ADAPTIVE MIMO-OFDM USING OSTBC WITH IMPERFECT CQI FEEDBACK

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ABSTRACT

In this paper, closed form expressions are derived for the average data rate and uncoded bit error rate of a multi-user Multiple Input Multiple Output (MIMO) Orthogonal Frequency Division Multiple Access (OFDMA) system, where two transmission modes are considered, both transmitting the data of the scheduled user using Orthogonal Space-Time Block Coding (OSTBC) at the transmitter and Maximum Ratio Combining (MRC) at the receiver to exploit spatial diversity. Applying the adaptive transmission mode, an adaptive subcarrier allocation and modulation is performed based on channel quality information (CQI) fed back digitised over a feedback channel to the transmitter. As CQI the resulting instantaneous signal-to-noise ratio (SNR) at the output of the maximum ratio combiner of the different subcarriers is applied to exploit multi-user diversity. The CQI available at the transmitter side is assumed to be imperfect due to time delay, estimation and feedback errors. Applying the non-adaptive transmission mode, no instantaneous CQI is required. A transmission scheme exploiting frequency diversity in addition to the spatial diversity using OSTBC and MRC is employed. The data rates are compared in order to identify the optimal transmission mode depending on the grade of CQI imperfectness, where a given target bit error rate has to be fullfilled.

1. INTRODUCTION

The multicarrier scheme Orthogonal Frequency Division Multiplexing (OFDM) [1] is regarded as groundwork for future mobile radio systems, supporting very high data rates. In a multicarrier scheme like OFDM, the overall channel can be divided in several subchannels in time and frequency dimension, so called subcarriers, which can be allocated to different connections. In a multi-user system the subcarriers can be adaptively allocated to different users in order to exploit multi-user diversity [2, 3], where knowledge about the channel quality of the subcarriers has to be available at the transmitter side. Having perfect channel knowledge at the transmitter, adaptive subchannel allocation schemes achieve very good performances [4, 5, 6].

However, in a realistic scenario perfect instantaneous channel knowledge is not available at the transmitter which leads to a performance degradation using adaptive techniques compared to the performance having perfect channel knowledge. In this case the use of diversity techniques can be beneficial. The exploitation of diversity, which does not require instantaneous channel knowledge at the transmitter, leads to an averaging of the channel qualities of the different subchannels resulting in a performance enhancement. Applying frequency hopping [7] or applying a DFT-precoding of the data [8, 9] together with interleaved carrier allocation are examples for techniques exploiting frequency diversity in an OFDM system. However, the achievable performance using diversity is worse compared to the performance using adaptive schemes with perfect channel knowledge. In a system with imperfect channel knowledge available at the transmitter the question arises which of the two transmission strategies adaption and diversity provides the better performance depending on the considered scenario and the grade of channel knowledge. In [11], special orthogonal space-timeblock codes with partial channel knowledge are analysed. In [12], combinations of frequency and spatial based diversity techniques for a multi-user scenario with limited feedback are discussed. In [10] a first comparison between adaptivity and diversity is drawn for a Multiple Input Multiple Output (MIMO) system, where space-time coding is compared to adaptive bit- and power loading. For perfect channel knowledge, the adaptive scheme provides the better performance as expected. In [13], the throughput of an adaptive SISO-OFDM system performing adaptive subcarrier allocation and adaptive modulation with imperfect channel quality information (CQI) is compared to the throughput using frequency diversity techniques. As CQI, the instantaneous signal-to-noise ratio (SNR) of the different subcarriers of the different users is used.

In this paper, we extend the SISO case to a MIMO scenario, where we use Orthogonal Space-Time Block Coding (OSTBC) at the transmitter side and Maximum Ratio Combining (MRC) at the receiver side. We apply the resulting SNR at the receiver as CQI fed back to the base station (BS), where we make the following assumptions for the CQI as also done in [13]:

The authors are indebted to the Deutsche Forschungsgemeinschaft (DFG) for sponsoring this work in the framework of the TakeOFDM project and to their project partners Prof. Weber from University of Rostock and Prof. Rohling from Technical University of Hamburg-Harburg for the yielding exchange of ideas.

978-1-4244-1757-5/08/$25.00

- The CQI available at the BS is outdated due to time delays.
- The CQI measured at the mobile station (MS) is only an estimate with a certain estimation error.
- The CQI is digitised before it is fed back over a feedback channel to the BS.
- During the transmission of the feedback errors may occur.

