Amah and Klein *EURASIP Journal on Wireless Communications and Networking* 2011, **2011**:29
http://jwcn.eurasipjournals.com/content/2011/1/29

EURASIP Journal on
Wireless Communications and Networking
a SpringerOpen Journal

RESEARCH **Open Access**

Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying

Aditya Umbu Tana Amah[1*] and Anja Klein[2]

Abstract

We consider non-regenerative multi-group multi-way (MGMW) relaying. A half-duplex non-regenerative multi-antenna relay station (RS) assists multiple communication groups. In each group, multiple half-duplex nodes exchange messages. In our proposal, the required number of communication phases is equal to the maximum number of nodes among the groups. In the first phase, all nodes transmit simultaneously to the RS. Assuming perfect channel state information is available at the RS, in the following broadcast (BC) phases the RS applies transceive beamforming to its received signal and transmits simultaneously to all nodes. We propose three BC strategies for the BC phases: unicasting, multicasting and hybrid uni/multicasting. For the multicasting strategy, network coding is applied to maintain the same number of communication phases as for the other strategies. We address transceive beamforming maximising the sum rate of non-regenerative MGMW relaying. Due to the high complexity of finding the optimum transceive beamforming maximising the sum rate, we design generalised low complexity transceive beamforming algorithms for all BC strategies: matched filter, zero forcing, minimisation of mean square error and BC-strategy-aware transceive beamforming. It is shown that the sum rate performance of non-regenerative MGMW relaying depends both on the chosen BC strategies and the applied transceive beamforming at the RS.

Keywords: Multi-way relaying, Non-regenerative, Multi-antenna, Analog network coding, Transceive beamforming

Introduction

Two-way relaying is a spectrally efficient protocol to establish bidirectional communication between two half-duplex nodes via a half-duplex relay station (RS) [1-3]. It was shown in [1,3] that two-way relaying outperforms the traditional one-way relaying due to its smaller number of communication resources. In two-way relaying, two communication phases are needed. The first phase is the multiple access (MAC) phase where the two communicating nodes send their data streams simultaneously to the RS. The second phase is the broadcast (BC) phase where the RS sends the processed signals simultaneously to both nodes. Consequently, the nodes need to cancel their self-interference.

Regarding the signal processing at the RS, it can be either regenerative, cf. [1,2] or non-regenerative, cf. [1,3]. A regenerative RS regenerates (decodes and re-encodes) the data streams of all nodes while a non-regenerative RS performs linear signal processing to the received signals and transmits the output to the nodes.

The use of multiple antennas can improve the spectral efficiency and/or the reliability of communication networks [4,5]. For two-way relaying, a multi-antenna RS that serves one bidirectional pair was considered in [6-8] for a regenerative RS and in [3,9-11] for a non-regenerative RS. For the non-regenerative case, while [3,9] assume multi-antenna nodes, [10,11] assume single antenna nodes. Their works consider optimal transceive beamforming at the RS maximising the sum rate as well as linear transceive beamforming based on Zero Forcing (ZF) [3,9-11], Minimisation of Mean Square Error (MMSE) [3,9,10], Maximisation of Signal to Noise Ratio (MSNR) [3] and Matched Filter (MF) criteria [10,11].

Multi-user two-way relaying, where an RS serves multiple bidirectional pairs, is treated in [12-14] for a regenerative RS and in [15,16] for a non-regenerative RS. In [12], all bidirectional pairs are separated using Code Division Multiple Access. Every two nodes in a

* Correspondence: a.amah@nt.tu-darmstadt.de
[1]Graduate School of Computational Engineering and Communications Engineering Lab, Technische Universität Darmstadt, Darmstadt 64283 Germany
Full list of author information is available at the end of the article

bidirectional pair have their own code which is different from the other pairs' codes. In contrast to [12], having multiple antennas at the RS and assuming perfect detection in MAC phase, in [13,14] the separation of the pairs in the BC phase is done spatially using transmit beamforming employed at the RS. In [15], ZF and MMSE transceive beamforming for multi-user non-regenerative two-way relaying is designed to separate the nodes. In [16], block-diagonalisation-singular-value-decomposition (BD-SVD) transceive beamforming is designed for separating the two-way pairs by extending BD-SVD transmit beamforming proposed in [17].

In applications such as video conference and multi-player gaming, multiple nodes exchange messages. In [18], the multi-way relay channel is considered, where an RS assists multiple communication groups. In each group, each member node exchanges messages with other member nodes, but not with other nodes from the other groups. A full-duplex communication is assumed and time division is used to separate the multi-way groups. The full duplex assumption, however, is not yet practical and half-duplex nodes and relays are more of practical importance [1,19]. Therefore, communication protocols for multi-way relaying for half-duplex nodes and a half-duplex RS are needed.

Multi-way relaying protocols for one communication group, where a half-duplex multi-antenna RS assists N half-duplex nodes to exchange messages, is proposed by the authors of this paper in [20] for a non-regenerative RS and in [21] for a regenerative RS. The required number of communication phases is only N, consisting of one MAC phase and N - 1 BC phases. In [20,21], in each BC phase, the RS sends N data streams to N nodes simultaneously. Thus, each node receives an intended data stream from a specific node, while seeing other data streams as interference. Nevertheless, the interference can be canceled by performing successive interference cancellation or by applying linear transceive beamforming, e.g., ZF, that nullifies the interference [20]. In [20], it was also shown that instead of transmitting N different data streams per BC phase, the RS can transmit one data stream simultaneously to all nodes in each BC phase. This data stream is a superposition of two different data streams. Consequently, each node has to perform self- and known-interference cancellation. Since in each BC phase the RS transmits only one superposed data stream to all nodes, there is no inter-stream interference and, thus, the performance is improved.

In this paper, we consider non-regenerative multi-group multi-way (MGMW) relaying. A half-duplex multi-antenna RS assists L multi-way groups where each group consists of half-duplex single antenna nodes. We consider non-regenerative relaying where the RS performs transceive beamforming. Non-regenerative, compared to regenerative, has three advantages: no decoding error propagation, no delay due to decoding and deinterleaving, and transparency to the modulation and coding schemes that are used at the nodes [3].

In each l-th multi-way group, $l \in \mathcal{L}$, $\mathcal{L} = \{1, \cdots, L\}$, there are $N_l \geq 2$ nodes that exchange messages. In our proposal, the required number P of communication phases is equal to $\max_l N_l$. Our work is a generalisation of some of the above mentioned publications. If $L = 1$ and $N_1 = 2$, we have a non-regenerative two-way relaying as in [3,10,22]. If $L > 1$ and $N_l = 2$, $\forall l$, $l \in \mathcal{L}$, we have a non-regenerative multi-user two-way relaying as in [15,16], and if $L = 1$ and $N_1 \geq 2$, we have a non-regenerative single-group multi-way relaying as in [20].

We propose three BC strategies for the BC phases, namely, unicasting, multicasting and hybrid uni/multicasting. The proposed strategies are designed in such a way that the number of communication phases remains $P = \max_l N_l$. We derive the sum rate expression for non-regenerative MGMW relaying with the proposed BC strategies for asymmetric and symmetric traffic. In asymmetric traffic all nodes in each group may communicate with different rate, while in symmetric traffic all nodes in each group communicate with the same rate.

We address the sum rate maximisation which requires optimum transceive beamforming. Due to the high complexity of finding the optimum transceive beamforming maximising the sum rate, we design generalised low complexity transceive beamforming algorithms for all proposed BC strategies, namely, ZF, MMSE, MF and BC-strategy-aware (BCSA) transceive beamforming. BCSA transceive beamforming is designed by suppressing unwanted signals using either block diagonalisation (BD) [17] or regularised BD proposed in [23].

This paper is organised as follows. Section II explains the proposed broadcast strategies and the system model of non-regenerative MGMW relaying. Section III explains the sum rate expression. The transceive beamforming algorithms are explained in Section IV. The simulation results are given in Section V. Finally, Section VI provides the conclusion.

Notations

Boldface lower and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operators $\mathrm{mod}_N(x)$, $\mathrm{E}\{\mathbf{X}\}$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the modulo N of x, the expectation and the trace of $\mathbf{X}$, respectively, and $\mathcal{CN}(0, \sigma^2)$ denotes the circularly symmetric zero-mean complex normal distribution with variance σ^2.

Broadcast Strategies And System Model

We consider L multi-way communication groups. It is assumed that there are no direct links among the nodes and the MGMW communication can only be performed with the assistance of a half-duplex multi-antenna RS with M antenna elements. In the lth group, $l \in \mathcal{L}$, $\mathcal{L} = \{1, \cdots, L\}$, there are N_l nodes which exchange messages through an RS. For simplicity of notations, we consider the same number of nodes in all groups, i.e., $N_l = N_{\text{mw}}$, $\forall l \in \mathcal{L}$. However, the extension to the case of different numbers of nodes in the groups is straightforward. The total number N of nodes in the network is $N = \sum_{l \in \mathcal{L}} N_l = LN_{\text{mw}}$.

Assuming that the RS already knows which nodes belong to which communication group, the RS makes the indexing of all nodes according to their group membership. Nodes in group one are indexed within the set $\{0,..., N_1 - 1\}$, nodes in group two are indexed within the set $\{N_1,..., (N_1 + N_2) - 1\}$, and so on. In general, it can be given as follows. The lth group consists of nodes $\mathrm{S}i_l$, $i_l \in \mathcal{I}_l$, where $\mathcal{I}_l$ is the set of node indices given by $\mathcal{I}_l = \{a_l, \cdots, b_l\}$, with $a_l = (l - 1)N_{\text{mw}}$, $b_l = lN_{\text{mw}} - 1$. Each node only exchanges messages with the other nodes in its group and each node belongs only to one multi-way group, i.e., $\mathcal{I}_l \cap \mathcal{I}_k = \emptyset$, $\forall l \neq k$ and $\mathcal{I} = \bigcup_{l=1}^{L} \mathcal{I}_l = \{0, \cdots, N-1\}$.

A. Broadcast Strategies

In this subsection, the broadcast strategies for non-regenerative MGMW relaying are described. The number P of communication phases to perform MGMW communication is given by the maximum number of nodes among all groups, i.e., $P = \max_l N_l = N_{\text{mw}}$. In the first phase, the MAC phase, all nodes transmit simultaneously to the RS. In the following $P - 1$ BC phases, the RS transmits to the nodes. Let p, $p \in \mathcal{P}$, $\mathcal{P} = \{2, \ldots, P\}$, denote the index of the BC phase. In pth phase, in group l, receiving node $r_l \in \mathcal{I}_l$ is intended to receive the data stream of transmitting node $t_l \in \mathcal{I}_l \backslash \{r_l\}$.

1) Unicasting Strategy

Using unicasting strategy, in each BC phase, the RS transmits different data streams to different nodes. Each data stream is intended only for one receiving node. Consequently, in each BC phase each node sees the other data streams transmitted by the RS to the other nodes as interference. The data stream transmitted from the RS to each particular node is changed in each BC phase, such that within $P - 1$ BC phases, each node receives the data streams from all other nodes in its group.

The relationship of the parameters p, r_l and t_l is given by

$$t_l = a_l + \operatorname{mod}_{N_l}(r_l + p - a_l - 1). \quad (1)$$

Using such strategy, assuming each node knows its index and all other nodes' indices in its group, there is no signalling required in the network. The proposed unicasting strategy is a generalization of the work in [3,10] for $L = 1$ and $N_1 = 2$, in [15] for $L > 1$ and $N_l = 2$, $\forall l$, and in [20] for $L = 1$ and $N_1 \geq 2$ with multiplexing transmission. Figure 1a shows an example of the unicasting strategy for MGMW relaying with $L = 2$ communication groups and $N_1 = N_2 = 3$ nodes.

2) Hybrid Uni/Multicasting Strategy

For each served group, one data stream is transmitted to one node exclusively (unicast transmission) and one data stream is transmitted to the other $N_l - 1$ nodes (multicast transmission). In each BC phase, the unicasted data stream is fixed and is transmitted to a different node in the group. Consequently, the multicasted data stream has to be changed in each BC phase to ensure that each node in each group receives all data streams of the other nodes in its group within P phases. Compared to the unicasting strategy, intra-group interference in each BC phase is reduced since only two data streams are transmitted simultaneously.

The procedure can be described as follows. For each group l, the RS chooses one data stream out of N_l data streams. This data stream will be unicasted to different nodes in different BC phases. Therefore, in each BC phase, to ensure that each node receives the $N_l - 1$ data streams from the other $N_l - 1$ nodes, the multicasted data stream is the transmitted data stream of the node who will receive the unicasted data stream. In the following, we derive the mathematical formulation of the procedure for hybrid uni/multicasting.

