International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Sep 2008, Cannes, France

Blind Channel Estimation based on Second Order Statistics for IFDMA

Anja Sohl and Anja Klein
Technische Universität Darmstadt, Communications Engineering Lab, Merckstr. 25, 64283 Darmstadt, Germany
Email: {a.sohl, a.klein}@nt.tu-darmstadt.de

***Abstract*—In this paper, the Interleaved Frequency Division Multiple Access (IFDMA) scheme is considered as a special case of Discrete Fourier Transform (DFT)-precoded Orthogonal Frequency Division Multiple Access (OFDMA) with interleaved subcarrier allocation. The redundancy that is characteristic for an IFDMA-signal is exploited in order to estimate the channel without pilot symbols and, thus, overcome the drawback of increased pilot symbol overhead that arises for IFDMA-systems. Two well-known blind channel estimation algorithms based on Second Order Statistics (SOS) are adapted to the application in an IFDMA-system and compared in terms of performance and convergence. Further on, the influence of multi-user transmission is illustrated for the two proposed blind channel estimation algorithms and the differences between multi-user uplink and downlink transmission in terms of channel estimation performance are investigated and emphasized by numerical results.**

I. Introduction

At present, research activities for beyond 3rd generation of mobile radio systems are in progress worldwide. Orthogonal Frequency Division Multiple Access (OFDMA) is a candidate Multiple Access (MA) scheme due to its favorable properties that have been described e.g. in [1]. The application of Discrete Fourier Transform (DFT) precoding to OFDMA leads to a combination of most of the advantages of OFDMA with a low Peak-to-Average Power Ratio (PAPR) of the transmit signal [2], [3]. In this work, the focus will be on DFT-precoded OFDMA with interleaved subcarrier allocation resulting in the well known Interleaved Frequency Division Multiple Access (IFDMA) scheme [4], [5].

For IFDMA, a set of subcarriers that are equidistantly distributed over the available bandwidth is assigned to each user. Due to the distributed subcarriers, IFDMA provides high frequency diversity [6]. In Time Domain (TD), the IFDMA signal can be described as a compression, repetition and subsequent user dependent phase rotation of blocks of modulated signals [4]. Thus, there exists a very efficient implementation for signal generation in TD for IFDMA. Compared to other DFT-precoded OFDMA schemes, IFDMA provides the lowest PAPR and, thus, enables the application of low cost amplifiers [7]. Nevertheless, in terms of Channel Estimation (CE) for IFDMA, especially for low data rates, interpolation in Frequency Domain (FD) between the distributed subcarriers allocated to a specific user is not possible due to the large distance between adjacent subcarriers [8]. This means, that one IFDMA-symbol has to be used for pilot transmission in order to get an estimate of the channel transfer factor of each subcarrier assigned to a specific user [8]. The missing possibility of interpolation in FD even for channels with low frequency selectivity, i.e. a small delay spread in TD, leads to an increasing pilot symbol overhead concerning CE for IFDMA [9].

In order to reduce the pilot symbol overhead, CE without using any pilot symbols, so called blind CE, is an appropriate approach. Several methods based on Second Order Statistics (SOS) have been introduced for single-carrier systems so far, e.g. [10]–[12]. Moreover, these methods have been extended to the application in an OFDM system, e.g. [13], [14]. SOS algorithms are principally based on redundancy existent in the receive signal. This redundancy can be introduced either at the transmitter via a specific precoding [11], [14], or at the receiver via an oversampling of the signal [10]. Due to the IFDMA signal generation by compression and repetition of modulated data blocks, redundancy is inherent to the IFDMA-signal. Thus, SOS algorithms are applicable without any modification of the transmitter or receiver in an IFDMA system.

In this paper, two SOS based blind CE algorithms based upon [11]–[13] are deduced for an IFDMA-signal and compared in terms of performance and convergence for the case that the signal of a single user is considered. Further on, the influence of multi-user transmission on the two proposed algorithms is derived and investigated for both an uplink and a downlink scenario. The differences between multi-user uplink and downlink transmission are identified and illustrated by numerical results.

This paper is organized as follows. In Section II, the IFDMA system model is described. In Section III, two SOS based algorithms are derived for IFDMA. Further on, the influence of multi-user transmission is derived for uplink and downlink transmission. In Section IV, numerical results illustrating the differences between both algorithms and the influence of multi-user transmission are discussed. Section V concludes the work.