In the following, we consider two transmission modes. First, an adaptive transmission mode applying adaptive subcarrier allocation and adaptive modulation based on CQI values. Second, a non-adaptive mode exploiting frequency and spatial diversity without any instantaneous channel knowledge at the transmitter. We provide closed form expressions of the average data rate and uncoded bit error rate performance applying the adaptive transmission mode for the MIMO case using OSTBC and MRC under the assumption of imperfect CQI. Furthermore, we derive closed form expressions of the average data rate and uncoded bit error rate using the non-adaptive transmission mode. We compare these data rate values in order to identify the optimal transmission mode depending on the grade of CQI imperfectness, where we introduce a target bit error rate, which both transmission modes have to fullfill. The remainder of this paper is organised as follows. Section 2 describes the system model. In Section 3, the adaptive and non-adaptive transmission mode are presented. In Section 4, the different sources of errors of the CQI are introduced together with the parameters describing the CQI imperfectness. In Section 5, the average data rate and bit error rate using the adaptive transmission mode is derived analytically for the case of imperfect CQI. Section 6 provides the analytical derivation of the average data rate and bit error rate using the non-adaptive transmission mode. Section 7 presents how to identify the optimal transmission mode based on the available channel knowledge. In Section 8, the achievable data rates for the adaptive and non-adaptive transmission are illustrated and compared for a realistic OFDM scenario. Finally, conclusions are drawn in Section 9.

2. SYSTEM MODEL

In this work, we consider a one cell MIMO Orthogonal Frequency Division Multiple Access (OFDMA) downlink scenario in an Frequency Division Duplex (FDD) system with N subcarriers with index $n = 1, \cdots, N$, where one BS and U MS with user index $u = 1, \cdots, U$ are located in the cell. The BS is equipped with n_T transmit antennas and each MS with n_R receive antennas. Furthermore, it is assumed that all user have the same requirements in terms of data rate. The entries of the $n_T \times n_R$ MIMO channel of each subcarrier are assumed to be uncorrelated as well as the channel realisations of different users and adjacent subcarriers. The transfer factor $H_u^{(i,j)}(n,k)$ of the channel from transmit antenna i with $i = 1, \cdots, n_T$ to receive antenna j with $i = 1, \cdots, n_R$ of each user u at subcarrier with index n at each time slot $k \in \mathbf{N}$ is modeled as a complex Gaussian distributed random process with variance one. Assuming a perfect power control, which compensates the effects of path loss and shadowing [14], all users experience the same average SNR $\bar{\gamma}$. From this, it follows that the instantaneous SNR $\gamma_u^{(i,j)}(n,k)$ of user u of subcarrier with index n in time slot k from transmit antenna i to receive antenna j is given by

$$\gamma_u^{(i,j)}(n,k) = \bar{\gamma} \cdot \left| H_u^{(i,j)}(n,k) \right|^2 . \qquad (1)$$

3. TWO MODE TRANSMISSION

The considered adaptive MIMO-OFDM system has the ability to switch between an adaptive and a non-adaptive transmission mode, as it is proposed in [15]. The adaptive transmission mode exploits spatial and multi-user diversity, where with the non-adaptive mode spatial and frequency diversity is exploited. Depending on the grade of the available channel knowledge, the mode that provides the highest data rate is selected for transmission, which will be presented in Section 7 later on. As CQI, the digitised instantaneous SNR of the different subcarriers of the different users is applied. In the following, the two transmission modes are presented.

3.1. Adaptive transmission mode

Applying the adaptive transmission mode, an adaptive subcarrier allocation and modulation based on the CQI available at the BS is performed. By doing so, one can benefit from multi-user diversity [2, 3]. In this paper, a Max-SNR Scheduler is employed that allocates the subcarriers to the users with the best SNR conditions, where one subcarrier is allocated to only one user exclusively. If several users have the best channel condition, the subcarrier is allocated randomly to one of these users. By assuming that each user experiences the same average SNR $\bar{\gamma}$, the probability of getting access to a subcarrier is equal for all users and given by $P_a = \frac{1}{U}$, i.e. the scheduler is long-term fair [16]. After performing the subcarrier allocation, a modulation scheme is selected for each subcarrier based on the CQI value, where it is assumed that the transmit power for each subcarrier is equal. In this work, the following modulation schemes are considered: BPSK, QPSK, 8-PSK, 16-QAM, 32-QAM, 64-QAM, 128-QAM, 256-QAM and 512-QAM. Finally, the data of the scheduled user is transmitted over the MIMO channel using OSTBC at the transmitter and MRC at the receiver. Applying OSTBC leads to an averaging of the SNR of the n_T transmit antennas at each receive antenna. These averaged SNR values are then superimposed performing the MRC. The resulting SNR $\gamma_u(n,k)$

of the subcarrier with index n of user u at time index k is then given by

$$\gamma_u(n,k) = \sum_{j=1}^{n_R} \sum_{i=1}^{n_T} \frac{1}{n_T} \gamma_u^{(i,j)}(n,k). \tag{2}$$

3.2. Non-adaptive transmission mode

Using the non-adaptive transmission mode no instantaneous channel knowledge for the different subcarriers of a user is required. All N subcarriers are now allocated to one user u exclusively at each time slot, i.e. each user gets the same amount of channel access. A MIMO-OFDM transmission scheme is used to exploit spatial and frequency diversity, like it is presented in [17], where DFT precoded OFDM is combined with Space-time Coding and MRC. Finally, one modulation scheme is selected for all subcarriers, where it is assumed that the average SNR $\bar{\gamma}$ is known to the transmitter.

