has to be r times the instantaneous data rate from S2 to S1, $1 \leq r \leq \infty$. The main objective is to consider this asymmetric rate constraint in the sum rate maximization with a limited transmit power at each node. In one-way relaying, the asymmetric rate constraint can be fulfilled by varying the corresponding time slot durations. Furthermore, transceive filters which are optimized for each unidirectional transmission can be applied at RS. In two-way relaying, the asymmetric rate constraint has to be fulfilled by optimizing the bidirectional transceive filter at RS and by reducing the power of one node. Two-way relaying overcomes the multiplexing loss of one-way relaying, but additional optimizations are required to fulfill the asymmetric rate constraint as described above. To enable the

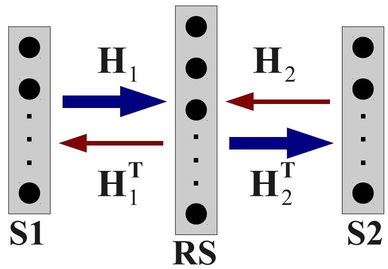

Fig. 1. Bidirectional two-hop relaying scenario.

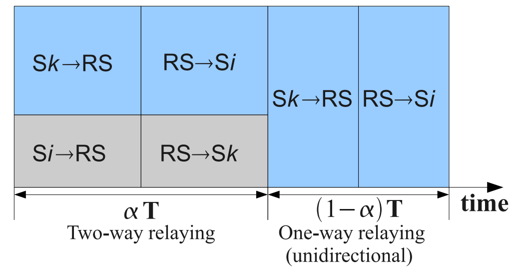

Fig. 2. Hybrid one-way / two-way relaying scheme.

consideration of suboptimal low-complexity approaches which cannot fulfill the asymmetric rate constraint by pure two-way relaying, a hybrid one-way / two-way scheme is suggested which ensures that the asymmetric rate constraint can always be fulfilled.

In this paper, the focus is on scenarios where the number of antennas at RS is lower than the total number of antennas at the nodes and, therewith, lower than the number of data streams which shall be multiplexed during the two-way relaying phase. This is possible, because perfect CSI is assumed and the nodes can subtract the self-interference in the receive signal. Transceive filters which are based on the combined relay channel [3] cannot be applied in the considered scenario. Therefore, a different transceive filter design method is proposed, which is based on singular value decomposition (SVD) of the individual channels.

The paper is organized as follows. In Section II, the system model and the hybrid one-way / two-way scheme are given. The problem of maximizing the sum rate under the asymmetric rate constraint and the transceive filter and power optimization approaches are presented in Section III. Simulation results in Section IV confirm the analytical investigations and Section V concludes the paper.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^\mathrm{T}$, $(\cdot)^*$ and $(\cdot)^\mathrm{H}$ stand for matrix or vector transpose, complex conjugate and complex conjugate transpose, respectively. The operator $[x]^+$ returns x if $x \geq 0$ and returns 0 if $x < 0$.

II. System Model & Hybrid Relaying Scheme

As shown in Figure 1, the two-hop bidirectional communication between the half-duplex nodes S1 and S2 via a half-duplex relay RS is considered. The transmit power at each node and at RS is limited by P_{Node} and P_{RS}, respectively. Furthermore, it is assumed that each node uniformly allocates its power over its antenna elements. The number of antenna elements at S1 and S2 is assumed to be equal and is given by M and the number of antenna elements at RS is given by L. Each node transmits M data streams at a time and perfect CSI is assumed. To support the multiplexing of M data streams, the number L of antenna elements at RS has to be at least equal to M. In this paper, $L \geq M$ is considered. Typical two-way relaying algorithms require $L \geq 2M$ antenna elements, e.g., [3],[4], because the combined $2M \times L$ channel is considered at RS. However, if perfect CSI is assumed, the nodes can subtract the self-interference in the receive signal and only M data streams have to be separated which only requires M antenna elements at RS even for two-way relaying.

