A PERFORMANCE-COMPLEXITY ANALYSIS OF FOUR SUBOPTIMAL SDMA ALGORITHMS

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ABSTRACT

Space Division Multiple Access (SDMA) is a promising solution to improve the spectral efficiency of future mobile radio systems. However, finding the group of Mobile Stations (MSs) that maximizes the system capacity using SDMA is a complex combinatorial problem, which can only be assuredly solved through an Exhaustive Search (ES). Because an ES is usually too complex, several suboptimal SDMA algorithms have been proposed. SDMA algorithms mainly differ on the grouping metrics they employ to quantify the spatial compatibility among MSs and on the grouping algorithm used to build the SDMA groups while avoiding ESs. In this work, the performance-complexity trade-off of four SDMA algorithms is investigated in terms of the average system capacity they achieve and on the number of operations they require. Expressions for the computational complexity of the algorithms are presented and it is shown that the algorithms proposed by the authors in [1, 2] attain almost the same average system capacity with comparable or lower complexity than other algorithms considered for benchmarking.

1. INTRODUCTION

Multiple Input Multiple Output (MIMO) techniques are a promising solution for high throughput provision in future mobile radio systems [3]. In the Downlink (DL) of Multi-User (MU) MIMO systems, if Channel State Information (CSI) is available at the transmitter, a group of Mobile Stations (MSs) can be multiplexed in space using Space Division Multiple Access (SDMA). In the following, such a group of MSs is termed an SDMA group.

The MSs in an SDMA group share the same resource in frequency and time but are separated in space, e.g., using transmit linear Zero-Forcing (ZF) precoding [4, 5]. Through SDMA, the system can serve more MSs without needing extra radio resources and, therefore, its spectral efficiency can be increased. Indeed, if MSs' spatial channels are close to orthogonal, SDMA gains are obtained by placing these MSs in the same SDMA group. Oppositely, placing MSs with spatially correlated channels in the same SDMA group can even lead to spectral efficiency losses. MSs with correlated channels must belong to different SDMA groups, which are multiplexed on different resources in frequency or time. Therefore, the SDMA algorithm must determine whether MSs are spatially compatible, i.e., whether they can efficiently share the same radio resource in space.

The problem of finding the SDMA group that maximizes the system capacity is a complex combinatorial problem. It is similar to the well-known knapsack problem and is a Non-deterministic Polynomial time Complete (NP-C) problem [6–8]. Its optimum solution is assuredly found through an Exhaustive Search (ES). However, an ES has exponential complexity and may lead to prohibitive computational costs even for a moderate number of MSs. Indeed, in order to compare different candidate SDMA groups in terms of their capacity, one needs to compute precoding vectors and allocated powers, thus increasing the complexity of each step in the ES. Therefore, suboptimal SDMA algorithms able to find an efficient SDMA group with reduced complexity are attractive.

Two relevant elements can be usually recognized in most suboptimal SDMA algorithms:

1. A **grouping metric**, which measures the spatial compatibility among MSs in an SDMA group, i.e., which quantifies how efficiently the MSs in an SDMA group can be separated in space. Additionally, the grouping metric can also be used to compare different SDMA groups.
2. A **grouping algorithm**, which, based on the grouping metric, builds and compares SDMA groups composed of spatially compatible MSs without needing to perform an ES.

Several suboptimal SDMA algorithms that are able to reach a considerable fraction of the average system capacity obtained through an ES have been proposed, as e.g. those in [1, 2, 9–12]. The SDMA algorithms in [1, 2, 9–12] employ different grouping metrics. Concerning the grouping algorithm, the SDMA algorithms in [1, 2] consider the Convex Grouping Algorithm (CGA), which is based on convex optimization [13], and the SDMA algorithms in [9–12] consider the Best Fit Algorithm (BFA) [7, 14], which is a greedy algorithm. Therefrom, these SDMA algorithms present considerably different performance-complexity trade-offs.

In this work, the performance-complexity trade-off will be studied for four suboptimal SDMA algorithms, which combine different grouping metrics and grouping algorithms, and it will be shown that even low complexity algorithms can obtain a high fraction of the average capacity achieved through an ES. The remainder of this work is organized as follows. Section 2 describes the considered system model. Section 3 introduces the SDMA algorithms studied in this work. Section 4 presents and analyzes the obtained performance and complexity results. Finally, section 5 draws some conclusions.

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2. SYSTEM MODEL

In this section, the system model considered in this work is described. The DL of a single Base Station (BS) equipped with an M-element Antenna Array (AA) is considered in the modeling. Gaussian signaling is considered. The interference from other BSs is assumed as Gaussian and is incorporated directly as part of the Additive White Gaussian Noise (AWGN) in the system. An average noise power σ^2 on each subcarrier is assumed for the AWGN in the system. The BS serves a number K of single-antenna MSs.