II. IFDMA System Model

In this section, a system model for IFDMA will be derived in TD. In the following, all signals are represented by their discrete time equivalents in the complex baseband. Vectors and matrices are denoted by lower and upper case boldfaced letters, respectively. Further notations used throughout this work are given in Table I.

Assuming a system with U users, let

$$\mathbf{d}^{(u)}(k) = [d_0^{(u)}(k), \cdots, d_{Q-1}^{(u)}(k)]^{\mathrm{T}} \tag{1}$$

denote the kth block, $k = 1, \cdots, K$, of Q data symbols $d_q^{(u)}(k),\ q = 0, \cdots, Q-1$, transmitted at symbol rate $1/T_s$ by a user with index u, $u = 0, \cdots, U-1$. The data symbols $d_q^{(u)}(k)$ can be taken from the alphabet of a modulation scheme like Phase Shift Keying (PSK) and are assumed to be i.i.d. with zero-mean. An IFDMA-symbol is obtained by L_u-fold compression of the block $\mathbf{d}^{(u)}(k)$, with $L_u = C/Q$ and C the number of available subcarriers in the system. The block of compressed data symbols is denoted by $\mathbf{w}^{(u)}(k) = [w_0^{(u)}(k), \cdots, w_{Q-1}^{(u)}(k)]$ with $\mathrm{E}\{|w_q^{(u)}(k)|^2\} = \sigma_w^2$. Subsequently, $\mathbf{w}^{(u)}(k)$ is repeated L_u-times. In order to avoid inter-block and inter-carrier interference, each IFDMA-symbol is preceded by a Cyclic Prefix (CP) that corresponds to an L_g-fold repetition of the compressed block with $(L_g{\cdot}Q) \in \mathbb{Z}$ [4]. The vector of $L = (L_u + L_g)$-times repeated blocks is multiplied by a user dependent phase shift matrix $\mathbf{\Phi}^{(u)} = \mathrm{diag}(\exp\{-j\cdot 0\cdot\varphi^{(u)}\}, \exp\{-j\cdot 1\cdot\varphi^{(u)}\}, \cdots, \exp\{-j\cdot(LQ-1)\cdot\varphi^{(u)}\})$, with $\varphi^{(u)} = u\cdot 2\pi/C$. Thus, the resulting kth IFDMA-symbol of user u including CP is given by

$$\mathbf{x}^{(u)}(k) \quad = \quad \mathbf{\Phi}^{(u)} \cdot \underbrace{[\mathbf{w}^{(u)}(k), \cdots, \mathbf{w}^{(u)}(k)]^{\mathrm{T}}}_{L-\text{times}} . \qquad (2)$$

The IFDMA signal $\mathbf{x}^{(u)}(k)$ is transmitted over a channel $\mathbf{h}^{(u)}$ with $M < Q$ non-zero coefficients $h_m^{(u)}, m = 0, \cdots, M-1$, at rate L_u/T_s. The channel is assumed to be time-invariant. The kth IFDMA-symbol $\mathbf{x}^{(u)}(k) = [x_0^{(u)}(k), \cdots, x_{L\cdot Q-1}^{(u)}(k)]$ of user u after transmission over the channel $\mathbf{h}^{(u)}$ and distortion by the Additive White Gaussian Noise (AWGN) $\mathbf{n}^{(u)}(k) = [n_0^{(u)}(k), \cdots, n_{L\cdot Q-1}^{(u)}(k)]$ with variance σ_n^2 can be split into L blocks with Q elements, i.e. $\mathbf{r}^{(u)}(k) = [\mathbf{r}_0^{(u)}(k), \cdots, \mathbf{r}_{L-1}^{(u)}(k)]^{\mathrm{T}}$, and is given by

$$\mathbf{r}^{(u)}(k) \quad = \quad \mathbf{H}^{(u)} \cdot \mathbf{\Theta}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} + \mathbf{n}^{(u)}(k) \ , \qquad (3)$$

with

$$\mathbf{H}^{(u)} = \begin{bmatrix} \mathcal{H}_1^{(u)} & \mathcal{H}_0^{(u)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_1^{(u)} & \mathcal{H}_0^{(u)} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathcal{H}_1^{(u)} & \mathcal{H}_0^{(u)} \end{bmatrix} . \qquad (4)$$