In the lth group, given the index $t_{l_{\text{u}}} \in \mathcal{I}_l$ of the transmit node whose data stream is unicasted by the RS and the index $t_{l_{\text{m}}} \in \mathcal{I}_{l_{\text{M}}}$, $\mathcal{I}_{l_{\text{M}}} = \mathcal{I}_l \backslash \{t_{l_{\text{u}}}\}$, of the transmit node whose data stream is multicasted, the relationship between r_l, t_l, and p is defined by

$$t_l = \begin{cases} t_{l_{\text{u}}}, & \text{if } r_l = t_{l_{\text{m}}} \\ t_{l_{\text{m}}}, & \text{otherwise} \end{cases} \quad (2)$$

where

$$t_{l_{\text{m}}} = \begin{cases} (p + a_l) - 1, & \text{for } (p + a_l) \geq t_{l_{\text{u}}} + 2 \\ (p + a_l) - 2, & \text{for } (p + a_l) \leq t_{l_{\text{u}}} + 1 \end{cases}. \quad (3)$$

The relationships in (2) and (3) are defined after choosing the data stream to be unicasted for group l, $t_{l_{\text{u}}}$, which remains the same in all $N_l - 1$ BC phases. In the pth phase, node $r_l = t_{l_{\text{m}}}$, whose data stream is multicasted by the RS, receives the unicasted data stream from $t_{l_{\text{u}}}$. The other nodes, r_l, $r_l \in \mathcal{I}_l \backslash \{t_{l_{\text{m}}}\}$, receive the data stream from node $t_{l_{\text{m}}}$ which is multicasted by the

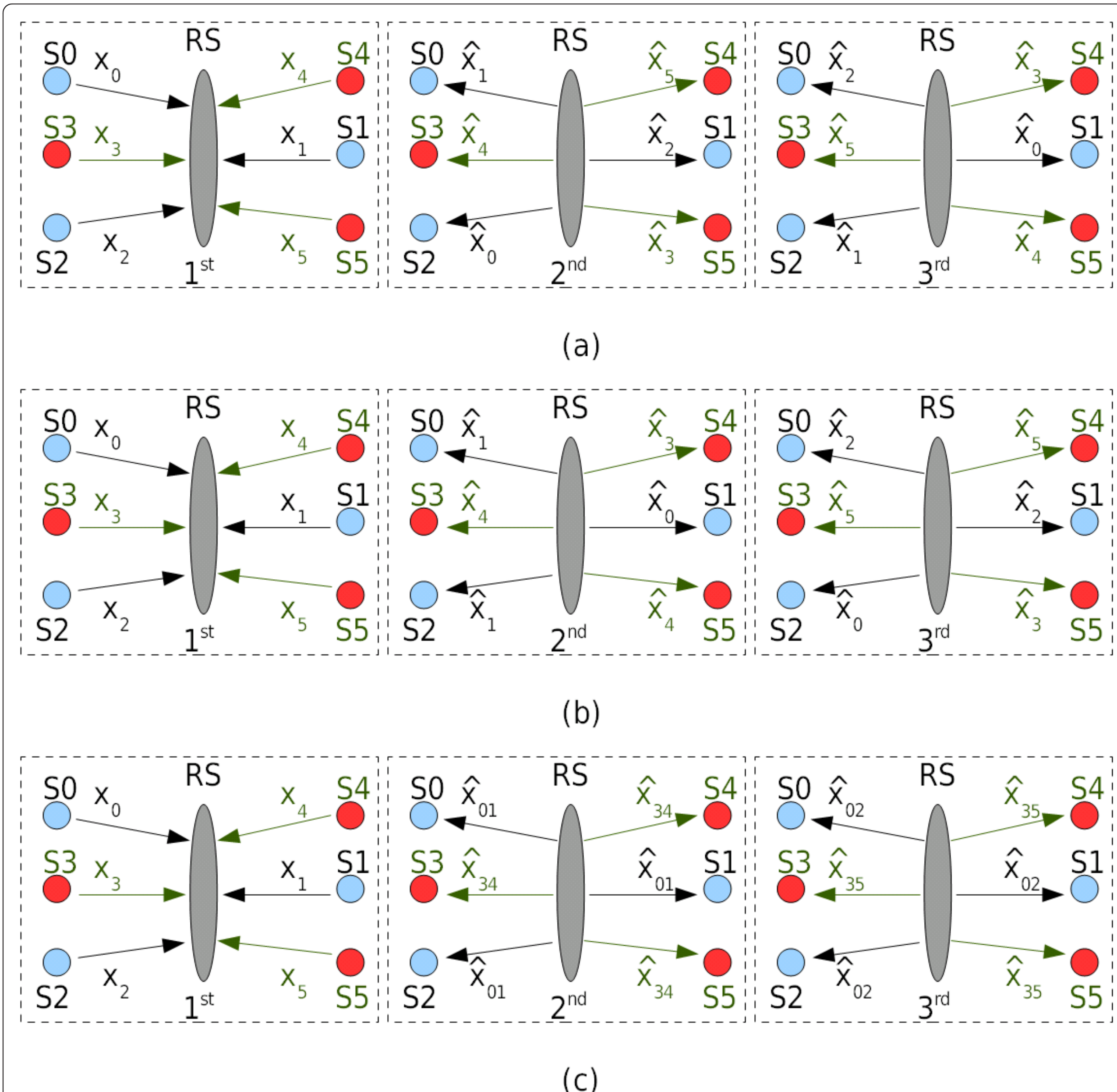

Figure 1 Multi-group multi-way relaying with L = 2, N_1 = N_2 = 3: (a) unicasting; (b) hybrid uni/multicasting; (c) multicasting.

RS to these N_l - 1 nodes. The multicasted data stream is changed in every BC phase as defined in Equation (3).

Using hybrid uni/multicasting strategy, the nodes need to know which data stream is unicasted and which data stream is multicasted by the RS in the pth phase. However, given (2) and (3), and by choosing the unicasted data stream from the node with the lowest index, that is, $t_{l_u} = a_l$, there is no signalling effort needed. The RS is then multicasting the data streams in the P - 1 BC phases starting from the lowest index in the set $\mathcal{I}_l \backslash \{t_{l_u} = a_l\}$. In case of one-pair two-way relaying and multi-user two-way relaying, the hybrid uni/multicasting strategy is the same as the unicasting strategy. The proposed hybrid uni/multicasting strategy is a generalisation of the work in [3,10] for L = 1 and N_1 = 2, and in [15] for L > 1 and N_l = 2, $\forall l$. Figure 1b shows an example of the proposed hybrid uni/multicasting strategy for MGMW relaying when L = 2 and N_1 = N_2 = 3 nodes.

3) Multicasting Strategy

Using multicasting strategy, the RS transmits only one data stream for each served group in each BC phase. The RS transmits $\hat{x}_{v_l w_l}$, i.e., the superposition of the data streams of nodes $\mathrm{S}v_l$, $v_l \in \mathcal{I}_l$, and $\mathrm{S}w_l$, $w_l \in \mathcal{I}_l\backslash\{v_l\}$, in the lth group, to all N_l nodes in group l. Prior to detection, each node has to cancel the self- and known-interference from each of the received data streams using the available side information. The side information can be its own transmitted data stream or a data stream which has been decoded in one of the previous BC phases. The general rule for selecting the two data streams for each group in each BC phase is that we have to ensure that the data stream of each node in each group is selected at least once.

The relationship between r_l, t_l, and p can be written as

$$t_l = \begin{cases} v_l, & \text{for } r_l = w_l, \\ w_l, & \text{otherwise.} \end{cases} \tag{4}$$

Given the general rule, we may have several options to define the superposed data stream. However, each option will lead to different signaling requirement, since the RS has to inform the nodes about the indices v_l and w_l in each BC phase. In this work, we are interested in an option that does not need any signalling. Therefore, we extend the proposal in [20] to the case of MGMW relaying. We always choose $v_l = a_l$, and, consequently, w_l is changed in each BC phase and is selected successively based on the relationship defined by $w_l = v_l + p - 1$.

Using such relationships, node $\mathrm{S}r_l = a_l$ always performs self-interference cancellation, i.e., $x_{v_l w_l} - x_{v_l = a_l}$, to obtain all N_l - 1 data streams from other nodes $t_l = w_l$, $\forall w_l \in \mathcal{I}_l\backslash\{a_l\}$. Regarding the other nodes $r_l \in \mathcal{I}_l\backslash\{a_l\}$, they have to be able to decode x_{a_l} and, afterwards, use x_{a_l} to perform known-interference cancellation. Each node $r_l \in \mathcal{I}_l\backslash\{a_l\}$ has to wait until its own data stream is superposed with $x_{v_l=a_l}$, that is in the pth phase which leads to $r_l = (p + a_l)$ -1. In this corresponding pth phase, node $r_l = (p + a_l)$ - 1 performs self-interference cancellation, i.e., $x_{v_l w_l} - x_{w_l=(p+a_l)-1}$ to obtain $x_{v_l=a_l}$. Afterwards, using $x_{v_l=a_l}$, it performs known-interference cancellation $x_{v_l w_l} - x_{v_l=a_l}$ to obtain the other data stream from the other nodes $t_l = w_l$, $\forall w_l \in \mathcal{I}_l\backslash\{a_l, r_l\}$ received in the other BC phases. The proposed multicasting strategy is a generalisation of the work in [1,22,24] for $L = 1$ and $N_1 = 2$, in [16] for $L > 1$ and $N_l = 2$, $\forall l$, and in [20] for $L = 1$ and $N_1 \geq 2$ with analog network coding transmission. Figure 1c shows an example of MGMW relaying using the multicasting strategy when $L = 2$ and $N_1 = N_2 = 3$ nodes.

In this subsection, we have mathematically formulated the BC strategies. With the assumption of time-invariant channels within P phases, for unicasting strategy, any other relationship that one may derive will lead to the same performance. The relationships given in Section II-A1 has an advantage that it does not require any signaling in the network. For hybrid uni/multicasting, one may also find other relationship than the relationships given in (2) and (3). However, with the time-invariant channels assumption and given the same t_{l_u}, the same performance will be obtained. In Section II-A2, we choose $t_{l_u} = a_l$ such that there will be no signaling in the network. This is sub-optimum and one may improve the performance by exhaustively searching the best t_{l_u}, $\forall l$. This will lead to a higher computational complexity and also requires signalling in the network since the RS needs to inform all nodes in group l about the chosen t_{l_u}. Regarding multicasting strategy, in this work we propose $v_l = a_l$ and $w_l = p + a_l$ -1 which does not require any signaling in the network. This is sub-optimum and one may improve the performance by exhaustively searching v_l and w_l which optimises the performance while fulfilling the general rule for multicasting strategy as described in Section II-A3. However, the computational complexity will be higher and there are signaling needed since the RS needs to inform the nodes in group l about v_l and w_l.

Note that all the relationships of parameters which are described in this subsection can also be directly applied for the case when the numbers of nodes are not equal in all groups. If the numbers of nodes are not equal in all groups, in each pth phase, the RS serves only the groups with $N_l \geq p$.

B. Generalised System Model

In this subsection, we explain the generalised system model for non-regenerative MGMW relaying which is valid for all BC strategies. In order to have non-regenerative MGMW relaying with a specific BC strategy, the relationship of p, r_l, and t_l as described in the previous subsection has to be set accordingly.

The overall channel matrix from the nodes to the RS is given by $\mathbf{H} = [\mathbf{h}_0, \ldots, \mathbf{h}_{N-1}] \in \mathbb{C}^{M \times N}$, with $i \in \mathcal{I}$, $i \in \mathcal{I}$, the channel vector between node $\mathrm{S}i$ and the RS. The channel coefficient $h_{i,\ \ m}, m \in \mathcal{M}$, $\mathcal{M} = \{1, \ldots, M\}$, follows $\mathcal{CN}(0, \sigma_x^2)$. The vector $\mathbf{x} \in \mathbb{C}^{N \times 1}$ is equal to $(x_0, \ldots, x_{N-1})^{\mathrm{T}}$, with x_i the transmit signal of node $\mathrm{S}i$ that follows $\mathcal{CN}(0, \sigma_x^2)$. The AWGN noise vector at the RS is denoted as $\mathbf{z}_{\mathrm{RS}} = (z_{\mathrm{RS}1}, \ldots, z_{\mathrm{RS}M})^{\mathrm{T}} \in \mathbb{C}^{M \times 1}$ with $z_{\mathrm{RS}m}$ following $\mathcal{CN}(0, \sigma_{z_{\mathrm{RS}}}^2)$. In this work, we assume that all nodes transmit with fixed and equal transmit power.

In the first phase, all nodes transmit simultaneously to the RS and the received signal at the RS is given by

$$\mathbf{y}_{\mathrm{RS}} = \mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}. \tag{5}$$

Assuming reciprocal and time-invariant channels in P phases, the downlink channel from the RS to the nodes is simply the transpose of the uplink channel $\mathbf{H}$. In the pth phase, the RS performs transceive beamforming, denoted by matrix $\mathbf{G}^p$, to the received signals and transmits to the nodes. Therefore, $\mathbf{G}^p$ has to be designed to ensure that the MGMW relaying is performed according to the chosen BC strategy. It is assumed that there is a transmit power constraint at the RS. The received signal vector of all nodes in the pth phase can be written as

$$\mathbf{y}^p_{\text{nodes}} = \mathbf{H}^{\mathrm{T}}\mathbf{G}^p(\mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}) + \mathbf{z}^p_{\text{nodes}}, \quad (6)$$

where $\mathbf{z}^p_{\text{nodes}} = (z^p_0, \cdots, z^p_{N-1})^{\mathrm{T}}$, with $z^p_{r_l}$ the noise at receiving node r_l which follows $\mathcal{CN}(0, \sigma^2_{z_{\text{node}}})$. Accordingly, the received signal at node $\mathrm{S}r_l$, $r_l \in \mathcal{I}_l$, while receiving the data stream from node $\mathrm{S}t_l$, $t_l \in \mathcal{I}_l\backslash\{r_l\}$, in the pth phase is given by

$$\gamma^p_{r_l,t_l} = \underbrace{\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_{t_l}x_{t_l}}_{\text{useful signal}} + \underbrace{\sum_{\substack{j=0\\ j\neq t_l}}^{N-1} \mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_j x_j}_{\text{interference signals}} + \underbrace{\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{z}_{\mathrm{RS}}}_{\text{RS's propagated noise}} + \underbrace{z^p_{r_l}}_{\text{node } r_l\text{'s noise}}. \quad (7)$$

Sum Rate Expression

In this section, we derive the sum rate expression of non-regenerative MGMW relaying. We start by defining the signal to interference and noise ratio (SINR) for the BC strategies. The achievable sum rate of MGMW relaying for both asymmetric and symmetric traffic are explained afterwards. The achievable sum rate is the sum of the rates received at all nodes. Asymmetric traffic refers to the situation where we allow all nodes in the group to transmit with different rates. Each node transmits with a rate that ensures that in the following N_l - 1 consecutive BC phases, all N_l - 1 nodes in its group can decode its data stream correctly. Symmetric traffic is when all nodes in group l have to transmit simultaneously with the same rate that is defined by the lowest rate among all possible link combinations of receive and transmit node (r_l, t_l) in group l.