The main objective of this paper is to consider an asymmetric rate constraint in the system. The instantaneous data rate from S1 to S2 has to be r times the instantaneous data rate from S2 to S1 which is indicated by the thickness of the arrows in Figure 1, $1 \leq r \leq \infty$. As explained in the introduction, a combination of one-way and two-way relaying is proposed to enable the consideration of suboptimal algorithms which cannot fulfill the asymmetric rate constraint by pure two-way relaying. Therefore, a hybrid one-way / two-way scheme is suggested and explained in the following.

The overall bidirectional communication is subdivided into two phases as shown in Figure 2. In the first phase, a bidirectional communication between S1 and S2 is enabled by using the two-way relaying scheme. Therefore, two time slots are required in the first phase. The superimposed signal of S1 and S2 is received by RS in the first time slot and is transmitted after linear signal processing in the second time slot. The overall duration of the first phase is given by αT, $0 \leq \alpha \leq 1$, where T is the overall duration of both phases. In the first phase, the transceive filter at RS can be optimized for the bidirectional transmission and the powers of the nodes can be varied to influence the achievable instantaneous data rates. The second phase conduces to fulfilling the asymmetric rate constraint. In the second phase, a unidirectional communication is established using half of the typical one-way relaying scheme. This phase can also be separated into two time slots. In the first time slot, RS receives the signal from Sk, $k = 1, 2$, which has to transmit additional symbols after the first phase to fulfill the asymmetric rate constraint. In the second time slot, the signal is transmitted from RS to Si, $i = 1, 2, i \neq k$. The overall duration of the second phase is given by $(1 - \alpha)T$ and the duration is zero if the asymmetric rate constrained was already fulfilled in the first phase. During the second phase, the transceive filter at RS can be optimized for the unidirectional transmission and the maximum transmit power is used at Sk and zero transmit power is used at Si. The overall sum rate is finally given by the sum of the rates in the first and in the second phase.

The channels $\mathbf{H}_1 \in \mathbb{C}^{L\times M}$ and $\mathbf{H}_2 \in \mathbb{C}^{L\times M}$ from S1 to RS and S2 to RS, respectively, are assumed to be constant during the two phases and channel reciprocity is assumed. The system equations for the two-way relaying phase are presented in the following where both nodes are simultaneously transmitting to RS. The transmitted symbols of S1 and S2 are contained in the vectors $\mathbf{s}_1$ and $\mathbf{s}_2$, respectively. Using the transmit matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$ and using the transmit powers P_1 and P_2 at S1 and S2, respectively, the received baseband signal at RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}_1\sqrt{P_1}\mathbf{Q}_1\mathbf{s}_1 + \mathbf{H}_2\sqrt{P_2}\mathbf{Q}_2\mathbf{s}_2 + \mathbf{n}_{\mathrm{RS}}, \qquad (1)$$

where $\mathbf{n}_{\mathrm{RS}}$ represents the complex white Gaussian noise vector at RS.

RS linearly processes the received signal and the relay operation can be split into three parts, namely, receive filtering expressed by $\mathbf{G}_{\mathrm{R}}$, weighting expressed by $\mathbf{W}$ and transmit filtering expressed by $\mathbf{G}_{\mathrm{T}}$. Therefore, the transceive filter at RS is given by

$$\mathbf{G} = \gamma\mathbf{G}_{\mathrm{T}}\mathbf{W}\mathbf{G}_{\mathrm{R}}, \qquad (2)$$

where γ is a scalar value to satisfy the relay power constraint. It is given by

$$\gamma = \sqrt{\frac{P_{\mathrm{RS}}}{\sum_{i=1}^{2}||\mathbf{G}_{\mathrm{T}}\mathbf{W}\mathbf{G}_{\mathrm{R}}\mathbf{H}_i\mathbf{Q}_i||_2^2 P_i + ||\mathbf{G}_{\mathrm{T}}\mathbf{W}\mathbf{G}_{\mathrm{R}}||_2^2\sigma_{\mathrm{n,RS}}^2}}. \qquad (3)$$

