MULTI-GROUP MULTICAST BEAMFORMING FOR MULTI-USER TWO-WAY RELAYING

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ABSTRACT

In this work, we consider a multi-user two-way relaying protocol. A multi-antenna relay station (RS) serves $2K$ nodes where K pairs of nodes would like to perform bidirectional communication. In the first phase, the $2K$ nodes transmit simultaneously and the RS spatially separates and decodes all the $2K$ bit sequences. In the second phase, after performing bit-wise XOR network coding for each bidirectional pair, which results in only K bit sequences, the RS sends all the K XOR-ed bit sequences simultaneously. At each node, having received its intended XOR-ed bit sequence, it cancels the self-interference by XOR-ing its received bit sequence with its transmitted one to obtain the bit sequence sent by its partner. Considering the second phase, as each node in a specific pair expects the same XOR-ed bit sequence from the RS, while seeing other pairs' XOR-ed bit sequences as interference, the RS has to separate the different pairs' XOR-ed bit sequences spatially. Thus, in the second phase, multi-group multicast beamforming can be applied with the node pairs being the multicast group. With this new perspective on the second phase, we propose to apply multi-group multicast beamforming algorithms which provide fairness to all pairs and have low computational complexity. We investigate several multi-group multicast beamforming algorithms, namely Zero Forcing (ZF), Multicast Aware ZF (MAZF) and SINR Balancing with Bisection Search (SINRB-Bisec), for the second phase transmission of the multi-user two-way relaying. We consider also two different XOR network coding approaches, namely with and without zero padding (ZP). The overall two-phase sum rate analysis is given which shows that both MAZF and SINRB-Bisec have similar performance and outperform ZF. Nevertheless, MAZF has lower computational complexity compared to SINRB-Bisec.

1. INTRODUCTION

In two-hop communication, a relay station (RS) assists the communication between a source node and a destination node. Due to the half-duplex constraint, two orthogonal resources are needed, one resource for the transmission from the source node to the RS and another one for the transmission from the RS to the destination node. This leads to a loss in capacity by a factor of 2 compared to single-hop communication [1, 2].

One way to mitigate this capacity loss is by allowing the RS to assist bidirectional communication between two nodes with the so-called two-way relaying protocol [2]. The communication in two-way relaying is performed in two phases [2]. In the first phase, the multiple access (MAC) phase, both nodes transmit at the same time to the RS. In the second phase, the broadcast (BC) phase, the RS forwards the superimposed signal to both nodes. Since each node a priori knows its own transmitted signal, it can subtract it from the received signal to obtain the signal from the other node.

At the RS, different types of signal processing can be performed. In Amplify and Forward (AF), the RS forwards the received signal after amplifying it [2–5]. In Decode and Forward (DF), the RS decodes the information of both nodes and forwards the re-encoded information [2, 6, 7].

Two-way relaying with two multi-antenna nodes and a multi-antenna RS has been treated in [8, 9] for AF and in [10] for DF. DF two-way relaying has the advantage that the noise at the RS is not propagated to the receive nodes. Its performance depends on the coding which is used to re-encode the information in the second phase. In [10,11], for the DF multi-antenna two-way relaying case, it has been shown that for the re-encoding of the decoded information of both nodes, bit-wise XOR coding outperforms superposition coding.

The aforementioned contributions considered a scenario where an RS assists two nodes (single pair) to perform bidirectional communication with each other. Recently, two-way relaying for multi-user scenarios has attracted more attention and has been considered in [12–14]. In [12], a multi-antenna RS assists a Base Station to perform bidirectional communication with multiple nodes. A different multi-user scenario is treated in [13, 14] where two sets of nodes perform two-

way relaying communication with the assistance of an RS. Here, each node in each set communicates bidirectionally to only one specific node in the other set. While [13] assumes a single antenna RS which separates the users using Code Division Multiple Access, a multi-antenna RS is assumed in [14] to separate the users spatially. The work in [14] considers a DF protocol using XOR network coding where an optimization algorithm for the precoding matrix to maximize the overall two-phase sum rate is proposed and possible extensions to max-min fairness optimization are addressed.

In this work, we also consider multi-user two-way relaying where the multi-antenna DF RS serves multiple pairs of nodes using spatial separation and applies XOR network coding as in [14]. In the first phase, all nodes transmit simultaneously to the RS and the RS spatially separates and decodes the bit sequences of all nodes. In the second phase, the RS performs XOR network coding to each pair's bit sequences, which halves the number of bit sequences which are transmitted. We consider two different XOR network coding approaches, namely with and without zero padding (ZP). The use of XOR network coding at the RS needs self-interference cancellation at each node. Each node obtains its partner's bit sequence by XOR-ing its received bit sequence with its transmitted one.

In the second phase, each node in a specific pair, due to the XOR-ing of two bit sequences of the two nodes from the first phase at the RS, expects the same XOR-ed bit sequence from the RS. Thus, the RS multicasts the same bit sequence to this pair. However, the RS has to send all bit sequences to all pairs simultaneously and, consequently, each pair sees other pairs' bit sequences as interference. Since there are multiple bit sequences for multiple pairs, the bit sequences transmission for the second phase can be performed using multi-group multicast beamforming. With this new perspective on the second phase, we can apply multi-group multicast beamforming solutions known from single-hop transmission in the two-way relaying case.