A. Signal to Interference and Noise Ratio

It is assumed that x_i, $\forall i$, z_{RS_m}, $\forall m$, and z_i, $\forall i$, are all statistically independent. Therefore, given the received signal in (7), the SINR for the link between receiving node $\mathrm{S}r_l$ and transmitting node $\mathrm{S}t_l$ is given by

$$\gamma^p_{r_l,t_l} = \frac{s_{r_l}}{I_{r_l} + Z_{\mathrm{RS}_{r_l}} + Z_{r_l}}, \quad (8)$$

with the useful signal power at node $\mathrm{S}r_l$

$$S_{r_l} = \mathrm{E}\{|\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_{t_l}x_{t_l}|^2\} = |\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_{t_l}|^2\sigma^2_x, \quad (9)$$

the RS's propagated noise power which appear at node $\mathrm{S}r_l$

$$Z_{\mathrm{RS}_{r_l}} = \mathrm{E}\{|\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{z}_{\mathrm{RS}}|^2\} = |\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p|^2\sigma^2_{z_{\mathrm{RS}}} \quad (10)$$

and the node $\mathrm{S}r_l$'s noise power

$$Z_{r_l} = \mathrm{E}\{|z_{r_l}|^2\} = \sigma^2_{z_{\text{node}}}. \quad (11)$$

The interference power at receiving node $\mathrm{S}r_l$, is given by

$$I_{r_l} = I_{\mathrm{sg}_{r_l}} + I_{\mathrm{og}_{r_l}}, \quad (12)$$

with $I_{\mathrm{sg}_{r_l}}$ the same group interference power and $I_{\mathrm{og}_{r_l}}$ the other group interference power. While $I_{\mathrm{sg}_{r_l}}$ depends on the applied BC strategy, $I_{\mathrm{og}_{r_l}}$ does not depend on the BC strategy and it is given by

$$I_{\mathrm{og}_{r_l}} = \sum_{d\notin\mathcal{I}_l} \mathrm{E}\{|\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_d x_d|^2\} = \sum_{d\notin\mathcal{I}_l} |\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_d|^2\sigma^2_x. \quad (13)$$

At each receiving node $\mathrm{S}r_l$, $I_{\mathrm{sg}_{r_l}}$ includes the interference power caused by its own data stream and other data streams that have been decoded in the previous BC phases. These a priori known data streams can be canceled by each receiving node prior to detection by performing self- and known-interference cancellation. If self- and known-interference cancellation is performed, the remaining interference power which is not canceled by the receiving node, $I_{\text{not - canc}_{r_l}}$, is given by

$$I_{\text{not - canc}_{r_l}} = I_{\mathrm{sg}_{r_l}} - I_{\mathrm{canc}_{r_l}} \quad (14)$$

with $I_{\mathrm{canc}_{r_l}}$ the interference power caused by the data streams which are a priori known by the receiving node $\mathrm{S}r_l$ and is canceled. With interference cancellation, the interference power in (12) can be rewritten as

$$I_{r_l} = I_{\text{not - canc}_{r_l}} + I_{\mathrm{og}_{r_l}}. \quad (15)$$

In the following, we explain $I_{\mathrm{sg}_{r_l}}$ and $I_{\text{not - canc}_{r_l}}$ for each BC strategy.

1) Unicasting

The same group interference power is given by

$$I^{\mathrm{u}}_{\mathrm{sg}_{r_l}} = \sum_{\substack{j=a_l\\ j\neq t_l}}^{b_l} \mathrm{E}\{|\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_j x_j|^2\} = \sum_{\substack{j=a_l\\ j\neq t_l}}^{b_l} |\mathbf{h}^{\mathrm{T}}_{r_l}\mathbf{G}^p\mathbf{h}_j|^2\sigma^2_x. \quad (16)$$

In every pth phase, node $\mathrm{S}r_l$ may perform interference cancellation. It subtracts the a priori known self-interference as well as the a priori known same group other-stream interference from the previous BC phases. Once the nodes have decoded other nodes' data streams in the previous BC phases, they may use it to perform known-interference cancellation in a similar fashion to self-interference cancellation. Using interference cancellation, $I_{\text{not - canc}_{r_l}}$ for unicasting strategy is given by

$$I^{\text{u}}_{\text{not - canc}_{r_l}} = \sum_{\substack{j = a_l \\ j \neq \{r_l, t_l\} \\ j \notin \mathcal{B}_{r_l}}}^{b_l} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2 \tag{17}$$

with $\mathcal{B}_{r_l}$ the set of the nodes' indices whose data streams have been decoded by receiving node r_l in the previous BC phases.

2) Hybrid uni/multicasting

The same group interference power can be decoupled into two parts. The first part is the interference caused by the unicasted or the multicasted data stream, denoted by $I_{\text{u/m}_{r_l}}$. The second part is the interference caused by other data streams which can only appear at the receiving node r_l if the transceive beamforming applied at the RS cannot fully suppress it. The same group interference power is given by

$$I^{\text{u/m}}_{\text{sg}_{r_l}} = I_{\text{u/m}_{r_l}} + \sum_{\substack{j = a_l \\ j \neq \{t_{l_\text{u}}, t_{l_\text{m}}\}}}^{b_l} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2, \tag{18}$$

with t_{l_u} the index of the transmitting node whose data stream is unicasted by the RS, t_{l_m} the index of the transmitting node whose data stream is multicasted by the RS, and

$$I_{\text{u/m}_{r_l}} = \begin{cases} \text{E}\{|\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{r_l} x_{r_l}|^2\} = ||\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{r_l}|^2 \sigma_x^2, & \text{if } r_l = t_{l_\text{m}}, \\ \text{E}\{|\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{t_{l_\text{u}}} x_{t_{l_\text{u}}}|^2\} = |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{t_{l_\text{u}}}|^2 \sigma_x^2, & \text{otherwise}, \end{cases} \tag{19}$$

the interference at the nodes which only can be either from the unicasted data stream (at N_l - 1 nodes which are intended to receive the multicasted data stream) or from the multicasted data stream (at the node which receives the unicasted data stream). Similar to the unicasting strategy, interference cancellation at the nodes can also be applied. For hybrid uni/multicasting transmission, $I_{\text{not - canc}_{r_l}}$ is defined by

$$I^{\text{u/m}}_{\text{not - canc}_{r_l}} = I_{\text{u}_l} + \sum_{\substack{j = a_l \\ j \neq \{r_l, t_{l_\text{m}}\} \\ j \notin \mathcal{B}_l}}^{b_l} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2, \tag{20}$$

with $\mathcal{B}_l$ the sets of nodes' indices whose data streams have been multicasted by the RS in the previous BC phases and

$$I_{\text{u}_l} = \begin{cases} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{t_{l_\text{u}}}|^2 \sigma_x^2, & \text{if } r_l \neq t_{l_\text{u}} \text{ and } r_l \notin \mathcal{B}_l \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

3) Multicasting

The same group interference power can be decoupled into two parts. The first part is the inherent interference within the superposed data stream which can only be either self- or known-interference, denoted by $I_{\text{s|k}}$. The second part is the interference caused by other data streams which can only appear at the receiving node r_l if the transceive beamforming applied at the RS cannot fully suppress it. The same group interference power is given by

$$I^{\text{m}}_{\text{sg}_{r_l}} = I_{\text{s|k}_{r_l}} + \sum_{\substack{g = a_l \\ j \neq \{v_l, w_l\}}}^{b_l} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2, \tag{22}$$

with $\{v_l, w_l\}$ the indices of the two nodes in group l whose data streams are superposed by the RS in the pth phase.

$I_{\text{s|k}_{r_l}}$ is the self- or known-interference power, which can only be either self-interference power at nodes $r_l = w_l$ and $r_l = v_l$ given by

$$I_{\text{s|k}_{r_l}} = I_{\text{s}_{r_l}} = \text{E}\{|\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{r_l} x_{r_l}|^2\} = |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{r_l}|^2 \sigma_x^2, \tag{23}$$

or known-interference power at nodes $r_l \neq w_l \neq v_l$ given by

$$I_{\text{s|k}_{r_l}} = I_{\text{k}_{r_l}} = \text{E}\{|\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{v\tilde{w}_l} x_{v\tilde{w}_l}|^2\} = |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_{v\tilde{w}_l}|^2 \sigma_x^2, \tag{24}$$

with $v\tilde{w}_l$ the index of the known-interference which can only be either w_l or v_l. As explained in Section II-A3, $I_{\text{s|k}_{r_l}}$ can be cancelled and, thus, $I_{\text{s|k}_{r_l}} = 0$. Moreover, once the nodes have decoded other nodes' data streams from the previous BC phases, they may use them to reduce the amount of interference in the second summand in (22). For the multicasting strategy, $I_{\text{not - canc}_{r_l}}$ is defined by

$$I^{\text{m}}_{\text{not - canc}_{r_l}} = \sum_{\substack{j = a_l \\ j \neq \{v_l, w_l\} \\ j \notin \mathcal{B}_{r_l}}}^{b_l} |\mathbf{h}_{r_l}^{\text{T}} \mathbf{G}^p \mathbf{h}_j|^2 \sigma_x^2 \tag{25}$$

with $\mathcal{B}_{r_l}$ the set of the nodes' indices whose data streams have been decoded by receiving node r_l in the previous BC phases.

B. Sum Rate for Asymmetric Traffic

Given the SINR as in (8), the information rate at receiving node r_l when it receives from transmitting node t_l in the pth phase is given by

$$R_{r_l, t_l} = \log_2(1 + \gamma^p_{r_l, t_l}). \tag{26}$$

Since in MGMW relaying there is only one MAC phase, the transmitting node t_l has to ensure that its data stream can be decoded correctly by all N_l - 1 intended receiving nodes. Consequently, we have

$$R_{t_l} = \min_{r_l \in \mathcal{I}_l \setminus \{t_l\}} (R_{r_l,t_l}), \tag{27}$$

which is the minimum rate among all receiving nodes r_l in group l when they receive the data stream from a certain transmitting node t_l. The achievable sum rate of non-regenerative MGMW relaying is given by

$$\mathrm{SR}_{\mathrm{asym}} = \frac{1}{P} \sum_{l=1}^{L} \left((N_l - 1) \sum_{t_l \in \mathcal{I}_l} R_{t_l} \right). \tag{28}$$

The factor N_l - 1 is since in group l there are N_l - 1 nodes that receive the same data stream from a certain transmitting node t_l. The scaling factor $\frac{1}{P}$ is due to P channel uses for MGMW relaying.

One important note regarding (27) is that by taking the minimum, we ensure each node Si transmits x_i with the rate that can be decoded correctly by all other nodes in its group. Thus, knowing x_i, all other nodes in the group can use it to perform known-interference cancellation in a similar fashion to their self-interference cancellation.

C. Sum Rate for Symmetric Traffic

In certain scenarios, there may be a requirement to have a symmetric traffic between all nodes in group l. All nodes communicate with the same data rate defined by the minimum of $R_{t_l}, \forall t_l,\ t_l \in \mathcal{I}_l$. The achievable sum rate for symmetric traffic for all BC strategies is given by

$$\mathrm{SR}_{\mathrm{symm}} = \frac{1}{P} \sum_{l=1}^{L} (N_l - 1) N_l \left(\min_{t_l \in \mathcal{I}_l} R_{t_l} \right). \tag{29}$$

Transceive Beamforming

In this section, first, we formulate the optimisation problem of finding the optimum transceive beamforming maximising the sum rate. Afterwards, we explain the design of generalised low complexity transceive beamforming algorithms for all BC strategies. It is assumed that perfect channel state information is available at the RS whose number of antennas is higher than or equal to the total number of the nodes, i.e., $M \geq N$.

A. Sum Rate Maximisation

The optimisation problem of finding the optimum transceive beamforming maximising the sum rate of non-regenerative MGMW relaying for asymmetric traffic can be written as

$$\begin{aligned} &\max_{\mathbf{G}^p} \sum_{i} \sum_{f(i,p)} R_{f(i,p),i} \\ &\text{s.t. } \mathrm{tr}\{\mathbf{G}^p (\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}) \mathbf{G}^{p\mathrm{H}}\} = E_{\mathrm{RS}}, \end{aligned} \tag{30}$$

with $\mathbf{R}_{\mathbf{x}} = \mathrm{E}\{\|\mathbf{x}\mathbf{x}^{\mathrm{H}}\|_2^2\}$, $\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}} = \mathrm{E}\{\|\mathbf{z}_{\mathrm{RS}}\mathbf{z}_{\mathrm{RS}}^{\mathrm{H}}\|_2^2\}$, and $f(i, p)$ the receiving node index, which is a function of transmitting index i and BC phase index p, and depends on the applied BC strategy.

In this work, we assume that the transmit powers at the nodes are fixed and equal. In order to improve the sum rate, one could have the transmit powers at the nodes as variables to be optimised subject to a power constraint at each node. However, since there is only one MAC phase, one has to find the optimum transmit power at each node and, simulateneously, the transceive beamforming for all BC phases, i.e., $\mathbf{G}^p, \forall p,\ p \in \mathcal{P}$. This joint optimisation problem would further increase the computational effort.

The optimisation problem in (30) is non-convex and it requires high computational complexity to find the global optimum solution. Thus, in the following, we propose generalised low complexity transceive beamforming algorithms for all proposed BC strategies.

As mentioned in Section II-B and as seen in (30), the transceive beamforming $\mathbf{G}^p$ depends on the BC strategy applied at the RS. In order to design generalised transceive beamforming for all BC strategies and to make the problem more tractable, we decouple $\mathbf{G}^p$ into transmit beamforming $\mathbf{G}_{\mathrm{T}}^p$, BC-strategy-defining permutation matrix $\mathbf{\Pi}^p$ and receive beamforming $\mathbf{G}_{\mathrm{R}}^p$, such that $\mathbf{G}^p = \mathbf{G}_{\mathrm{T}}^p \mathbf{\Pi}^p \mathbf{G}_{\mathrm{R}}^p$.

In the following, we explain specially designed transceive beamforming for MGMW relaying. First, we explain the generalised linear transceive beamforming based on three different optimisation criteria, namely, MF, ZF, and MMSE. Afterwards, we explain the generalised BC-strategy-aware (BCSA) transceive beamforming.

B. Linear Transceive Beamforming

In this subsection, we explain the design of three low complexity generalised linear transceive beamforming algorithms, namely, MF, ZF, and MMSE. Since we have only one MAC phase, the receive beamforming is computed only once, i.e., $\mathbf{G}_{\mathrm{R}}^p = \mathbf{G}_{\mathrm{R}}, \forall p \in \mathcal{P}$. The BC-strategy-defining permutation matrix $\mathbf{\Pi}^p$ defines the transmission from the RS according to the BC strategies. Table 1 shows $\mathbf{\Pi}^p$ for the example in Figure 1 for MF, ZF, and MMSE for all BC strategies. One important note is that, even though the derivation for the MF, ZF, and MMSE generalised transceive beamforming appears to be similar with the three-step transceive beamforming for two-way relaying in [9]; however, our generalised transceive beamforming algorithms have a different approach and

Table 1 BC-strategy-defining permutation matrices of L = 2, $N_1 = N_2 = 3$

	$\boldsymbol{\Pi}^2$	$\boldsymbol{\Pi}^3$
Unicasting	$\begin{pmatrix} 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&0&0&1&0&0 \end{pmatrix}$	$\begin{pmatrix} 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0 \end{pmatrix}$
Uni/multicasting	$\begin{pmatrix} 0&1&0&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0 \end{pmatrix}$	$\begin{pmatrix} 0&0&1&0&0&0\\ 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&1\\ 0&0&0&1&0&0 \end{pmatrix}$
Multicasting	$\begin{pmatrix} 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 1&1&0&0&0&0\\ 0&0&0&1&1&0\\ 0&0&0&1&1&0\\ 0&0&0&1&1&0 \end{pmatrix}$	$\begin{pmatrix} 1&0&1&0&0&0\\ 1&0&1&0&0&0\\ 1&0&1&0&0&0\\ 0&0&0&1&0&1\\ 0&0&0&1&0&1\\ 0&0&0&1&0&1 \end{pmatrix}$

are based on different motivations. In [9], the downlink (from the RS to the nodes) channel matrix is a permuted matrix of the uplink (from the nodes to the RS) channel matrix. Therefore, $\boldsymbol{\Pi}^p$ in [9] is a diagonal matrix with weighting factors in each of its diagonal elements. Such approach as in [9] is only suitable for unicasting strategy. Hence, our generalised transceive beamforming is a generalisation of the three-step transceive beamforming for two-way relaying in [9].