The relay transmits the linearly processed version of $\mathbf{y}_{\mathrm{RS}}$ to S1 and S2. The received signal at Sk, $k = 1, 2$, is given by

$$\begin{aligned}\mathbf{y}_{Sk} &= \mathbf{H}_k^{\mathrm{T}}\mathbf{G}(\mathbf{H}_k\mathbf{Q}_k\mathbf{s}_k\sqrt{P_k} + \mathbf{H}_i\mathbf{Q}_i\mathbf{s}_i\sqrt{P_i} + \mathbf{n}_{\mathrm{RS}}) + \mathbf{n}_{\mathrm{S}k},\\ &\quad i = 1, 2, i \neq k, \end{aligned} \qquad (4)$$

where $\mathbf{n}_{\mathrm{S}k}$ represents the complex white Gaussian noise vector at Sk. Assuming that $\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_k$ is perfectly known at Sk, the back-propagated self-interference can be canceled [10] and the received signal at Sk reduces to

$$\mathbf{y}_{Sk} = \mathbf{H}_k^{\mathrm{T}}\mathbf{G}(\mathbf{H}_i\mathbf{Q}_i\sqrt{P_i}\mathbf{s}_i + \mathbf{n}_{\mathrm{RS}}) + \mathbf{n}_{\mathrm{S}k}. \qquad (5)$$

The system equations for the second phase where unidirectional one-way relaying is used are identical to the equations in the first phase except that in the second phase the power P_1 or P_2 of one node S1 or S2, respectively, is set to zero and no self-interference cancellation has to be performed at the nodes. The transceive filter matrix $\mathbf{G}$ is also different in both phases, but this does not affect the equations, because up to now a general definition of $\mathbf{G}$ is used.

Assuming that the noise at RS and at the nodes is additive white Gaussian with the variances $\sigma_{\mathrm{n,RS}}^2$ and σ_n^2, respectively, the matrices

$$\mathbf{A}_{i\to k} = \mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{H}_i\mathbf{Q}_i, \qquad (6)$$

$$\mathbf{B}_k = \sigma_{\mathrm{n,RS}}^2\mathbf{H}_k^{\mathrm{T}}\mathbf{G}\mathbf{G}^{\mathrm{H}}\mathbf{H}_k^* + \sigma_n^2\mathbf{I}_M, \qquad (7)$$

can be defined, describing the overall channel from Si to Sk and the noise autocorrelation matrix at Sk, respectively. Similarly to the considerations in [9], the information-theoretic limits shall be considered. Therefore, it is assumed that optimal Gaussian codebooks are used at S1 and S2. Under these assumptions, the rate from Si to Sk in the first or in the second phase is given by

$$\begin{aligned}C_{\mathrm{1st/2nd\ phase}}^{\mathrm{S}i\to\mathrm{S}k} &= \frac{1}{2}\log_2(\det(\mathbf{I}_M + P_i\mathbf{A}_{i\to k}\mathbf{A}_{i\to k}^{\mathrm{H}}\mathbf{B}_k^{-1})),\\ &\quad i, k = 1, 2, i \neq k,\end{aligned} \qquad (8)$$

where the factor $1/2$ is needed because two time slots are used in each phase. The overall sum rate for the hybrid scheme is given by the weighted sum of the two-way rates from S1 to S2 and from S2 to S1 in the first phase and the unidirectional one-way rate from Sk to Si in the second phase as shown in Figure 2. It is dependent on which node has to transmit additional symbols after the first phase to fulfill the asymmetric rate constraint. In case that Sk has to transmit additional symbols, it is given by

$$C_{\mathrm{sum}} = \alpha\left(C_{\mathrm{1st\ phase}}^{\mathrm{S1}\to\mathrm{S2}} + C_{\mathrm{1st\ phase}}^{\mathrm{S2}\to\mathrm{S1}}\right) + (1-\alpha)\left(C_{\mathrm{2nd\ phase}}^{\mathrm{S}k\to\mathrm{S}i}\right). \qquad (9)$$