Optimizing the precoding matrix to maximize the sum rate of multi-user two-way relaying as in [14] may lead to unfairness as the RS may distribute more power to pairs with good channel conditions and, consequently, cause very low data rate to other pairs. Thus, we propose to apply multi-group multicast beamforming that provides fairness to the pairs. For the second phase we investigate Multicast Aware Zero Forcing (MAZF) which was introduced in [15] for single-hop communication and consider as well the SINR balancing multi-group multicast beamforming with Bisection search (SINRB-Bisec) as introduced in [16] for single-hop. As computational complexity is a practical issue, we investigate as well the Zero Forcing (ZF) multi-group multicast beamforming given in [15] for single-hop.

This paper is organized as follows. Section 2 provides the system and signal model of the multi-user two-way relaying. The multi-group multicast beamforming algorithms under consideration are explained in Section 3. Section 4 gives the achievable overall two-phase sum rate. The performance analysis is given in Section 5. Finally, Section 6 provides the conclusion.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose, complex conjugate, and complex conjugate transpose, respectively. The operators $\mathrm{E}\{\mathbf{X}\}$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the expectation and the trace of $\mathbf{X}$, respectively, and $\mathcal{CN}(0, \sigma^2)$ denotes the zero-mean complex normal distribution with variance σ^2.

2. SYSTEM AND SIGNAL MODEL

We consider a multi-user two-way relaying scenario where a multi-antenna RS supports multiple single antenna node pairs. Each pair of nodes would like to perform an exclusive bidirectional communication. It is assumed that there exists no direct link between the two nodes in each pair, such that the bidirectional communication may happen only through the assistance of an RS. The K pairs, which consist of $N = 2K$ single antenna nodes, will be served by a multi-antenna RS which has $M \geq N$ antennas. A number $M \geq N$ of antennas at the RS is required as we need to decode N bit sequences in the first phase and we are using multi-group multicast beamforming addressed to N users for the second phase.

The RS applies a DF protocol. In the first phase, all N nodes transmit simultaneously to the RS and the RS decodes the bit sequences of all nodes. It then performs bit-wise XOR network coding to each pair's bit sequences. Thus, at the RS, there exist K XOR-ed bit sequences which need to be transmitted to K different pairs. In the second phase, to separate the pairs spatially from one another, the RS uses multi-group multicast beamforming and forwards each of the K bit sequences to its intended pair. Each node performs self-interference cancellation by XOR-ing its received bit sequence with its transmitted one to obtain the bit sequence from its bidirectional communication partner.

In the following, let us put each node S_n, $n \in \{1, \cdots, N\}$, into one of two sets of nodes, namely the set of odd numbered nodes $\mathcal{S}_{\mathrm{odd}}$ and the set of even numbered nodes $\mathcal{S}_{\mathrm{even}}$. Each node in $\mathcal{S}_{\mathrm{odd}}$ would like to perform bidirectional communication using the two-way relaying protocol with a specific node in $\mathcal{S}_{\mathrm{even}}$, where each node in each set can only have one bidirectional partner node in the other set. Let $G^{(k)}$, $k \in \{1, \cdots, K\}$ denote the k-th pair of nodes consisting of nodes $\mathrm{S}_{a_k} \in \mathcal{S}_{\mathrm{odd}}$ and $\mathrm{S}_{b_k} \in \mathcal{S}_{\mathrm{even}}$, where $\{a_k \in \mathbb{Z}_+ | a_k = 2k - 1, \ k \in \{1, \cdots, K\}\}$ and $\{b_k \in \mathbb{Z}_+ | b_k = 2k, \ k \in \{1, \cdots, K\}\}$.

Each node S_n wants to send to its partner a bit sequence denoted as $\mathbf{x}_n$. In the first phase, all N nodes transmit simultaneously, i.e., S_1 sends $\mathbf{x}_1$, S_2 sends $\mathbf{x}_2$, and so on. The RS decodes all the N nodes' bit sequences. In the second phase,
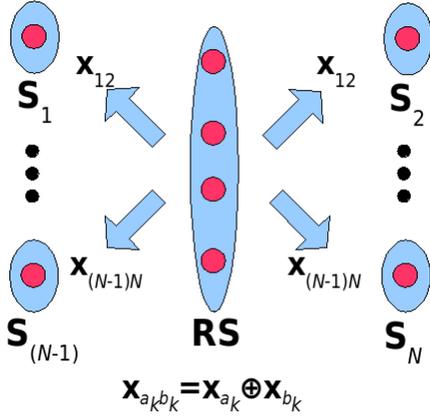
Fig. 1. Example of multi-group multicast beamforming and the XOR Network Coding for multi-user two-way relaying.

the RS performs bit-wise XOR operation to the bit sequences of each pair. The XOR operation of pair $G^{(k)}$ is defined by $\mathbf{x}_{a_k b_k} = \mathbf{x}_{a_k} \oplus \mathbf{x}_{b_k}$, for example $\mathbf{x}_{12} = \mathbf{x}_1 \oplus \mathbf{x}_2$. All the K XOR-ed bit sequences of all pairs are transmitted simultaneously by the RS using multi-group multicast beamforming. All nodes perform self-interference cancellation by XOR-ing the received information with their transmitted information to obtain the information being sent by their partner, for example S_1 performs $\mathbf{x}_{12} \oplus \mathbf{x}_1$ to receive $\mathbf{x}_2$ from node S_2. Figure 1 shows an example how the N nodes are divided into two sets, $\mathcal{S}_{\text{odd}}$ on the left and $\mathcal{S}_{\text{even}}$ on the right, and how the RS forwards the K bit sequences by spatially separating each pair using multi-group multicast beamforming.