1) Matched Filter

Given the received signal at RS as in (5), the output of the receive filtering is given by

$$\hat{\mathbf{X}}_{\mathrm{RS}} = \mathbf{G}_{\mathrm{R}}\mathbf{y}_{\mathrm{RS}} = \mathbf{G}_{\mathrm{R}}(\mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}). \tag{31}$$

The MF optimisation problem for receive beamforming can be written as

$$\mathrm{G}_{\mathrm{R}_{\mathrm{MF}}} = \arg\max_{\mathbf{G}_{\mathrm{R}}} \frac{|\mathrm{E}\{\mathbf{x}^{\mathrm{H}}\hat{\mathbf{x}}_{\mathrm{RS}}\}|^2}{\mathrm{E}\{||\mathbf{x}||_2^2\}\mathrm{E}\{||\mathbf{G}_{\mathrm{R}}\mathbf{z}_{\mathrm{RS}}||_2^2\}}. \tag{32}$$

The objective function in (32) can be written as

$$\frac{|\mathrm{E}\{\mathbf{x}^{\mathrm{H}}\hat{\mathbf{x}}_{\mathrm{RS}}\}|^2}{\mathrm{E}\{||\mathbf{x}||_2^2\}\mathrm{E}\{||\mathbf{G}_{\mathrm{R}}\mathbf{z}_{\mathrm{RS}}||_2^2\}} = \frac{|\mathrm{tr}(\mathbf{G}_{\mathrm{R}}\mathbf{H}\mathbf{R}_{\mathrm{x}})|^2}{\mathrm{tr}(\mathbf{R}_{\mathrm{x}})\mathrm{tr}(\mathbf{G}_{\mathrm{R}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\mathbf{G}_{\mathrm{R}}^{\mathrm{H}})}. \tag{33}$$

By taking the derivative of (33) with respect to $\mathbf{G}_{\mathrm{R}}$ and setting it equal to zero, we have [see, e.g, [25,26]]

$$\mathbf{G}_{\mathrm{R}_{\mathrm{MF}}} = \mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}^{-1}. \tag{34}$$

The received signal at the nodes in (6) can now be rewritten as

$$\mathbf{y}_{\mathrm{nodes}}^{p} = \mathbf{H}^{\mathrm{T}}\mathbf{G}_{\mathrm{T}}^{p}\prod\nolimits^{p}\hat{\mathbf{x}}_{\mathrm{RS}} + \mathbf{z}_{\mathrm{nodes}}^{p} = \mathbf{H}^{\mathrm{T}}\mathbf{G}_{\mathrm{T}}^{p}\tilde{\mathbf{x}}_{\mathrm{RS}}^{p} + \mathbf{z}_{\mathrm{nodes}}^{p}, \tag{35}$$

with $\tilde{\mathbf{x}}_{\mathrm{RS}}^{p} = \boldsymbol{\Pi}^p\hat{\mathbf{x}}_{\mathrm{RS}}$ the transmitted signals from the RS in the pth phase. The MF optimization problem for transmit beamforming can be written as

$$\begin{aligned} \mathbf{G}_{\mathrm{T}_{\mathrm{MF}}}^{p} &= \underset{\mathbf{G}_{\mathrm{T}}^{p}}{\mathrm{argmax}} \frac{|\mathrm{E}\{\tilde{\mathbf{x}}_{\mathrm{RS}}^{p\mathrm{H}}\mathbf{y}_{\mathrm{nodes}}^{p}\}|^2}{\mathrm{E}\{||\tilde{\mathbf{x}}_{\mathrm{RS}}^{p}||_2^2\}\mathrm{E}\{||\mathbf{z}_{\mathrm{nodes}}^{p}||_2^2\}} \\ &\quad \text{s.t. } \mathrm{E}\{||\mathbf{G}_{\mathrm{T}}^{p}\tilde{\mathbf{x}}_{\mathrm{RS}}^{p}||^2\} = E_{\mathrm{RS}}. \end{aligned} \tag{36}$$

Let $\mathbf{R}_{\tilde{\mathbf{x}}_{\mathrm{RS}}}^{p} = \mathrm{E}\{||\tilde{\mathbf{x}}_{\mathrm{RS}}^{p}\tilde{\mathbf{x}}_{\mathrm{RS}}^{p^{\mathrm{H}}}||_2^2\}$ and $\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}}^{p} = \mathrm{E}\{||\mathbf{z}_{\mathrm{nodes}}^{p}\mathbf{z}_{\mathrm{nodes}}^{p^{\mathrm{H}}}||_2^2\}$. Using the same steps as in [26] by deriving the Lagrangian function and solving the Karush-Kuhn-Tucker (KKT) conditions, we have

$$\mathbf{G}_{\mathrm{T}_{\mathrm{MF}}}^{p} = \beta_{\mathrm{MF}}^{p}\mathbf{H}^{*}, \tag{37}$$

where $\beta_{\mathrm{MF}}^{p} \in \mathbb{R}_{+}$ is needed to fulfill the power constraint and given by

$$\beta_{\mathrm{MF}}^{p} = \sqrt{\frac{E_{\mathrm{RS}}}{\mathrm{tr}\left(\mathbf{H}^{*}\mathbf{R}_{\tilde{\mathbf{x}}_{\mathrm{RS}}}^{p}\mathbf{H}^{\mathrm{T}}\right)}}. \tag{38}$$

2) Zero Forcing

Given (31), the ZF optimisation problem for receive beamforming can be written as

$$\begin{aligned} \mathbf{G}_{\mathrm{R}_{\mathrm{ZF}}} &= \underset{\mathbf{G}_{\mathrm{R}}}{\mathrm{argmin}}\, \mathrm{E}\{||\mathbf{x} - \hat{\mathbf{x}}_{\mathrm{RS}}||_2^2\} \\ &\quad \text{s.t. } \hat{\mathbf{x}}_{\mathrm{RS}} = \mathbf{x}|_{\mathbf{z}_{\mathrm{RS}}=0}. \end{aligned} \tag{39}$$

where $\hat{\mathbf{x}}_{\mathrm{RS}} = \mathbf{x}|_{\mathbf{z}_{\mathrm{RS}}=0}$ is the ZF constraint which implies that $\mathbf{G}_{\mathrm{R}}\mathbf{H} = \mathbf{I}_M$. Due to the ZF constraint, the objective function in (39) can be written as

$$\mathrm{E}\{||\mathbf{x} - \hat{\mathbf{x}}_{\mathrm{RS}}||_2^2\} = \mathrm{tr}\left(\mathbf{G}_{\mathrm{R}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\mathbf{G}_{\mathrm{R}}^{\mathrm{H}}\right). \tag{40}$$

Using the same steps as in [26] by deriving the Lagrangian function and solving the KKT conditions, we have [see, e.g., [25]]

$$\mathbf{G}_{\mathrm{R}_{\mathrm{ZF}}} = \left(\mathbf{H}^{\mathrm{H}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{H}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}^{-1}. \tag{41}$$

Given (35), the ZF optimisation problem for transmit beamforming can be written as

$$\begin{aligned} \mathbf{G}_{\mathrm{T}_{\mathrm{ZF}}}^{p} &= \underset{\mathbf{G}_{\mathrm{T}}^{p}}{\mathrm{argmin}}\, \mathrm{E}\{||\tilde{\mathbf{x}}_{\mathrm{RS}}^{p} - \mathbf{y}_{\mathrm{nodes}}^{p}||_2^2\} \\ &\quad \text{s.t. } \mathrm{E}\{||\mathbf{G}_{\mathrm{T}}^{p}\tilde{\mathbf{x}}_{\mathrm{RS}}^{p}||^2\} = E_{\mathrm{RS}}, \\ &\quad \mathbf{y}_{\mathrm{nodes}} = \tilde{\mathbf{x}}_{\mathrm{RS}}|_{\mathbf{z}_{\mathrm{nodes}}=0}. \end{aligned} \tag{42}$$

where $\mathbf{y}_{\mathrm{nodes}} = \tilde{\mathbf{x}}_{\mathrm{RS}}|_{\mathbf{z}_{\mathrm{nodes}}=0}$ is the ZF constraint which implies that $\mathbf{H}^{\mathrm{T}}\mathbf{G}_{\mathrm{T}}^{p} = \mathbf{I}_N$. Due to the ZF constraint, the objective function in (42) can be written as

$$\mathrm{E}\{||\tilde{\mathbf{x}}_{\mathrm{RS}}^{p} - \mathbf{y}_{\mathrm{nodes}}^{p}||_2^2\} = \mathrm{tr}\left(\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}}^{p}\right). \quad (43)$$

Using the same steps as in [26] by deriving the Lagrangian function and solving the KKT conditions, we have

$$\mathbf{G}_{\mathrm{T_{ZF}}}^{p} = \beta_{\mathrm{ZF}}^{p}\mathbf{H}^{*}(\mathbf{H}^{\mathrm{T}}\mathbf{H}^{*})^{-1}, \quad (44)$$

where $\beta_{\mathrm{ZF}}^{p} \in \mathbb{R}_{+}$ is needed to fulfill the power constraint and given by

$$\beta_{\mathrm{MF}}^{p} = \sqrt{\frac{E_{\mathrm{RS}}}{\mathrm{tr}\left((\mathbf{H}^{*}\mathbf{H}^{\mathrm{T}})\mathbf{R}_{\tilde{\mathbf{x}}_{\mathrm{RS}}}^{p}\right)}}. \quad (45)$$

3) Minimisation of Mean Square Error

Given (31), the MMSE optimisation problem for receive beamforming can be written as

$$\mathbf{G}_{\mathrm{R_{MMSE}}} = \underset{\mathbf{G}_{\mathrm{R}}}{\mathrm{argmin}}\,\mathrm{E}\{||\mathbf{x} - \hat{\mathbf{x}}_{\mathrm{RS}}||_2^2\}. \quad (46)$$

The objective function in (46) can be written as

$$\mathrm{E}\{||\mathbf{x} - \hat{\mathbf{x}}_{\mathrm{RS}}||_2^2\} = \mathrm{tr}\left(\mathbf{R}_{\mathbf{x}} - 2\mathbb{R}(\mathbf{G}_{\mathrm{R}}\mathbf{H}\mathbf{R}_{\mathbf{x}}) + \mathbf{G}_{\mathrm{R}}\mathbf{H}\mathbf{R}_{\mathbf{x}}\mathbf{H}^{\mathrm{H}}\mathbf{G}_{\mathrm{R}}^{\mathrm{H}} + \mathbf{G}_{\mathrm{R}}\mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\mathbf{G}_{\mathrm{R}}^{\mathrm{H}}\right). \quad (47)$$

By taking the derivative of (47) with respect to $\mathrm{G_R}$ and setting it equal to zero, we have [see, e.g., [25,26]]

$$\mathbf{G}_{\mathrm{R_{MMSE}}} = \mathbf{R}_{\mathbf{X}}\mathbf{H}^{\mathrm{H}}\left(\mathbf{H}\mathbf{R}_{\mathbf{X}}\mathbf{H}^{\mathrm{H}} + \mathbf{R}_{\mathbf{z}_{\mathrm{RS}}}\right)^{-1}. \quad (48)$$

Given (35), the MMSE optimisation problem for transmit beamforming can be written as

$$\{\mathbf{G}_{\mathrm{T_{MMSE}}}^{p},\ \beta_{\mathrm{MMSE}}^{p}\} = \underset{\mathbf{G}_{\mathrm{T}}^{p},\beta^{p}}{\mathrm{argmin}}\mathrm{E}\,||\tilde{\mathbf{x}}_{\mathrm{RS}}^{p} - \frac{1}{\beta^{p}}\mathbf{y}_{\mathrm{nodes}}^{p}||_2^2\} \quad \text{s.t. } \mathrm{E}\{||\mathbf{G}_{\mathrm{T}}^{p}\tilde{\mathbf{x}}_{\mathrm{RS}}^{p}||^2\} = E_{\mathrm{RS}}. \quad (49)$$

where $1/\beta^{p}$ is introduced to modify the mean square error as in [27,28]. Using the same steps as in [28] by deriving the Lagrangian function and solving the KKT conditions, we have

$$\mathbf{G}_{\mathrm{T_{MMSE}}}^{p} = \beta_{\mathrm{MMSE}}^{p}\left(\mathbf{H}^{*}\mathbf{H}^{\mathrm{T}} + \frac{\mathrm{tr}\left(\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}}^{p}\right)}{E_{\mathrm{RS}}}\mathbf{I}_{M}\right)^{-1}\mathbf{H}^{*}, \quad (50)$$

where $\beta_{\mathrm{MMSE}}^{p} \in \mathbb{R}_{+}$ is needed to fulfill the power constraint and given by

$$\beta_{\mathrm{MMSE}}^{p} = \sqrt{\frac{E_{\mathrm{RS}}}{\mathrm{tr}\left(\left(\mathbf{H}^{*}\mathbf{H}^{\mathrm{T}} + \frac{\mathrm{tr}\left(\mathbf{R}_{\mathbf{z}_{\mathrm{nodes}}}^{p}\right)}{E_{\mathrm{RS}}}\mathbf{I}_{M}\right)^{-2}\mathbf{H}^{*}\mathbf{R}_{\tilde{\mathbf{x}}_{\mathrm{RS}}}^{p}\mathbf{H}^{\mathrm{T}}\right)}}. \quad (51)$$

Finally, for MF, ZF, and MMSE the transceive beamforming is given by

$$\mathbf{G}^{p} = \beta_{\mathrm{algorithm}}^{p}\mathbf{G}_{\mathrm{T_{algorithm}}}^{p}\Pi^{p}\mathbf{G}_{\mathrm{R_{algorithm}}}^{p}, \quad (52)$$

where the subscript $(\cdot)_{\mathrm{algorithm}}$ refers to either MF, ZF, or MMSE.