III. Sum Rate Maximization

The maximization of the sum rate under the asymmetric rate constraint can be achieved by optimizing the transceive filter at RS for each phase and by adapting the power of one node in the first phase. The transmit filters which are assumed to be unitary matrices as well as the receive filters at the nodes do not affect the information-theoretic limits of the sum rate. A general notation of the optimization problem is given by

$$\begin{aligned}&\max_{\alpha, P_1, P_2, \mathbf{G}_{\mathrm{1st}}, \mathbf{G}_{\mathrm{2nd}}} C_{\mathrm{sum}}\\ &\text{subject to:}\\ &P_i \leq P_{\mathrm{Node}}, i = 1, 2,\\ &\sum_{i=1}^{2}\|\mathbf{G}\mathbf{H}_i\|_2^2 P_i + \mathrm{tr}(\mathbf{G}\mathbf{G}^{\mathrm{H}}) \leq P_{\mathrm{RS}},\\ &C^{\mathrm{S1}\to\mathrm{S2}} = r\cdot C^{\mathrm{S2}\to\mathrm{S1}},\end{aligned} \qquad (10)$$

where $C^{\mathrm{S}i\to\mathrm{S}k}$ is the sum of the rates from Si to Sk during both phases. The parameter α has to be calculated for every transceive filter and power combination in a way that the considered combination fulfills the asymmetric rate constraint.

The maximization of the overall sum rate is split into three parts. First, the second phase is considered and the transceive filter at RS is optimized for the unidirectional transmission. Second, the bidirectional transmission in the first phase is considered and the transceive filter at RS and the node powers are optimized. Third, the overall sum rate is calculated by a weighted combination of the rates in the first phase with the unidirectional rate in the second phase. The maximization of the sum rate is performed by numerical optimization of the transceive filter at RS and of the node powers in the first phase.

A. Transceive filter optimization in the second phase

The optimization of the transceive filter matrix in the second phase is already solved in [5] and [6] based on singular value decomposition (SVD) of the first hop and second hop channel. The SVD of the channels is given by

$$\mathbf{H}_i = \mathbf{U}_i \boldsymbol{\Lambda}_i^{1/2} \mathbf{V}_i^{\mathrm{H}}, i = 1, 2, \tag{11}$$

where $\mathbf{U}_i$ contains the left singular vectors, $\mathbf{V}_i$ contains the right singular vectors and $\boldsymbol{\Lambda}_i$ contains the corresponding eigenvalues of the channel $\mathbf{H}_i$ in decreasing order. In this paper, the eigenmodes of the first and of the second hop channel are sorted in decreasing order and are pairwise combined. The power is assigned to each channel eigenmode by the diagonal weighting matrix $\mathbf{W}_i$ which results in the transceive filter

$$\begin{aligned} \mathbf{G}_{\mathrm{2nd}} &= \mathbf{G}_{i \to k} = \gamma \mathbf{U}_k^* \mathbf{W}_i \mathbf{U}_i^{\mathrm{H}}, \\ & i, k = 1, 2, i \neq k, \end{aligned} \tag{12}$$

for the unidirectional transmission from Si to Sk with γ from (3) for $P_i = P_{\mathrm{Node}}$ and $P_k = 0$. The mth component in the diagonal weighting matrix $\mathbf{W}_i$ given by [6] is

$$\begin{aligned} w_m &= \sqrt{\frac{\left[\sqrt{\mu\kappa\frac{\lambda_{i,m}}{\lambda_{k,m}} + \left(\kappa\frac{\lambda_{i,m}}{2\lambda_{k,m}}\right)^2} - \kappa\frac{\lambda_{i,m}}{2\lambda_{k,m}} - \frac{\sigma_n^2}{\lambda_{k,m}}\right]^+}{(P_{\mathrm{Node}}/M)\lambda_{i,m} + \sigma_{n,RS}^2}}, \\ \kappa &= \frac{P_{\mathrm{Node}}\sigma_n^2}{M\sigma_{n,RS}^2}, \\ m &= 1, 2, ..., M, \end{aligned} \tag{13}$$

where μ is a constant to fulfill the power constraint at RS for $\gamma = 1$ and $\lambda_{i,m}$ is the mth element on the diagonal of $\boldsymbol{\Lambda}_i$. The unidirectional rate is calculated by (8).