$\mathcal{H}_0^{(u)}$ denotes the $Q \times Q$ Toeplitz matrix with first column $[h_0^{(u)}, ..., h_{M-1}^{(u)}, 0, ..., 0]^T$ and first row $[h_0^{(u)}, 0, ..., 0]$. $\mathcal{H}_1^{(u)}$ denotes the $Q \times Q$ Toeplitz matrix with first column $[0, ..., 0]^T$ and first row $[0, ..., 0, h_{M-1}^{(u)}, ..., h_1^{(u)}]$. The matrix $\mathbf{\Theta}^{(u)}$ contains the user dependent phase shift and is given by

$$\mathbf{\Theta}^{(u)} = \begin{bmatrix} \mathbf{\Phi}_{L-1}^{(u)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_0^{(u)} \\ \mathbf{0} & \mathbf{\Phi}_1^{(u)} \\ \vdots & \vdots \\ \mathbf{0} & \mathbf{\Phi}_{L-1}^{(u)} \end{bmatrix} , \qquad (5)$$

with $\mathbf{\Phi}_i^{(u)} = \mathrm{diag}(\phi_{i,0}^{(u)}, ..., \phi_{i,Q-1}^{(u)}) = \mathrm{diag}(\exp\{-j\cdot(i\cdot Q)\cdot\varphi^{(u)}\}, ..., \exp\{-j\cdot(i\cdot Q+Q-1)\cdot\varphi^{(u)}\})$ for $i = 0, ..., L-1$.

TABLE I
NOTATIONS

$(\cdot)^*$	Conjugate complex of a vector / matrix
$(\cdot)^{\mathrm{T}}$	Transpose of a vector / matrix
$(\cdot)^{\mathrm{H}}$	Hermitian of a vector / matrix
$\mathrm{E}\{\cdot\}$	Expectation of a vector / matrix
$\mathrm{diag}\{\mathbf{a}\}$	Diagonal matrix having the vector $\mathbf{a}$ as its main diagonal
$\mathbf{I}_\nu$	$\nu \times \nu$ identity matrix

In a system with U users, the total received signal is given by the superposition of the U users' signals. In an uplink scenario the received signal at the base station is given by

$$\mathbf{y}(k) = \sum_{u=0}^{U-1} \left(\mathbf{H}^{(u)} \cdot \mathbf{\Theta}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} \right) + \mathbf{n}(k) \ . \qquad (6)$$

In a downlink scenario, the received signal at the mobile station of user u_1 is given by

$$\mathbf{y}^{(u_1)}(k) = \mathbf{H}^{(u_1)} \cdot \sum_{u=0}^{U-1} \left(\mathbf{\Theta}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} \right) + \mathbf{n}^{(u_1)}(k) \ , \qquad (7)$$

as the superposition of signals transmitted to different users experiences the same Channel Impulse Response (CIR) in downlink considering the received signal at the mobile station of a given user.

For $U \leq L_u$ the U users' signals are orthogonal to each other and due to CP insertion, maintain the orthogonality even for transmission over a multipath channel [4]. Thus, the signal of user u can be separated from the other users' signals at the receiver in FD by extracting the Q subcarriers allocated to user u. The CP of the received signal cannot be separated into U parts due to lack of orthogonality.

III. SOS based blind CE for IFDMA

In this section, the application of two SOS based blind CE algorithms in an IFDMA system is investigated. The first one, denoted as Correlation Based Approach in the following, is based on [11] and exploits the information about the CIR that is inherent to the autocorrelation function of the received signal. The second one, denoted as Subspace Based Approach, is rested on [12] and [13] and takes advantage of the orthogonality between signal and noise subspace. Both approaches are introduced for a single-user scenario ($U = 1$) in order to emphasize the principle of the approach. Further on, the influence of multiple users ($U > 1$) on the Correlation and Subspace Based Approach is derived for an up- and a downlink scenario, respectively. In the following, the CP is assumed to consist of $L_g = 1$ repetition of $\mathbf{w}^{(u)}(k)$ as the channel contains $M \leq Q$ non-zero coefficients. This restriction is made, as for $M > Q$, the CIR is only identifiable with ambiguity for both approaches. In the following, $\square$ denotes elements of no relevance for the sequel and the user index u is omitted for the single-user case due to simplicity matters.