A single frequency resource block is considered, which is composed of Q adjacent subcarriers. Coherence grouping [15] is considered so that the channel transfer function over all the Q subcarriers in the frequency resource block can be efficiently represented by a single subcarrier. Let $\mathbf{h}_{qk} \in \mathbb{C}^{1\times M}$ denote the vector channel of the k^{th} MS on subcarrier $q \in \{1,\ldots,Q\}$. Denoting vector/matrix transposition by $(\cdot)^{\mathrm{T}}$, the channel matrix $\mathbf{H}_q \in \mathbb{C}^{K\times M}$ containing the channel coefficients of all MSs on the q^{th} subcarrier can be written as

$$\mathbf{H}_q = \begin{bmatrix}\mathbf{h}_{1q}^{\mathrm{T}} & \mathbf{h}_{2q}^{\mathrm{T}} & \ldots & \mathbf{h}_{Kq}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}. \tag{1}$$

Let $\lceil\cdot\rceil$ denote the nearest integer larger than or equal to the argument. Herein, it is assumed that the vector channel of the MSs on the middle subcarrier $\bar{q} = \lceil Q/2 \rceil$ is estimated and used to represent the whole frequency resource block, i.e., $\mathbf{H}_{\bar{q}}$ is estimated in order to provide the SDMA algorithms with the CSI they require. Herein, the system is supposed to employ Time Division Duplexing (TDD) and $\mathbf{H}_{\bar{q}}$ can be estimated at the BS by exploiting the reciprocity between Uplink (UL) and Downlink (DL) channels.

In fact, the CSI obtained at the BS is usually not perfect. Channel estimation errors and errors due to delays are often modeled by an additive Zero Mean Circularly Symmetric Complex Gaussian (ZMCSCG) error term [16–19]. Let $\sigma_{\mathbf{H}}^2$ denote the variance of the entries of $\mathbf{H}_{\bar{q}}$, which is considered to be known and equal to one. This is a common assumption, e.g., when ZMCSCG fading is considered [16–19]. Let $\mathbf{E} \in \mathbb{C}^{K\times M}$ be a ZMCSCG error term, whose entries have a variance $\sigma_{\mathbf{E}}^2 = \sigma_{\mathbf{H}}^2$. Under these assumptions, the estimated channel matrix $\hat{\mathbf{H}}$ can be modeled as

$$\hat{\mathbf{H}} = \sqrt{1-\nu}\,\mathbf{H}_{\bar{q}} + \sqrt{\nu}\,\mathbf{E}, \tag{2}$$

where $0 \le \nu \le 1$ is a model parameter. Perfect CSI at the BS is obtained when $\nu = 0$ in (2), while increasing ν increases the amount of error in the CSI. Using (2), one can define a measure

$$\gamma_{\text{CSI}} = \frac{1-\nu}{\nu}, \tag{3}$$

which expresses the relationship between the magnitude of the term due to the actual channel matrix $\mathbf{H}_{\bar{q}}$ and the magnitude of the additive estimation error matrix $\mathbf{E}$ present in the estimated channel matrix $\hat{\mathbf{H}}$ under the assumption that all entries in $\mathbf{H}_{\bar{q}}$ are identically distributed [20]. Thus, γ_{CSI} describes the quality of the CSI available at the BS.

In order to evaluate the average system capacity that can be achieved by the SDMA algorithms described in Section 3, both perfect and erroneous CSI at the transmitter will be considered. This will allow to investigate how the performance of the SDMA algorithms degrades with the quality of the CSI and whether any of the considered algorithms is particularly more robust against CSI imperfections than the others.

Denoting by $\mathcal{G}$ the SDMA group allocated on the considered frequency resource block, the channel group matrix $\mathbf{G}_q \in \mathbb{C}^{G\times M}$ on the subcarrier q is composed by the $G \le M$ out of the K rows of $\mathbf{H}_q$ corresponding to the vector channels of the MSs in the group. Let p_g and $\mathbf{w}_g$ denote the allocated power and precoding vector associated to the g^{th} MS in $\mathcal{G}$, respectively. Denoting by $\|\cdot\|_2$ the 2-norm of a vector, it is considered here that $\|\mathbf{w}_g\|_2 = 1$, for $g = 1,\ldots,G$. The precoding vectors $\mathbf{w}_g$ and allocated powers p_g are determined according to the precoding and power allocation algorithms employed in the system and are applied to all the Q subcarriers of the frequency resource block. Denoting by $\mathbf{g}_{gq}$ the g^{th} row of $\mathbf{G}_q$, the capacity $C(\mathcal{G})$ of the system when transmitting to group $\mathcal{G}$ is given by

$$C(\mathcal{G}) = \sum_{q=1}^{Q}\sum_{g=1}^{G}\log_2\left(1 + \frac{p_g \mathbf{w}_g^{\mathrm{H}}\mathbf{g}_{gq}^{\mathrm{H}}\mathbf{g}_{gq}\mathbf{w}_g}{\sigma^2 + \sum\limits_{j=1,j\neq g}^{G} p_j \mathbf{w}_j^{\mathrm{H}}\mathbf{g}_{gq}^{\mathrm{H}}\mathbf{g}_{gq}\mathbf{w}_j}\right), \tag{4}$$

which is used later to calculate the average system capacity.

For any two MSs i and j in the system, the degree of orthogonality between their channels $\hat{\mathbf{h}}_i$ and $\hat{\mathbf{h}}_j$ can be measured by the maximum normalized scalar product [1, 2, 6, 7, 11]. Let $|\cdot|$ denote the absolute value of a complex scalar, $\mathcal{D}\{\cdot\}$ denote a diagonal matrix whose diagonal elements are given in the vector argument, and $[\,\cdot\,]_{ij}$ denote the element on the i^{th} row and j^{th} column of a matrix. Then, using (2), a matrix $\mathbf{C}$ can be written as

$$\mathbf{C} = \left|\sqrt{\mathcal{D}\{\mathbf{a}\}}\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}}\sqrt{\mathcal{D}\{\mathbf{a}\}}\right| \in \mathbb{R}_{+}^{K\times K}, \text{ with} \tag{5a}$$

