A. Fernekeß, A. Klein, B. Wegmann, K. Dietrich, "Analysis of Cellular Mobile Radio Networks using Fair Throughput Scheduling," in *Proc. of International Symposium on Personal, Indoor and Mobile Radio Communications*, Tokyo, Japan, Sep. 2009

Analysis of Cellular Mobile Networks using Fair Throughput Scheduling

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Abstract— **Providing quality of service in broadband wireless access systems is an important issue. Fair throughput scheduling is one scheduling algorithm that is able to guarantee equal throughput in the long term for all users within one cell by allocating a resource to the user with the lowest throughput averaged over a past interval. The throughput that can be obtained by a user depends on the signal to interference plus noise ratio achieved by the users within the cell. Usually, system level simulations are necessary to get results on the user throughput for a specific cell layout, user distribution and the scheduling algorithm. This paper presents an analytical performance analysis for cellular systems using fair throughput scheduling. Derivations are presented for the number of allocated resources and the user throughput that can be achieved at a certain position within the network as well as the cell throughput that can be obtained. The investigation can be done for arbitrary user distributions and different network layouts. Results from system level simulations show that the analytical estimates achieve good accuracy with only a fraction of the computational complexity compared to a system level simulation.**

I. Introduction

Broadband wireless access (BWA) has emerged over the past years and it is expected that the importance will further increase with the upcoming fourth generation networks. A promising candidate for fourth generation networks is orthogonal frequency division multiple access (OFDMA). OFDMA is currently deployed in networks like IEEE 802.16, which is also know as Worldwide Interoperability for Microwave Access (WiMAX), and Long Term Evolution (LTE) of the Universal Mobile Telecommunications System (UMTS). OFDMA allows a flexible resource allocation such that scheduling algorithms gain importance [1, 2].

The scheduling algorithm in combination with link adaptation schemes has impact on the user throughput and on the cell throughput and they should therefore be considered during the planning process. Resources can be allocated to users considering, e.g., channel state information or user throughput conditions. Scheduling algorithms that utilize multiuser diversity [3] and provide high cell throughput are, e.g., scheduling algorithms that maximize the sum rate by allocating a resource to the user with the highest signal to interference plus noise ratio (SINR) [4]. Proportional fair or round robin scheduling algorithms provide fairness to the users by allocating equal amount of resources [5-8]. Scheduling algorithms like fair throughput scheduling are able to fulfill user throughput constraints by allocating resources to the user with the lowest average user throughput so that all users within one cell will achieve equal user throughput [9, 10].

Usually user and cell throughput results for different scheduling algorithms are obtained using system level simulations, e.g., [11-13]. However, system level simulations are very complex and therefore not applicable during the network planning process when user and cell throughput results for a given network layout should be available fast. In [14], a methodology is proposed to get cell throughput without performing system level simulations but only round robin scheduling is assumed. An analytical cell throughput estimation for scheduling algorithms that allocate resources fair among the users is presented in [15]. However, equal Quality of Service (QoS) cannot be guaranteed by these algorithms so that fair throughput scheduling has to be taken into account in scenarios where equal QoS is required

Throughout this paper, an analytical performance analysis of cellular mobile networks using fair throughput scheduling is presented. The fair throughput scheduling algorithm allocates resources to the user with the lowest average throughput in a past time interval. This leads to a higher amount of resources allocated to users close to the cell border compared to users located in the cell center due to the different modulation and coding schemes that are utilized. Users located at the cell border transmit with a robust modulation scheme and a low coding rate while users in the cell center can utilize higher order modulation schemes with high coding rates. Results are given for the number of allocated resources depending on the location within the network. The probability density function (pdf) of the average user throughput and the average cell throughput are derived. The investigation can be applied to arbitrary user distributions and network layouts and is therefore applicable in the network planning process.