In the following, we assume a frequency-flat block fading channel between all nodes and the RS where $\mathbf{h}_n \in \mathbb{C}^{M \times 1}$ denotes the channel vector from node S_n to the RS with identically and independently distributed $\mathcal{CN}(0, \sigma_{\mathbf{h}_n}^2)$ entries. Assuming reciprocity and stationarity of the channel for both phases, the channel vector from the RS to node S_n is given by $\mathbf{h}_n^{\text{T}}$.

2.1. First Phase - Multiple Access Phase

In the first phase, all nodes transmit simultaneously to the RS. The mapping process of the bit sequence of node S_n onto its corresponding transmitted symbol is defined by $\mathbf{x}_n \rightarrow t_n \in \mathbb{C}$. Each node S_n has power constraint $\mathrm{E}\{|t_n|^2\} \leq P_n, \forall n, n \in \{1, \cdots, N\}$. The received vector at the RS is given by

$$\mathbf{r}_{\text{RS}} = \sum_{k=1}^{K} (\mathbf{h}_{a_k} \cdot t_{a_k} + \mathbf{h}_{b_k} \cdot t_{b_k}) + \mathbf{n}_{\text{RS}}, \qquad (1)$$

which can be rewritten as

$$\mathbf{r}_{\text{RS}} = \mathbf{H} \cdot \mathbf{t} + \mathbf{n}_{\text{RS}}, \qquad (2)$$

where the channel matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_N] \in \mathbb{C}^{M \times N}$ and the transmit vector $\mathbf{t} = [t_1, t_2, \cdots, t_N]^{\text{T}} \in \mathbb{C}^{N \times 1}$ are the matrix consisting of all nodes' channel vectors and the vector consisting of all nodes' transmit symbols, respectively, and $\mathbf{n}_{\text{RS}} \in \mathbb{C}^{M \times 1} \sim \mathcal{CN}(0, \sigma_{\text{RS}}^2 \mathbf{I}_M)$ is the complex additive white Gaussian noise (AWGN) vector at the RS, with $\mathbf{I}_M$ denoting an identity matrix of size $M \times M$.

Assuming that the RS has the ability to perform perfect decoding, there exist N bit sequences of all N nodes at the RS, i.e., $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N$.

2.2. Second Phase - Broadcast Phase

Having $N = 2K$ bit sequences available, the RS performs XOR operation of the bit sequences $\mathbf{x}_{a_k}$ and $\mathbf{x}_{b_k}$ of pair $G^{(k)}$, $\forall k, k \in \{1, \cdots, K\}$ resulting in $\mathbf{x}_{a_k b_k} = \mathbf{x}_{a_k} \oplus \mathbf{x}_{b_k}$. This XOR operation reduces the number of bit sequences to be transmitted from $2K$ to K, which means halving the number of interfering bit sequences for the BC phase. The next task for the RS is to forward these K bit sequences to the K pairs such that each pair can receive its intended bit sequence while receiving only small interference from unintended bit sequences. Multi-group multicast beamforming algorithms are used to spatially separate the pairs (and consequently the bit sequences), which will be explained in Section 3.

Assuming reciprocal and stationary channels for both phases and having transmit symbol $t_{a_k b_k} \in \mathbb{C}$ as the result of the mapping process $\mathbf{x}_{a_k b_k} \rightarrow t_{a_k b_k}$, the received vector of nodes S_{a_k} and S_{b_k} in pair $G^{(k)}$ can be written as

$$r_{a_k} = \mathbf{h}_{a_k}^{\text{T}} \cdot \mathbf{m}_k \cdot t_{a_k b_k} + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{h}_{a_k}^{\text{T}} \cdot \mathbf{m}_l \cdot t_{a_l b_l} + n_{a_k}$$

and $\qquad (3)$

$$r_{b_k} = \mathbf{h}_{b_k}^{\text{T}} \cdot \mathbf{m}_k \cdot t_{a_k b_k} + \sum_{\substack{l=1 \\ l \neq k}}^{K} \mathbf{h}_{b_k}^{\text{T}} \cdot \mathbf{m}_l \cdot t_{a_l b_l} + n_{b_k},$$

where $\mathbf{m}_k \in \mathbb{C}^{M \times 1}$ and $\mathbf{m}_l \in \mathbb{C}^{M \times 1}$ are the precoding vectors for pairs $G^{(k)}$ and $G^{(l)}, \forall l, l \in \{1, \cdots, K\}, l \neq k$, respectively, and $n_n \sim \mathcal{CN}(0, \sigma_n^2)$ is the AWGN at node n. In matrix formulation, the received vector is given by

$$\mathbf{r}_{\text{nodes}} = \mathbf{H}^{\text{T}} \cdot \mathbf{M}_{\text{nodes}} \cdot \mathbf{t}_{\text{RS}} + \mathbf{n}_{\text{nodes}}, \qquad (4)$$

