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Estimating Cell Throughput of OFDMA Systems with Scheduling

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ABSTRACT

To dimension cellular wireless networks, accurate estimates of the cell throughput have to be available. Consideration of all requirements of future packet-switched networks, especially the impact of scheduling, leads to extensive system level simulations that have to be performed in order to obtain the cell throughput. A semi-analytical methodology is proposed in this paper that is able to give average cell throughput estimates for given network layouts without performing extensive system level simulations. The signal to interference ratio (SIR) probability density function (pdf) which reflects the behavior that is achieved after scheduling of users is derived. Based on the SIR pdf, probabilities for the usage of modulation and coding schemes are obtained and the average cell throughput is calculated.

Results are presented for well known scheduling algorithms and a channel model that is valid for networks utilizing frequency diversity by distributing the subcarriers over the whole system bandwidth like the partial usage of subchannels (PUSC) mode in IEEE 802.16e. It is shown that the average cell throughput estimates are within 10 % of the average cell throughput that is obtained by system level simulations which model all the properties including scheduling in detail. Due to the usage of analytical expressions, results can be obtained in less than a minute for the proposed methodology while a system level simulation usually takes several hours.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Packet-switching networks, Wireless communication*

General Terms

Algorithms, Performance

Keywords

radio network planning, OFDMA, user scheduling

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1. INTRODUCTION

Planning of wireless networks comprises two main aspects: coverage planning and network dimensioning [12, 10]. During coverage planning the cellular network setup is defined and the transmit power is adjusted so that a predefined signal power is guaranteed at each position within the network.

Afterwards, the network is dimensioned so that the expected amount of traffic can be handled. In circuit-switched networks like the Global System for Mobile Communications (GSM), e.g. the amount of required resources could be derived using the Erlang B formula. The Erlang B formula describes the relationship between the traffic load, i.e. the amount of active users, the amount of available resources and the blocking probability [12]. Current and future wireless networks like the Universal Terrestrial Mobile System (UMTS) and its Long Term Evolution (LTE) or systems according to IEEE 802.16e [7] which is also known as Worldwide Interoperability for Microwave Access (WiMAX) are packet-switched. Resources will be no longer dedicated to one user during the whole session but shared among all active users. If multimedia traffic is used, e.g. download traffic, blocking is not critical. Sophisticated scheduling algorithms can be used to distribute the resources optimally among the users [11] and the cell throughput can be increased utilizing multiuser diversity [9]. Furthermore, different modulation and coding schemes are used to adapt the transmission to the link conditions.

For network dimensioning, an accurate estimate of the cell throughput has to be available. Among others, the cell throughput strongly depends on the used scheduling algorithm and the data traffic models [4]. The Erlang B formula is no longer applicable. Therefore, extensive system level simulations have to be performed to obtain cell throughput results for a specific network setup [18, 15] modeling different properties like generation of data traffic, scheduling, link adaptation or a fading channel in detail. Those system level simulations are time consuming due to their complexity. Simulations have to be repeated if the network setup is changed during the planning process.

In [2], a method is provided to derive the average cell throughput without the need of long system level simulations. The cumulative distribution function (cdf) of the signal to interference plus noise ratio (SINR) due to pathloss is considered to derive the probabilities for the utilization of the different modulation and coding schemes. The average cell throughput is calculated by multiplication of the probabilities and the data rate of the different modulation and coding schemes. However, the impact of scheduling is not taken into account.

In this paper, a new methodology is presented that gives a fast estimate of the average cell throughput considering the impact of scheduling. The pdf of the SINR due to pathloss and shadowing which is, e.g. the outcome of a pathloss prediction tool [19], is needed as input for the methodology. The probability density function (pdf) of the SINR due to fast fading and scheduling is then calculated analytically by taking statistical properties of the fading channel and the scheduling algorithm into account. Afterwards, probabilities for the utilization of a specific modulation and coding scheme are derived from the pdf and the average cell throughput is given. Analytical expressions are presented for two state-of-the-art scheduling algorithms. The methodology can be applied to different wireless systems, e.g. WiMAX, and is not restricted to a specific network layout. Due to the usage of analytical expressions, results can be obtained within minutes and no time-consuming system level simulations are necessary.