The paper is organized as follows. In Section II, the system model used for the investigation is described. Section III gives a short outline of the assumed fair throughput scheduling algorithm. In Section IV, the asymptotic performance of fair throughput scheduling in cellular networks is analyzed. Numerical results are presented in Section V. Finally, conclusions are drawn in Section VI.

II. System Model

In this Section, the system model used for the investigation is described. A cellular system in the downlink is assumed. Without loss of generality base stations are located in the cell center and equipped with omnidirectional antennas. Users are distributed uniformly over the cell area. The network is assumed to be interference limited so that additional noise can be omitted. Assuming N_{I} interfering base stations, the signal to interference ratio (SIR) due to pathloss and shadowing can be calculated as

$$\gamma_{\mathrm{PL}} = \frac{S}{I} = \frac{P \cdot \alpha_{\mathrm{S}}}{\sum\limits_{j=1}^{N_{\mathrm{I}}} P \cdot \alpha_{\mathrm{I},j}} \quad (1)$$

with α_{S} and $\alpha_{\mathrm{I},j}$ the distance dependent attenuation including shadowing between the considered mobile station and the serving base station and interfering base station $j,\ j = 1, ..., N_{\mathrm{I}}$, respectively. It is assumed that all base stations transmit with maximum transmit power P. Within each cell, N_{act} users are active. No power control is used and the transmit power per subcarrier is constant. To avoid border effects, the investigation area is mapped onto a torus.

The network uses OFDMA. For resource allocation, resource units are defined which represent the smallest granularity that can be allocated to one user. A resource unit is defined by a time duration T_{RU} and an amount of subcarriers N_{SC} in frequency domain. During one second, N_{res} resource units are available for allocation. The subcarriers of one resource unit are distributed over the total system bandwidth so that frequency diversity is achieved as in the partial usage of subchannels (PUSC) mode of IEEE 802.16 [16].

Link adaptation is performed based on effective channel state information [17]. Within an SIR interval $[\gamma_k\ \ \gamma_{k+1}]$ modulation and coding scheme $k,\ k = 1, ..., N_{\mathrm{MCS}}$ is used for transmission. The amount of possible modulation and coding schemes is given by N_{MCS}. With modulation and coding scheme k, $d_{\mathrm{MCS},k}$ bits per resource unit can be transmitted.

It is assumed that the users have always enough data available for transmission. This is often referred as full buffer traffic, e.g. [18]. The full buffer traffic model leads to optimistic throughput values but it is similar to a scenario with download or streaming traffic when the system is fully loaded [19].

III. Fair Throughput Scheduling

In this Section, the fair throughput scheduling is described. With fair throughput scheduling a resource unit is allocated to the user with index l' where

$$l' = \arg\min_{l} (r_l) \quad (2)$$

with r_l the user throughput of user l averaged over a past time interval. Actual SIR conditions are not considered during scheduling decision. After allocation of the resource unit, the average throughput is updated for all users. The amount of bits that can be transmitted per resource unit depends on the SIR.

Fair throughput scheduling provides a high degree of fairness among the users. In the long term, equal amount of throughput will be achieved by all active users. The distribution of the users and the SIR that is achieved by the active users will have impact on the user throughput. The performance will be limited by the user with the lowest SIR conditions.

IV. Asymptotic Analysis

In this Section, the performance of wireless networks using fair throughput scheduling is analyzed analytically. The throughput that is achieved by user i is given by

$$r_i = n_{\mathrm{alloc},i} \cdot d_{\mathrm{res},i} \quad (3)$$

with $n_{\mathrm{alloc},i}$ the amount of resource units allocated to user i within a time interval of one second and $d_{\mathrm{res},i}$ the average amount of bits transmitted per resource unit depending on the SIR achieved by user i. With fair throughput scheduling, equal throughput is guaranteed to each user such that (3) results into

$$r_i = r = \mathrm{const} \quad \text{for all } i, i = 1, \ldots, N_{\mathrm{act}}. \quad (4)$$