where $\mathbf{t}_{\text{RS}} = [t_{12}, t_{34}, \cdots, t_{(N-1)N}]^{\text{T}} \in \mathbb{C}^{K \times 1}$ is the transmitted symbols vector at the RS, $\mathbf{n}_{\text{nodes}} \in \mathbb{C}^{N \times 1} \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I}_N)$ is the stacked vector of the complex AWGN at all nodes and $\mathbf{M}_{\text{nodes}} = [\mathbf{m}_1, \cdots, \mathbf{m}_K] \in \mathbb{C}^{M \times K}$ is the precoding matrix according to the multi-group multicast beamforming algorithm under consideration. It is assumed that the transmitted symbols are identically and independently distributed with $\mathrm{E}\{\mathbf{t}_{\text{RS}} \mathbf{t}_{\text{RS}}^{\text{H}}\} = \sigma_{\mathbf{t}_{\text{RS}}}^2 \mathbf{I}_K$ and the RS has a power constraint defined by $\sigma_{\mathbf{t}_{\text{RS}}}^2 \mathrm{tr}(\mathbf{M}_{\text{nodes}} \mathbf{M}_{\text{nodes}}^{\text{H}}) \leq P_{\text{RS}}$.

After each node decodes its intended bit sequence, it performs self-interference cancelation by applying XOR operation of its received bit sequence and its a priori transmitted bit sequence to obtain its partner's bit sequence, such that at node S_{a_k}, $\mathbf{x}_{b_k} = \mathbf{x}_{a_k b_k} \oplus \mathbf{x}_{a_k}$ and at node S_{b_k}, $\mathbf{x}_{a_k} = \mathbf{x}_{a_k b_k} \oplus \mathbf{x}_{b_k}$ is obtained.

3. MULTI-GROUP MULTICAST BEAMFORMING

The capacity region for the MAC phase has been well investigated and may be found in the literature such as in [22]. In this work, we concentrate on the BC phase of multi-user two-way relaying, for which we propose the use of multi-group multicast beamforming for the second phase of multi-user two-way relaying. As we have multiple pairs, multi-group multicast beamforming algorithms which provide fairness to all pairs are desirable. An optimization objective which promotes fairness and is adequate for multicast beamforming is the maximization of the minimum SINR among the nodes [15].

In this work, we extend several single-hop multi-group multicast beamforming algorithms to be applied to the multi-user two-way relaying scenario. As computational complexity is a practical issue, we investigate ZF multi-group multicast beamforming, which was introduced in [15] for single-hop communication. For fairness consideration, we investigate the so-called MAZF, which provides fairness by performing fair power loading to all nodes and was proposed in [15] for single-hop communication and the SINRB-Bisec, which tries to solve the aforementioned optimization objective and was proposed in [16, 17] for single-hop communication.

In the following, we will only briefly explain these three algorithms, starting with ZF followed by MAZF and SINRB-Bisec. For more details, the interested reader is refered to the references directly. The corresponding precoding matrix $\mathbf{M}_{\text{nodes}}$, which is needed in (4), is derived for the three algorithms in the subsequent subsections.

3.1. Zero Forcing

The ZF optimization problem for multi-group multicast beamforming [15] can be rewritten for multi-user two-way relaying scenario as

$$\mathbf{M}_{\text{ZF}} = \underset{\mathbf{M}}{\arg\min} \, \mathrm{E}\{\|\mathbf{r}_{\text{nodes}} - \mathbf{U}^+ \mathbf{t}_{\text{RS}}\|^2\},$$

$$\text{subject to}: \quad \mathrm{E}\{\|\mathbf{M}\mathbf{t}_{\text{RS}}\|^2\} \leq P_{\text{RS}} \quad , \tag{5}$$

$$\mathbf{r}_{\text{nodes}}|_{\mathbf{n}_{\text{nodes}}=\mathbf{0}} = \mathbf{U}^+ \mathbf{t}_{\text{RS}}$$

where the second constraint corresponds to the ZF constraint and $\mathbf{U}^+ \in \mathbb{R}^{N \times K}$ has its a_k-th and b_k-th rows given by $\mathbf{u}_{a_k}^+ = \mathbf{u}_{b_k}^+ = \mathbf{e}_k$, with $\mathbf{e}_k$ corresponding to the k-th column of an identity matrix of dimension K. For example, the

$\mathbf{U}^+$ matrix for two pairs, $G^{(1)}$ and $G^{(2)}$, which consist of four nodes, S_1, S_2, S_3 and S_4, is given by $\mathbf{U}^+ = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$.

Using the optimization procedure in [18], the solution of (5) is given by

$$\mathbf{M}_{\text{ZF}} = \beta \mathbf{H}^* (\mathbf{H}^{\mathrm{T}} \mathbf{H}^*)^{-1} \mathbf{U}^+, \tag{6}$$

where the scalar factor

$$\beta = \sqrt{\frac{P_{\text{RS}}}{\sigma_{\mathbf{t}_{\text{RS}}} \mathrm{tr}((\mathbf{H}^{\mathrm{T}} \mathbf{H}^*)^{-1} \mathbf{U}^+ \mathbf{U}^{+^{\mathrm{T}}})}} \tag{7}$$

is needed to fulfill the power constraint in (5).

In ZF multi-group multicast beamforming, the interference is cancelled by inverting the gram matrix of the channel. Thus, energy is unnecessarily spent on cancelling interference of the own pair's partner.

3.2. Multicast Aware Zero Forcing

The idea of MAZF is to avoid unnecessarily energy spending to cancel the interference of the own group's partner and to provide fairness to the nodes [15]. The MAZF uses Block Diagonalization [19,20] to suppress only the interference among data streams of different pairs and then applies a fair power loading to all nodes. By having a Block Diagonalization, only the interference of other pairs' channels is cancelled and not of the pair partner's channel. Thus, the MAZF provides higher power when transmitting to the intended node compared to the aforementioned ZF. In the following, the extension to the multi-user two-way relaying scenario is given.