$$\mathbf{a} = \begin{bmatrix}\|\hat{\mathbf{h}}_1\|_2^{-2} & \|\hat{\mathbf{h}}_2\|_2^{-2} & \ldots & \|\hat{\mathbf{h}}_K\|_2^{-2}\end{bmatrix}^{\mathrm{T}} \in \mathbb{R}_{+}^{K\times 1}, \tag{5b}$$

where $|\cdot|$ is applied to $\mathbf{C}$ element-wise. The elements $[\mathbf{C}]_{ij}$ measure the degree of orthogonality between the spatial channels $\hat{\mathbf{h}}_i$ and $\hat{\mathbf{h}}_j$ of MSs i and j, respectively [1, 2, 11]. The more orthogonal the spatial channels of MS i and j are, the lower $[\mathbf{C}]_{ij}$ is. In order to quantify how orthogonal the channels of the MSs in an SDMA group $\mathcal{G}$ are, a total orthogonality measure can be obtained, e.g., by adding up $[\mathbf{C}]_{ij}$ for all MSs $i, j \in \mathcal{G}$. In the next section, (5) will be used to build input data to the SDMA algorithms based on spatial orthogonality and channel gains.

3. SDMA ALGORITHMS

In this section, the SDMA algorithms studied in this work are described. Section 3.1 describes the grouping metrics used by the considered SDMA algorithms. Section 3.2 describes the BFA [7, 14] and the CGA [2], which are the grouping algorithms considered in this work. Finally, Section 3.3 defines the SDMA algorithms by combining a grouping metric and a grouping algorithm from Sections 3.1 and 3.2, respectively.

3.1. Grouping metrics

The first grouping metric considered in this work is an estimate of the capacity of the SDMA group $\mathcal{G}$ on the frequency resource block. This metric will be denoted by $f_{\text{CAP}}(\mathcal{G})$. Considering the models in Section 2, it can be defined as

$$f_{\text{CAP}}(\mathcal{G}) = Q\sum_{g=1}^{G}\log_2\left(1+\frac{p_g\mathbf{w}_g^{\text{H}}\hat{\mathbf{g}}_g^{\text{H}}\hat{\mathbf{g}}_g\mathbf{w}_g}{\sigma^2+\sum\limits_{j=1,j\neq g}^{G}p_j\mathbf{w}_j^{\text{H}}\hat{\mathbf{g}}_g^{\text{H}}\hat{\mathbf{g}}_g\mathbf{w}_j}\right), \quad (6)$$

where $\hat{\mathbf{g}}_g$ is the g^{th} row of the estimated group channel matrix $\hat{\mathbf{G}}$, which is derived from $\hat{\mathbf{H}}$ in (2) in the same way as $\mathbf{G}_q$ from $\mathbf{H}_q$ in Section 2. For every possible SDMA group $\mathcal{G}$, $f_{\text{CAP}}(\mathcal{G})$ can be computed using the available CSI and provides an estimate of the actual group capacity given by (4).

Assuming adequate precoding and power allocation, the more spatially compatible the MSs in an SDMA group $\mathcal{G}$ are, the higher its estimated group capacity $f_{\text{CAP}}(\mathcal{G})$ is. Since $f_{\text{CAP}}(\mathcal{G})$ reflects the capacity of the group including the effective precoding and power allocation, it is a reliable grouping metric [9, 11, 21].

The second grouping metric considered herein is the sum of the MSs' channel gains with Successive Projections (SPs). It will be denoted by $f_{\text{SP}}(\mathcal{G})$. Using SPs, the channel $\hat{\mathbf{g}}_g$ of MS g being admitted to the SDMA group $\mathcal{G}$ is projected onto the null space of the channels of all MSs $g' < g$ already admitted to $\mathcal{G}$. Consequently, all the MSs $g' = 1, 2, \ldots, g-1$, see no interference from MS g. Let $\mathbf{I}_M$ denote an $M \times M$ identity matrix and let $\mathbf{T}_g \in \mathbb{C}^{M\times M}$ denote the matrix that projects the channel of MS g onto the null space of the channels of MSs g' in the group $\mathcal{G}$. Then, $\mathbf{T}_g$ is defined as

$$\mathbf{T}_g = \begin{cases}\mathbf{I}_M, & g=1,\\ \mathbf{T}_{g-1}-\dfrac{\mathbf{T}_{g-1}^{\text{H}}\hat{\mathbf{g}}_{g-1}^{\text{H}}\hat{\mathbf{g}}_{g-1}\mathbf{T}_{g-1}}{\|\hat{\mathbf{g}}_{g-1}\mathbf{T}_{g-1}\|_2^2}, & g>1,\end{cases} \quad (7)$$

where for $g = 1$ no projections are needed and $\mathbf{T}_1 = \mathbf{I}_M$ [10, 21, 22]. Then, using (7), $f_{\text{SP}}(\mathcal{G})$ is written as

$$f_{\text{SP}}(\mathcal{G}) = \sum_{g=1}^{G_t}\|\hat{\mathbf{g}}_g\mathbf{T}_g\|_2^2, \quad (8)$$

where G_t is the target SDMA group size, which can assume values between 1 and M.