4. MODELLING IMPERFECT CQI

In this section, the four different sources of error for imperfect CQI considered in this work are presented, where the parameters describing the imperfectness are introduced as also done in [13].

4.1. Outdated CQI

Between the instant of measuring the channel quality of a given subcarrier and the actual transmission of the data using this subcarrier there is certain time delay, i.e. the available CQI at the transmitter is outdated. This can be modelled by correlation, i.e. the outdated channel and the actual channel are two complex Gaussian distributed random variables with a correlation coefficient ρ, where the correlation coefficient ρ is given by

$$\rho = J_0\left(2\pi\, v_{MS}\, \frac{f_c}{c}\, T\right), \tag{3}$$

assuming a Jakes' scattering model with the MS velocity v_{MS}, the carrier frequency f_c, the speed of light c and the delay time T between the outdated and the actual channel realisation. In the following, we assume that the correlation coefficient ρ is equal for all users.

4.2. CQI with an estimation error

Since in a real system the channel at the receiver is not perfectly known, the measured channel is only an estimate of the actual channel. Assuming minimum mean square error (MMSE) estimation, and skipping the user, subcarrier, antenna and time-slot indices, the estimation error is given by $E = H - \hat{H}$, where $\hat{H}$ denotes the MMSE estimate. The estimation error E is complex Gaussian distributed with variance σ_E^2 and independent from $\hat{H}$, which is also complex Gaussian distributed with variance $1-\sigma_E^2$. The error variance $\sigma_E^2 \in [0,1]$ depends on the conditions of the channel and the applied estimation scheme and is according to [18] given by

$$\sigma_E^2 = \frac{1}{1+\frac{T_\tau P_\tau}{n_T}} \tag{4}$$

where T_τ is the number of training symbols per coherence time and P_τ the SNR during the training phase. In the following, it is assumed that σ_E^2 is known both to the transmitter and the receiver.

4.3. Digitised CQI

Using digitised CQI, the analog CQI of each subcarrier n in each time slot k is digitised at each MS u. As a consequence, the scheduler at the BS can not distinguish between the channel qualities of different users as precise as with analog CQI, since there is only a limited numbers of CQI levels. Each measured SNR value is now quantised in $W = 2^{N_Q}$ quantisation levels with $W+1$ quantisation bounds γ_l with $l = 0, \cdots, W$, where $\gamma_0 = 0, \gamma_W = \infty$ and N_Q denoting the number of quantisation bits per subcarrier. The quantised CQI values are then digitised according to a certain bit coding scheme which is defined by a $W \times W$ matrix $\mathbf{B}$. The (i,j)-th element $b_{i,j}$ of matrix $\mathbf{B}$ with $i,j = 1, \cdots, W$ determines the number of bits which differ comparing the bit coding of the i-th quantisation level $[s_{i-1}, s_i]$ with the bit coding of the j-th $[s_{j-1}, s_j]$ quantisation level.

4.4. Digitised CQI with feedback errors

Since the feedback channel can not be assumed to be error-free, bit errors may occur during the transmission of the digitised CQI with a certain average bit error rate p_b,which depends on the condition of the feedback channel and the used modulation and coding scheme. In the following, we assume that the average feedback bit error rate p_b is the same for all users. Now an SNR value, which is measured to be in the i-th quantisation level $[\gamma_{i-1}, \gamma_i]$ might be assumed to be in the j-th quantisation level $[\gamma_{j-1}, \gamma_j]$. It can be shown [13] that the probability of this event is calculated using the bit coding matrix $\mathbf{B}$ according to

$$d_{i,j} = (1-p_b)^{N_Q - b_{i,j}} \cdot p_b^{b_{i,j}}, \tag{5}$$

resulting in a $W \times W$ matrix $\mathbf{D}$ with the elements $d_{i,j}$.

5. DATA RATE AND BIT ERROR RATE USING THE ADAPTIVE TRANSMISSION MODE

In this section, we analytically derive expressions for the average data rate and the average uncoded bit error rate performance using the adaptive transmission mode taking into account all types of imperfect CQI introduced in Section 4.

5.1. Average data rate using imperfect CQI

The average data rate $\bar{R}_A$ using OSTBC at the transmitter and MRC at the receiver and applying adaptive modulation is defined as the number of transmitted data bits and given by

$$\bar{R}_A = r_{n_T} \cdot \sum_{m=1}^{card(\mathcal{M})} c_m \cdot \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}(\hat{\gamma}) \, d\hat{\gamma}, \tag{6}$$

with $\mathcal{M}$ denoting a certain selection of modulation schemes, where $card(x)$ denotes the cardinality of the x. The interval in which a particular modulation scheme is applied is determined by the bounds γ_{m-1} and γ_m, with $m = 1, ..., card(\mathcal{M})$. Note that these modulation bounds are identical to the quantisation bounds introduced in Section 4. The number of bits per symbol corresponding to the modulation scheme is given by c_m, where r_{n_T} denotes the data rate of the Space-Time Block Code as a function of n_T. The probability density function (PDF) $p_{\hat{\gamma}}(\hat{\gamma})$ determines the distribution of the SNR value of a scheduled subcarrier, signaled from the MSs to the BS. In order to determine the PDF of $\hat{\gamma}$, the $W \times 1$ vector $\mathbf{z}$ is introduced with its elements

