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A Transceive Strategy for Regenerative Multi-Antenna Multi-Way Relaying

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***Abstract*—In this paper, firstly, we introduce regenerative multi-way relaying. A half-duplex multi-antenna relay station (RS) assists multiple nodes which want to communicate to each other. Each node has a message and wants to decode the messages from all other nodes. The number of communication phases is equal to the number of nodes, *N*, such that when $N = 2$, we have the well known two-way relaying. In the first phase, multiple access (MAC) phase, all nodes transmit simultaneously to the RS and the RS decodes the data streams of all nodes. In the following $(N - 1)$ broadcast (BC) phases, the RS transmits to each node the intended $(N - 1)$ data streams from the other $(N - 1)$ nodes. Secondly, we propose a transceive strategy at the RS which couples the MAC and the BC phases. The strategy exploits the information about the achievable MAC signal to interference and noise ratio at the RS which enables the RS to transmit with the achievable MAC rate in each BC phase. As the strategy involves transmit beamforming at the RS, we intend to minimise the power at the RS. Since the power minimisation problem is NP-hard, we rewrite the problem into a semidefinite optimisation to solve it in reasonable time.**

I. INTRODUCTION

The two-way (bidirectional) communication channel between two nodes which want to communicate to each other is introduced in [1]. Recently, as relay communication becomes an interesting topic of research, the work of [1] is extended by other works, such as [2]–[4], to the case of bidirectional communication using a relay station (RS).

Bidirectional communication using an RS can be realised by one-way relaying or two-way relaying protocol. In [2] and [3], it is shown that two-way relaying outperforms one-way relaying in terms of sum rate performance as it uses the resources more efficiently.

The work in [2] and [4] consider regenerative two-way relaying where the RS regenerates the data streams of the nodes. The RS decodes the bit information from the two communicating nodes, reencodes them and forwards the reencoded data streams to both nodes simultaneously. The regenerative relaying has the advantage that the noise from the RS is not propagated to the nodes. Moreover, in the decoding process at the nodes, they need to know only their own channel to the RS. Hence, in comparison to the non-regenerative in [3], the number of resources for channel estimation or for feedback from the RS to the nodes is reduced.

A multi-antenna regenerative RS supporting two-way relaying for one bidirectional node pair is considered in [5] and [6], while for more than one bidirectional pair is treated in [7] and [8]. The bidirectional pairs separation in [7] and [8] is done spatially by applying beamforming at the RS.

A different scenario where more than two nodes are in the same communication group may appear, such as in video conference or multi-player gaming. In such applications, multiple nodes want to communicate to each other. Until now, there are only few works on such multi-way channel, e.g., the work in [9] and [10], in which [1] is a special case when the number of nodes is $N = 2$.

There is different notion in [9] and [10] about a multi-way channel. Here, we are using the same notion as in [10], *an N-node multi-way channel is one in which each node has a message and wants to decode the messages from all other nodes*. An example scenario is given in [10] where members of an emergency response team, each equipped with a wireless enabled device, wants to transmit and receive data to and from all other responders at a disaster site. In a disaster site where the communication infrastructures are down, an alternative multi-way communication using a newly installed RS may help the emergency response team members to communicate to each other.

In multi-way communication, if all N nodes are half-duplex and there are direct links between them, the required number of communication phases in order for each node to obtain the information from all other nodes is N, as depicted in Figure 1.(a) for the case of 3 nodes, namely, S0, S1 and S2.

During the preparation of this paper, we found out very recently that [11] also considers multi-way relaying. While [11] considers full-duplex communication and symmetric traffic, in [12] both symmetric and asymmetric traffic are considered for non-regenerative multi-way relaying with half-duplex communication which requires N communication phases.

In this paper, we introduce an N-phase regenerative multi-way relaying. Figure 1.(b) shows the proposed 3-phase multi-way relaying for 3 nodes. In the first phase, S0 sends x_0, S1 sends x_1 and S2 sends x_2 simultaneously to the RS. In the second phase, the RS forwards x_0 to S2, x_1 to S0 and x_2 to S1. In the third phase, the RS forwards x_0 to S1, x_1 to S2 and x_2 to S0. After completing 3 communication phases, each node receives the data streams from all other nodes. Our work is a generalisation of regenerative two-way relaying, i.e., if $N = 2$, we have the regenerative two-way relaying.