If all resource units are allocated, the sum over all allocated resource units yields

$$N_{\mathrm{res}} = \sum_{i=1}^{N_{\mathrm{act}}} n_{\mathrm{alloc},i} = \sum_{i=1}^{N_{\mathrm{act}}} \frac{r}{d_{\mathrm{res},i}} \quad (5)$$

which can be rewritten as

$$\frac{N_{\mathrm{res}}}{r} = \sum_{i=1}^{N_{\mathrm{act}}} \frac{1}{d_{\mathrm{res},i}}. \quad (6)$$

The reciprocal of the amount of data that can be transmitted per resource unit $\frac{1}{d_{\mathrm{res},i}}$ can be treated as the sample of a random variable $\frac{1}{d_{\mathrm{res}}}$. The distribution of $\frac{1}{d_{\mathrm{res}}}$ depends on the distribution of possible user locations within the cell and the network setup, i.e. the SIR that is achieved at a certain location. The average value μ_{FT} and the standard deviation σ_{FT} of $\frac{1}{d_{\mathrm{res}}}$ can be derived, e.g., using the pdf of the SIR and a methodology described in [15, 14]. If N_{act} is sufficiently large, the central limit theorem [20] can be applied and the ratio of the available resource units and the user throughput as given in (6)

$$z = \frac{N_{\mathrm{res}}}{r} \quad (7)$$

can be approximated by a normally distributed random variable. The pdf of z is then given by

$$f_z(z) = \frac{1}{\sqrt{2\pi N_{\mathrm{act}}}\sigma_{\mathrm{FT}}} \exp\left(-\frac{(z - N_{\mathrm{act}}\mu_{\mathrm{FT}})^2}{2N_{\mathrm{act}}\sigma_{\mathrm{FT}}^2} \right) \quad (8)$$

where exp() denotes the exponential function.

Using (3) and (7), the number of allocated resource units to user i that is able to transmit $d_{\mathrm{res},i}$ bits per resource unit is given by the random variable

$$n_{\mathrm{alloc},i} = \frac{N_{\mathrm{res}}}{d_{\mathrm{res},i} \cdot z}. \quad (9)$$

Applying random variable transformation [20] to the pdf given in (8), the conditional pdf of the number of allocated resource units assuming that $d_{\text{res},i}$ bits per resource unit can be transmitted by the user is given by

$$f_{n_{\text{alloc}}}(n_{\text{alloc}}|d_{\text{res}} = d_{\text{res},i}) = \frac{\zeta_i}{n_{\text{alloc}}^2} \exp\left(-\frac{\left(\frac{N_{\text{res}}}{n_{\text{alloc}} \cdot d_{\text{res},i}} - N_{\text{act}}\mu_{\text{FT}}\right)^2}{2 \cdot N_{\text{act}}\sigma_{\text{FT}}^2}\right) \quad (10)$$

with

$$\zeta_i = \frac{N_{\text{res}}}{d_{\text{res},i}\sqrt{2\pi N_{\text{act}}}\sigma_{\text{FT}}}. \quad (11)$$

With round robin scheduling, the amount of available resource units is divided equally among the active users so the each user gets

$$n_{\text{alloc,rr}} = \frac{N_{\text{res}}}{N_{\text{act}}}. \quad (12)$$

Comparing (9) with (12) it can be seen that with fair throughput scheduling the number of allocated resource units depends on the amount of bits a user can transmit with one resource unit. The ratio between the allocated resource units with fair throughput scheduling and round robin scheduling is given by

$$\Delta_{n,i} = \frac{n_{\text{alloc},i}}{n_{\text{alloc,rr}}}. \quad (13)$$

The user throughput is given by (3). The conditional pdf of the throughput that is achieved by a user that is able to transmit $d_{\text{res},i}$ bits per resource unit can be obtained of (10) by applying random variable transformation. The conditional pdf is given by

$$f_{r_i}(r_i|d_{\text{res}} = d_{\text{res},i}) = \frac{f_{n_{\text{alloc}}}\left(\frac{r_i}{d_{\text{res},i}}|d_{\text{res}} = d_{\text{res},i}\right)}{d_{\text{res},i}} \quad (14)$$

with the conditional pdf of the number of allocated resource units as given by (10).