The more orthogonal and the higher the channel gains of the MSs in the group $\mathcal{G}$ are, the higher the values that $f_{\text{SP}}(\mathcal{G})$ assumes. The same is also valid for $f_{\text{CAP}}(\mathcal{G})$. In spite of effectively capturing the spatial compatibility among MSs based on the available CSI, $f_{\text{CAP}}(\mathcal{G})$ and $f_{\text{SP}}(\mathcal{G})$ have some potential drawbacks. For $f_{\text{CAP}}(\mathcal{G})$, whenever the composition of the group changes, new precoding vectors and a new power allocation have to be computed for all MSs in the group. $f_{\text{SP}}(\mathcal{G})$ does not have this requirement, but it explicitly depends on the encoding order of the MSs in the SDMA group. Nevertheless, depending on the design of the SDMA algorithm, $f_{\text{CAP}}(\mathcal{G})$ and $f_{\text{SP}}(\mathcal{G})$ can be efficiently used as grouping metric [10, 11, 21, 22].

The third metric considered here employs a Convex Combination (CC) of the total spatial orthogonality and channel gains of the MSs in an SDMA group $\mathcal{G}$ to quantify their spatial compatibility [1, 2, 11]. It will be denoted by $f_{\text{CC}}(\mathcal{G})$ and is described in the sequel. The motivation for this metric is to favor sets containing MSs whose channels have high gain and are close to orthogonal. Let the binary selection vector $\mathbf{u}$ be defined as

$$\mathbf{u} = [u_1 \;\; u_2 \;\; \ldots \;\; u_K]^{\text{T}}, \text{ with } u_k \in \{0, 1\}. \quad (9)$$

Thus, for any group $\mathcal{G}$, one has the variables $u_k = 1, \forall k \in \mathcal{G}$, and $u_k = 0$, otherwise. Denoting by $\|\cdot\|_{\text{F}}$ the Frobenius-norm of a matrix or vector, one can formulate $f_{\text{CC}}(\mathcal{G})$ by combining (5) and (9) as

$$f_{\text{CC}}(\mathcal{G}) = \frac{(1-\beta)}{\|\mathbf{C}\|_{\text{F}}}\mathbf{u}^{\text{T}}\mathbf{C}\mathbf{u} + \frac{\beta}{\|\mathbf{a}\|_{\text{F}}}\mathbf{a}^{\text{T}}\mathbf{u}, \quad (10)$$

where $0 \le \beta \le 1$ is a control parameter establishing the trade-off between spatial orthogonality and channel gain. The factors $\frac{1}{\|\mathbf{C}\|_{\text{F}}}$ and $\frac{1}{\|\mathbf{a}\|_{\text{F}}}$ are normalization factors introduced to tentatively balance $\mathbf{C}$ and $\mathbf{a}$, i.e., to compensate for their absolute difference and have an unbiased β. Denoting by $[\,\cdot\,]_i$ the i^{th} element of a vector, (10) can also be written as

$$f_{\text{CC}}(\mathcal{G}) = \frac{(1-\beta)}{\|\mathbf{C}\|_{\text{F}}}\sum_{i\in\mathcal{G}}\sum_{j\in\mathcal{G}}[\mathbf{C}]_{ij} + \frac{\beta}{\|\mathbf{a}\|_{\text{F}}}\sum_{i\in\mathcal{G}}[\mathbf{a}]_i\,. \quad (11)$$

Thus, the first and the second terms in (10) and (11) relate to the total spatial orthogonality and to the gains of the spatial channels of the MSs in the SDMA group $\mathcal{G}$, respectively. The more orthogonal and the less attenuated the channels of the MSs' in $\mathcal{G}$ are, the more spatially compatible they are.

3.2. Grouping algorithms

The first grouping algorithm considered in this work is the BFA, which has been proposed in [7, 14] and is described in the sequel. Its basic idea is, starting with an SDMA group containing a single MS, to sequentially extend the size of the SDMA group by admitting to the group the MS that most increases the grouping metric. This procedure is then repeated with the extended group until a target group size G_t is reached.

Let $\mathcal{G} = \{k'\}$ be the initial SDMA group containing only MS k' and let G be the size of $\mathcal{G}$, i.e., the number of MSs in $\mathcal{G}$. Then, the BFA temporarily admits one MS $k \notin \mathcal{G}$ to the SDMA group and computes the grouping metric $f_{(\cdot)}(\mathcal{G})$ for this extended group. This MS k is removed from $\mathcal{G}$ and the process is repeated with the next MS. After the grouping metrics for the groups built by temporarily admitting each of the $K-G$ MSs have been computed, the MS which resulted in the highest value for the grouping metric when admitted to $\mathcal{G}$ is permanently inserted into the group. Then, the same procedure is repeated with the remaining MSs for the extended group $\mathcal{G}$ until the group size G reaches the target group size G_t or until no more MSs able to increase the grouping metric exist. The BFA is shown in Table 1.

An example of the BFA is shown in Fig. 1 for a total number $K = 9$ of single-antenna MSs and $M = 4$ antennas at the BS. G_t is set to the maximum allowed group size, i.e., $G_t = M = 4$. In Fig. 1, MS 1 is selected as the MS for the initial SDMA group $\mathcal{G}$. Then, MSs 2 to 9 are temporarily admitted to $\mathcal{G}$. The grouping metric is computed and one finds out that MS 3 is the best MS, i.e., the MS that at most increases the grouping metric when admitted to $\mathcal{G}$. MS 3 is then permanently added to $\mathcal{G}$, and the process is repeated now for MSs 2,

Table 1. Best Fit Algorithm.

1. Select an initial MS k' and build $\mathcal{G} = \{k'\}$.
2. While $G < G_t$
   (a) Set $\mathcal{G}' = \mathcal{G} \cup \left\{ \arg\max_{k \notin \mathcal{G}} \left\{ f_{(\cdot)}(\mathcal{G} \cup \{k\}) \right\} \right\}$.
   (b) If $f_{(\cdot)}(\mathcal{G}') > f_{(\cdot)}(\mathcal{G})$, set $\mathcal{G} = \mathcal{G}'$, otherwise stop.
3. Define the best group as $\mathcal{G}^\star = \mathcal{G}$.