For regenerative two-way relaying with multi-antenna RS, the work in [5]–[8] decouple the MAC and BC phases. They

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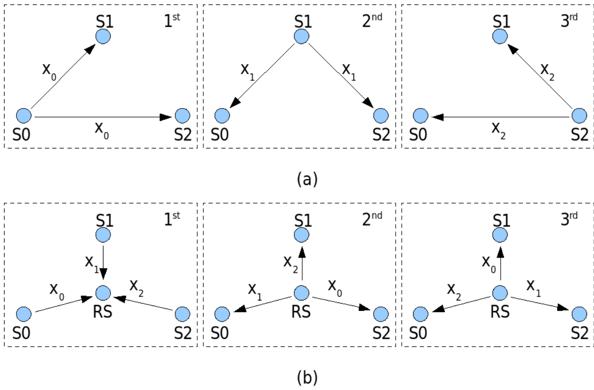

Fig. 1. Multi-way communication: (a). Direct link; (b). No direct link, with a multi-antenna RS applying multi-way protocol

assume a perfect MAC and concentrate the work on the BC phase. Nevertheless, the overall sum rate is defined by the minimum between the rate of MAC phase and the rate of BC phase. It is shown in [5] and [7] that, even though the BC phase is optimised under a power constraint, the overall sum rate is still below the rate of the MAC phase. In this paper, we propose a transmit-receive (transceive) strategy at the RS for regenerative multi-way relaying by coupling the MAC and BC phases. The strategy ensures that we transmit with the achievable MAC rate in each BC phase.

The contribution of this paper can be summarised as follows: 1. We propose an N-phase regenerative multi-way relaying for N nodes, 2. We propose a transceive strategy at the RS which enables us to transmit in each BC phase with the achievable MAC rate while having minimum power, and 3. As the power minimisation problem is non-convex, we rewrite the problem into a semidefinite optimisation to solve the 2nd contribution.

This paper is organised as follows. Section II describes the protocol and the system model of the N-phase regenerative multi-way relaying. Section III explains the transceive strategy and the semidefinite optimisation. The performance analysis is given in Section IV. Finally, Section V provides the conclusion.

Throughout this paper, boldface lower case and upper case letters denote vectors and matrices, respectively, while normal letters denote scalar values. The superscripts $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ stand for matrix or vector transpose and complex conjugate transpose, respectively. The operators $\mathrm{mod}_N(x)$ and $\mathrm{tr}\{\mathbf{X}\}$ denote the modulo N of x and the trace of $\mathbf{X}$, respectively, and $\mathcal{CN}(0,\sigma^2)$ denotes the zero-mean complex normal distribution with variance σ^2.

II. Protocol and System Model

In this section, the communication protocol and the system model of N-phase regenerative multi-way relaying are described. There are N single-antenna nodes which want to communicate to each other through a multi-antenna RS with M antenna elements. In order to reliably separate the received data streams, we require $M \geq N$ [13]. Hence, we assume in this work that the RS has a number of antenna elements higher than or equal to the number of nodes. Although in this paper we only consider single-antenna nodes, our work can be readily extended to the case of multi-antenna nodes.

In the first phase, MAC phase, all nodes transmit simultaneously to the RS and the RS decodes all the data streams from all nodes. In the following $(N-1)$ BC phases, the RS sends to each node the intended $(N-1)$ data streams from $(N-1)$ other nodes. In each BC phase, the RS sends simultaneously N data streams from the MAC phase to N different nodes, where each data stream is intended for one particular node. The separation of the data streams transmission in BC phases is done spatially at the RS by applying transmit beamforming.

In the n-th broadcast (BC) phase, $n, n \in \mathcal{P}, \mathcal{P} = \{2, \cdots, N\}$, if a node $k, k \in \mathcal{N}, \mathcal{N} = \{0, \cdots, (N-1)\}$, receives the data stream from node $i, i \in \mathcal{N}$, their relationship is defined by $i = \mathrm{mod}_N(k+n-1)$. A pictorial example of the BC phases transmission is given in the middle and right part of Figure 1.(b) for the 2nd and 3rd phase of multi-way relaying with 3 nodes.