C. Broadcast-Strategy-Aware Transceive Beamforming

In the following, we explain the design of BCSA transceive beamforming. Based on the chosen BC strategy, the RS separates the data streams which are going to be transmitted in the BC phase and transmits to the corresponding node or nodes. For unicasting strategy, the RS separates all data streams and transmits each data stream to each corresponding receiving node. For hybrid uni/multicasting, for each group, the RS separates the unicasted data stream from the other data streams and transmits it to the corresponding node whose data stream is multicasted. The RS also separates the multicasted data stream from the other data streams and transmits it to the remaining nodes in the corresponding group. For multicasting strategy, the RS separates the superposition of two data streams from the others and transmits the superposed data stream to all nodes in the group.

In order to compute the transceive beamforming, we first compute the equivalent channels for receive beamforming and transmit beamforming. The equivalent channels are needed to ensure that there will be no interstream interference received at the unintended receiving node or nodes. In order to find the equivalent channel, BD as proposed in [17] can be applied. Several works have considered BD for separation of data streams, e.g., [16,20,29,30]. In this work, we also consider regularised BD (RBD) as proposed in [23]. RBD avoids the drawbacks of BD which has a quite poor performance if the subspaces of the users channel matrices overlap significantly [23].

Equivalent channel

Without loss of generality, in the following we omit the BC phase index p. Let $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} \in \mathbb{C}^{\eta_{\mathrm{in}} \times M}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} \in \mathbb{C}^{(N-\eta_{\mathrm{in}}) \times M}$ denote the channel matrix of the intended nodes and the channel matrix of the other unintended nodes, respectively, with η_{in} the number of intended nodes. Both channel matrices are parts of the overall channel matrix, i.e., $\mathbf{H}^{\mathrm{T}} = \mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} \cup \tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$. Since the steps of computing the equivalent channel for receive beamforming and transmit beamforming are similar, we generally explain the methods for finding the equivalent channel using $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$. In order to relate them with the receive beamforming and transmit beamforming for

BC strategies, we have to set $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$ accordingly. Table 2 shows the corresponding $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$ for all BC strategies. Given the singular value decomposition (SVD) of the unintended nodes' channels as

$$\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \tilde{\mathbf{U}}_{\mathrm{u_n}} \tilde{\mathbf{\Sigma}}_{\mathrm{u_n}} \underbrace{\left[\tilde{\mathbf{V}}_{\mathrm{u_n}}^{(1)}, \tilde{\mathbf{V}}_{\mathrm{u_n}}^{(0)}\right]}_{\tilde{\mathbf{V}}_{\mathrm{u_n}}}, \tag{53}$$

we compute the equivalent channel for the intended nodes $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}}$. The equivalent channel is given by

$$\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}} = \mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} \mathbf{F}_{\mathrm{null}}, \tag{54}$$

where $\mathbf{F}_{\mathrm{null}}$ is the null-space matrix which can be computed either using BD or RBD.

Using BD, $\mathbf{F}_{\mathrm{null}} = \tilde{\mathbf{V}}_{\mathrm{u_n}}^{(0)} \in \mathbb{C}^{M \times (N - \tilde{r}_{\mathrm{u_n}})}$ with $\tilde{r}_{\mathrm{u_n}}$ denoting the rank of matrix $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$. The BD approach can be used directly for receive and transmit beamforming, since it only deals with the channels without considering the noise. Using RBD, however, the equivalent channels for receive and transmit beamforming need to be computed differently. RBD for transmit beamforming has been derived in [23] and in this work, we provide the derivation of RBD for receive beamforming in the appendix. Using RBD, $\mathbf{F}_{\mathrm{null}} = \tilde{\mathbf{V}}_{\mathrm{u_n}} \left(\tilde{\mathbf{\Sigma}}_{\mathrm{u_n}}^{\mathrm{T}} \tilde{\mathbf{\Sigma}}_{\mathrm{u_n}} + \kappa \mathbf{I}_M \right) \in \mathbb{C}^{M \times M}$ with $\kappa = \frac{N \sigma_{\mathrm{node}}^2}{E_{\mathrm{RS}}}$ for transmit beamforming [23] and $\kappa = \frac{\sigma_{\mathrm{RS}}^2}{\sigma_x^2}$ for receive beamforming, see appendix.

Having $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}}$, we can now compute the receive beamforming and transmit beamforming. In the following, when computing the receive beamforming, $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}}$ and $\eta_{\mathrm{i_n}}$ relate to $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$ as defined in Table 2 for receive beamforming, while when computing transmit beamforming, $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}}$ and $\eta_{\mathrm{i_n}}$ relate to $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$ as defined in Table 2 for transmit beamforming.

In this work, we consider signal processing algorithms which do not deal with interference since $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{eq}}$ is free from unwanted data streams, namely, MF, SVD, and semidefinite relaxation (SDR) of maximising the minimum SNR. MF and SVD for single-pair two-way relaying have been investigated in [3,10]. The BD-MF and BD-SDR have been designed in [20] for single-group multi-way relaying for multicasting strategy. BD-SVD has been designed in [16] only for multi-user two-way relaying with multicasting strategy. In this work, BCSA transceive beamforming is designed for non-regenerative MGMW relaying for all proposed BC strategies. Due to the requirement to make generalised BCSA transceive beamforming also suitable for non-regenerative MGMW relaying with multicasting strategy, it has a slight difference to [3,10,16]. Using BCSA for multicasting strategy, for each group l, the RS has to transmit one data stream, which is a superposition of two data streams, to N_l nodes in the group where N_l can be any number higher than two. For that reason, in the design of receive beamforming using MF and SVD, we have to do a superposition of two data streams. This makes the proposed BCSA not a direct generalisation of [3,10,16]. However, for cases of one-pair two-way relaying and multi-user two-way relaying, if the superposition is not performed, BCSA is a generalisation of [3,10,16].

Table 2 The corresponding $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}}$ and $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}}$ for all BC strategies

	Receive beamforming	Transmit beamforming
UC	$\forall t_l$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{t_l}^{\mathrm{T}} \in \mathbb{C}^{1 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{t_l\}}^{\mathrm{T}} \in \mathbb{C}^{(N-1) \times M}$	$\forall r_l$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{r_l}^{\mathrm{T}} \in \mathbb{C}^{1 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{r_l\}}^{\mathrm{T}} \in \mathbb{C}^{(N-1) \times M}$
U/MC	For $t_{l_{\mathrm{u}}}$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{t_{l_{\mathrm{u}}}}^{\mathrm{T}} \in \mathbb{C}^{1 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathrm{H}_{\mathcal{I} \backslash \{t_{l_{\mathrm{u}}}\}}^{\mathrm{T}} \in \mathbb{C}^{(N-1) \times M}$ For $t_{l_{\mathrm{m}}}$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{t_{l_{\mathrm{m}}}}^{\mathrm{T}} \in \mathbb{C}^{1 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{t_{l_{\mathrm{m}}}\}}^{\mathrm{T}} \in \mathbb{C}^{(N-1) \times M}$	For $r_l = t_{l_{\mathrm{m}}}$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{r_l}^{\mathrm{T}} \in \mathbb{C}^{1 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{r_l = t_{l_{\mathrm{m}}}\}}^{\mathrm{T}} \in \mathbb{C}^{(N-1) \times M}$ $\forall r_l,\ r_l \in \mathcal{I}_l \backslash t_{l_{\mathrm{m}}}$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I}_l \backslash \{r_l = t_{l_{\mathrm{m}}}\}}^{\mathrm{T}} \in \mathbb{C}^{(N_l-1) \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{t_{l_{\mathrm{m}}}\}}^{\mathrm{T}} \in \mathbb{C}^{(N-(N_l-1)) \times M}$
MC	$\forall l$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{v_l w_l}^{\mathrm{T}} \in \mathbb{C}^{2 \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \{v_l w_l\}}^{\mathrm{T}} \in \mathbb{C}^{(N-2) \times M}$	$\forall l$: $\mathbf{H}_{\mathrm{i_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I}_l}^{\mathrm{T}} \in \mathbb{C}^{N_l \times M}$ $\tilde{\mathbf{H}}_{\mathrm{u_n}}^{\mathrm{T}} = \mathbf{H}_{\mathcal{I} \backslash \mathcal{I}_l}^{\mathrm{T}} \in \mathbb{C}^{(N-N_l) \times M}$

UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting
$t_{l_{\mathrm{u}}}$: index of transmitting node whose data stream is unicasted, $t_{l_{\mathrm{u}}} \in \mathcal{I}_l$
$t_{l_{\mathrm{m}}}$: index of transmitting node whose data stream is multicasted, $t_{l_{\mathrm{m}}} \in \mathcal{I}_l \backslash \{t_{l_{\mathrm{u}}}\}$

Matched filter The receive beamforming vector is given by

$$\mathbf{m} = \Gamma_{\mathrm{i_n}} \underbrace{1^{\mathrm{T}}_{\eta_{\mathrm{i_n}}} \mathbf{H}^{\mathrm{eq}*}_{\mathrm{i_n}} \mathbf{F}^{\mathrm{T}}_{\mathrm{null}}}_{\tilde{\mathbf{m}}}, \tag{55}$$

where $\mathbf{1}_{\eta \mathrm{i_n}}$ a vector of ones of length $\eta_{\mathrm{i_n}}$. $\mathbf{1}_{\eta \mathrm{i_n}}$ superposes (adds) two-data streams from two nodes in each group for multicasting strategy. $\Gamma_{\mathrm{i_n}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}^{\mathrm{T}}|)$ can be seen as receive power loading where the modulus operator $|\cdot|$ is assumed to be applied element-wise and the mean function returns the mean of a vector.

The transmit beamforming vector is given by

$$\mathbf{m}_{\mathrm{DL}} = \underbrace{\mathbf{F}_{\mathrm{null}} \mathbf{H}^{\mathrm{eqH}}_{\mathrm{i_n}} \mathbf{1}_{\eta \mathrm{i_n}}}_{\tilde{\mathbf{m}}_{\mathrm{DL}}} \Gamma_{\mathrm{i_n DL}}, \tag{56}$$

with $\Gamma_{\mathrm{i_n DL}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}_{\mathrm{DL}}|)$ the transmit power loading. $\mathbf{1}_{\eta \mathrm{i_n}}$ is a vector of ones with size $\eta_{\mathrm{i_n}} \times 1$. For multicasting strategy, $\mathbf{1}_{\eta \mathrm{i_n}}$ replicates the superposed data stream $\eta_{\mathrm{i_n}}$ times.

Singular value decomposition Let the SVD of the equivalent channel be given by

$$\mathbf{H}^{\mathrm{eq}}_{\mathrm{i_n}} = \mathbf{U}^{\mathrm{eq}}_{\mathrm{i_n}} \mathbf{\Sigma}^{\mathrm{eq}}_{\mathrm{i_n}} [\mathbf{V}^{\mathrm{eq}(1)}_{\mathrm{i_n}}, \mathbf{V}^{\mathrm{eq}(0)}_{\mathrm{i_n}}], \tag{57}$$

The receive beamforming vector is given by

$$\mathbf{m} = \Gamma_{\mathrm{i_n}} \underbrace{\mathbf{1}^{\mathrm{T}}_{\eta_{\mathrm{i_n}}} \mathbf{V}^{\mathrm{eq}(1)^{\mathrm{T}}}_{\mathrm{i_n}} \mathbf{F}^{\mathrm{T}}_{\mathrm{null}}}_{\tilde{\mathbf{m}}}, \tag{58}$$

with $\Gamma_{\mathrm{i_n}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}^{\mathrm{T}}|)$ the receive power loading.

The transmit beamforming vector is given by

$$\mathbf{m}_{\mathrm{DL}} = \underbrace{\mathbf{F}_{\mathrm{null}} \mathbf{V}^{\mathrm{eq}(1)}_{\mathrm{i_n}} 1_{\eta_{\mathrm{i_n}}}}_{\tilde{\mathbf{m}}_{\mathrm{DL}}} \Gamma_{\mathrm{i_n DL}}, \tag{59}$$

with $\Gamma_{\mathrm{i_n DL}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}_{\mathrm{DL}}|)$ the transmit power loading.

Semidefinite Relaxation

Since in MGMW relaying all member nodes in each group exchange messages, we are also interested in a fair beamforming algorithm which aims at balancing the SNRs at the RS as well as at the receiving nodes in each group.

The SNR balancing problem for receive beamforming can be written as

$$\mathbf{m}_{\mathrm{sdr}} = \arg\max_{\mathbf{m}} \min_{i_{\mathrm{in}} \in \mathcal{I}_{\mathrm{i_n}}} \left| \frac{\mathbf{m}\mathbf{h}^{\mathrm{eq}}_{i_{\mathrm{in}}}}{\sigma^2_{z_{\mathrm{RS}}}} \right|^2, \quad \text{s.t. } ||\mathbf{m}||_2^2 \le 1 \tag{60}$$

with $\mathcal{I}_{\mathrm{i_n}}$ the set of intended nodes with cardinality equal to $\eta_{\mathrm{i_n}}$. i_{in} is the index of a member node in $\mathcal{I}_{\mathrm{i_n}}$ and $\mathbf{h}^{\mathrm{eq}}_{i_{\mathrm{in}}} \in \mathbf{H}^{\mathrm{eq}}_{\mathrm{i_n}}$. The receive beamforming is given by

$$\mathbf{m} = \Gamma_{\mathrm{i_n}} \underbrace{\mathbf{m}^{\mathrm{T}}_{\mathrm{sdr}} \mathbf{F}^{\mathrm{T}}_{\mathrm{Null}}}_{\tilde{\mathbf{m}}_{\mathrm{sdr}}}, \tag{61}$$

with $\Gamma_{\mathrm{i_n}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}^{\mathrm{T}}_{\mathrm{sdr}}|)$ the receive power loading.

Equation (60) is a non-convex quadratically constrained quadratic program. A similar optimisation is also considered in [31]. It is proved to be NP-hard in [31]. Nonetheless, it can be approximately solved using SDR techniques [31,32]. We will not go further into this relaxation and the interested reader may find more detailed derivation in [20,31,32].