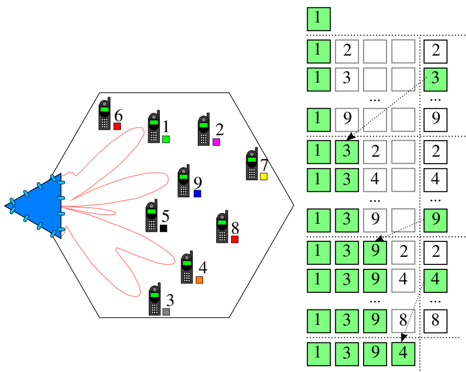


Fig. 1. Example of BFA.

and 4 to 9, with MS 9 being added in this next run of the BFA, followed by MS 4, when the size of $\mathcal{G}$ reaches the target group size G_t.

The second grouping algorithm considered in this work is the CGA. The CGA has been designed together with $f_{\text{CC}}(\mathcal{G}_g)$ in [1]. Let $\|\cdot\|_1$ denote the 1-norm of a vector and let $\tilde{u}_k \in [0, 1]$ and $\tilde{\mathbf{u}}$ denote the continuous relaxed versions of u_k and $\mathbf{u}$ in (9), respectively. Then, using (10), the CGA is formulated as the quadratic optimization problem

$$\tilde{\mathbf{u}}^\star = \arg\min_{\tilde{\mathbf{u}}} \left\{ \frac{(1-\beta)}{\|\mathbf{C}\|_{\text{F}}} \tilde{\mathbf{u}}^{\text{T}} \mathbf{C} \tilde{\mathbf{u}} + \frac{\beta}{\|\mathbf{a}\|_{\text{F}}} \mathbf{a}^{\text{T}} \tilde{\mathbf{u}} \right\}, \quad \text{(12a)}$$

$$\text{s.t.: } \|\tilde{\mathbf{u}}\|_1 = G_t, \quad \text{(12b)}$$

$$\tilde{u}_k \in [0, 1], \forall k, \quad \text{(12c)}$$

$$\tilde{u}_{k'} = 1, \quad \text{(12d)}$$

which is the relaxation of the corresponding integer problem in which $\mathbf{u}$ from (9) is used instead of $\tilde{\mathbf{u}}$ [8, 13]. The CGA tries to find an SDMA group of size G_t containing MSs that are close to orthogonal and have low total channel attenuation. It also considers a target group size G_t and the constraint (12d) allows to force an initial MS k' into the optimum group $\mathcal{G}^\star$, like the BFA. Then, using (12), the CGA can be defined as shown in Table 2.

Table 2. Convex Grouping Algorithm.

1. Solve the problem (12).
2. Determine $\mathcal{G}^\star$ by rounding to one the G_t largest and to zero the other $K - G_t$ components $\tilde{u}_k^\star$ of $\tilde{\mathbf{u}}^\star$.

The components $\tilde{u}_k^\star$ of the obtained continuous solution $\tilde{\mathbf{u}}^\star$ can be interpreted as the probability of the corresponding MS k belonging to the best group $\mathcal{G}^\star$. Thus, in order to determine $\mathcal{G}^\star$, the solution $\tilde{\mathbf{u}}^\star$ of (12) has to be converted into an integer solution $\mathbf{u}^\star$. This is done by rounding to one the G_t largest and to zero the other $K - G_t$ components of $\tilde{\mathbf{u}}^\star$, thus determining which MSs belong to $\mathcal{G}^\star$.

3.3. SDMA algorithm definition

In this section, combinations of the grouping metrics from Section 3.1 and grouping algorithms from Section 3.2 are defined as the SDMA algorithms to be investigated in this work. They are listed in Table 3 and are named after the grouping metric and grouping algorithm they employ.

Table 3. SDMA algorithms.

SDMA algorithm	Grouping metric	Grouping algorithm
CAP-BFA	$f_{\text{CAP}}(\mathcal{G})$ cf. (6)	BFA cf. Table 1
SP-BFA	$f_{\text{SP}}(\mathcal{G})$ cf. (8)	
CC-BFA	$f_{\text{CC}}(\mathcal{G})$ cf. (11)	
CC-CGA	$f_{\text{CC}}(\mathcal{G})$ cf. (10)	CGA cf. Table 2

The CAP-BFA uses the capacity of the SDMA group as grouping metric and the BFA [7, 14] as grouping algorithm. This algorithm has been independently studied by the authors of [9, 11, 12].

The SP-BFA employs the BFA as grouping algorithm. However, it projects the channels of MSs to be admitted in the SDMA group onto the null space of the channels of the MSs previously admitted to the group and uses as grouping metric the sum of the gains of the MS's channels after such SPs. This algorithm is considered in [10] and is also used for MS ordering in the non-linear precoding strategies considered in [9, 22, 23].

The CC-BFA has been proposed by the authors in [2]. It uses the BFA as grouping algorithm and a convex combination of the spatial orthogonality and channel gains of the MSs as grouping metric. The CC-BFA is a greedy variant of the CC-CGA proposed by the authors in [1], which employs the CGA to build the SDMA groups. Differently from the other SDMA algorithms, the CC-CGA builds the SDMA group at once by solving (12).