The statistic of the amount of bits transmitted per resource unit depends on the user distribution and network layout. The statistic of the fading channel for fair throughput scheduling is the same as for round robin scheduling due to no actual channel conditions being considered during scheduling decision. For this investigation the probability α_k that modulation and coding scheme k is used and $d_{\text{MCS},k}$ bits can be transmitted per resource unit is obtained as described in [15]. The pdf of d_{res} is approximated by

$$f_{d_{\text{res}}}(d_{\text{res}}) = \sum_{k=0}^{N_{\text{MCS}}-1} \alpha_k \cdot \delta(d_{\text{res}} - d_{\text{MCS},k}) \quad (15)$$

with $\delta()$ the Dirac delta. Therefore, the pdf of the user throughput is given by

$$f_{r_i}(r_i) = \sum_{k=1}^{N_{\text{MCS}}} \alpha_k \cdot f_{r_i}(r_i|d_{\text{res}} = d_{\text{MCS},k}). \quad (16)$$

The cell throughput that is achieved with fair throughput scheduling is given by

$$r_{\text{cell}} = \sum_{i=1}^{N_{\text{act}}} r_i \quad (17)$$

with r_i the throughput of user i with the pdf given in (16). If N_{act} is sufficiently large, the central limit theorem can be applied and the pdf of the cell throughput tends towards a Gaussian distributed random variable with pdf given by

$$f_{r_{\text{cell}}}(r_{\text{cell}}) = \frac{1}{\sqrt{2\pi N_{\text{act}}}\sigma_r} \exp\left(-\frac{(r_{\text{cell}} - N_{\text{act}}\mu_r)^2}{2N_{\text{act}}\sigma_r^2}\right) \quad (18)$$

with μ_r and σ_r the average value and standard deviation of the user throughput as given by (16), respectively.

V. Results

In this Section, numerical results are provided to show the accuracy of the asymptotic analysis.

The utilization of the different modulation and coding schemes is obtained from the SIR pdf as described in [14]. The amount of data that can be transmitted per resource unit for the different modulation and coding schemes and the frequency of utilization can be found in Table I. The remaining parameters are given in Table II. System level simulations are performed modeling aspects like the scheduling and the fading channel in detail as benchmark for the analytical investigation.

Fig. 1 shows the amount of resource units that are allocated to a specific user depending on the SIR the user experiences due to pathloss. 20 users are active in each cell. The average number of allocated resource units as well as the 10-%ile are given in Fig. 1 for fair throughput scheduling. The number of allocated resource units for round robin scheduling which is independent of the location of the user is given for comparison reason. It can be seen that the number of allocated resource units decreases if the SIR due to pathloss increases. For the analytical investigation users are treated equal if the SIR due to pathloss is within one modulation and coding scheme interval, while in the system level simulation properties like the fading channel and packet errors during transmission are modeled in detail so that user with an SIR close to the upper border of an interval achieves higher throughput than users close to the lower border. Nevertheless, the analytical results for the average value and the 10-%ile approximate the simulation results for an SIR higher than 7 dB very well. For low SIR values the analytical results are much higher than the simulation results due to another property which is considered in the system level simulation, i.e., no resource unit is allocated

TABLE II

Simulation Parameters

Parameter	value
Site to site distance	1000 m
Number of Cells	25
Antenna pattern	omnidirectional, gain 0 dBi
pathloss coefficient	3.5
Channel Model	ITU Vehicular A
User speed	30 km/h
T_{RU}	1.4 ms
N_{SC}	12
System bandwidth	1.25 MHz
N_{res}	5714 resource units per second
Traffic Model	Full Buffer