B. Transceive filter and node power optimization in the first phase

For the transceive filter optimization in the first phase, the basic idea is to use a weighted combination of unidirectional transceive filters at RS. To solve this problem, the receive and transmit filters $\mathbf{G}_{\mathrm{R}}$ and $\mathbf{G}_{\mathrm{T}}$ at RS are designed independent of the rate constraint and the adaptation to the rate constraint is performed by the weighting matrix $\mathbf{W}$. The receive and transmit filters are chosen to exploit the eigendirections of the channels $\mathbf{H}_1$ and $\mathbf{H}_2$ similar to the above mentioned filter for the second phase. The relay receive and transmit matrices in the first phase are given by

$$\begin{aligned} \mathbf{G}_{\mathrm{R}} &= \begin{pmatrix} \mathbf{U}_1^{\mathrm{H}} \\ \mathbf{U}_2^{\mathrm{H}} \end{pmatrix}, \\ \mathbf{G}_{\mathrm{T}} &= \left(\mathbf{U}_2^*, \mathbf{U}_1^*\right). \end{aligned} \tag{14}$$

The singular vectors contained in $\mathbf{U}_1$ are generally not orthogonal to the singular vectors contained in $\mathbf{U}_2$. The eigenmodes of the first and of the second hop channel are sorted in decreasing order and are pairwise combined for each direction. Furthermore, the power which is allocated to the eigenmodes at RS is given by a weighted combination of the weighting matrices which were described in (12), (13). Therefore, the weighting matrix reduces to a diagonal matrix with the structure

$$\mathbf{W} = \begin{pmatrix} w_1\mathbf{W}_1 & \mathbf{0}_{M\times M} \\ \mathbf{0}_{M\times M} & (1-w_1)\mathbf{W}_2 \end{pmatrix}, \tag{15}$$

where w_1 is the weight to perform the adaptation to the asymmetric rate constraint, $0 \leq w_1 \leq 1$. The overall transceive filter $\mathbf{G}_{\mathrm{1st}}$ at RS is given by (2) with γ from (3). The effect of varying w_1 on the rates $C^{S1\to S2}$ and $C^{S2\to S1}$ depends on the relation between the channels $\mathbf{H}_1$ and $\mathbf{H}_2$. If the singular vectors of the channels are parallel to each other, the variation of w_1 has only a small influence on the rates which depends on the difference between the power allocations by $\mathbf{W}_1$ and $\mathbf{W}_2$.

Furthermore, the optimization of the powers P_1 and P_2 is considered in the first phase. Only the power P_i of one node has to be adapted in the first phase, because the power of the node whose corresponding rate has to be increased to fulfill the asymmetric rate constraint by pure two-way relaying is kept at P_{Node}. Decreasing P_i decreases the rate $C^{Si\to Sk}$ and it also increases $C^{Sk\to Si}$, because more power can be used at RS for the amplification of the signal received from Sk. The bidirectional rates are calculated by (8) using a fixed weight w_1 and fixed transmit powers at the nodes. The maximization of the sum rate is performed by numerical optimization of the weight w_1 and of the node power P_i.