The paper is structured as follows. In Section 2, the assumed system model is presented. Section 3 describes the SINR gain that is achieved due to scheduling. Section 4 outlines how the pdf of the SINR due to scheduling can be derived. Performance results are given in Section 5. Finally, conclusions are drawn in Section 6.

2. SYSTEM MODEL

A cellular wireless network in the downlink using Orthogonal Frequency Division Multiple Access (OFDMA) like WiMAX is assumed for the investigation. The system is interference limited so that the noise is omitted for the investigation. It is assumed that the signal to interference ratio (SIR) at each position within the network can be derived based on a distance dependent pathloss and a random variable indicating a shadow fading component. The statistics of the SIR over the investigation area are defined by the locations of the base stations and the pathloss and shadowing model.

Resource units are used for transmission of data to the users. A resource unit is the smallest granularity that can be allocated to one user and is defined by time duration and frequency bandwidth. In WiMAX [7], a resource unit represents one slot in the OFDMA physical layer. Subcarriers of one resource unit are distributed over the whole system bandwidth as for instance in the partial usage of subchannels (PUSC) mode in WiMAX [7]. Therefore, frequency diversity and interference averaging can be obtained and the interference within the cellular network can be approximated as a Gaussian distributed random variable [1]. Resource units still experience different SIR conditions so that scheduling is beneficial. It is assumed that the channel state information is known perfectly.

Within the slot duration T_{S}, N_{res} resource units are available for scheduling. A full buffer traffic model [6] is used so that the system is fully loaded. All N_{res} resource units are utilized during each timeslot and base stations transmit with maximum transmit power. This gives an upper bound of the average cell throughput. In case of data traffic models, e.g. download traffic, it may happen that a base station runs out of data for a certain time duration so that resource units are not utilized for data transmission. This will reduce the average cell throughput.

3. SIR GAIN

In this section, the SIR gain which can be obtained due to scheduling is derived. A frequency selective and time-variant fading channel is assumed. The amplification of the signal power due to the fading process is called fading power gain in the following. The fading power gain is assumed to be constant within one subcarrier bandwidth and one symbol duration. If subcarriers are separated by the coherence bandwidth or the coherence time [14], the fading power gain can be assumed as uncorrelated. Each resource unit consists of N_{sc} subcarriers that experience independent and identically distributed (i.i.d.) fading due to the interleaved subcarrier allocation. For one resource unit, the effective fading power gain a_{ff} is obtained using the average mutual information of the subcarriers taking each single fading power gain into account [1, 5]. According to the central limit theorem [13], a_{ff} can be approximated by a Gaussian distributed random variable if N_{sc} is sufficiently large. This is valid for any kind of fading channel distribution. The pdf of a_{ff} is given by

$$f_{a_{\mathrm{ff}}}(a_{\mathrm{ff}}) = \frac{1}{\sqrt{2\pi N_{\mathrm{sc}}}\sigma} e^{-\frac{(a_{\mathrm{ff}} - N_{\mathrm{sc}}\mu)^2}{2N_{\mathrm{sc}}\sigma^2}} \quad (1)$$

with μ and σ the average value and standard deviation of the fading process.

With a_{pl} and a_{sf} the amplification due to pathloss and shadow fading, respectively, the SIR due to pathloss and shadowing at a specific location within the network is given by

$$\gamma_{\mathrm{sf}} = \frac{a_{\mathrm{pl}} \cdot a_{\mathrm{sf}}}{I} \quad (2)$$

with I the interference power normalized to the transmit power which can be approximated by the expectation value of the sum of all interfering signals [8]. If fast fading is considered, the SIR at a specific location within the network and for a specific resource unit results into

$$\gamma_{\mathrm{ff}} = a_{\mathrm{ff}} \cdot \gamma_{\mathrm{sf}}. \quad (3)$$

Performing scheduling, the statistics of γ_{ff} for a specific user can be changed by allocating resource units only if certain requirements regarding for instance the value of a_{ff} are fulfilled. The SIR that is experienced after scheduling is called γ_{eff} in the following. The SIR gain due to scheduling is given by

$$\Delta_{\gamma} = \frac{\gamma_{\mathrm{eff}}}{\gamma_{\mathrm{sf}}}. \quad (4)$$

Two well know scheduling algorithms are considered in the following that allocate asymptotically equal amount of resource units to all users. These scheduling algorithms under consideration are fair resource scheduling [3] and proportional fair scheduling [17].