A. Correlation Based Approach

1) Single-User Scenario: The proposed Correlation Based Approach is applied to the IFDMA signal of one specific user. The autocorrelation matrix $\mathbf{R}_r = \mathrm{E}\{\mathbf{r}(k)\cdot\mathbf{r}(k)^{\mathrm{H}}\}$ of the received kth IFDMA-symbol $\mathbf{r}(k)$ is given by

$$\begin{aligned}\mathbf{R}_r &= \mathbf{H}\cdot\mathbf{\Theta}\cdot\sigma_w^2\cdot\mathbf{I}_{2Q}\cdot\mathbf{\Theta}^{\mathrm{H}}\cdot\mathbf{H}^{\mathrm{H}} + \sigma_n^2\cdot\mathbf{I}_{LQ} \\ &= \sigma_w^2\cdot\begin{bmatrix} \mathcal{H}_0\mathcal{H}_0^{\mathrm{H}} + \mathcal{H}_1\mathcal{H}_1^{\mathrm{H}} & \square\cdots\square \\ (\mathcal{H}_0\mathbf{\Phi}_1 + \mathcal{H}_1\mathbf{\Phi}_0)\cdot\mathbf{\Phi}_0^{\mathrm{H}}\mathcal{H}_0^{\mathrm{H}} & \square\cdots\square \\ \vdots & \vdots\ddots\vdots \\ (\mathcal{H}_0\mathbf{\Phi}_{L-1} + \mathcal{H}_1\mathbf{\Phi}_{L-2})\cdot\mathbf{\Phi}_0^{\mathrm{H}}\mathcal{H}_0^{\mathrm{H}} & \square\cdots\square \end{bmatrix} \\ &\quad + \sigma_n^2\cdot\mathbf{I}_{LQ}\ . \end{aligned} \tag{8}$$

The autocorrelation matrix contains $(L-1)$-times the matrices

$$(\mathcal{H}_0\mathbf{\Phi}_i + \mathcal{H}_1\mathbf{\Phi}_{i-1})\cdot\mathbf{\Phi}_0^{\mathrm{H}}\mathcal{H}_0^{\mathrm{H}} = \begin{bmatrix} h_0\cdot h_0^*\cdot\phi_{i,0}\cdot\phi_{0,0}^* & \square & \cdots & \square \\ h_1\cdot h_0^*\cdot\phi_{i,0}\cdot\phi_{0,0}^* & \square & \cdots & \square \\ \vdots & \vdots & \cdots & \vdots \\ h_{M-1}\cdot h_0^*\cdot\phi_{i,0}\cdot\phi_{0,0}^* & \square & \cdots & \square \\ 0 & \square & \cdots & \square \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \square & \cdots & \square \end{bmatrix} \tag{9}$$

for $i = 1, ..., L-1$ and, thus, $(L-1)$-times the information about $\mathbf{h}$. The CIR can be calculated by

$$\mathbf{h} = \frac{(L-1)^{-1}}{\sigma_w^2 h_0^*}\cdot\mathrm{E}\left\{\sum_{i=1}^{L-1}[\phi_{i,0}\cdot\phi_{0,0}^*]^{-1}\cdot r_0(k)^*\cdot\mathbf{r}_i(k)\right\}\ . \tag{10}$$

Thus, an estimate for the CIR within a complex scalar ambiguity, that is inherent to all blind estimation techniques, is given by

$$\hat{\mathbf{h}}_c = \frac{(L-1)^{-1}}{K\sigma_w^2 h_0^*}\cdot\sum_{k=0}^{K-1}\sum_{i=1}^{L-1}[\phi_{i,0}\cdot\phi_{0,0}^*]^{-1}\cdot r_0^*(k)\cdot\mathbf{r}_i(k)\ , \tag{11}$$

with K the number of IFDMA-symbols used to estimate the autocorrelation function at the receiver.

2) Multi-User Uplink Scenario: For $U > 1$ in the uplink, the received signal of the considered user u_1 is separated from the other users' signals at the base station in order to estimate the corresponding CIR $\mathbf{h}^{(u_1)}$. As the CP of user u_1 cannot be separated from the other users' CP, the signal of user u_1 at the base station is given by

$$\begin{aligned}\tilde{\mathbf{r}}^{(u_1)}(k) &= \mathbf{H}^{(u_1)}\cdot\mathbf{\Theta}^{(u_1)}\cdot[\mathbf{w}^{(u_1)}(k-1), \mathbf{w}^{(u_1)}(k)]^T \\ &+ \sum_{\substack{u=0\\u\neq u_1}}^{U-1}\left(\begin{bmatrix} \mathcal{H}_1^{(u)} & \mathcal{H}_0^{(u)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\right. \\ &\left.\cdot\mathbf{\Theta}^{(u)}\cdot\left[\mathbf{w}^{(u)}(k-1), \mathbf{w}^{(u)}(k)\right]^T\right) + \mathbf{n}(k)\ . \end{aligned} \tag{12}$$