C. Numerical maximization of the sum rate under the asymmetric rate constraint

For every value of the weight w_1 combined with fixed transmit powers at the nodes the bidirectional rates in the first phase can be calculated. These rates are combined with the unidirectional rate of the second phase as described in (9) to fulfill the asymmetric rate constraint. The weighting factor α for $C_{\text{1st phase}}^{\text{S1}\to\text{S2}} \leq r \cdot C_{\text{1st phase}}^{\text{S2}\to\text{S1}}$ is given by

$$\alpha = \frac{C_{\text{2nd phase}}^{\text{S1}\to\text{S2}}}{r \cdot C_{\text{1st phase}}^{\text{S2}\to\text{S1}} - C_{\text{1st phase}}^{\text{S1}\to\text{S2}} + C_{\text{2nd phase}}^{\text{S1}\to\text{S2}}}, \tag{16}$$

and in all other cases it is given by

$$\alpha = \frac{C_{\text{2nd phase}}^{\text{S2}\to\text{S1}}}{C_{\text{1st phase}}^{\text{S1}\to\text{S2}}/r - C_{\text{1st phase}}^{\text{S2}\to\text{S1}} + C_{\text{2nd phase}}^{\text{S2}\to\text{S1}}}. \tag{17}$$

The combination of the rates in the first phase with the rate in the second phase results in the sum rate which shall be maximized. The maximum is computed by a two-dimensional search over the weight w_1 and the transmit power P_i of one node. The corresponding algorithm is named HybridRateMax in the following. To reduce the computational complexity, it is also possible to perform the optimization of the weight and the optimization of the node power separately. An algorithm which first performs the optimization of w_1 and optimizes the node power P_i afterwards is named HybridRateMaxDecoupled.

D. Upper bounds for the considered algorithms

In the considered scenario, the transmit powers of S1 and S2 have to be adapted to enable an optimal transmit power distribution at RS. The adaptation can only be performed by transmit power reduction at the nodes, which reduces the overall power used in the system. Therefore, a sum rate upper bound is derived for the proposed algorithm which corresponds to the sum rate in the case that the full power can be used in the system and the relay can optimally distribute its power to both directions. The HybridRateMax bound is given by

$$\begin{aligned} C_{\text{sum,HRM bound}} =& \alpha \left[C^{\text{S1}\rightarrow\text{S2}}_{\text{1st phase}}(\mathbf{G}(\beta P_{\text{RS}})) \right. \\ & \left. + C^{\text{S2}\rightarrow\text{S1}}_{\text{1st phase}}(\mathbf{G}((1-\beta)P_{\text{RS}})) \right] \\ & +(1-\alpha) C^{\text{S}i\rightarrow\text{S}k}_{\text{2nd phase}}(\mathbf{G}_{i\rightarrow k}, P_{\text{RS}}), \end{aligned} \tag{18}$$

where $C^{Si\rightarrow Sk}_{\text{1st phase}}(\mathbf{G}(\beta P_{\text{RS}}))$ is the capacity from Si to Sk for a transceive filter $\mathbf{G}$ which uses the contribution βP_{RS} of the overall relay power P_{RS} to support the transmission from Si to Sk. Therefore, the transceive filter $\mathbf{G}$ fulfills the following condition

$$\beta P_{\text{RS}} = ||\mathbf{G}\mathbf{H}_i\mathbf{Q}_i||_2^2 P_i + \beta ||\mathbf{G}||_2^2 \sigma^2_{\text{n,RS}}, \tag{19}$$

where β is also included in the term of the amplified noise to guarantee that the overall noise amplification is identical to that in the bidirectional transmission.