TABLE I

LINK ADAPTATION PARAMETERS

Index k	1	2	3	4	5	6	7	8	9	10	11
$d_{\mathrm{MCS},k}$ in bit	48	96	144	192	240	288	384	480	576	672	864
α_k	18.2 %	16.6 %	13.2 %	5.4 %	4.2 %	8.1 %	6.0 %	3.5 %	3.5 %	3.4 %	5.1 %

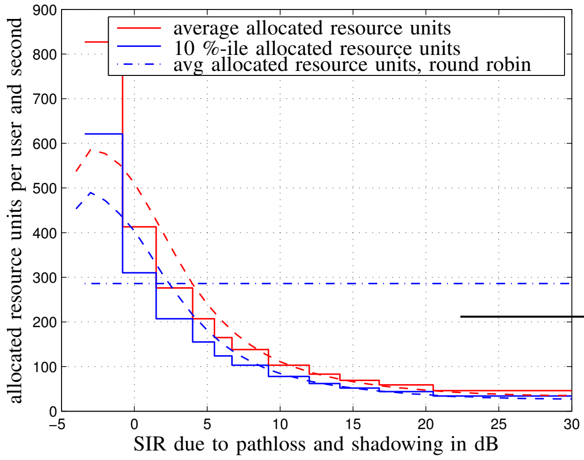
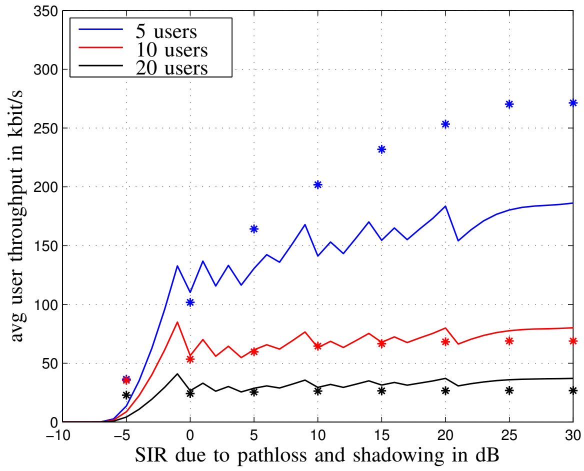
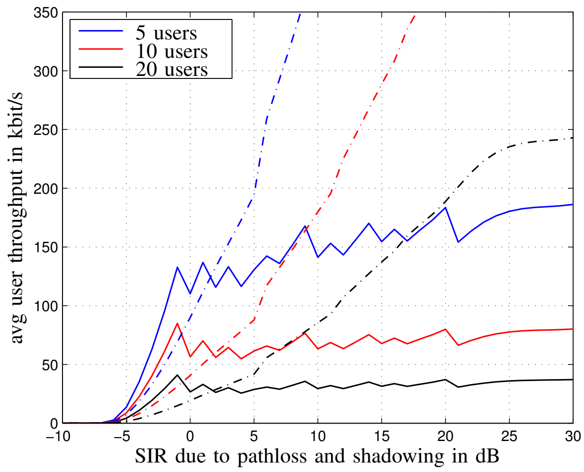


Fig. 1. Number of allocated resource units depending on the SIR, $N_{\mathrm{act}} = 20$, solid line: analytical results, dashed line: simulation results.



(a) Fair throughput scheduling (solid line: analytical results, marker: simulation results).



(b) Fair throughput scheduling (solid line) and round robin scheduling (dash-dotted line).

Fig. 2. Average user throughput as function of the SIR

if a transmission with the lowest modulation and coding scheme is only possible with a block error probability higher than 10 %. This happens for an effective SIR below -3.5 dB. Users with low SIR values will be below this threshold very often so that the number of allocated resource units in the system level simulation is reduced compared to the analytical investigation.