With fair resource scheduling, resource units are allocated to the users successively. Channel state information is not considered for the scheduling decision. Therefore, the effective SIR due to fair resource scheduling results into

$$\gamma_{\mathrm{eff,FR}} = \gamma_{\mathrm{ff}}. \quad (5)$$

Using (4) and (5) the SIR gain due to fair resource scheduling is given by

$$\Delta_{\gamma,\mathrm{FR}} = \frac{\gamma_{\mathrm{eff,FR}}}{\gamma_{\mathrm{sf}}} = a_{\mathrm{ff}}. \quad (6)$$

The pdf of $\Delta_{\gamma,\mathrm{FR}}$ is given by (1).

Proportional fair scheduling on the other hand considers channel state information for the scheduling decision. A resource unit is allocated to the user that has the highest actual SIR normalized to the SIR due to pathloss and shadowing [3, 16]. Normalizing (3) to γ_{sf}, it can be seen that a proportional fair scheduler allocates the resource unit to the user that has the highest a_{ff} for the resource unit under consideration. The SIR gain for proportional fair scheduling is given by

$$\Delta_{\gamma,\mathrm{PF}} = \max(a_{\mathrm{ff},1},\ a_{\mathrm{ff},2},\ a_{\mathrm{ff},3},\ \cdots\ a_{\mathrm{ff},\mathrm{N_{active}}}) \quad (7)$$

with N_{active} the amount of active users and $a_{\mathrm{ff,i}}$ the fading gain for user i for the resource unit under consideration. $a_{\mathrm{ff,i}}$ are i.i.d. random variables with the pdf given by (1). It can be shown that the pdf of $\Delta_{\gamma,\mathrm{PF}}$ for $N_{\mathrm{active}} = n$ is given by

$$f_{\Delta_{\gamma,\mathrm{PF}}}(\Delta_\gamma) = n \cdot f_{a_{\mathrm{ff}}}(\Delta_\gamma) \cdot F_{a_{\mathrm{ff}}}(\Delta_\gamma)^{n-1} \quad (8)$$

[13] with $f_{a_{\mathrm{ff}}}(\Delta)$ the pdf as given by (1) and $F_{a_{\mathrm{ff}}}(\Delta)$ the cdf of $f_{a_{\mathrm{ff}}}(\Delta)$.

4. ADJUSTMENT OF THE SIR PDF

The SIR at a specific position within the network after the scheduling in linear scale can be obtained from (4). In logarithmic scale, it is given by

$$\gamma_{\mathrm{eff,dB}} = 10 \cdot \log_{10}(\Delta_\gamma \cdot \gamma_{\mathrm{sf}}) = 10 \cdot \log_{10}(\Delta_\gamma) + \gamma_{\mathrm{sf,dB}} \quad (9)$$

with $\gamma_{\mathrm{sf,dB}}$ the SIR due to pathloss and shadowing in dB and Δ_γ the SIR gain due to scheduling.