The autocorrelation matrix $\tilde{\mathbf{R}}_r = \mathrm{E}\{\tilde{\mathbf{r}}^{(u_1)}(k)\cdot\tilde{\mathbf{r}}^{(u_1)}(k)^{\mathrm{H}}\}$ of $\tilde{\mathbf{r}}^{(u_1)}(k)$ yields

$$\tilde{\mathbf{R}}_r = \mathbf{R}_r + \sum_{\substack{u=0\\u\neq u_1}}^{U-1}\begin{bmatrix} \sigma_w^2\cdot(|\mathcal{H}_1^{(u)}|^2 + |\mathcal{H}_0^{(u)}|^2) & \mathbf{0}\cdots\mathbf{0} \\ \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \\ \vdots & \vdots\ddots\vdots \\ \mathbf{0} & \mathbf{0}\cdots\mathbf{0} \end{bmatrix}, \tag{13}$$

as $\mathrm{E}\{\mathbf{w}^{(u_1)}(k)\cdot\mathbf{w}^{(u_2)}(k)^{\mathrm{H}}\} = \mathbf{0}$, for $u_1 \neq u_2$. Thus, the additive term solely influences the part of the autocorrelation matrix $\mathbf{R}_r$ that is not used for the estimation of $\mathbf{h}^{(u_1)}$ according to (11) and the estimate $\hat{\mathbf{h}}_c^{(u_1)}$ is not disturbed by multi-user transmission in the uplink.

3) Multi-User Downlink Scenario: For $U > 1$ in the downlink, the signals of the U users experience the same CIR $\mathbf{h}^{(u_1)}$. As in the uplink scenario, the U users' signals can be separated except the CP. As shown in (13), the additive term caused by multi-user transmission does not affect the CE for the user under consideration. This is also valid for a downlink scenario.

B. Subspace Based Approach

1) Single-User Scenario: In order to apply the proposed Subspace Based Approach to the IFDMA signal of a user under consideration, three blocks of length Q of the received signal are considered. The last block $\mathbf{r}_{L-1}(k-1)$ of the previous IFDMA-symbol and the first two blocks $\mathbf{r}_0(k)$ and $\mathbf{r}_1(k)$ of the current IFDMA-symbol can be expressed as

$$\begin{aligned}\mathbf{r}_S(k) &= [\mathbf{r}_{L-1}(k-1), \mathbf{r}_0(k), \mathbf{r}_1(k)]^{\mathrm{T}} \\ &= \underbrace{\begin{bmatrix} \mathcal{H}_1 & \mathcal{H}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_1 & \mathcal{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathcal{H}_1 & \mathcal{H}_0 \end{bmatrix}}_{\tilde{\mathbf{H}}}\cdot\underbrace{\begin{bmatrix} \mathbf{\Phi}_{L-2} & \mathbf{0} \\ \mathbf{\Phi}_{L-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_0 \\ \mathbf{0} & \mathbf{\Phi}_1 \end{bmatrix}}_{\tilde{\mathbf{\Theta}}} \\ &\quad\cdot[\mathbf{w}(k-1), \mathbf{w}(k)]^{\mathrm{T}} + \mathbf{n}_S(k)\ . \end{aligned} \tag{14}$$

The autocorrelation matrix $\mathbf{R}_{r_s} = \mathrm{E}\{\mathbf{r}_S(k)\cdot\mathbf{r}_S(k)^{\mathrm{H}}\}$ of $\mathbf{r}_S(k)$ is given by

$$\mathbf{R}_{r_s} = \tilde{\mathbf{H}}\cdot\tilde{\mathbf{\Theta}}\cdot\sigma_w^2\cdot\mathbf{I}_{2Q}\cdot\tilde{\mathbf{\Theta}}^{\mathrm{H}}\cdot\tilde{\mathbf{H}}^{\mathrm{H}} + \sigma_n^2\cdot\mathbf{I}_{3Q}\ . \tag{15}$$