In MAC phase, the received signal at the RS is given by

$$\mathbf{r}_{\mathrm{RS}} = \mathbf{H}\mathbf{x} + \mathbf{z}_{\mathrm{RS}}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{M\times N} = [\mathbf{h}_0, \cdots, \mathbf{h}_{(N-1)}]$ is the overall channel matrix, with $\mathbf{h}_i \in \mathbb{C}^{M\times 1} = (h_{i,1}, \cdots, h_{i,M})^{\mathrm{T}}$ the channel vector between node i and the RS. The channel coefficient $h_{i,m}, m \in \mathcal{M}, \mathcal{M} = \{1, \cdots, M\}$, follows $\mathcal{CN}(0, \sigma_h^2)$. The vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is a vector of $(x_0, \cdots, x_{(N-1)})^{\mathrm{T}}$, with x_i the signal of node i which follows $\mathcal{CN}(0, \sigma_{x_i}^2)$. The AWGN noise vector at the RS is denoted as $\mathbf{z}_{\mathrm{RS}} \in \mathbb{C}^{M\times 1} = (z_{\mathrm{RS}1}, \cdots, z_{\mathrm{RS}M})^{\mathrm{T}}$ with $z_{\mathrm{RS}m}$ follows $\mathcal{CN}(0, \sigma_{z_{\mathrm{RS}}}^2)$.

Assuming that the RS decodes correctly and reencodes the data streams with the same codebook used at the nodes, we have once again the transmitted signal vector $\mathbf{x}$ at the RS. Moreover, assuming reciprocal and stationary channels in N phases, the downlink channel matrix from the RS to the nodes is simply the transpose of the uplink channel matrix $\mathbf{H}$. Thus, the received signal vector of all nodes in the n-th BC phase can be written as

$$\mathbf{r}_{\mathrm{nodes}}^n = \mathbf{H}^{\mathrm{T}}\mathbf{G}^n\mathbf{x} + \mathbf{z}_{\mathrm{nodes}} \tag{2}$$

and, accordingly, the received signal at node k while receiving the data streams from node i in the n-th BC phase is given by

$$r_{k,i}^n = \underbrace{\mathbf{h}_k^{\mathrm{T}}\mathbf{g}_i^n x_i}_{\text{useful signal}} + \underbrace{\sum_{\substack{j=0\\ j\neq i}}^{N-1} \mathbf{h}_k^{\mathrm{T}}\mathbf{g}_j^n x_j}_{\text{BC's interference signals}} + z_k, \tag{3}$$

where $\mathbf{G}^n \in \mathbb{C}^{M\times N} = [\mathbf{g}_0^n, \cdots, \mathbf{g}_{(N-1)}^n]$ is the n-th phase transmit beamforming matrix and $\mathbf{z}_{\mathrm{nodes}} = \left(z_0, \cdots, z_{(N-1)}\right)^{\mathrm{T}}$ with z_k follows $\mathcal{CN}(0, \sigma_{z_k}^2)$. Note, however, if the channels are not stationary, we simply need to change $\mathbf{h}_k^{\mathrm{T}}$ with the instantaneous channel coefficients from the RS to node k.

III. Transceive Strategy

In this section, we explain the transceive strategy for regenerative multi-way relaying. In order to transmit in the BC phases with the achievable MAC rate, we need to couple the MAC and BC phases. Therefore, we need parameters from the MAC phase to be used as an optimisation parameter at BC phases. Here, we use the received signal to interference and noise ratio (SINR) of each node at the RS as the MAC-BC coupling parameters.

A. Signal to Interference and Noise Ratio

1) MAC Phase: In order to obtain the required MAC-BC coupling parameters, we propose to apply a decoding process at the RS which achieves the optimum rate region of the MAC phase. It is shown in [14] that Minimum Mean Square Error (MMSE) with Successive Interference Cancellation (SIC) receivers are information theoretically optimal for uplink MAC scenario.

First, we compute the SINR achieved by the MMSE receiver for each node i, γ_i, which is defined by

$$\gamma_i = \sigma_{x_i}^2 \mathbf{h}_i^{\mathrm{H}} \left(\sum_{\substack{j=0 \\ j\neq i}}^{N-1} \mathbf{h}_j \mathbf{h}_j^{\mathrm{H}} \sigma_{x_j}^2 + \sigma_{z_{\mathrm{RS}}}^2 \mathbf{I}_M \right)^{-1} \mathbf{h}_i \tag{4}$$

[14] and, afterwards, we sort γ_i and perform the SIC starting from the highest γ_i to the lowest γ_i. Finally, we obtain the MAC SINR of node i's after SIC, γ_i^{MAC}, which is given by

$$\gamma_i^{\mathrm{MAC}} = \begin{cases} \gamma_i, & \textit{if } i \textit{ has the highest } \gamma_i \\ \frac{|\mathbf{h}_i|^2 \sigma_{x_i}^2}{\sigma_{z_{\mathrm{RS}}}^2}, & \textit{if } i \textit{ has the lowest } \gamma_i \\ \gamma_i^{\mathrm{sic}}, & \textit{otherwise} \end{cases} \tag{5}$$