The SNR balancing problem for transmit beamforming can be written as

$$\mathbf{m}_{\mathrm{sdr\,DL}} = \arg\max_{\mathbf{m}} \min_{i_{\mathrm{in}} \in \mathcal{I}_l} \left| \frac{\mathbf{m}\mathbf{h}^{\mathrm{eq}}_{i_{\mathrm{in}}}}{\sigma^2_{z_{\mathrm{node}}}} \right|^2 \quad \text{s.t. } ||\mathbf{m}_{\mathrm{DL}}||_2^2 \le 1, \tag{62}$$

with $\mathbf{h}^{\mathrm{eq}}_{i_{\mathrm{in}}} \in \mathbf{H}^{\mathrm{eq}}_{\mathrm{i_n}}$. The transmit beamforming is given by

$$\mathbf{m}_{\mathrm{DL}} = \underbrace{\mathbf{F}_{\mathrm{Nu11}} \mathbf{m}_{\mathrm{sdr\,DL}}}_{\tilde{\mathbf{m}}_{\mathrm{sdr_{DL}}}} \Gamma_{\mathrm{i_n DL}}, \tag{63}$$

with $\Gamma_{\mathrm{i_n DL}} = \mathrm{mean}(|\mathbf{H}^{\mathrm{T}}_{\mathrm{i_n}} \tilde{\mathbf{m}}_{\mathrm{sdr_{DL}}}|)$ the transmit power loading. Note that to compute the transmit beamforming with SDR, we assume that the information of the noise power at the nodes is available at the RS. Similar to (60), (62) can be approximately solved with semidefinite relaxation techniques using a solver such as SEDUMI [33].

In the following, we use again the BC phase index p to describe the BCSA transceive beamforming. For unicasting strategy, the receive beamforming matrix is given by

$$\mathbf{G}^{p}_{\mathrm{R}} = [\mathbf{m}^{p}_{1}, \ldots, \mathbf{m}^{p}_{N}], \tag{64}$$

where $\mathbf{m}^{p}_{t_l}$, $\forall t_l \in \mathcal{I}$, is the receive beamforming as in (55), (58) or (61) given the equivalent channel of node t_l. The transmit beamforming matrix is given by

$$\mathbf{G}^{p}_{\mathrm{T}} = \left[\mathbf{m}^{p}_{\mathrm{DL}_1}, \ldots, \mathbf{m}^{p}_{\mathrm{DL}_1}\right], \tag{65}$$

where $\mathrm{m}^{p}_{\mathrm{DL}_{r_l}}$, $\forall r_l \in \mathcal{I}$, is the transmit beamforming as in (56), (59), or (63) given the equivalent channel of node r_l.

For hybrid uni/multicasting strategy, the receive beamforming matrix is given by

$$\mathbf{G}_{\mathrm{R}}^{p}=\left[\mathbf{m}_{t_{1\mathrm{u}}}^{p},\mathbf{m}_{t_{1\mathrm{m}}}^{p},\ \ldots,\ \mathbf{m}_{t_{L\mathrm{u}}}^{p},\ \mathbf{m}_{t_{L\mathrm{m}}}^{p}\right], \tag{66}$$

where m $\mathbf{m}_{t_{l\mathrm{u}}}^{p}$, $\forall l \in \mathcal{L}$, and $\mathbf{m}_{t_{l\mathrm{m}}}^{p}$, $\forall l \in \mathcal{L}$ are the receive beamforming as in (55), (58), or (61) given the equivalent channels of nodes $t_{l\mathrm{u}}$ and $t_{l\mathrm{m}}$, respectively. The transmit beamforming matrix is given by

$$\mathbf{G}_{\mathrm{T}}^{p}=\left[\mathbf{m}_{\mathrm{DL}_{r_1=t_{1\mathrm{m}}}}^{p},\mathbf{m}_{\mathrm{DL}_{\mathcal{I}_1\backslash\{t_{1\mathrm{m}}\}}}^{p},\ldots,\mathbf{m}_{\mathrm{DL}_{r_L=t_{L\mathrm{m}}}}^{p},\mathbf{m}_{\mathrm{DL}_{\mathcal{I}_L\backslash\{t_{L\mathrm{m}}\}}}^{p}\right], \tag{67}$$

where $\mathbf{m}_{\mathrm{DL}_{r_l=t_{l\mathrm{m}}}}^{p}$, $\forall l \in \mathcal{L}$, and $\mathbf{m}_{\mathrm{DL}_{\mathcal{I}_l\backslash\{t_{l\mathrm{m}}\}}}^{p}$, $\forall l \in \mathcal{L}$, are the transmit beamforming as in (56), (59), or (63) given the equivalent channel of node $r_l = t_{l\mathrm{m}}$ and the equivalent channel of all other nodes in group l, $\forall r_l \in \mathcal{I}_l\backslash\{t_{l\mathrm{m}}\}$, respectively.

For multicasting strategy, the receive beamforming matrix is given by

$$\mathbf{G}_{\mathrm{R}}^{p}=[\mathbf{m}_{l}^{p},\ldots,\ \mathbf{m}_{L}^{p}], \tag{68}$$

where $\mathbf{m}_{l}^{p}$, $\forall l \in \mathcal{L}$, is the receive beamforming as in (55), (58), or (61) given the equivalent channels of two nodes v_l and w_l whose data streams are superposed. The transmit beamforming matrix is given by

$$\mathbf{G}_{\mathrm{T}}^{p}=\left[\mathbf{m}_{\mathrm{DL}_1}^{p},\ldots,\ \mathbf{m}_{\mathrm{DL}_L}^{p}\right], \tag{69}$$

where $\mathbf{m}_{\mathrm{DL}_l}^{p}$, $\forall l \in \mathcal{L}$, is the transmit beamforming as in (56), (59), or (63) given the equivalent channels of all nodes in group l.

Finally, BCSA transceive beamforming is given by

$$\mathbf{G}^{p}=\beta^{p}\mathbf{G}_{\mathrm{T}}^{p}\Pi^{p}\mathbf{G}_{\mathrm{R}}^{p}. \tag{70}$$

where β^p is needed in order to satisfy the transmit power constraint at the RS, with

$$\beta^{p}=\sqrt{\frac{E_{\mathrm{RS}}}{\operatorname{tr}\left\{\mathrm{G}_{\mathrm{T}}^{p}\Pi^{p}\mathbf{G}_{\mathrm{R}}^{p}(\sigma_x^2\mathbf{H}\mathbf{H}^{\mathrm{H}}+\sigma_{\mathrm{RS}}^2\mathbf{I})\mathbf{G}_{\mathrm{R}}^{p^{\mathrm{H}}}\Pi^{p^{\mathrm{H}}}\mathbf{G}_{\mathrm{T}}^{p^{\mathrm{H}}}\right\}}}. \tag{71}$$

Note that $\mathbf{\Pi}^p$ is not the same for all BC strategies. For unicasting strategy, $\mathbf{\Pi}^p$ is the same as for MF, ZF, and MMSE, where an example for $L = 2$, $N_1 = N_1 = 3$ is given in Table 1. For hybrid uni/multicasting strategy, $\mathbf{\Pi}^p = \mathbf{I}_{2L}$ and for multicasting strategy, $\mathbf{\Pi}^p = \mathbf{I}_L$.

Simulation Results

In this section, the sum rate performance is analysed based on simulation results. We set $\sigma_{z_{\mathrm{RS}}}^2 = \sigma_{z_{\mathrm{node}}}^2 = 1$, $\sigma_x^2 = 1$, and $E_{\mathrm{RS}} = 1$. The channel coefficients are i.i.d. $\mathcal{CN}(0,\ \sigma_x^2)$, i.e., Rayleigh fading. Hence, the SNR value is given by $\frac{\sigma_x^2}{\sigma_{z_{\mathrm{node}}}^2}|h_{i,m}|^2 = \frac{\sigma_x^2}{\sigma_{z_{\mathrm{RS}}}^2}|h_{i,m}|^2$.

We consider three scenarios. The first two scenarios are the well-known scenarios, namely, one-pair two-way relaying and multi-user two-way relaying. We consider both scenarios to show that the proposed BC strategies and the generalised transceive beamforming designed in this work are valid for both well-known scenarios. The third scenario is the two-group multi-way case, where each group consists of three nodes.

A. First Scenario: L = 1 and N_1 = 2

In one-pair two-way relaying, unicasting and hybrid uni/multicasting are the same. Figure 2 shows the sum rate performance of one-pair two-way relaying with MF, ZF, and MMSE transceive beamforming for both asymmetric traffic and symmetric traffic. Regarding symmetric traffic, to reduce the number of lines in the figure, we only plot the result for MF transceive beamforming. The approximation of maximum sum rate is also provided for two cases, i.e., with optimised and with fixed transmit power at the nodes. Both optimum transceive beamforming solutions maximising the sum rate were computed using fmincon from MATLAB to provide performance bounds for two-way relaying. We use the value of MMSE transceive beamforming as the initial value. In general, using MF, ZF and MMSE transceive beamforming, unicasting and hybrid uni/multicasting outperform multicasting strategy. A direct superposition of the output of receive beamforming for the multicasting strategy doubles the amount of the RS's filtered noise. Moreover, the RS transmit power is distributed within the superposed data stream and after the self-interference cancellation, each node only receives half of the power. Since each node performs self-interference cancellation, no interference appears at the nodes, and thus, for all BC strategies MF outperforms MMSE and ZF. At low SNR, MMSE converges to MF and in high SNR, ZF converges to MMSE. In this work, we assume fixed transmit power at all nodes and the performance of unicasting and hybrid uni/multicasting with MF is close to the approximation of maximum sum rate with fixed transmit power. If the nodes can optimise their transmit power, the sum rate is improved with a penalty of having higher computational complexity. It can also be seen that asymmetric traffic leads to a higher rate compared to symmetric traffic since the rate for symmetric traffic is defined by the weakest link among all available links. Therefore, in the following, we only consider asymmetric traffic.

Figure 3 shows the sum rate performance of two-way relaying with BCSA transceive beamforming. For multicasting strategy, since there is no separation needed both for receive beamforming and transmit beamforming, BD and RBD are the same. For unicasting and

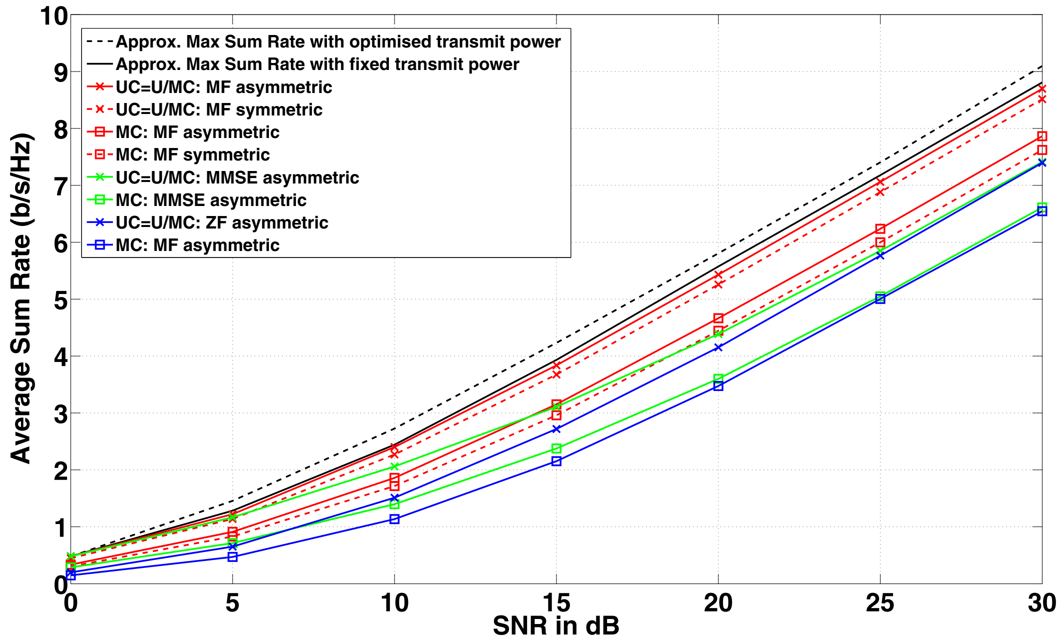

Figure 2 Sum rate performance of first scenario with MF, ZF, and MMSE; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

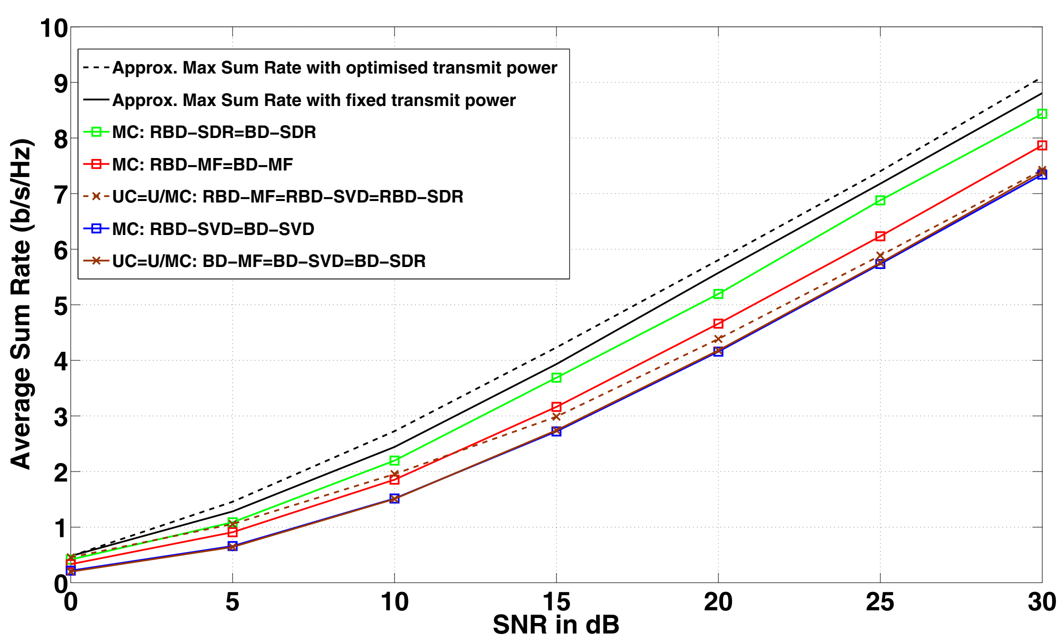

Figure 3 Sum rate performance of first scenario with BCSA; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

hybrid uni/multicasting strategies, BD-MF, BD-SVD, and BD-SDR perform the same and they have similar performance to multicasting strategy with SVD. Different to multicasting strategy, for unicasting and hybrid uni/multicasting, since there is a stream separation both in receive beamforming and transmit beamforming, RBD improves the performance in low SNR region. In high SNR region, BD converges to RBD. For unicasting and hybrid uni/multicasting strategies, since the equivalent channels (which are free from interference) always correspond only to one intended node for both receive beamforming and transmit beamforming, MF, SVD, and SDR will always have the same performance. It can be seen that multicasting strategy with SDR performs best. Hence, having a suitable transceive beamforming, one can exploit the benefit of beamforming-based physical layer network coding for non-regenerative single group multi-way relaying as proposed in [20].