It is straightforward that the grouping metrics in (6), (8), and (11) can be easily combined with the BFA by changing maximization to minimization whenever necessary and by considering the MSs outside of the SDMA group $\mathcal{G}$ as candidates for admission to the group.

The three last SDMA algorithms in Table 3 are unaware of the precoding and power allocation employed in the system. The groups they build may contain MSs whose removal could increase the system capacity. Consider e.g. the case in which linear ZF precoding and the Water Filling Algorithm (WFA) are used by the system. Whenever the WFA allocates null power to an MS, this MS does not contribute to enhance the group capacity anymore. On the contrary, since other MSs are projected on the null space of this one MS, its removal from the group can only improve the group capacity [6, 11]. In order to cope with this issue, the Sequential Removal Algorithm (SRA) proposed by the authors in [1] and shown in Table 4 is applied to the groups formed by these SDMA algorithms.

The SRA sequentially removes MSs from $\mathcal{G}$ and computes the capacity of the resulting groups. At the end, only the group $\mathcal{G}^\star$ with the best capacity is kept. Since at most $G_t < K$ capacity calculations are done by the SRA, it adds only slightly

Table 4. Sequential Removal Algorithm.

1. Set $\mathcal{G} = \mathcal{G}^\star$ and compute the group capacity $f_{\text{CAP}}(\mathcal{G})$ using (6).
2. While the size G of $\mathcal{G}$ is greater than one.
   (a) Remove the MS $g^\star = \arg\min_{g\in\mathcal{G}} \{|\hat{\mathbf{g}}_g \mathbf{w}_g|\}$ from $\mathcal{G}$, i.e., set $\mathcal{G} = \mathcal{G} \setminus \{g^\star\}$.
   (b) Compute the group capacity $f_{\text{CAP}}(\mathcal{G})$ of $\mathcal{G}$ using (6).
   (c) If $f_{\text{CAP}}(\mathcal{G}) > f_{\text{CAP}}(\mathcal{G}^\star)$, define $\mathcal{G}^\star = \mathcal{G}$.

to the complexity of SDMA algorithms.

4. ANALYSIS AND RESULTS

In order to evaluate the performance of the described SDMA algorithms, a scenario considering a single hexagonal cell with the BS located on a corner is considered. A Uniform Linear Array (ULA) with $M = 4$ omnidirectional elements separated by half wavelength is used at the BS. A total number $K = 16$ of single-antenna MSs are associated with the BS. An average speed of 10 km/h is assumed for the MSs.

A center frequency of 5 GHz is considered. A single frequency resource block composed of $Q = 6$ subcarriers is considered for data transmission. Subcarriers are spaced of $\Delta_f \approx 9.766$ kHz. The BS is assumed to have a total power $P \cdot Q$, which is equally divided among the Q subcarriers, thus leading to an average Signal-to-Noise Ratio (SNR) of $\gamma = P/\sigma^2$ on each subcarrier. Only fast fading is considered, which is generated using the WINNER Channel Model (WIM) [24].

In order to separate the MSs in space, the precoding vectors $\mathbf{w}_g$ are determined using linear ZF precoding [4,5], while the allocated powers p_g are determined according to the WFA [4,25].

A target group size $G_t = M = 4$ is adopted for all the algorithms, as well as the initial MS k' is set as the one with the highest channel gain $\|\hat{\mathbf{h}}_k\|_2^2$. For the CC-BFA and CC-CGA, the parameter β is set to 0.4, which has been found experimentally, as it has been done in [1]. The most relevant simulation parameters are summarized in Table 5.

Initially, the average capacity of the system in bits/s/Hz achieved by each of the SDMA algorithms is investigated. For comparison purposes, the average capacity achieved by an Exhaustive Search Algorithm (ESA) which searches for the best SDMA group using the group capacity as grouping metric is also considered. This algorithm will be termed CAP-ESA.

In Fig. 2 the average capacity achieved by the SDMA algorithms considering perfect CSI is shown as a function of the average SNR γ in dB. As it can be seen, all the four considered SDMA algorithms are able to achieve almost the same capacity as an ES for the best SDMA group, i.e., the group that maximizes the sum capacity of the system. Indeed, all the suboptimal algorithms obtain at least 95% of the capacity of CAP-ESA while being considerably less complex, since the ESA examines all the $\sum_{G=1}^{M} \binom{K}{G}$ possible candidate groups and applies for each of them the considered precoding and power allocation. It is also observed that the CAP-BFA and the SP-BFA only slightly surpass the performance of the CC-BFA and CC-CGA for high average SNR values.

Table 5. System parameters.

Parameter	Value
Center frequency	5.0 GHz
Subcarrier spacing	9.766 kHz
# of subcarriers/resource	6
# of resources	1
Frame duration	1 ms
Channel model	WIM, scenario C2
BS AA	ULA with 4 omnidirectional elements separated by half wavelength
# of MSs	16 single-antenna MSs
Average MS speed	10 km/h
Precoding	Linear ZF
Power allocation	WFA
Target SDMA group size	$G_t = M = 4$
Initial MS	$k' = \arg\max_k \left\{\|\hat{\mathbf{h}}_k\|_2^2\right\}$
Parameter β of (12)	0.4

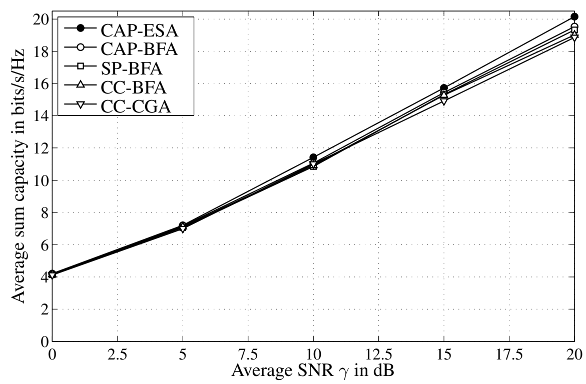


Fig. 2. Average capacity achieved by the SDMA algorithms.