Furthermore, a two-way upper bound is considered for comparison during the simulations. The two-way upper bound is given by the sum rate of the two unidirectional rates from S1 to S2 and S2 to S1. Therefore, unidirectional transceive filters are used at RS as described in (12). These transceive filters optimally share the overall transmit power at RS to fulfill the asymmetric rate constraint. The sum rate is given by

$$\begin{aligned} C_{\text{sum,2way bound}} =& C^{\text{S1}\rightarrow\text{S2}}_{\text{2nd phase}}(\mathbf{G}_{1\rightarrow 2}(\beta P_{\text{RS}})) \\ & + C^{\text{S2}\rightarrow\text{S1}}_{\text{2nd phase}}(\mathbf{G}_{2\rightarrow 1}((1-\beta)P_{\text{RS}})). \end{aligned} \tag{20}$$

IV. Simulation Results

In this section, numerical results on the achievable sum rates for different two-hop relaying algorithms are compared. For the simulations, i.i.d. Rayleigh fading channels are considered with an average path loss $p_{\text{loss},k}$ between Sk and RS. Therefore, the signal to noise ratio (SNR) for Sk is defined as

$$\text{SNR}k = \frac{p_{\text{loss},k} P_{\text{Node}}}{\sigma^2_{\text{RS}}}. \tag{21}$$

It is assumed that $P_{\text{Node}} = P_{\text{RS}}$ and $\sigma^2_{\text{RS}} = \sigma^2_{\text{n}}$. Each node is equipped with $M = 4$ antenna elements and the relay is equipped with $L = 6$ antenna elements. The algorithms which are compared during the simulations are the HybridRateMax algorithm together with its upper bound and the version HybridRateMaxDecoupled. Furthermore, an algorithm which only optimizes for w_1 and uses the maximum transmit power at each node is considered. This algorithm is named Opt(w_1) in the following. The lower bound is given by pure one-way relaying and the upper bound is given by the two-way upper bound described in the last paragraph of Section III-D.

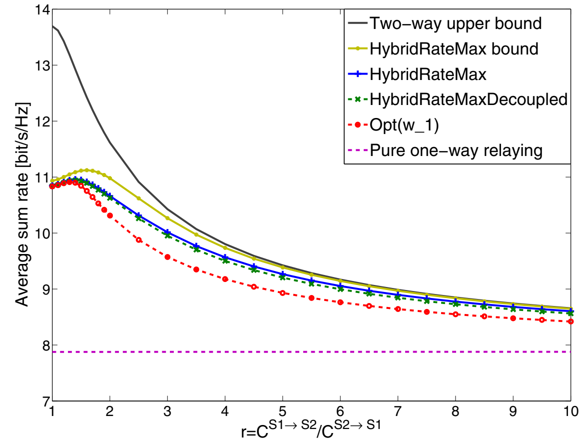

Fig. 3. Average sum rates for different rate constraints, $M = 4$, $L = 6$, SNR1=SNR2=15dB.

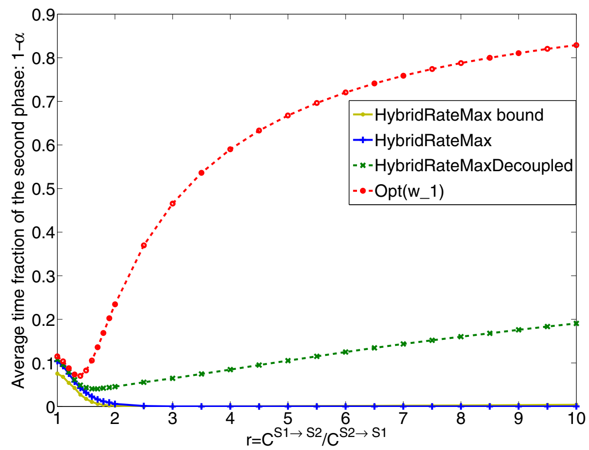

Fig. 4. Average time fractions $(1-\alpha)$ of the second phase for different rate constraints, $M = 4$, $L = 6$, SNR1=SNR2=15dB.