It is straightforward from the derivations in [12], that $\tilde{\mathbf{H}}$ is full column rank if no channel zero is located on the subcarriers in FD. Provided this, the matrix $\tilde{\mathbf{H}}\tilde{\mathbf{\Theta}}\sigma_w^2\mathbf{I}_{2Q}\tilde{\mathbf{\Theta}}^{\mathrm{H}}\tilde{\mathbf{H}}^{\mathrm{H}}$ has rank $2Q$ and describes the noise-free case. The matrix $\mathbf{R}_{r_s}$ has dimension $3Q\times 3Q$ and, thus, in the noise-free case, cannot be full rank. I.e., for the noisy case, signal and noise subspace can be separated by an eigenvalue decomposition of $\mathbf{R}_{r_s}$. The Q eigenvectors corresponding to the Q smallest eigenvalues are assumed to span the noise subspace. A detailed derivation of identifying signal and noise subspace can be found in [12]. Let $\mathbf{g}_0, \cdots, \mathbf{g}_{Q-1}$ denote the eigenvectors of $\mathbf{R}_{r_s}$ that span the noise subspace. Then, due to orthogonality between signal and noise subspace, the following equation must hold:

$$\mathbf{g}_n^{\mathrm{H}}\cdot\tilde{\mathbf{H}}\cdot\tilde{\mathbf{\Theta}} = \mathbf{0}, \qquad 0 \leq n \leq Q-1\ , \tag{16}$$

as the signal subspace is spanned by $\tilde{\mathbf{H}}\cdot\tilde{\mathbf{\Theta}}$. Since there are only estimates $\hat{\mathbf{g}}_n$ of $\mathbf{g}_n$ available, (16) is solved in the least-square sense and, thus, the following equation is to be minimized:

$$\sum_{n=0}^{Q-1} \| \hat{\mathbf{g}}_n^{\mathrm{H}} \cdot \tilde{\mathbf{H}} \cdot \tilde{\mathbf{\Theta}} \|^2 \tag{17}$$

In order to explicitly represent (17) as a function of $\mathbf{h}$, the matrices $\hat{\mathcal{G}}_n$ composed of $\hat{\mathbf{g}}_n$, that fulfill

$$\mathbf{h}^{\mathrm{H}} \cdot \hat{\mathcal{G}}_n \cdot \tilde{\mathbf{\Theta}} = \mathbf{0}, \qquad 0 \leq n \leq Q-1\ , \tag{18}$$

have to be found. The transformation of $\hat{\mathbf{g}}_n^{\mathrm{H}}$ into the matrix $\hat{\mathcal{G}}_n$ will not be explained in this work, as the main principle is identical to the transformation explained in [12] or [13]. It has also been shown in principle in [12], that an estimate $\hat{\mathbf{h}}_S$ of $\mathbf{h}$ can be found by minimizing $\sum_{n=0}^{Q-1} \| \mathbf{h}^{\mathrm{H}} \cdot \hat{\mathcal{G}}_n \cdot \tilde{\mathbf{\Theta}} \|^2$ under the constraint $\| \mathbf{h} \|= 1$. This yields, that $\hat{\mathbf{h}}_S$ is (up to a scalar complex ambiguity) the unit-norm eigenvector corresponding to the smallest eigenvalue of $\sum_{n=0}^{Q-1} \hat{\mathcal{G}}_n \cdot \tilde{\mathbf{\Theta}} \cdot \tilde{\mathbf{\Theta}}^{\mathrm{H}} \cdot \hat{\mathcal{G}}_n^{\mathrm{H}}$.

2) Multi-User Uplink Scenario: In the case of a multi-user uplink scenario, the separated received signal of the considered user u_1 at the base station still including the CP parts of the other users' signals is given by

$$\begin{aligned}\tilde{\mathbf{r}}_S^{(u_1)}(k) &= \tilde{\mathbf{H}}^{(u_1)} \cdot \tilde{\mathbf{\Theta}}^{(u_1)} \cdot [\mathbf{w}^{(u_1)}(k-1), \mathbf{w}^{(u_1)}(k)]^{\mathrm{T}} \\ &+ \sum_{\substack{u=0\\u\neq u_1}}^{U-1} \left(\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathcal{H}_1^{(u)} & \mathcal{H}_0^{(u)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{\Phi}_{L-1}^{(u)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Phi}_0^{(u)} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \right. \\ &\left. \cdot \left[\mathbf{w}^{(u)}(k-1), \mathbf{w}^{(u)}(k)\right]^{\mathrm{T}} \right) + \mathbf{n}_S(k)\ .\end{aligned} \tag{19}$$