In the sequel, it is interesting to see if any of the SDMA algorithms is particularly more robust against imperfections on the CSI. For this purpose, the quality of the CSI is adjusted by varying γ_{CSI}.

In Fig. 3 the average capacity achieved by the SDMA algorithms listed in Table 3 considering erroneous CSI is shown as a function of γ_{csi} in dB. Moreover, an average SNR γ of 10 dB is considered in the system. Because all the SDMA algorithms in Table 3 strongly rely on the available CSI, they shown the same degradation when the amount of imperfection on the CSI increases. At this point, one can conclude that the studied SDMA algorithms have almost the same performance. However, because they involve different operations, the complexity of each algorithm considerably differs.

The number of operations required by each SDMA algorithm can be used to compare their complexity. For simplicity, only multiplications will be considered here as operations, since they usually require considerably more processing time than additions and logical operations. Also for simplicity, operations in $\mathbb{R}$ and in $\mathbb{C}$ will be assumed to have the same complexity. The complexity order $\mathcal{O}(\cdot)$ of each algorithm can be approximated by the dominant term in the expression describing its complexity. $\mathcal{O}(\cdot)$ is often more useful than having a

Table 6. Complexity of the SDMA algorithms.

SDMA algorithm	Number of operations	$\mathcal{O}(\cdot)$
CAP-ESA	$\sum_{G=1}^{M} \frac{K!}{G!(K-G)!} \left(\frac{G^3+7G^2(M+1)+G(9M+18)}{2} \right)$	Non-Polynomial
CAP-BFA	$\sum_{G=2}^{M} (K-G+1) \left(\frac{G^3+7G^2(M+1)+G(9M+18)}{2} \right) + (KM+6M+10)$	$\mathcal{O}\left(4KM^4\right)$
SP-BFA	$\sum_{G=2}^{M} (K-G+1) \left(\frac{5M^2+5M+2}{2} \right) + \left(KM + \frac{4M^3+12M^2+16M}{2} \right)$	$\mathcal{O}\left(\frac{5KM^3}{2}\right)$
CC-BFA	$\frac{M}{K} \frac{K^2(M+8)+K(M+2)+6}{2} + \left(KM + \frac{4M^3+12M^2+16M}{2} \right)$	$\mathcal{O}\left(\frac{KM(M+8)}{2}\right)$
CC-CGA	$I_{\text{CGA}}(2K^2+2K) + \frac{K^2(M+8)+K(M+2)+6}{2} + \left(KM + \frac{4M^3+12M^2+16M}{2} \right)$	$\mathcal{O}\left(\frac{K^2(M+8+4I_{\text{CGA}})}{2}\right)$

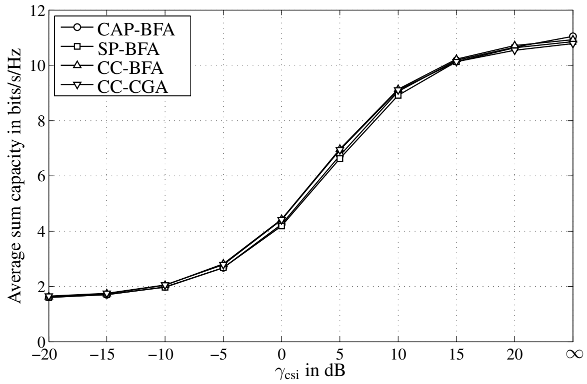


Fig. 3. Average capacity achieved by the SDMA algorithms with erroneous CSI. Average SNR $\gamma = 10$ dB.

complicated complexity expression [26]. Table 6 gives an estimation of the complexity of the SDMA algorithms considering $G_t = M$.

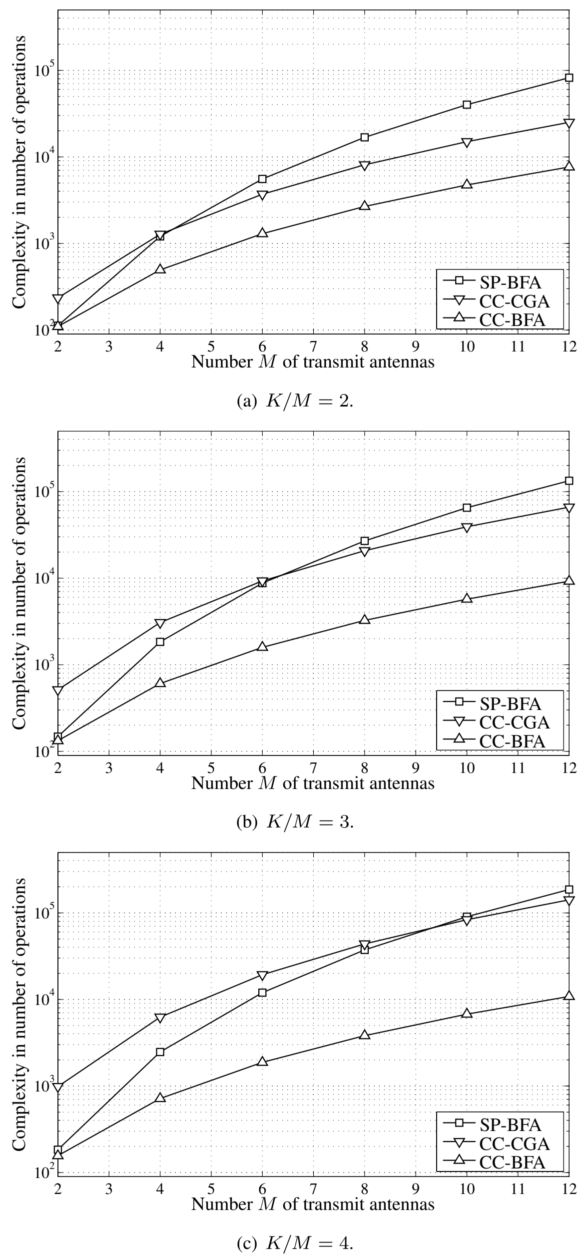
Determining the initial MS k' requires to compute the channel gain of the K MSs and involves KM operations common to all the algorithms other than CAP-ESA. The same is valid for the cost of performing precoding and power allocation for the final group generated by the SDMA algorithms.