$$\begin{aligned} z_i \;=\; & \sum_{k=0}^{n_T n_R - 1} \frac{1}{k!} \left[\exp\left(-\frac{n_T \gamma_{i-1}}{E\{\hat{\gamma}\}}\right) \left(\frac{n_T \gamma_{i-1}}{E\{\hat{\gamma}\}}\right)^k \right. \\ & \left. - \exp\left(-\frac{n_T \gamma_i}{E\{\hat{\gamma}\}}\right) \left(\frac{n_T \gamma_i}{E\{\hat{\gamma}\}}\right)^k \right], \end{aligned} \tag{7}$$

with $i = 1, \cdots, W-1$ and $E\{\hat{\gamma}\} = \bar{\gamma}(1 - \sigma_E^2)$. Each element z_i determines the probability that a measured SNR value at the MS is in the i-th quantisation level [γ_{i-1},γ_i] using OSTBC at the transmitter and MRC at the receiver. We further introduce the $W \times 1$ vector $\mathbf{p}$, which is calculated according to $\mathbf{p} = \mathbf{D} \cdot \mathbf{z}$, where the j-th element p_j of vector $\mathbf{p}$ with $j = 1, \cdots, W-1$ denotes the probability that the signalled SNR value is assumed to be in the j-th quantisation level [γ_{j-1},γ_j] at the BS. It can be shown that the PDF of the assumed SNR $\hat{\gamma}$ of the scheduled user applying an Max-SNR scheduling policy in case of digitised CQI with feedback errors using OSTBC at the transmitter and MRC at the receiver is then given by

$$\begin{aligned} p_{\hat{\gamma}}(\hat{\gamma}) \;=\; & \sum_{m=1}^{card(\mathcal{M})} \frac{\tilde{a}_m}{(n_T n_R - 1)!} \sum_{j=1}^{card(\mathcal{M})} d_{m,j} \\ & \cdot \left(\frac{n_T}{E\{\hat{\gamma}\}}\right)^{n_T n_R} \cdot \hat{\gamma}^{n_T n_R - 1} \cdot \exp\left(-\frac{n_T \hat{\gamma}}{E\{\hat{\gamma}\}}\right) \\ & \cdot [\sigma(\hat{\gamma} - \gamma_{j-1}) - \sigma(\hat{\gamma} - \gamma_j)] \end{aligned} \tag{8}$$

with

$$\tilde{a}_m = \frac{\left(\sum_{i=1}^{m} p_i\right)^U - \left(\sum_{i=1}^{m-1} p_i\right)^U}{p_m}. \tag{9}$$

Inserting (8) in (6) and using the identity

$$\int x^m e^{-ax} \, dx = -x^{m+1} e^{-ax} \sum_{k=0}^{m} \frac{m!}{k!} (ax)^{k-m-1}, \tag{10}$$

the average data rate $\bar{R}_a$ for outdated digitised CQI feedback with estimation and feedback errors is given by

$$\begin{aligned} \bar{R}_A \;=\; & r_{n_T} \cdot \sum_{m=1}^{card(\mathcal{M})} \tilde{a}_m \cdot c_m \sum_{j=1}^{card(\mathcal{M})} d_{m,j} \\ & \sum_{k=0}^{n_T n_R - 1} \frac{1}{k!} \cdot \left[\left(\frac{n_T \gamma_{j-1}}{E\{\hat{\gamma}\}}\right)^k \cdot \exp\left(-\frac{n_T \gamma_{j-1}}{E\{\hat{\gamma}\}}\right) \right. \\ & \left. - \left(\frac{n_T \gamma_j}{E\{\hat{\gamma}\}}\right)^k \cdot \exp\left(-\frac{n_T \gamma_j}{E\{\hat{\gamma}\}}\right) \right]. \end{aligned} \tag{11}$$

5.2. Average bit error rate using imperfect CQI

Using outdated digitised CQI with estimation and feedback errors, several effects lead to a degradation of the average bit error rate performance compared to perfect CQI. First, the CQI available at the BS is already outdated when transmitting the data to the scheduled users. Secondly, the SNR values are possibly quantised in the wrong quantisation level since the SNR values available at the MS are only estimates. Thirdly, the scheduler at the BS can not distinguish between users within the same quantisation level, i.e. the scheduler has to choose randomly between these users. Furthermore, the number of modulation schemes the scheduler can select from is limited by the number of quantisation levels. Finally, the SNR values are possibly assumed to be in the wrong quantisation level at the BS due to the feedback bit errors. Consequentially, the user and modulation scheme selection is based on possibly erroneous channel knowledge, i.e. a user with a weak channel can be selected for transmission using a modulation scheme, which is only suitable for high SNR channels resulting in an BER degradation. Taking these effects into account, the average bit error rate BER_A using imperfect CQI is determined by