where $\gamma_i^{\mathrm{sic}} = \sigma_{x_i}^2 \mathbf{h}_i^{\mathrm{H}} \left(\sum_{j\in\mathcal{B}_i} \mathbf{h}_j \mathbf{h}_j^{\mathrm{H}} \sigma_{x_j}^2 + \sigma_{z_{\mathrm{RS}}}^2 \mathbf{I}_M \right)^{-1} \mathbf{h}_i$ with $\mathcal{B}_i = \{j \mid \gamma_j < \gamma_i\}$ is the set of all nodes j which have lower SINR than the SINR of node i. The achievable sum rate for MAC phase is given by

$$R^{\mathrm{MAC}} = \sum_i R_i^{\mathrm{MAC}} \tag{6}$$

with $R_i^{\mathrm{MAC}} = \log_2\left(1 + \gamma_i^{\mathrm{MAC}}\right)$.

2) BC Phases: Given the received signal as in (3), the SINR of node k when receiving the data stream from node i in the n-th BC phase, $\gamma_{k,i}^n$, is given by

$$\gamma_{k,i}^n = \frac{|\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_i^n|^2 \sigma_{x_i}^2}{I_{\mathrm{s}}^n + I_{\mathrm{os}}^n + \sigma_{z_k}^2}, \tag{7}$$

with the self-interference

$$I_{\mathrm{s}}^n = |\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_k^n|^2 \sigma_{x_k}^2, \tag{8}$$

and the other-stream interference

$$I_{\mathrm{os}}^n = \sum_{\substack{j\neq k \\ j\neq i}}^{N-1} |\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_j^n|^2 \sigma_{x_j}^2. \tag{9}$$

In each n-th BC phase, node k may perform interference cancellation. Before it decodes the corresponding data stream, it subtracts the a priori self-interference and, assuming it has decoded correctly the received data streams from the previous BC phases, the a priori other-stream interference. Applying interference cancellation at all nodes, the SINR can be rewritten as

$$\gamma_{k,i}^n = \frac{|\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_i^n|^2 \sigma_{x_i}^2}{\sum_{\substack{j\neq k \\ j\notin\mathcal{L}^n}}^{N-1} |\mathbf{h}_k^{\mathrm{T}} \mathbf{g}_j^n|^2 \sigma_{x_j}^2 + \sigma_{z_k}^2}, \tag{10}$$

with $\mathcal{L}^n = \{l^n \mid l^n = \mathrm{mod}_N(k+q-1), \forall q, q \in \mathcal{Q}, \mathcal{Q} = \{2, \cdots, (n-1)\}\}$ the set of nodes which data streams have been decoded by node k in the previous BC phases.

B. Transmit Beamforming Strategy

Since R^{MAC} is known a priori before the BC phases, we intend to ensure that we transmit in the BC phases what we obtain in the MAC phase. However, as power is a limited resource, we have to minimise its use while achieving our intention. If $x_i, \forall i$, follows $\mathcal{CN}(0,1)$, the optimisation problem can be written as

$$\min \sum_i \|\mathbf{g}_i^n\|_2^2 \quad \text{subject to} \quad \gamma_{k,i}^n \geq \gamma_i^{\mathrm{MAC}}, \tag{11}$$

with $\gamma_{k,i}^n$ as in (7) or (10) by setting $\sigma_{x_i}^2 = \sigma_{x_j}^2 = 1$. The constraint in (11) shows the coupling of MAC and BC phases, where $\gamma_i^{\mathrm{MAC}}, \forall i$, are the MAC-BC coupling parameters. This constraint ensures the transmission of rate R^{MAC} in each n-th BC phase. Thus, by (11), we achieve our intention to transmit R^{MAC} in each BC phase and the achievable sum rate of regenerative multi-way relaying is given by

$$R_{\mathrm{m-way}} = \frac{1}{N} \cdot (N-1) \cdot R^{MAC}, \tag{12}$$

where the factor $\frac{1}{N}$ is due to the use of N orthogonal resources and the factor $(N-1)$ is the number of BC phases in which we transmit with the rate of R^{MAC} in each BC phase.