Figure 3 shows the sum rates for the variation of the asymmetric rate constraint r and Figure 4 shows the corresponding average time fractions $1-\alpha$ of the second phase. The average sum rates achieved by pure one-way relaying are independent of the rate constraint if SNR1 equals SNR2, because the asymmetric rate constraint is fulfilled by a variation of the time slots for each direction of transmission. The gap between HybridRateMax and the HybridRateMax bound is due to the node power reduction of HybridRateMax to achieve an optimal transmit power distribution at RS. The proposed transceive filter design in the first phase has its weakness if the channels and the rates which shall be supported are approximately symmetric. Therefore, the average time fractions of the second phase are not zero for small values of r and the gap between the algorithms and the two-way upper bound is large in the considered scenario. For $r \geq 2$, the sum rates of HybridRateMax and the HybridRateMax bound are given by pure two-way relaying. If r increases, the gap to the two-way upper bound decreases. For high values of r, HybridRateMax achieves sum rates close to the two-way upper bound, because one rate becomes more and more the limiting rate and the transceive filter is focused on this direction of transmission. Therefore, the amount of power which is received and transmitted through the eigenvectors of the opposite direction is decreased.

For HybridRateMaxDecoupled and Opt(w_1) the time fractions of the second phase increase with the asymmetric rate constraint r. HybridRateMaxDecoupled has a lower computational complexity and achieves slightly degraded sum rates compared to HybridRateMax. Opt(w_1) requires the highest time fractions of the second phase, because the node powers cannot be adapted to fulfill the rate constraint. The achieved sum rates do not significantly get closer to the two-way upper bound for an increase of r. The gain of adapting the node powers can be seen on the gap between Opt(w_1) and HybridRateMax.

Figure 5 shows the sum rates for an asymmetry between SNR1 and SNR2 for a fixed asymmetric rate constraint $r = 2.5$. Opt(w_1) performs close to HybridRateMax in the range where only small node power adaptation is required due to the channel asymmetry. HybridRateMaxDecoupled performs well in the region where the SNR imbalance supports the asymmetric rate constraint (SNR1 $>$ 15dB $>$ SNR2) and performs worse in the region where it counteracts r, because w_1 is optimized first. The HybridRateMax algorithm gets closer to the two-way upper bound if the asymmetry between the channels increases, which has the same reasons as explained above. The performance of the proposed transceive filter design improves if one rate becomes more and more the limiting rate. In this case, the combination of node power optimization with the proposed transceive filter design performs close to the two-way upper bound and HybridRateMax performs pure two-way relaying.

V. Conclusions

The optimization problem of maximizing the overall sum rate for a multi-antenna non-regenerative relaying scenario considering an asymmetric rate constraint is formulated. A hybrid one-way / two-way relaying scheme is introduced to ensure that the asymmetric rate constraint can always be fulfilled. Low-complexity transceive filter and power optimization approaches are derived which can be applied in scenarios where the number of antenna elements at the relay equals at least the number of antenna elements at each node. Simulation results for different link qualities and rate constraints show that the proposed low-complexity transceive filter and power optimization achieves sum rates close to the two-way upper bound in case of high asymmetric rate constraints or highly asymmetric channels.

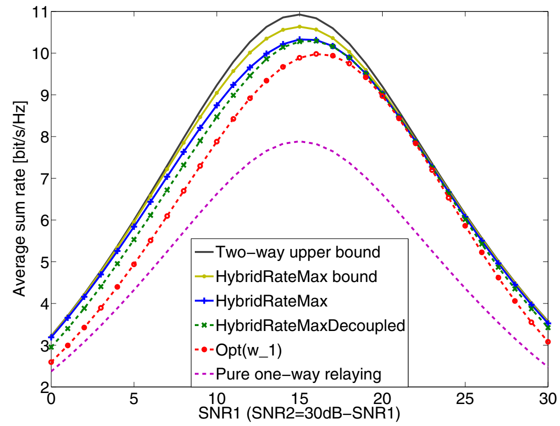

Fig. 5. Average sum rates for asymmetric SNRs (SNR1 + SNR2 = 30dB), $M = 4$, $L = 6$, $r = 2.5$.

Acknowledgment

The work of Holger Degenhardt is funded by the Deutsche Forschungsgesellschaft (DFG) and is in cooperation with the Graduate School of Computational Engineering, Technische Universität Darmstadt.

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