The pdf of $\gamma_{\mathrm{sf,dB}}$ within the network can be obtained analytically if for instance circular cell areas are assumed. For practical network planning problems cell areas are defined by the environment. Therefore, it is not within the scope of this paper to give an analytical expression for $\gamma_{\mathrm{sf,dB}}$. Instead, it is assumed that the pdf of $\gamma_{\mathrm{sf,dB}}$ is the result of for instance a coverage planning procedure and is obtained simulative or from measurements and is approximated by the discrete pdf

$$\hat{f}_{\gamma_{\mathrm{sf,dB}}}(\gamma_{\mathrm{sf,dB}}) \approx \sum_i \beta_i \cdot \delta(\gamma_{\mathrm{sf,dB}} - \gamma_i) \quad (10)$$

with

$$\beta_i = \int_{\gamma_{i-1}}^{\gamma_i} f_{\gamma_{\mathrm{sf,dB}}}(\gamma_{\mathrm{sf,dB}})\ d\gamma_{\mathrm{sf,dB}}. \quad (11)$$

$\delta(x)$ is the Dirac delta and $f_{\gamma_{\mathrm{sf,dB}}}(\gamma_{\mathrm{sf,dB}})$ the pdf of $\gamma_{\mathrm{sf,dB}}$ as obtained empirically. The integration limits γ_{i-1} and γ_i can be chosen arbitrarily. A small granularity $\gamma_i - \gamma_{i-1}$ will lead to a better approximation of $f_{\gamma_{\mathrm{sf,dB}}}(\gamma_{\mathrm{sf,dB}})$.

The pdf of Δ_γ in linear scale is derived in section 3. To get the pdf of $\gamma_{\mathrm{eff,dB}}$, considering only the scheduled users and transmitted resource units, the two random variables Δ_γ and $\gamma_{\mathrm{sf,dB}}$ have to be combined.

The SIR gain due to scheduling in logarithmic scale is given by

$$\Delta_{\gamma,\mathrm{dB}} = 10 \cdot \log_{10}(\Delta_\gamma). \quad (12)$$

Applying random variable transformation [13] to the pdfs derived in section 3, the pdf of $\Delta_{\gamma,\mathrm{dB}}$ is given by

$$f_{\Delta_{\gamma,\mathrm{dB}}}(\Delta_{\gamma,\mathrm{dB}}) = \frac{10^{\frac{\Delta_{\gamma,\mathrm{dB}}}{10}}}{10 \cdot \log_{10}(e)} \cdot f_{\Delta_\gamma}\left(10^{\frac{\Delta_{\gamma,\mathrm{dB}}}{10}}\right). \quad (13)$$

Using the relationship of (9), the pdf of $\gamma_{\mathrm{eff,dB}}$ is obtained by convolution of the pdfs of (10) and (13) and is given by

$$f_{\gamma_{\mathrm{eff,dB}}}(\gamma_{\mathrm{eff,dB}}) \approx \sum_i \beta_i \cdot \frac{10^{\frac{\gamma_{\mathrm{eff,dB}} - \gamma_i}{10}}}{10 \cdot \log_{10}(e)} \cdot f_{\Delta_\gamma}\left(10^{\frac{\gamma_{\mathrm{eff,dB}} - \gamma_i}{10}}\right). \quad (14)$$

As described in [2], the probability of utilizing modulation and coding scheme i is obtained from (14) by

$$\eta_i = \int_{\gamma_{\mathrm{lb},i}}^{\gamma_{\mathrm{ub},i}} f_{\gamma_{\mathrm{eff,dB}}}(\gamma_{\mathrm{eff,dB}})\ d\gamma_{\mathrm{eff,dB}} \quad (15)$$

with $\gamma_{\mathrm{lb},i}$ and $\gamma_{\mathrm{ub},i}$ the lower and upper signal to noise ratio (SNR) bounds for the usage of modulation and coding scheme i, respectively. With d_i bits per resource unit that can be transmitted using modulation and coding scheme i and N_{res} the amount of available resource units, the average cell throughput is given by

$$r_{\mathrm{cell}} = N_{\mathrm{res}} \cdot \sum_{i=1}^{N_{\mathrm{mcs}}} \eta_i \cdot d_i. \quad (16)$$

The amount of available modulation and coding schemes is given by N_{mcs}.

5. RESULTS

The simulation scenario consists of hexagonal cells. Base stations are located in the center of a cell and are equipped with omnidirectional antennas. To avoid border effect, the investigation area is wrapped onto a torus. Mobile stations are distributed uniformly in the investigation area. The system is interference limited. It is assumed that the base station is always able to transmit data to a mobile station if the mobile station is scheduled for transmission due to the full buffer traffic model.