C. Semidefinite Optimisation

The optimisation problem in (11) is similar to the problem treated in [15] and [16] for downlink beamforming with Quality of Service constraint. As it is a quadratic optimisation problem with quadratic nonconcex constraint, the problem is an NP-hard problem [15], [16]. However, by rewriting the problem we may obtain a semidefinite optimisation of the problem which can be solved efficiently using a solver such as SeDuMi [17]. By defining, $\mathbf{X}_i^n = \mathbf{g}_i^n \mathbf{g}_i^{n\mathrm{H}}$ and $\mathbf{Q}_k = \mathbf{h}_k^* \mathbf{h}_k^{\mathrm{T}}$, we can rewrite (11) into

$$\begin{aligned} &\min \sum_i \mathrm{tr}\left(\mathbf{X}_i^n\right) \\ &\text{s.t. } \mathrm{tr}\left(\mathbf{Q}_k \mathbf{X}_i^n\right) - \gamma_i^{\mathrm{MAC}} \sum_j \mathrm{tr}\left(\mathbf{Q}_k \mathbf{X}_j^n\right) \geq \gamma_i^{\mathrm{MAC}} \sigma_{z_k}^2 \\ &\mathbf{X}_i^n = \mathbf{X}_i^{n\mathrm{H}} \\ &\mathbf{X}_i^n \succeq \mathbf{0}, \end{aligned} \tag{13}$$

TABLE I
SOLUTION OF THE SEMIDEFINITE OPTIMISATION: 300 REALISATION

	0	5	10	15	20	25	30
2	2.9402	3.3635	2.1600	2.4514	1.9088	1.9231	1.8375
3	2.8181	2.4981	2.2604	2.2299	2.1113	2.2186	2.0996
4	2.7459	2.4571	2.4708	2.4362	2.4314	2.2973	2.2805

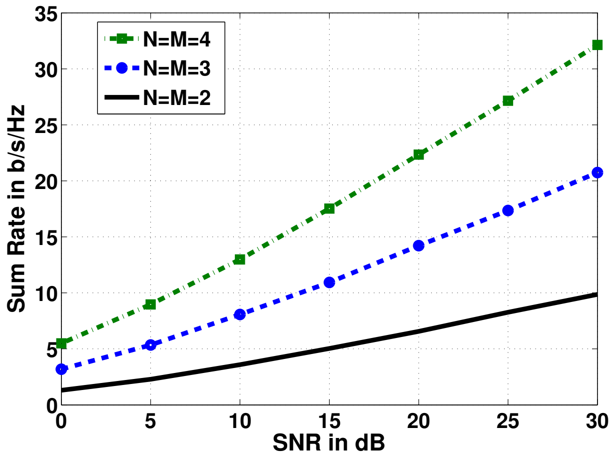

Fig. 2. Achievable sum rate of multi-way relaying: 2-, 3- and 4-way relaying

where $\mathbf{X}_i^n \succeq \mathbf{0}$ means $\mathbf{X}_i^n$ is positive semidefinite and j has two possibilities, $j \neq i$ if no interference cancellation and $\{j \neq k, j \notin \mathcal{L}\}$ if interference cancellation is applied at all nodes. By Lemma 2 in [15], (13) is not a strict relaxation but indeed the equivalent reformulation of (11) and we will have at least one optimum solution where all $\mathbf{X}_i$ have rank one.

IV. PERFORMANCE ANALYSIS

In this section, the sum rate performance is analysed. N single-antenna nodes communicate to each other with the help of an RS with $M = N$ antenna elements. We set $\sigma^2_{z_{\mathrm{RS}}} = \sigma^2_{z_k} = 1, \forall k, \sigma^2_{x_i} = 1, \forall i$, and $\mathrm{SNR} = \sigma^2_h$. Figure 2 shows the achievable sum rate of regenerative multi-way relaying for the case of $N = 2$, 3 and 4 number of nodes. It can be seen that the higher the number of nodes (and correspondingly the number of RS' antenna elements) the higher are the sum rate and the spatial multiplexing gain, which is shown by the slope of the curve. The achievable sum rate is obtained by transmitting in each BC phase with the achievable MAC rate using the proposed transceive strategy.

The solution of the power minimisation is tabulated in Table I. We tabulate only one solution in each table cell due to the fact that the solutions in all BC phases are the same. In general, for relatively low SNR, 0 and 5 dB, the solutions are higher than for relatively good SNR. A remark for table I is that we only consider the case without interference cancellation.

V. CONCLUSION

In this paper, we introduce regenerative multi-way relaying where N single-antenna nodes want to communicate to each other through the assistance of a multi-antenna RS. We propose an N-phase multi-way relaying protocol which consists of one MAC phase and $(N-1)$ BC phases. We propose a transceive strategy for regenerative multi-way relaying by coupling the MAC and the BC phases with an intention to transmit with the achievable MAC rate in each BC phase. The strategy involves transmit beamforming at the RS and we intend to achieve the intention while having minimum power at the RS. As the power minimisation problem is NP-hard, we rewrite the problem into semidefinite optimisation which can be solved efficiently.

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