Let $\mathbf{H}_k^{\mathrm{T}} \in \mathbb{C}^{2 \times M}$ and $\tilde{\mathbf{H}}_k^{\mathrm{T}} \in \mathbb{C}^{(N-2) \times M}$ denote the channel matrix of pair $G^{(k)}$ and the channel matrix of all other pairs $G^{(l)}$, $\forall l$, $l \in \{1, \cdots, K\}$, $l \neq k$, respectively. The latter can be written as

$$\tilde{\mathbf{H}}_k^{\mathrm{T}} = [\mathbf{H}_1^{\mathrm{T}}, \cdots, \mathbf{H}_{k-1}^{\mathrm{T}}, \mathbf{H}_{k+1}^{\mathrm{T}}, \cdots, \mathbf{H}_K^{\mathrm{T}}]^{\mathrm{T}}. \tag{8}$$

The channel matrix $\tilde{\mathbf{H}}_k^{\mathrm{T}}$ can be decomposed using Singular Value Decomposition as follows:

$$\tilde{\mathbf{H}}_k^{\mathrm{T}} = \tilde{\mathbf{U}}_k \tilde{\mathbf{S}}_k [\tilde{\mathbf{V}}_k^{(1)}, \tilde{\mathbf{V}}_k^{(0)}], \tag{9}$$

where $\tilde{\mathbf{S}}_k \in \mathbb{R}^{(N-2) \times M}$ is a diagonal matrix, a unitary matrix given by $\tilde{\mathbf{U}}_k \in \mathbb{C}^{(N-2) \times (N-2)}$, $\tilde{\mathbf{V}}_k^{(1)} \in \mathbb{C}^{M \times \tilde{r}_k}$ and $\tilde{\mathbf{V}}_k^{(0)} \in \mathbb{C}^{M \times (N-\tilde{r}_k)}$ contain the right singular vectors of $\tilde{\mathbf{H}}_k^{\mathrm{T}}$, with $\tilde{r}_k$ denoting the rank of matrix $\tilde{\mathbf{H}}_k^{\mathrm{T}}$. The matrix $\tilde{\mathbf{V}}_k^{(0)}$ can be used to specify a beamforming vector that cancels the interference from the other pairs.

Given $\mathbf{H}_k^{\mathrm{T(eq)}} = \mathbf{H}_k^{\mathrm{T}} \tilde{\mathbf{V}}_k^{(0)} \in \mathbb{C}^{2 \times (N-\tilde{r}_k)}$ as the equivalent channel matrix of pair $G^{(k)}$, one can assure that other pairs' interferences are suppressed. Based on this channel we can derive a single-group multicast beamforming vector $\mathbf{m}_k^{(\text{eq})} \in \mathbb{C}^{(N-\tilde{r}_k) \times 1}$ for pair $G^{(k)}$, which can be written as

$$\mathbf{m}_k^{(\text{eq})} = \mathbf{H}_k^{\mathrm{T(eq)H}} (\mathbf{H}_k^{\mathrm{T(eq)}} \mathbf{H}_k^{\mathrm{T(eq)H}})^{-1} \mathbf{1}, \tag{10}$$

where $\mathbf{1} = [1, 1]^{\mathrm{T}}$.

The resulting beamforming vector for pair $G^{(k)}$ is then given by

$$\mathbf{m}_k = \tilde{\mathbf{V}}_k^{(0)} \mathbf{m}_k^{(\mathrm{eq})}, \tag{11}$$

which leads to the resulting beamforming matrix

$$\mathbf{M} = [\tilde{\mathbf{V}}_1^{(0)} \mathbf{m}_1^{(\mathrm{eq})}, \cdots, \tilde{\mathbf{V}}_K^{(0)} \mathbf{m}_K^{(\mathrm{eq})}]. \tag{12}$$

In MAZF, different to ZF, the received power is not balanced among the nodes. For this reason, a fair power loading which balances the received power among the nodes is considered in [15]. The power loading matrix $\mathbf{\Gamma} \in \mathbb{R}^{K \times K}$ is given by

$$\mathbf{\Gamma} = \mathrm{diag}(\min(|\mathbf{H}_1^{\mathrm{T}} \mathbf{m}_1|), \cdots, \min(|\mathbf{H}_K^{\mathrm{T}} \mathbf{m}_K|))^{-1}, \tag{13}$$

where the modulus operator $| \cdot |$ is assumed to be applied element-wise and the $\min$ function in this case returns the minimum element of a vector. This power loading ensures that the same amount of power is given to the worst node in each multicast pair. In order to satisfy the transmit power constraint, a normalization factor $\beta \in \mathbb{R}$ is needed and is given by

$$\beta = \sqrt{\frac{P_{\mathrm{RS}}}{\sigma_{\mathbf{t}_{\mathrm{RS}}} \mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{M} \mathbf{\Gamma}^2)}}. \tag{14}$$