Fig. 2(a) shows the average user throughput as function of the SIR due to pathloss the user experiences for fair throughput scheduling. It can be seen that an equal average throughput is achieved for fair throughput scheduling if the SIR is greater than -2 dB. The average user throughput is approximately halved if the amount of active users is doubled due to less resource units that are allocated to each single user. Comparing the analytical results with the results obtained from the system level simulation, it can be seen that the analytical results are too pessimistic for small amount of users especially if the SIR of the user is higher than 10 dB. If the amount of users increases the user throughput is approximated quite well by the analytical derivations. Fig. 2(b) shows the average user throughput as function of the SIR due to pathloss for fair throughput and round robin scheduling. As expected, it can be seen that fair throughput scheduling provides better user throughput to users with SIR values below 2 dB compared to round robin scheduling. However, with round robin scheduling higher user throughput is achieved by users with high SIR values compared to fair throughput scheduling due to the higher number of allocated resource units. Of course, the user throughput of users with high SIR values can be further increased using opportunistic scheduling algorithms at the cost of users with low SIR values but this is not within the scope of this paper.

Fig. 3 shows the average cell throughput for fair throughput and round robin scheduling normalized to the average cell throughput that is achieved if only one user is active per cell occupying all resource units. It can be seen that the average cell throughput with fair throughput scheduling decreases with increasing number of active users. The user throughput and

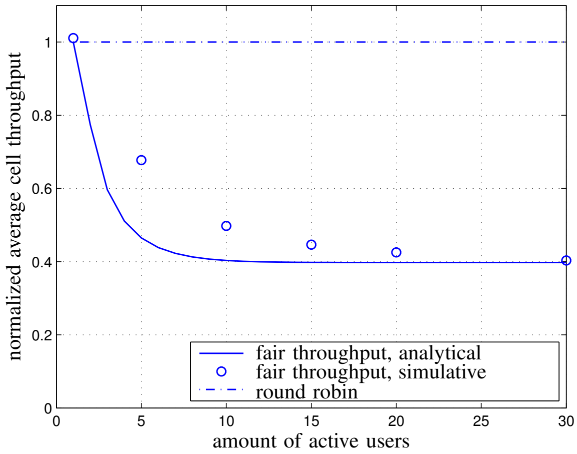


Fig. 3. Normalized average cell throughput depending on the amount of active users.

therefore also the cell throughput depends on the performance of the user with the lowest SIR. With increasing amount of active users the probability that at least one user is located close to the cell border becomes one and the average cell throughput converges to approximately 40 % of the average cell throughput that is possible with one user for the investigated scenario. The convergence is observed for an amount of active users higher than 10. With round robin scheduling the average cell throughput remains constant for varying number of active users due to not utilizing multi-user diversity [3]. It can be observed that for small number of active users the analytical results are too pessimistic due to the assumption of the central limit theorem in (7) which is not fulfilled for small amount of active users. For large amount of active users the average cell throughput estimate is accurate.

VI. Conclusion

An analytical performance analysis for cellular mobile networks with fair throughput scheduling is presented in this paper. Fair throughput scheduling is one scheduling algorithm under consideration for BWA networks if QoS is taken into account. Fair throughput scheduling is able to provide equal throughput to all users. Therefore, more resource units are allocated to users with low SINR values than users with high SINR values. Consequently, the throughput that is achieved by a user depends the total amount of active users and the locations of all active users. This is the different to, e.g., round robin scheduling, where the throughput only depends on the location of the considered users and the number of allocated resource units which depends on the number of active users but not on the location of these users.

Analytical derivations are given in this paper showing the number of allocated resource units and the user throughput depending on the SINR due to pathloss of the user as well as the cell throughput. The results of system level simulations show that the analytical results are quite accurate. Furthermore, the investigation is not limited to a specific network layout and can be performed for arbitrary user distributions. Therefore, the presented methodology is able to give a fast and accurate estimate of user and network performance for fair throughput scheduling based on the SINR distribution due to pathloss and shadowing without the need to perform complex system level simulations.

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