B. Second Scenario: $L = 2$ and $N_1 = N_2 = 2$

Figure 4 shows the sum rate performance of multi-user two-way relaying with MF, ZF, and MMSE transmit beamforming. In this scenario, unicasting and hybrid uni/multicasting are the same and they outperform multicasting strategy. The reason is the same as in the case of one-pair two-way relaying. Moreover, the direct superposition of the output of receive beamforming for multicasting strategy not only increases the amount of the RS's filtered noise but also increases the unwanted interference at the receiving nodes. For all strategies, MMSE performs best and in high SNR region, ZF converges to MMSE, while in low SNR region, MF converges to MMSE. Different to the case of one-pair two-way relaying, in multi-user two-way relaying MF performs worse since it does not cancel the interference from other pairs which appears at each node. The transceive beamforming maximising the sum rate was computed using fmincon from MATLAB to provide a bound for multi-user two-way relaying. We use the value of MMSE tranceive beamforming as initial value. It can be clearly seen that if the transmit power at the nodes can be optimised, the sum rate can be improved at the expense of computational complexity.

Figure 5 shows the sum rate performance of multi-user two-way relaying with BCSA transceive beamforming. In general, RBD outperforms BD in low SNR region and BD converge to RBD in high SNR. Only for multicasting strategy, BD-SVD outperforms RBD-SVD for all SNR values and it has similar performance as unicasting and hybrid uni/multicasting with BD-MF, BD-SVD, and BD-SDR. The gain of RBD compared to BD is obtained most for unicasting and hybrid uni/multicasting strategies, while for multicasting strategy (with MF and SDR), the gain is small. In medium to high SNR region,

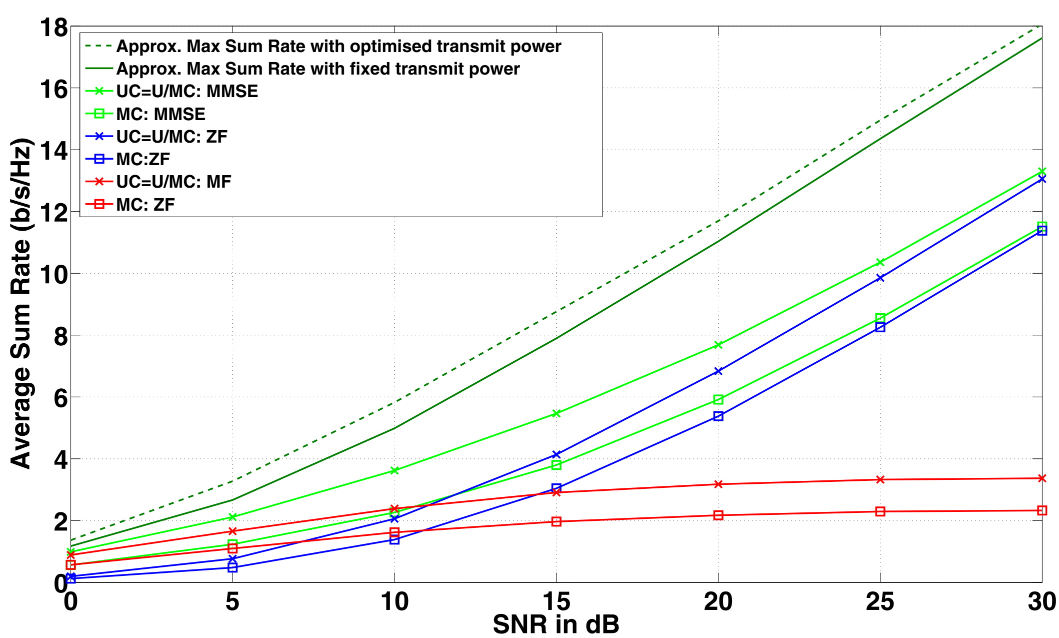

Figure 4 Sum rate performance of second scenario with MF, ZF, and MMSE; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

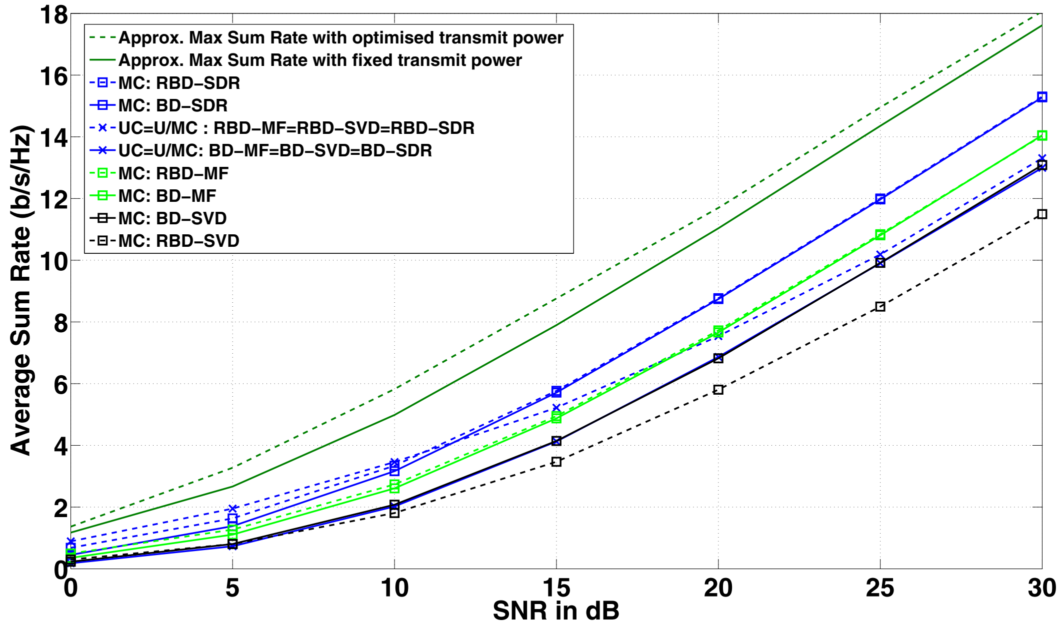

Figure 5 Sum rate performance of second scenario with BCSA; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

multicasting strategy with RBD-SDR or BD-SDR performs best. While for both unicasting and hybrid uni/multicasting strategies, the performance of MF, SVD, and SDR is the same, for multicasting strategy SDR always performs best followed by MF and SVD.

C. Third Scenario: L = 2 and $N_1 = N_2 = 3$

Figure 6 shows the sum rate performance of two-group three-way relaying using MF, ZF, and MMSE. In general hybrid uni/multicasting performs best followed by unicasting and multicasting strategies. While hybrid uni/

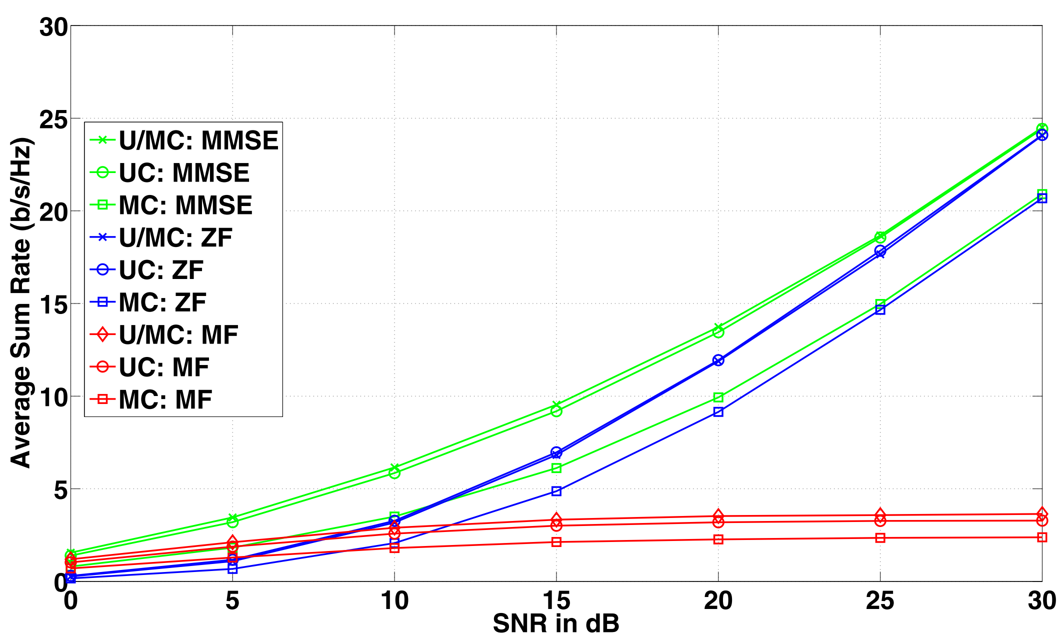

Figure 6 Sum rate performance of third scenario with MF, ZF, and MMSE; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

multicasting strategy with MMSE slightly ouperforms unicasting strategy with MMSE, both strategies have similar ZF performance. With MMSE, we find the trade off between the noise enhancement and the interference suppression. Since, hybrid uni/multicasting has smaller number of transmit data streams from the RS, it performs better than unicasting strategy both for MMSE and MF. ZF perfectly cancels the interference, and, thus, both unicasting and hybrid uni/multicasting perform similar. In general, for all strategies, ZF converges to MMSE in high SNR region and in low SNR region, MF converges to MMSE. It can be seen that multicasting strategy is outperformed by other strategies since it suffers from the increase of RS's filtered noise and the reduced received power at the nodes. This shows that analog network coding for non-regenerative MGMW relaying obtained by directly adding the output of receive beamforming (using MF, ZF, and MMSE receive beamforming) is not an efficient strategy and, thus, appropriate transceive beamforming is required.

Figs. 7 and 8 show the sum rate performance of two-group three-way relaying using BCSA transceive beamforming with BD and RBD, respectively. Comparing both the figures, in general, RBD outperforms BD, and they converge in high SNR. Only when using SVD, for both hybrid uni/multicasting and multicasting strategies, RBD-SVD performs worse than BD-SVD and BD-SVD does not converge to RBD-SVD in high SNR region. In medium to high SNR, multicasting strategy outperforms other strategies when using SDR and MF. However, if SVD is applied, unicasting strategy performs best. For unicasting strategy, MF, SVD and SDR have similar performance.

Comparing Figures 6, 7, and 8, one can clearly see that BCSA transceive beamforming improves the sum rate performance compared to MF, ZF, and MMSE transceive beamforming, especially for multicasting strategy. For the multicasting strategy, only RBD-SVD performs worse than ZF and MMSE. For the hybrid uni/multicasting strategy, BD-MF performs similar to ZF, RBD-MF performs similar to MMSE and both BD-SDR and RBD-SDR outperform MF, ZF, and MMSE. For the unicasting strategy, BD-MF, BD-SVD, and BD-SDR perform similar to ZF, while RBD-MF, RBD-SVD, and RBD-SDR perform similar to MMSE. The highest sum rate (especially in high SNR) is obtained by multicasting strategy with BCSA BD-SDR and RBD-SDR. Therefore, provided a suitable transceive beamforming is used which can exploit analog network coding, the sum rate of non-regenerative MGMW relaying can be improved.

Conclusion

In this paper, we consider non-regenerative MGMW relaying. A multi-antenna RS assists L communication groups, where N_l nodes in each group communicate to

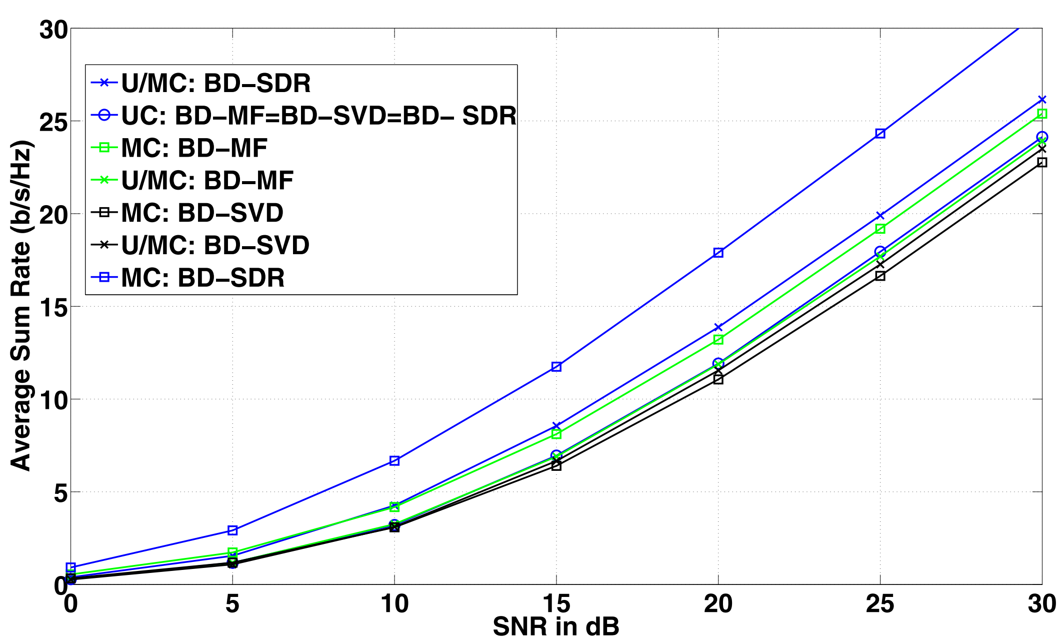

Figure 7 Sum rate performance of third scenario with BCSA BD; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