$$\begin{aligned} BER_A \;=\; & \sum_{m=1}^{card(\mathcal{M})} \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}(\hat{\gamma}) \\ & \cdot \left[\int_0^{\infty} BER_m(\gamma) p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) \, d\gamma \right] \, d\hat{\gamma}, \end{aligned} \tag{12}$$

with $p_{\hat{\gamma}}(\hat{\gamma})$ the PDF of the assumed SNR values of the scheduled users at the BS introduced in Section 5.1 and $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ the conditional PDF of the actual SNR γ and the assumed SNR $\hat{\gamma}$, which depends on the type of available channel knowledge and the number of antennas. For the case of outdated CQI with estimation errors using OSTBC at the transmit side and MRC at the receiver, it can be shown that the conditional

PDF is given by

$$p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{n_T}{\bar{\gamma}\sigma_r^2} \cdot \exp\left(-\frac{n_T(\rho^2 \cdot \hat{\gamma} + \gamma)}{\bar{\gamma}\sigma_r^2}\right) \cdot I_{n_T n_R - 1}\left(\frac{2n_T\rho\sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}\sigma_r^2}\right), \quad (13)$$

with $\sigma_r^2 = 1 - \rho^2(1 - \sigma_E^2)$ and $I_n(x)$ denoting the nth-order modified Bessel function of the first kind. The expression BER_m determines the bit error rate of the applied modulation scheme with index m. In the following, the approximation for the BER for M-QAM and M-PSK modulation introduced by [19] is applied, given by

$$BER_m(\gamma) = 0.2 \cdot \exp(-\beta_m \gamma) \quad (14)$$

with $\beta_m = \frac{1.6}{2^{c_m}-1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9c_m}+1}$ using M-PSK modulation respectively.
Inserting (8), (13) and (14) in (12) and using the identities [21, Eq. 1.111], [21, Eq. 6.643.4], [21, Eq. 8.406.3], [21, Eq. 8.970.1] and (10), the average bit error rate BER_A for outdated digitised CQI feedback with estimation and feedback errors can be determined shown on the top of the next page. This closed form expression determines the average bit error rate as a function of all types of imperfect CQI introduced in Section 4. The different sources of errors caused by time delay, channel estimation and feedback transmission can be switched off setting $\rho = 1$, $\sigma_E^2 = 0$ and $p_b = 0$ respectively.

6. DATA RATE AND BIT ERROR RATE USING THE NON-ADAPTIVE TRANSMISSION MODE

Using an OFDM transmission technique that exploit spatial and frequency diversity as presented in [17] leads to an averaging over the n_T different SNR conditions of a subcarrier at each receive antenna. Applying MRC at the receiver, these resulting SNRs are superimposed. Finally, due to the exploitation of frequency diversity, the combined SNRs of the N different subcarriers are averaged leading to the resulting subcarrier SNR γ_D given by

$$\gamma_D = \frac{1}{N}\sum_{n=1}^{N}\sum_{j=1}^{n_R}\sum_{i=1}^{n_T}\frac{1}{n_T}\gamma_n^{(i,j)}, \quad (16)$$

where $\gamma_n^{(i,j)}$ denotes the instantaneous SNR of the n-th subcarrier of the channel from the i-th transmit antenna to the j-th receive antenna. By assuming that the MIMO channel of each subcarrier is uncorrelated as well as the channels of adjacent subcarriers, Eq. (16) can be rewritten to

$$\gamma_D = \sum_{n=1}^{n_R \cdot n_T \cdot N}\frac{1}{N \cdot n_T}\gamma_n. \quad (17)$$

It can be shown that the PDF of the resulting SNR γ_D is a chi-square distribution with $2Nn_Tn_RN$ degrees of freedom [20] and given by

$$p_{\gamma_D}(\gamma_D) = \left(\frac{n_T \cdot N}{\bar{\gamma}}\right)^{n_T \cdot n_R \cdot N} \cdot \frac{\gamma_D^{n_T \cdot n_R \cdot N - 1}}{(n_T \cdot n_R \cdot N - 1)!} \cdot \exp\left(-\frac{n_T \cdot N\gamma_D}{\bar{\gamma}}\right). \quad (18)$$

Applying the non-adaptive transmission mode, one fixed modulation scheme with index m is used for all subcarriers, resulting in the data rate $\bar{R}_D$ given by

$$\bar{R}_D = r_{n_T} \cdot c_m. \quad (19)$$

The average BER using the modulation scheme with index m is then determined by

$$BER_D = \int_0^\infty BER_m(\gamma_D) \cdot p_{\gamma_D}(\gamma_D)\, d\gamma_D. \quad (20)$$

Inserting (18) and (14) in (20) and using the identities [21, Eq. 1.111] and [21, 3.351.3], the average BER using the non-adaptive transmission mode can be calculated according to

$$BER_D = 0.2 \cdot \left[\frac{n_T N}{n_T N + \beta_m \bar{\gamma}}\right]^{n_T n_R N}. \quad (21)$$