The autocorrelation matrix $\tilde{\mathbf{R}}_{r_s}^{(u_1)} = \mathrm{E}\{\tilde{\mathbf{r}}_S^{(u_1)}(k) \cdot \tilde{\mathbf{r}}_S^{(u_1)}(k)^{\mathrm{H}}\}$ is given by

$$\tilde{\mathbf{R}}_{r_s}^{(u_1)} = \mathbf{R}_{r_s}^{(u_1)} + \sigma_w^2 \cdot \sum_{\substack{u=0\\u\neq u_1}}^{U-1} \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & |\mathcal{H}_1^{(u)}|^2 + |\mathcal{H}_0^{(u)}|^2 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\ . \tag{20}$$

It becomes clear, that the signal subspace is no longer exclusively spanned by the matrix $\tilde{\mathbf{H}}^{(u_1)} \cdot \tilde{\mathbf{\Theta}}^{(u_1)}$ as in (15), but additionally by the sum of the U users autocorrelated CP part of the signal. I.e., the identification of $\mathbf{h}^{(u_1)}$ with the help of (16) is distorted by the signals of the additional users in the system.

3) Multi-User Downlink Scenario: In the case of a multi-user downlink scenario, the received signal of user u_1 prior to the application of user separation can be expressed as

$$\tilde{\tilde{\mathbf{r}}}_S^{(u_1)}(k) = \tilde{\mathbf{H}}^{(u_1)} \cdot \sum_{u=0}^{U-1} \left(\tilde{\mathbf{\Theta}}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} \right) + \mathbf{n}_S^{(u_1)}(k)\ . \tag{21}$$

The autocorrelation matrix $\tilde{\tilde{\mathbf{R}}}_{r_s}^{(u_1)} = \mathrm{E}\{\tilde{\tilde{\mathbf{r}}}_S^{(u_1)}(k) \cdot \tilde{\tilde{\mathbf{r}}}_S^{(u_1)}(k)^{\mathrm{H}}\}$ of $\tilde{\tilde{\mathbf{r}}}_S^{(u_1)}(k)$ is given by

TABLE II
SYSTEM PARAMETERS

Carrier Frequency	3.7 GHz
Bandwidth	40 MHz
No. of Subcarriers	1024
Modulation	QPSK

$$\begin{aligned}\tilde{\tilde{\mathbf{R}}}_{r_s}^{(u_1)} &= E\left\{ \tilde{\mathbf{H}}^{(u_1)} \cdot \left(\sum_{u=0}^{U-1} \tilde{\mathbf{\Theta}}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} \right) \right. \\ &\left. \cdot \left(\sum_{u=0}^{U-1} \tilde{\mathbf{\Theta}}^{(u)} \cdot \begin{bmatrix} \mathbf{w}^{(u)}(k-1) \\ \mathbf{w}^{(u)}(k) \end{bmatrix} \right)^{\mathrm{H}} \cdot \tilde{\mathbf{H}}^{(u_1)^{\mathrm{H}}} \right\} + \sigma_n^2 \cdot \mathbf{I}_{3Q} \\ &= \tilde{\mathbf{H}}^{(u_1)} \cdot \sigma_w^2 \cdot \sum_{u=0}^{U-1} \left(\tilde{\mathbf{\Theta}}^{(u)} \cdot \tilde{\mathbf{\Theta}}^{(u)^{\mathrm{H}}} \right) \cdot \tilde{\mathbf{H}}^{(u_1)^{\mathrm{H}}} + \sigma_n^2 \mathbf{I}_{3Q}\ .\end{aligned} \tag{22}$$

Thus, in a multi-user downlink scenario, the CIR $\mathbf{h}^{(u_1)}$ can be estimated as described in Section III-B1 by considering the received signal prior to user separation and replacing $\tilde{\mathbf{\Theta}}$ by the sum of user dependent phase shift matrices $\sum_{u=0}^{U-1} \tilde{\mathbf{\Theta}}^{(u)}$.