Because the number of SDMA groups increases exponentially when the number K of MSs increases and because for each of these groups the group capacity has to be estimated, which requires to compute precoding vectors and allocated powers, the complexity of CAP-ESA increases exponentially with K.

The CAP-BFA has a relatively higher computational cost than the SP-BFA, CC-BFA, and CC-CGA algorithms, since precoding vectors and allocated powers have to be computed for each candidate group. This can be clearly noted by observing the expressions in Table 6.

The CC-CGA is based on quadratic convex optimization and, according to [13], the complexity order of a quadratic optimization problem like this is roughly proportional to the cost of evaluating its cost function and its first derivative, which is $2K^2 + 2K$. The complexity of the CC-CGA in Table 6 has been determined assuming one evaluation of the cost function in (10) and of its first derivative per iteration and that a number I_{CGA} of iterations is required by the CGA to converge.

The CC-CGA still needs to compute the scaled versions of $\mathbf{C}$ and $\mathbf{a}$ as input data, a characteristic which is shared by the CC-BFA. However, the CC-BFA does not need to perform any further operation to build the SDMA group and requires only a fraction $\frac{M}{K}$ of the scaled versions of $\mathbf{C}$ and $\mathbf{a}$, thus having



(a) $K/M = 2$.

(b) $K/M = 3$.

(c) $K/M = 4$.

Fig. 4. Complexity of the SDMA algorithms SP-BFA, CC-CGA, and CC-BFA for different ratios between the number K of MSs and the number M of transmit antennas.

lower complexity than CC-CGA.

The SP-BFA strategy makes no use of $\mathbf{C}$ and $\mathbf{a}$, but employs null space SPs. Differently from CC-CGA, whose complexity increases quadratically with K and linearly with M, the complexity of SP-BFA increases only linearly with K, but cubically with M for $G_t = M$.

In the sequel, the complexity of CC-CGA, SP-BFA, and CC-BFA are compared. They have been selected, since they have very similar performance and achieve almost the same average capacity than the CAP-BFA, but are clearly less complex than CAP-BFA and CAP-ESA. Therefore, they offer a good trade-off between capacity and complexity. For the complexity comparison, the number M of antennas has been varied while the ratio between the number of MSs K and the number M of transmit antennas has been assumed as constant. Besides that, $G_t = M$ has been assumed. Moreover, an average number $\bar{I}_{\text{CGA}} = \frac{K}{2}$ of iterations has been assumed for the convergence of CC-CGA. This value has been measured in the simulations performed considering the parameters in Table 5 and has been extrapolated here to the other configurations. Fig. 4 shows the complexity of the algorithms for different ratios between K and M.

In Fig. 4, it can be noted that for small AA sizes the complexity of SP-BFA strategy is usually smaller than that of the CC-CGA for all the ratios K/M. However, the complexity of SP-BFA increases faster for larger array sizes and larger number of MSs and becomes larger than the complexity of CC-CGA. Anyway, the CC-BFA algorithm presents in particular the best performance-complexity trade-off and its complexity lies below the complexity of the other two algorithms in all the considered configurations.

Nevertheless, it must be mentioned that the complexity of the CC-CGA and CC-BFA can still be reduced. For the CC-CGA strategy, the tolerances for the optimum solution of the quadratic optimization algorithm can be adjusted as to find a solution in a shorter number of iterations. A maximum number of iterations can also be imposed, so that a suboptimal group can be found in a shorter time. However, this may affect the performance of the algorithm. Besides that, for both CC-CGA and CC-BFA the scaling factors $\frac{1}{\|\mathbf{C}\|_{\text{F}}}$ and $\frac{1}{\|\mathbf{a}\|_{\text{F}}}$ can be arbitrarily defined, thus avoiding the computation of the involved norms. The only consequence of this change would be a new set of optimum values for β. Therefore, the CC-CGA and CC-BFA are SDMA algorithms that also offer a good trade-off between achieved sum capacity for the system and computational complexity.

5. CONCLUSIONS

In this work, the performance-complexity trade-off of four suboptimal SDMA algorithms has been investigated. In terms of achieved average system capacity, the SDMA algorithms CC-BFA and CC-CGA proposed by the authors in [1, 2] have been shown to perform as good as the CAP-BFA and SP-BFA, which are relatively well-known algorithms. Expressions for the computational complexity of the algorithms have been provided and it has been shown that CC-BFA is considerably less complex than the other considered SDMA algorithms, thus offering a good trade-off between performance and complexity.

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