7. ADAPTIVE TRANSMISSION MODE WITH IMPERFECT CQI VS. NON-ADAPTIVE TRANSMISSION MODE

In the following, the data rates of the two transmission modes are compared, where we introduce a target bit error rate BER_T, which both transmission modes have to fullfill. In order to identify the optimal transmission mode, we further introduce the resulting average data rate $\bar{R}$ and the resulting bit error rate BER given by

$$\bar{R} = \kappa \cdot \bar{R}_A + (1 - \kappa) \cdot \bar{R}_D \quad (22)$$

$$BER = \kappa \cdot BER_A + (1 - \kappa) \cdot BER_D, \quad (23)$$

with $\kappa \in \{0, 1\}$, where $\kappa = 1$ corresponds to an active adaptive transmission mode and $\kappa = 0$ to an active non-adaptive transmission mode. As seen in Section 4, the parameters defining the quality of the channel knowledge are the correlation coefficient ρ between the actual and the outdated channel, the estimation error variance σ_E^2, and the average BER p_b of the feedback channel. The parameters defining the scenario, which are assumed to be known to both the transmitter and the receiver, are the number N of subcarriers, the average SNR $\bar{\gamma}$ the number U of active users and the number N_Q of quantisation bits per subcarrier. The parameters, which can be adaptively changed by the system, are the quantisation levels $[\gamma_{m-1}, \gamma_m]$ with $m = 1, ..., card(\mathcal{M})$, the bit coding scheme $\mathbf{B}$, the applied modulation schemes $\mathcal{M}$, the number of active antennas (n_T,n_R) and the switching parameter κ. For a given

$$\begin{aligned} BER_A &= 0.2 \cdot \sum_{m=1}^{card(\mathcal{M})} \left(\frac{n_T}{n_T + \beta_m \bar{\gamma}} \right)^{n_T n_R} \tilde{a}_m \sum_{j=1}^{card(\mathcal{M})} d_{m,j} \sum_{k=0}^{n_T n_R - 1} \frac{1}{k!} \left[\left(\frac{n_T \gamma_{j-1}(1+\beta_m \bar{\gamma})}{E\{\hat{\gamma}\}(n_T + \beta_m \bar{\gamma} \sigma_r^2)} \right)^k \right. \\ &\quad \left. \cdot \exp\left(- \frac{n_T \gamma_{j-1}(1+\beta_m \bar{\gamma})}{E\{\hat{\gamma}\}(n_T + \beta_m \bar{\gamma} \sigma_r^2)} \right) - \left(\frac{n_T \gamma_j (1+\beta_m \bar{\gamma})}{E\{\hat{\gamma}\}(n_T + \beta_m \bar{\gamma} \sigma_r^2)} \right)^k \cdot \exp\left(- \frac{n_T \gamma_j (1+\beta_m \bar{\gamma})}{E\{\hat{\gamma}\}(n_T + \beta_m \bar{\gamma} \sigma_r^2)} \right) \right] \end{aligned} \quad (15)$$

set of channel knowledge parameters and scenario parameters the maximum data rate can be identified optimising the resulting data rate (22) with regard to the set of adaptive parameters

$$\bar{R}_{opt} = \max_{[\gamma_{m-1},\gamma_m],\mathbf{B},\mathcal{M},n_T,n_R,\kappa} \left(\bar{R}\right) \quad (24)$$
$$\text{subject to}$$
$$BER \leq BER_T.$$

This optimisation can be done in three steps. Firstly, for the given set for scenario parameters, the modulation scheme with index m is selected, which maximises the data rate using the non-adaptive transmission mode, on condition that $BER_D \leq BER_T$ holds, resulting in the optimal data rate $\bar{R}_{D,opt}$. In the second step, the average data rate using the adaptive transmission mode is maximised subject to the target bit error rate for the given set of scenario and channel knowledge parameters with regard to the set of adaptive parameters resulting in the optimal data rate $\bar{R}_{A,opt}$. Finally, the resulting data rate is maximised by determing the optimal switching parameter κ

$$\bar{R}_{opt} = \max_{\kappa} \left(\kappa \cdot \bar{R}_{A,opt} + (1-\kappa) \cdot \bar{R}_{D,opt} \right). \quad (25)$$