Blocks of 4 resource units have to be allocated to users during scheduling. Link adaptation is performed based on actual channel state information. The simulation parameters are given in Table 1. The spectral efficiencies and the SNR thresholds of the different modulation and coding schemes can be found in Table 2.

For the analytical results, a regular grid is used covering the investigation area. The SIR due to pathloss and shad-

Table 1: Simulation Parameters

Parameter	value
Site to site distance	1000 m
Number of Cells	25
Pathloss coefficient	3.5
Standard deviation of the shadowing	8 dB
T_{s}	100 μs
N_{res}	8
System bandwidth	1.25 MHz
Channel Model	ITU Vehicular A
N_{sc}	4
μ in (1)	1
σ in (1)	1
Traffic Model	Full Buffer
Granularity of the pdf approximation in (14)	0.1 dB

Table 2: **Link adaptation properties**

SNR thresholds in dB	-3.4	-0.8	1.5	4.0	5.5	6.7	9.2	12.0	14.1	16.8	20.5
d_i in bit/resource unit	48	96	144	192	240	288	384	480	576	672	864

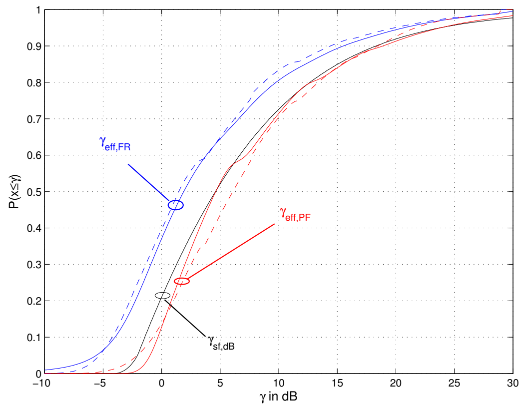

Figure 1: SIR cdf due to pathloss, shadowing and scheduling, $N_{\text{active}} = 20$, solid lines: analytical results, dashed lines: results of ONe-PS

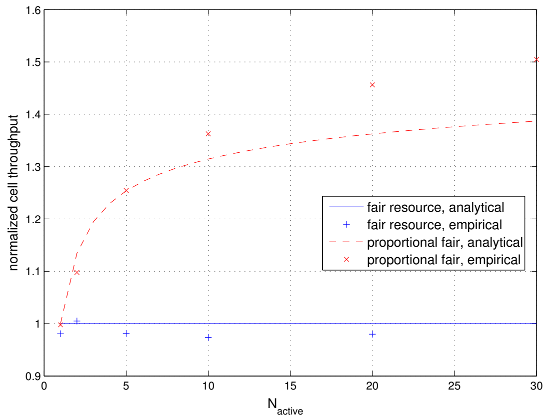

Figure 2: Normalized average cell throughput depending on the number of active users

owing is calculated for each grid point assuming a possible mobile station located at the grid position is allocated to the base station with the lowest pathloss. Afterwards, the SIR pdf is calculated according to (10). Simulation results are obtained with the OFDMA network performance simulator (ONe-PS). In ONe-PS, the same environment is assumed and mobile stations are located uniformly in the investigation area. The transmission including scheduling, frequency selective fading and block errors during decoding is modeled in detail.

Figure 1 shows the cdf of the SIR considering all users within the network. On average, 20 users are active in each cell. The cdf of $\gamma_{\text{sf,dB}}$ is equal for both the analytical and simulation results. Additionally, the cdfs of $\gamma_{\text{eff,FR}}$ after fair resource and $\gamma_{\text{eff,PF}}$ after proportional fair scheduling are presented as obtained by (14) and as measured in ONe-PS. Only the actual SIR conditions of the resource units after the scheduling decision are taken into account so that the impact of the scheduling algorithm can be observed.