The beamforming solution for MAZF is given by

$$\mathbf{M}_{\mathrm{MAZF}} = \beta \mathbf{M} \mathbf{\Gamma}. \tag{15}$$

3.3. SINRB-Bisec

The optimization problem for obtaining the optimum precoding matrix which maximizes the worst node SINR can be written as

$$\mathbf{M}_{\mathrm{opt}} = \underset{\mathbf{M}}{\mathrm{argmax}} \min_n \gamma_n, \quad n \in \{1, \cdots, N\}$$
$$\text{subject to} : \sigma_{\mathbf{t}_{\mathrm{RS}}}^2 \, \mathrm{tr}(\mathbf{M}^{\mathrm{H}} \mathbf{M}) \leq P_{\mathrm{RS}}, \tag{16}$$

with

$$\gamma_n = \frac{\sigma_{\mathbf{t}_{\mathrm{RS}}}^2 |\mathbf{h}_n^{\mathrm{T}} \mathbf{m}_k|^2}{\sum_{\substack{l=1 \\ l \neq k}}^{K} \sigma_{\mathbf{t}_{\mathrm{RS}}}^2 |\mathbf{h}_n^{\mathrm{T}} \mathbf{m}_l|^2 + \sigma_n^2}, \quad n \in \{1, \cdots, N\}, \tag{17}$$

being the SINR of node S_n. Note that the pair index k associated to node n can be obtained as $k = \lfloor (n+1)/2 \rfloor$. SINRB-Bisec is not adequate to maximize the sum rate but it provides equal SINR to all nodes such that it maximizes the fairness among nodes. Therefore, the performance comparison of SINRB-Bisec with MAZF and ZF is not that fair.

The contributions of [16, 17] have shown that (16) can be solved using bisection search method. First we need to specify the SINR interval where the optimal solution must lie and then determine the solution of (16) when considering the

middle point of the interval as the target SINR. The interval is then successively bisected based on whether the required amount of power P_{req} exceeds the transmit power constraint P_{RS} or not. For each interval middle point, the problem (16) is solved. The bisection proceeds until a desired precision is reached with regard to $|P_{\mathrm{req}} - P_{\mathrm{RS}}|$.

4. ACHIEVABLE SUM RATE

In this section, the rate expressions for each phase and the overall two-phase sum rate will be given. It is assumed that $\log_2$ denotes the logarithm of base 2.

4.1. MAC Phase

In this subsection, the MAC phase rate is given. Having received vector $\mathbf{r}_{\mathrm{RS}}$, it is assumed that the RS is able to perform perfect decoding. Assuming Gaussian codebooks, the MAC phase capacity region is defined by

$$\sum_{n=1}^{N} R_n^{\mathrm{MAC}} \leq R^{\mathrm{MAC}} = \log_2 \det \left(\mathbf{I}_M + \frac{P_n}{\sigma_{RS}^2} \sum_{n=1}^{N} \mathbf{h}_n \mathbf{h}_n^{\mathrm{H}} \right) \tag{18}$$

[22], where

$$R_n^{\mathrm{MAC}} \leq R_n = \log_2(1 + \gamma(\|\mathbf{h}_n\|^2)), \forall n, n \in \{1, \cdots, N\} \tag{19}$$

means that each node's rate cannot be larger than when it is the only node transmitting.

4.2. BC Phase

In this subsection, the rate of the BC phase followed by the overall two-phase sum rate are given. The rate of node n is given by

$$R_n^* = \log_2(1 + \gamma_n), \ \forall n, n \in \{1, \cdots, N\} \tag{20}$$

[22], where γ_n is given in (17) due to having multiple transmit antennas at the RS and using multi-group multicast beamforming as explained in Section 3. Since the RS sends the same bit sequence to each node in pair $G^{(k)}$, the minimum rate between both nodes in $G^{(k)}$ will define the rate that can be transmitted in the second phase. Thus, the achievable rate for the second phase at both nodes S_{a_k} and S_{b_k} in $G^{(k)}$ is defined by

$$R_k^{2\mathrm{nd}} = \min(R_{a_k}^*, R_{b_k}^*), \ \forall k, k \in \{1, \cdots, K\}. \tag{21}$$

The rate given in (21) considers only the second phase transmission when the RS multicasts the same bit sequences to both nodes. However, the information rate that can be received at each node depends on the available information rate of its partner in the first phase. Thus, $R_{a_k b_k}$ and $R_{b_k a_k}$, which
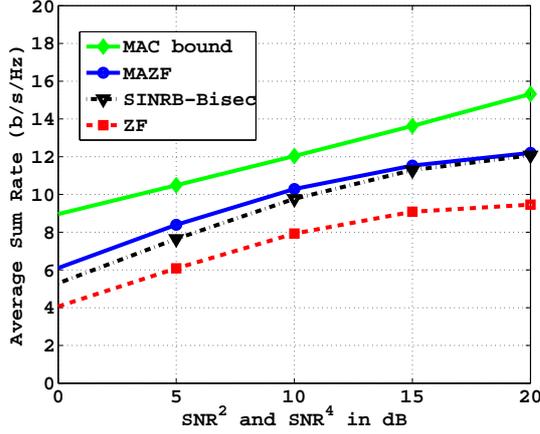
Fig. 2. Sum Rate performace of multi-group multicast beamforming: SNR^1 and SNR^3 fixed at 10 dB.