8. NUMERICAL RESULTS

In the following, the achievable data rates using the adaptive and non-adaptive transmission mode are illustrated for an OFDM system with $N = 1024$ subcarriers and a target bit error rate $BER_T = 10^{-3}$, where $N_Q = 1$ feedback bit per subcarrier is used to signal the CQI to the BS. We assume a MIMO system with $n_{T,max} = 2$ transmit antennas and $n_{R,max} = 2$ receive antennas, i.e. the well known Alamouti STBC with a data rate of $r_{n_T} = 1$ can be applied. First, we consider the case of perfect CQI ($\rho = 1$, $\sigma_E^2 = 0$ and $p_b = 0$) in a system with $U = 40$ users. Now, the average data rate using the adaptive mode has to be maximised for the given set of scenario and channel knowledge parameters with regard to the set of adaptive parameters ($[\gamma_{m-1}, \gamma_m]$,$\mathbf{B}$,$\mathcal{M}$,n_T,n_R) leading to the optimal adaptive parameters $\gamma_{1,opt}$, $n_{T,opt}$, $n_{R,opt}$, $\mathcal{M}_{opt}$ and $\mathbf{B}_{opt}$. In this work, this operation is done by numerical optimisation. In Fig. 1 the optimal quantisation bound $\gamma_{1,opt}$ is illustrated as a function of the average SNR $\bar{\gamma}$. For all SNR values, the optimal number of receive antennas is $n_{R,opt} = 2$ and the optimal bit coding scheme is $\mathbf{B}_{opt} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, where for $N_Q = 1$ quantisation bit only one possible bit coding matrix $\mathbf{B}$ exists. Note that for perfect CQI the transmission using only one transmit antenna $n_{T,opt} = 1$ achieves a higher data rate than using the two transmit antenna Alamouti STBC since the averaging effect of the STBC on the resulting SNR of a subcarrier decreases the probability of high SNR values, which leads to a lower multi-user diversity. The optimal modulation schemes $\mathcal{M}_{opt}$ for the first and second quantisation level are shown in Fig. 1 as well. In

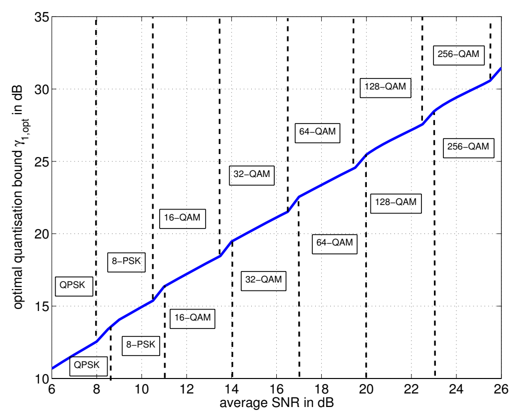

Fig. 1. Optimal quantisation bound $\gamma_{1,opt}$ for target bit error rate $BER_T = 10^{-3}$

Fig. 2, the maximal achievable data rates are illustrated for the adaptive and non-adaptive transmission mode with perfect CQI for $U = 40$ users as a function of the average SNR $\bar{\gamma}$. It can be seen, that having perfect CQI, the adaptive mode clearly outperforms the non-adaptive mode, where for both modes the data rate increases with increasing average SNR, since for higher SNR modulation schemes with a higher number of data bits can be applied.

Now, we consider the case of imperfect CQI in a system with $U = 40$ users and $\bar{\gamma} = 10$ dB average SNR. It is assumed that $T_\tau = 1$ training symbols are used with $P_\tau = \bar{\gamma}$, leading to $\sigma_E^2 = \frac{n_T}{n_T + \bar{\gamma}}$. In the following, we investigate the impact of the feedback bit errors on the data rate of the adaptive transmission mode. We assume, that the channel knowl-

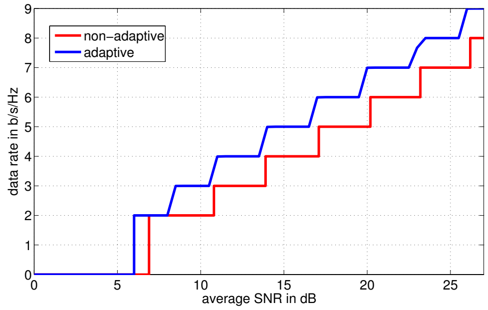

Fig. 2. Comparsion data rate adaptive mode vs. non-adaptive mode with perfect CQI for $N_Q = 1$ quantisation bit and target bit error rate $BER_T = 10^{-3}$

edge parameters are known to the BS. In Fig. 3 the average data rate of the adaptive transmission mode is depicted as a function of the average BER p_b of the feedback channel for different correlation coefficient ρ assuming a target bit error rate of $BER_T = 10^{-3}$. It appears, that being aware of the

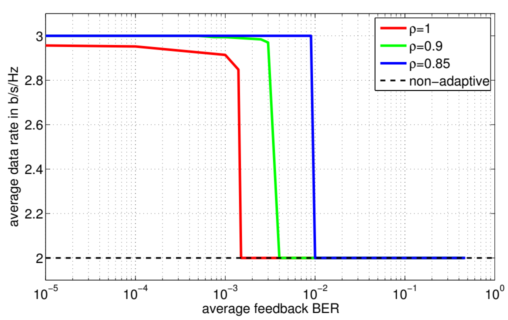

Fig. 3. Average data rate as a function of the average feedback BER p_b for $\bar{\gamma} = 10$ dB, $N_Q = 1$ and target bit error rate $BER_T = 10^{-3}$

fact, that the CQI feedback is not perfect, one can adapt to this imperfectness up to feedback BER of $p_b = 10^{-2}$ as long as the CQI is not too outdated ($\rho > 0.85$). For feedback BER $p_b > 10^{-2}$, the non-adaptive mode provides the same data rate without requiring any CQI.