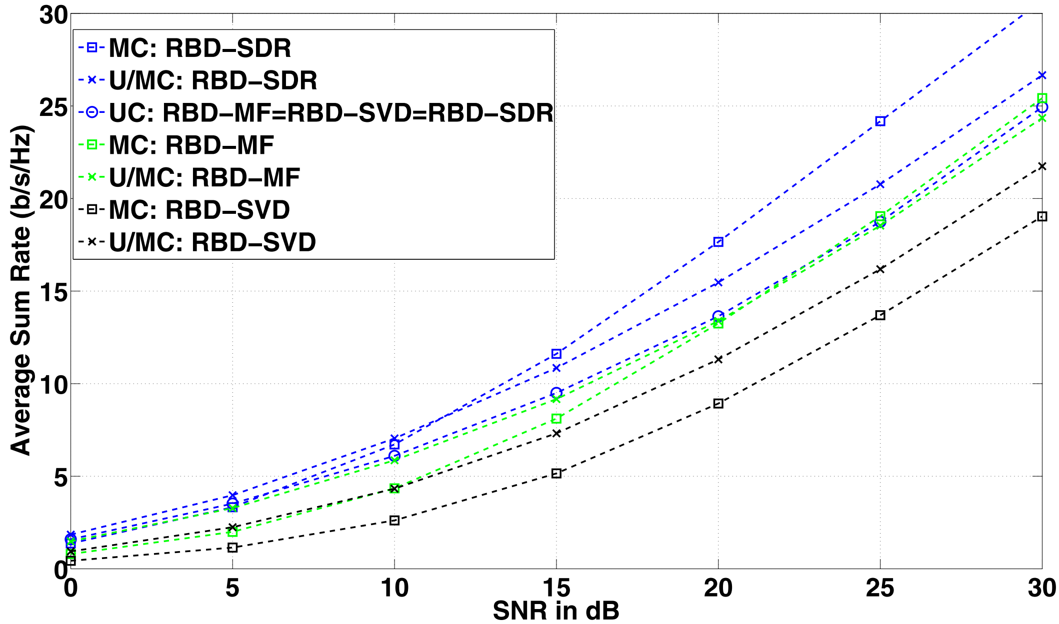

Figure 8 Sum rate performance of third scenario with BCSA RBD; UC, unicasting; U/MC, hybrid uni/multicasting; MC, multicasting.

each other but not with other nodes in other groups. The number P of communication phases is equal to $max_l\ N_l$. Three BC strategies are proposed, namely, unicasting, hybrid uni/multicasting, and multicasting. We derive the sum rate expression for non-regenerative MGMW relaying for asymmetric and symmetric traffic. We address the optimum transceive beamforming maximising the sum rate of non-regenerative MGMW relaying. We design generalised low complexity sub-optimum transceive beamforming for all BC strategies, namely, MF, ZF, MMSE, and BCSA transceive beamforming. It is shown that the performance of non-regenerative MGMW relaying depends on the BC strategy and the applied transceive beamforming. While multicasting strategy using BCSA SDR provides better sum rate performance compared to other strategies, however, if either MF, ZF, or MMSE are applied, multicasting strategy is outperformed by the other strategies.

Appendix

RBD for Receive Beamforming

For receive beamforming, the RS has to ensure that the interference from other users to the intended user i can be minimised while taking into consideration the appearance of noise at the RS. The matrix $\mathrm{F}_{\mathrm{Null}}$ is designed to achieve the aim and, by rewriting the optimisation problem for transmit beamforming in [23] Equation (9) we have the optimisation problem for receive beamforming,

$$\mathrm{F}_{\mathrm{Nu11}_{i,\forall i}} = \underset{\mathrm{F}_i,\forall i}{\arg\min}\, \mathrm{E}\left\{\sum_{i=1}^{N} ||\mathbf{F}_i\tilde{\mathbf{H}}_{i_{\mathrm{un}}}||^2 + \frac{||\mathbf{F}_i\mathbf{z}_{\mathrm{RS}}||^2}{\beta^{-1}}\right\}, \quad \text{s.t. } \beta\mathrm{E}\{\,||x_i x_i^{\mathrm{H}}||\} = P_{\mathrm{nodes}}, \tag{72}$$

where β is a scaling factor needed to fulfill the nodes' transmit power constraint. In this work, we assume that all nodes transmit with fixed and equal unit power and, thus, the constraint can be written as

$$\beta = \frac{P_{\mathrm{nodes}}}{\mathrm{E}\{||x_i x_i^{\mathrm{H}}||\}} = \frac{1}{\sigma_x^2}. \tag{73}$$

The objective function in (72), $f(\mathbf{F}_i)$, can be written as

$$f(\mathbf{F}_i) = \sum_{i=1}^{N}\left(\mathrm{tr}\left(\mathbf{F}_i\tilde{\mathbf{H}}_{i_{\mathrm{un}}}\tilde{\mathbf{H}}_{i_{\mathrm{un}}}^{\mathrm{H}}\mathbf{F}_i^{\mathrm{H}}\right) + \frac{\sigma_{\mathrm{RS}}^2\mathbf{I}_M}{\sigma_x^2}\mathrm{tr}\left(\mathbf{F}_i^{\mathrm{H}}\mathbf{F}_i\right)\right). \tag{74}$$

Let the SVD of $\tilde{\mathbf{H}}_{i_{\mathrm{un}}}$ be given by

$$\tilde{\mathbf{H}}_{i_{\mathrm{un}}} = \tilde{\mathbf{U}}_{i_{\mathrm{un}}}\tilde{\Sigma}_{i_{\mathrm{un}}}\tilde{\mathbf{V}}_{i_{\mathrm{un}}}, \tag{75}$$

(74) can be rewritten as

$$f(\mathbf{F}_i) = \sum_{i=1}^{N}\left(\mathrm{tr}\left(\mathbf{F}_i\tilde{\mathbf{U}}_{i_{\mathrm{un}}}\left(\tilde{\Sigma}_{i_{\mathrm{un}}}\tilde{\Sigma}_{i_{\mathrm{un}}}^{\mathrm{T}} + \frac{\sigma_{\mathrm{RS}}^2\mathbf{I}_M}{\sigma_x^2}\right)\tilde{\mathbf{U}}_{i_{\mathrm{un}}}^{\mathrm{H}}\mathbf{F}_i^{\mathrm{H}}\right)\right). \tag{76}$$

Let $\mathbf{F}_i = \mathbf{Fa}_i\mathbf{Fb}_i$ and let $\mathbf{Fb}_i = \widetilde{\mathbf{U}}_{i_{\mathrm{un}}}^{\mathrm{H}}$, the optimisation problem reduces to

$$\underset{\mathbf{Fa}_i,\forall i}{\operatorname{argmin}} \sum_{i=1}^{N} \left(\operatorname{tr} \left(\left(\tilde{\Sigma}_{i_{\mathrm{un}}} \tilde{\Sigma}_{i_{\mathrm{un}}}^{\mathrm{T}} + \frac{\sigma_{\mathrm{RS}}^2 \mathbf{I}_M}{\sigma_x^2} \right) \mathbf{Fa}_i^2 \right) \right) \tag{77}$$

where $\mathbf{Fa}_i$ needs to be positive definite in order to find a nontrivial solution [23]. Using the results from [23,34], we have

$$\mathbf{Fa}_i = \left(\tilde{\Sigma}_{i_{\mathrm{un}}} \tilde{\Sigma}_{i_{\mathrm{un}}}^{\mathrm{T}} + \kappa \mathbf{I}_M \right)^{-1/2}, \tag{78}$$

with $\kappa = \frac{\sigma_{\mathrm{RS}}^2}{\sigma_x^2}$.

Acknowledgements
The work of Aditya U. T. Amah is supported by the 'Excellence Initiative' of the German Federal and State Governments and the Graduate School of Computational Engineering, Technische Universität Darmstadt. The authors would like to thank the anonymous reviewers whose review comments are very helpful in improving this article.
Some parts of this paper have been presented at the IEEE ICASSP 2010, Dallas, USA, and IEEE WCNC 2010, Sydney, Australia.

Author details
[1]Graduate School of Computational Engineering and Communications Engineering Lab, Technische Universität Darmstadt, Darmstadt 64283 Germany [2]Communications Engineering Lab, Technische Universität Darmstadt, Darmstadt 64283 Germany

Competing interests
The authors declare that they have no competing interests.

Received: 23 December 2010 Accepted: 5 July 2011
Published: 5 July 2011

References

1. B Rankov, A Wittneben, Spectral efficient protocols for half-duplex relay channels. IEEE Journal on Selected Areas in Communications, **25**(2), 379–389 (Feb. 2007)
2. T Oechtering, Spectrally Efficient Bidirectional Decode-and-Forward Relaying for Wireless Networks. Ph.D dissertation, TU Berlin (2007)
3. T Unger, Multi-antenna two-hop relaying for bi-directional transmission in wireless communication systems. Ph.D dissertation, TU Darmstadt (2009)
4. E Biglieri, R Calderbank, A Constantinides, A Goldsmith, HV Poor, *MIMO Wireless Communication* (Cambridge University Press, 2007)
5. D Tse, P Viswanath, *Fundamentals of Wireless Communication* (Cambridge University Press, 2005)
6. I Hammerström, M Kuhn, C Esli, J Zhao, A Wittneben, G Bauch, MIMO two-way relaying with transmit CSI at the relay. in *Proc IEEE Signal Processing Advances in Wireless Communications*, Helsinki, 1–5 (June 2007)
7. TJ Oechtering, RF Wyrembelski, H Boche, Multiantenna bidirectional broadcast channels - optimal transmit strategies. IEEE Transactions on Signal Processing, **57**(5), 1948–1958 (May 2009)
8. TJ Oechtering, EA Jorswieck, RF Wyrembelski, H Boche, On the optimal transmit strategy for the mimo bidirectional broadcast channel. IEEE Transactions on Communications, **57**(12), 3817–3826 (2009)
9. T Unger, A Klein, Duplex schemes in multiple-antenna two-hop relaying. EURASIP Journal on Advances in Signal Processing **2008**. Article ID 128592 (2008)
10. Y-C Liang, R Zhang, Optimal analogue relaying with multiantennas for physical layer network coding, in *Proc IEEE International Conference on Communications, Beijing*. 3893–3897 (2008)
11. R Zhang, Y-C Liang, CC Chai, S. Cui, Optimal beamforming for two-way multi-antenna relay channel with analogue network coding. IEEE Journal on Selected Areas in Communications, **27**(5), 699–712 (June 2009)
12. M Chen, A Yener, Multiuser two-way relaying for interference limited systems. in *Proc IEEE International Conference on Communications*, Beijing, 3883–3887 (2008)
13. C Esli, A Wittneben, One- and two-way decode-and-forward relaying for wireless multiuser mimo networks. in *Proc IEEE Global Communications Conference*, New Orleans, 1–6 (2008)
14. AUT Amah, A Klein, YCB. Silva, A Fernekeß, Multi-group multicast beamforming for multiuser two-way relaying. in *Proc International ITG Workshop on Smart Antennas*, Berlin. (2009)
15. J Joung, AH Sayed, Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems. IEEE Transactions on Signal Processing, **58**(3), 1833–1846 (March 2010)
16. E Yilmaz, R Zakhour, D Gesbert, R Knopp, Multi-pair two-way relay channel with multiple antenna relay station. in *Proc IEEE International Conference on Communications*, Cape Town, (May 2010)
17. QH Spencer, AL Swindlehurst, M Haardt, Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels. IEEE Transaction on Signal Processing, **52**, 461–471 (Feb 2004). doi:10.1109/TSP.2003.821107
18. D Gündüz, A Yener, A Goldsmith, HV Poor, Multi-way relay channel. Proc IEEE International Symposium on Information Theory, Seoul. (2009)
19. S Simoens, Cooperative MIMO Communications, Information Theoretical Limits and Practical Coding Strategies. Ph.D dissertation, TU Catalonia (UPC) (2009)
20. AUT Amah, A Klein, Beamforming-based physical layer network coding for non-regenerative multi-way relaying. EURASIP Journal on Wireless Communications and Networking, Special Issue on Physical Layer Network Coding for Wireless Cooperative Networks **2010**. Article ID 521571 2010.
21. A transceive strategy for regenerative multi-antenna multi-way relaying, Proc. IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing, Aruba. (Dec. 2009)
22. S Katti, S Gollakota, D Katabi, Embracing wireless interference: analog network coding. Proc ACM Special Interest Group on Data Communication, Kyoto, 397–408 (2007)
23. V Stankovic, M Haardt, Generalized design of multi-user mimo precoding matrices. IEEE Transactions on Wireless Communications, **7**, 953–961 (2008)
24. P Popovski, H Yomo, Wireless network coding by amplify and forward for bi-directional traffic flows. IEEE Communications Letters, **11**(1), 16–18 (2007)
25. JA Nossek, M Joham, W Utschick, Transmit processing in mimo wireless systems. Proc IEEE International Symposium on Emerging Technologies, Shanghai (May 2004)
26. M Joham, Optimization of linear and non-linear transmit signal processing. Ph.D dissertation, TU München (2004)
27. M Joham, K Kusume, MhH Gzara, W Utschick, JA Nossek, Transmit wiener filter for downlink of tddds-cdma systems. Proc IEEE International Symposium on Spread-Spectrum Techniques and Applications (May 2002)
28. M Joham, W Utschick, JA Nossek, Linear transmit processing in mimo communications systems. IEEE Transactions on Signal Processing, **53**(8), 2700–2712 (Aug. 2005)
29. YCB Silva, Adaptive beamforming and power allocation in multi-carrier multicast wireless network. Ph.D dissertation, TU Darmstadt (2008)
30. YCB Silva, A Klein, Linear transmit beamforming techniques for the multi-group multicast scenario. IEEE Transactions on Vehicular Technology, **58**, 4353–4367 (Oct. 2009)
31. ND Sidiropoulos, TN Davidson, ZQ Luo, Transmit beamforming for physical-layer multicasting. IEEE Transaction on Signal Processing. **54**, 2239–2251 (2006)
32. E Karipidis, ND Sidiropoulos, ZQ Luo, Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups. IEEE Transaction on Signal Processing. **56**(3), 1268–1279 (2008)
33. JF Sturm, Using sedumi 1.02, a matlab toolbox for optimisation over symmetric cones. Optimisation Methods and Software. **11-12**, 625–653 (June 1999)
34. A Scaglione, P Stoica, S Barbarosa, G Giannakis, H Sampath, Optimal designs for space-time linear precoders and decoders. IEEE Transactions on Signal Processing, **50**(5), 1051–1064 (May 2002). doi:10.1109/78.995062

doi:10.1186/1687-1499-2011-29
Cite this article as: Amah and Klein: **Non-Regenerative Multi-Antenna Multi-Group Multi-Way Relaying.** *EURASIP Journal on Wireless Communications and Networking* 2011 **2011**:29.