Fig. 3. Sum Rate performace of multi-group multicast beamforming: equal SNR for all nodes.

are the achievable rates for transmission from node S_{a_k} to node S_{b_k} and vice versa, respectively, are given by

$$R_{a_k b_k} = \min(R_{a_k}, R_k^{2\text{nd}})$$
$$\text{and} \tag{22}$$
$$R_{b_k a_k} = \min(R_{b_k}, R_k^{2\text{nd}}).$$

The achievable BC rate for each pair is given by

$$R_k^{\text{BC}} = R_{a_k b_k} + R_{b_k a_k} \tag{23}$$

and the overall two-phase sum-rate is, thus, given by

$$R_{sum} = \frac{1}{2} \min\left(\sum_{k=1}^{K} R_k^{\text{BC}}, R^{\text{MAC}} \right), \tag{24}$$

where the pre-log factor $1/2$ is due to the use of two orthogonal resources. In [14], it is proposed to perform ZP such that a higher rate can be transmitted in the second phase. As an example, it is assumed that the RS is able to decode correctly the bit sequence $\mathbf{x}_{a_k}$ from S_{a_k} and the bit sequence $\mathbf{x}_{b_k}$ from S_{b_k}, which have lengths L_{a_k} and L_{b_k}, respectively. Assuming that the transmission rate from the first phase is sufficient to support the second phase, if for the second phase we have $R_{a_k}^* > R_{b_k}^*$ then the corresponding length of bit sequences that can be transmitted in the second phase is $L_{b_k} > L_{a_k}$, as node S_{a_k} expects to receive $\mathbf{x}_{b_k}$, which has the length of L_{b_k}, and vice versa. Using ZP, we append $L_{b_k} - L_{a_k}$ zeros to the bit sequence $\mathbf{x}_{a_k}$. At the second phase, the RS sends $\mathbf{x}_{a_k b_k}$, which is the XOR applied to the zero appended $\mathbf{x}_{a_k}$ and $\mathbf{x}_{b_k}$ and which is encoded with a codebook of rate $\max(R_{a_k}^*, R_{b_k}^*) = R_{a_k}^*$.

For the decoding process, node S_{a_k} employs a codebook with rate $R_{a_k}^*$ and obtains $\mathbf{x}_{a_k b_k}$ perfectly. On the other hand, node S_{b_k} knows a priori that there is ZP being used, so that it decodes only the beginning part of $\mathbf{x}_{a_k b_k}$ according to the rate
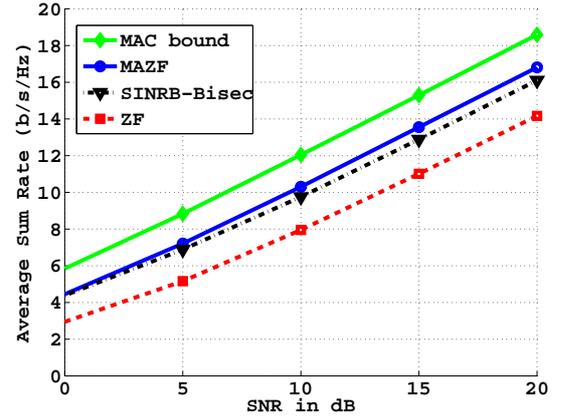
$R_{b_k}^*$. Thus, as stated in [14], with ZP and a priori knowledge at the corresponding nodes, different rates are supported and the broadcast capacity as given in [21] is achieved.

Using the ZP approach, (22) needs to be adjusted according to

$$R_{a_k b_k} = \min(R_{a_k}, R_{b_k}^*)$$
$$R_{b_k a_k} = \min(R_{b_k}, R_{a_k}^*). \tag{25}$$

The overall two-phase sum rate with ZP remains the same as in (24) when exchanging (22) by (25).

5. PERFORMANCE ANALYSIS

As in [14], the average signal to noise ratio (SNR) is defined as $\text{SNR} = P_s / \sigma_r^2$, where P_s is the transmit signal power at the transmitter and σ_r^2 is the noise variance at the receiver. In the first phase, the nodes are the transmitters and the RS is the receiver, and for the second phase it is vice versa. In the following, the RS has $N = 4$ antennas , so that it can support four nodes, namely S_1, S_2, S_3 and S_4. Two bidirectional pairs are considered, namely $G^{(1)}$ consisting of S_1 and S_2 and $G^{(2)}$ consisting of S_3 and S_4, respectively. All nodes and the RS are assumed to have the same noise variance and the RS is assumed to have the sum transmit power of all nodes, i.e., $P_{\text{RS}} = N P_n$. We assume reciprocal flat fading channels with unit average gain for both phases. The notation SNR^x denotes the SNR of the link between node x and the RS.