Next, we investigate the influence of the correlation between the actual channel and the assumed channel for a fixed feedback bit error rate of $p_b = 10^{-3}$. We now consider a system with different numbers of users ($U = 5, 10, 20, 40$), where the carrier frequency is assumed to be $f_c = 5$ GHz and the time delay $T = 0.843$ ms [15]. In Fig. 4, the optimal data rate is depicted as a function of the MS velocity. For $U = 40$ users, one can adapt to the CQI imperfectness up to MS velocity of $v_{MS} = 36$ km/h still providing a higher data rate compared to the non-adaptive mode. For larger velocities, the non-adaptive transmission mode provides the same data rate. For the case of less users in the system, the velocity, where the adaptive mode no longer outperforms the non-adaptive mode, decreases, since less multi-user diversity can be exploited. In Fig. 5, the optimal quantisation bound $\gamma_{1,opt}$ is

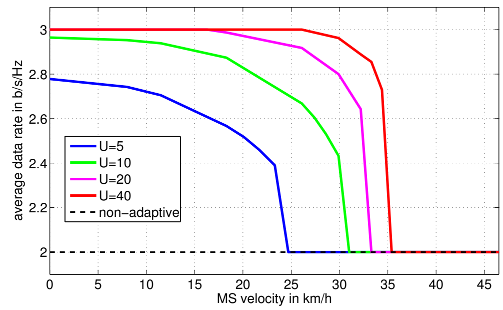

Fig. 4. Average data rate as a function of the MS velocity v_{MS} for $\bar{\gamma} = 10$ dB, $N_Q = 1$ and target bit error rate $BER_T = 10^{-3}$

shown as a function of the MS velocity v_{MS} for a sytem with $U = 20$ users. The optimal modulation schemes $\mathcal{M}_{opt}$ and optimal number of transmit antennas $n_{T,opt}$ are depicted as well. It appears, that with increasing MS velocity which corresponds to a decreasing correlation coefficient ρ, the quantisation bound increases. This behaviour can be explained by the fact, that with decreasing correlation, the probability increases, that a selected user assumed to be in the second quantisation level $[\gamma_1,\infty[$ is actually in the first quantisation level $[0,\gamma_1]$, causing an BER degradation due to a wrong user and modulation scheme selection. Increasing the first quantisation level $[0,\gamma_1]$ decreases the probability of this event. At an MS velocity of $v_{MS} = 16$ km/h, a discontinuity can be observed corresponding to the change of the modulation schemes and number of transmit antennas. In this example, the optimal modulation schemes $\mathcal{M}_{opt}$={8-PSK,8-PSK} and number of transmit antennas $n_{T,opt} = 1$ change to $\mathcal{M}_{opt}$={QPSK,8-PSK} and $n_{T,opt} = 2$, i.e. at an MS velocity of $v_{MS} = 16$ km/h one achieves a higher data rate using the more robust two transmit antenna Alamouti STBC and the more robust modulation scheme for the second quantisation level. The same can be observed at an MS velocity of $v_{MS} = 32$ km/h, where the optimal modulation schemes change to $\mathcal{M}_{opt}$={QPSK,QPSK}.

9. CONCLUSIONS

In this paper, we derive closed form expressions for the average data rate and uncoded bit error rate of a multi-user MIMO-OFDM system with two transmission modes using OSTBC at the transmitter and MRC at the receiver to exploit spatial diversity. Applying the adaptive transmission mode, an

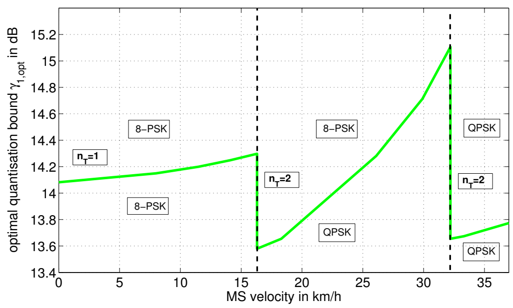

Fig. 5. Optimal quantisation bound $\gamma_{1,opt}$ as a function of the MS velocity v_{MS}

adaptive subcarrier allocation and adaptive modulation based on CQI values is performed to exploit multi-user diversity, where the CQI is assumed to be outdated and digitised with estimation and feedback errors. Applying the non-adaptive mode, a transmission scheme exploiting additional frequency diversity is employed which does not require instantaneous CQI. By comparing the achievable data rates using the adaptive transmission mode with imperfect CQI and the data rate achievable by the non-adaptive transmission the optimal transmission mode can be identified depending on the grade of CQI imperfectness. Being aware of the CQI imperfectness and the parameters describing the imperfectness, one can adapt to this imperfectness for feedback bit error rates $p_b < 10^{-2}$ and MS velocities up to 36 km/h in a system with one bit feedback and an average SNR of 10 dB. For feedback bit error rates $p_b > 10^{-2}$ and larger MS velocities, the non-adaptive transmission mode is the better choice.

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