It can be seen that due to the fading channel, the cdf of $\gamma_{\text{eff,FR}}$ is shifted to the left by approximately 3 dB compared to the cdf of $\gamma_{\text{sf,dB}}$. Due to the fading channel statistics approximately 50 % of the realizations have $a_{\text{ff}} \leq -3$ dB. This is independent of the location of the mobile station. With proportional fair scheduling, resource units are only allocated if the channel conditions are good compared to the average conditions. Therefore, the cdf of $\gamma_{\text{eff,PF}}$ is improved compared to fair resource scheduling. Comparing the results obtained with the proposed methodology and the measurements of the simulations, it can be seen that the approximation of the SIR cdf is accurate over a large region of SIR values.

Figure 2 shows the average cell throughput that is obtained in the assumed network as a function of the number N_{active} of active users in the cell. The scope of this paper is to evaluate the performance and accuracy of the analyitcal results. Therefore, relative performance results are presented in Figure 2. The average cell throughput is normalized to the average cell throughput that is obtained analytically if only one user is active per cell. In case of fair resource scheduling the cell throughput performance cannot be improved if the amount of active users is increased as can be seen in Figure 2. Fair resource scheduling is not able to utilize multiuser diversity [9] due to not considering channel state information during scheduling decision.

With proportional fair scheduling, channel state information is considered during scheduling decision. It can be seen that the cell throughput is increased with increasing number of active users. Comparing the results obtained analyitcally and with ONe-PS, it can be seen that the analyitcal results closely approximate the average cell throughput. For fair resource scheduling, the difference between the results obtained analyitcally and with ONe-PS is within 2 %. For proportional fair scheduling, the simulation results are well approximated up to 5 active users. If the amount of active users increases further, the analytical results lead to lower average cell throughput results than obtained with ONe-PS and the difference increases to approximately 10 % between the analytical and simulative results for 30 active users. The reason is in limiting number of modulation and coding schemes so that the fading gains of the different users are not identically distributed as assumed in section 3.

Table 3 shows the impact of the granularity on the approximation of the SIR pdf as given by (10). Equidistant values of γ_i are used and the spacing between γ_i and γ_{i-1} is varied. The pdf of $\gamma_{\text{sf,dB}}$ is approximated between -20 dB and 80 dB. A scenario with 20 active users and fair resource scheduling is investigated. The relative error in average cell throughput between the analytical and simulation results is given as well as the amount of summands in (14). It can be seen that a larger granularity reduces the number of summands.

The relative error is nearly constant down to a granularity of 1 dB. If the granularity of the pdf approximation is further decreased, the relative error increases.

Table 3: Impact of the granularity of the pdf approximation of $\gamma_{\mathrm{sf,dB}}$

granularity of pdf	relative error in r_{cell} compared to results with ONe-PS	number of summands in (14)
0.1 dB	2.6 %	1000
0.5 dB	2.7 %	200
1 dB	2.6 %	100
5 dB	3.0 %	20
10 dB	4.2 %	10

6. CONCLUSION

This paper presents a new methodology to estimate the average cell throughput considering the impact of scheduling without performing extensive system level simulations. The statistics of the SIR due to pathloss and shadowing as obtained for the network setup are needed in form of a pdf. The proposed methodology derives the pdf of the SIR so that the impact of scheduling is reflected. Based on the pdf of the SIR including the impact of scheduling, the average cell throughput is calculated by deriving the probability that a certain modulation and coding scheme is used.

The pdf of the SIR gain for two well known scheduling algorithms are derived in this paper. A channel model is assumed that is valid for transmission of resource units with subcarriers that are distributed over the whole system bandwidth. It is also possible to consider systems where subcarriers are allocated block-wise to resource units by adapting the effective fading power in (1). Further scheduling algorithms can be included easily into the proposed methodology. Due to usage of analytical expressions, the methodology is able to give a fast estimation of the average cell throughput and for instance different network layouts or scheduling strategies can be compared with each other. The proposed methodology provides more accurate average cell throughput results than it is possible to get from a static scenario due to considering the impact of scheduling. Additionally, the proposed methodology simplifies the network dimensioning process by avoiding extensive system level simulations.

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