Figures 2 and 3 show the overall two-phase sum rate performance and the MAC bound when multi-group multicast beamforming is applied for the second phase and the XOR operation is performed without ZP. In Figure 2, SNR^1 and SNR^3 are fixed at 10 dB while SNR^2 and SNR^4 are varied. Figure 3 shows the average sum rate when all SNR values are equal. The MAZF outperforms the non-multicast aware ZF multi-group multicast beamforming as it does not perform null-space projection to the channel of the node within the
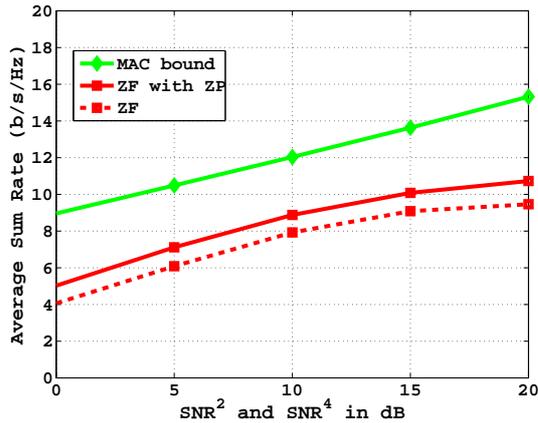
Fig. 4. Performance comparison of different XOR approaches for ZF: SNR^1 and SNR^3 fixed at 10 dB.
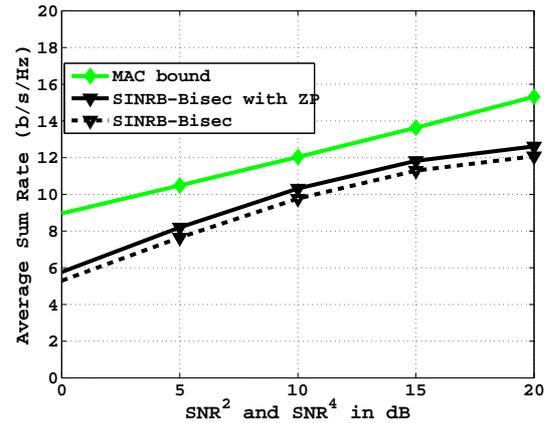

Fig. 6. Performance comparison of different XOR approaches for SINRB-Bisec: SNR^1 and SNR^3 fixed at 10 dB.
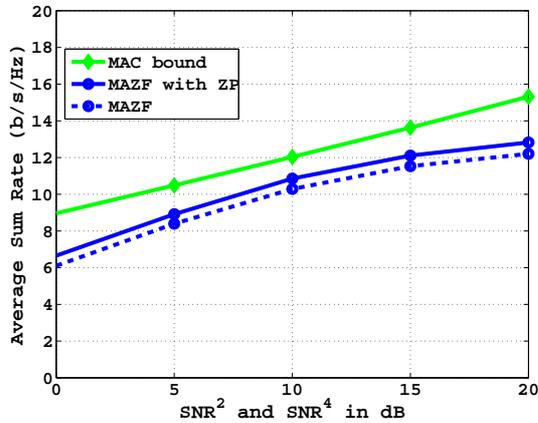

Fig. 5. Performance comparison of different XOR approaches for MAZF: SNR^1 and SNR^3 fixed at 10 dB.

same pair. The performance improvement of MAZF comes at the price of a higher computational complexity compared to the multi-group multicast ZF beamforming. The MAZF performs slightly better than the SINRB-Bisec. This happens since SINRB-Bisec provides higher fairness by guaranteeing equal SINR to all nodes, which is not the case when using MAZF. MAZF provides fairness by balancing the received power among the nodes, which cannot assure that the SINR is equal among all nodes. Furthermore, in [15] it is shown that MAZF has a lower complexity compared to SINRB-Bisec. It can be seen that the average sum rate will saturate when the SNR of one node in a pair is fixed while the other node's SNR is increased. This is due to the fact that the lower SNR limits the overall two-phase sum rate.

Figures 4, 5 and 6 show the performance improvement of all three multi-group multicast beamforming algorithms when using ZP. The usage of ZP is beneficial in the case of un-balanced SINR among two nodes in a pair. We measure the

performance of multi-user two-way relaying when SNR^1 and SNR^3 are fixed at 10 dB while SNR^2 and SNR^4 are varied. The use of ZP at the RS when performing the XOR operation for each pair helps to achieve the two-way relaying broadcast capacity of each pair, resulting in a higher sum rate compared to the case without ZP, which is only able to transmit the min-imum broadcast rate between the two nodes in a pair. From these three figures, it can be seen that ZF obtains the high-est gain compared to the other multi-group multicast beam-forming algorithms when the ZP approach is used. Without ZP, the rate is defined by the worst node in each pair. Both MAZF and SINRB-Bisec consider fairness between groups and, in the case of SINRB-Bisec, balances the SINR. Thus, the difference between the worst SINR and the highest SINR in a pair is not high and the performance improvement of ZP approach is not as much as for ZF, which does not include fairness.

In contrast to [14], we do not intend to maximize the sum rate of the multi-user two-way relaying but to provide fairness among the pairs. Therefore, algorithms maximizing the sum rate as provided in [14] will outperform the ZF, MAZF and SINRB-Bisec at the cost of fairness.

Point to point Multiple Input Multiple Output (MIMO) communication is expected to achieve higher sum rate than the multi-user two-way relaying. However, in multi-user two-way relaying it is assumed that there exists no direct link such that point to point communication among nodes in a pair can not be performed, e.g., due to high attenuation. To perform a fair comparison between point to point MIMO and multi-user two-way relaying, the attenuation factor must be taken into account.

6. CONCLUSION

In this work, we propose to apply multi-group multicast beamforming for the second phase transmission in a multi-user two-way relaying protocol. We extend three multi-group multicast beamforming algorithms, namely ZF, MAZF and SINRB-Bisec to the case of multi-user two-way relaying. The overall two-phase sum rate expressions are derived for both cases, with and without ZP, as a way to measure the performance of the multi-user two-way relaying. From the simulation analysis, the ZP approach outperforms the one without ZP. Comparing the three multicast beamforming strategies, the fairness oriented MAZF and SINRB-Bisec algorithms outperform the non multicast aware ZF, which requires the lowest computational complexity. Although MAZF and SINRB-Bisec provide similar performances, MAZF requires less computational complexity compared to SINRB-Bisec.

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