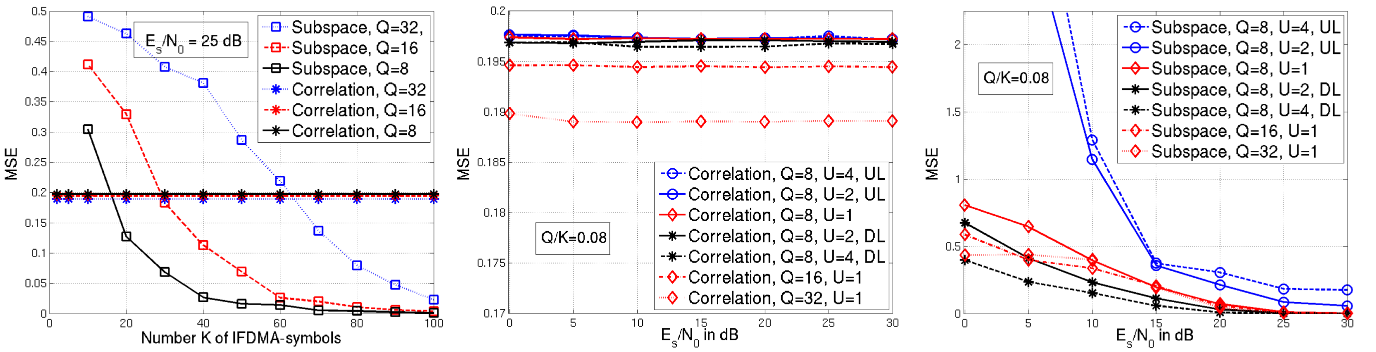
IV. Simulation Results

In this section, simulation results are given for the Correlation and the Subspace Based Approach. For both approaches, the influence of multiple users is investigated for the up- and downlink of an IFDMA-system. The parameters used for the simulations are given in Table II. The signal is transmitted over a time-invariant multi-path channel with $M = 5$ coefficients with a maximum delay $\tau_{max} = 125$ ns and an exponential decrease of power. The results are valid for the assumption that the maximum delay τ_{max} and the scalar ambiguity inherent to the estimates $\hat{\mathbf{h}}_{C/S}^{(u)}$ are known at the receiver. In Fig. 1, the Mean Square Error (MSE) between the estimated CIR $\hat{\mathbf{h}}_{C/S}^{(u)}$ and the true CIR $\mathbf{h}^{(u)}$ normalized by the number of channel coefficients, i.e. MSE $=\| \hat{\mathbf{h}}_{C/S}^{(u)} - \mathbf{h}^{(u)} \|^2/M$, is given as a result of 500 channel realizations. Further on, $\sigma_w^2 = 1$ and $Q = 8$ corresponds to a bit rate of 0.625 Mbit/s, $Q = 16$ to 1.25 Mbit/s and $Q = 32$ to 2.5 Mbit/s.

In Fig. 1(a), the convergence of the MSE is compared for the Correlation and Subspace Based Approach for $E_s/N_0 = 25$ dB in case of a single-user scenario. For the Correlation Based Approach, the MSE converges to MSE ≈ 0.19 for $K < 10$ IFDMA-symbols used to estimate the autocorrelation of the received signal. This is due to the additional averaging over L repetitions as described in Section III-A1. The bit rate, only slighty influences the MSE. For the Subspace Based Approach it is obvious, that the lower Q the faster the convergence. E.g., for $Q = 8$, the MSE reduces to MSE $= 0.001$ for $K = 100$ IFDMA-symbols. For $K < 10$, the MSE is not converging at all and, therefore, the results are omitted. Compared to the Correlation Based Approach, lower MSE-values are achievable for the Subspace Based Approach at the expense of the speed of convergence for $E_s/N_0 = 25$ dB.

In Fig. 1(b), the MSE is depicted in dependency of E_s/N_0 for the Correlation Based Approach for different data rates. Additionally, for $Q = 8$, the MSE is given for $U = 2, 4$ users for up- and downlink transmission, respectively. The number K of IFDMA-symbols is chosen such that the relation

International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Sep 2008, Cannes, France



(a) (b) (c)

Fig. 1. (a) MSE vs. K for Correlation and Subspace Based Approaches in dependency of Q at $E_s/N_0 = 25$ dB, (b) MSE vs. E_s/N_0 for the Correlation Based Approach in dependency of Q with $U = 1$ and in dependency of U for uplink (UL) and downlink (DL) scenarios with $Q = 8$ (c) MSE vs. E_s/N_0 for the Subspace Based Approach in dependency of Q with $U = 1$ and in dependency of U for uplink (UL) and downlink (DL) with $Q = 8$.

$Q/K = 0.08$ holds true for each value of Q and the MSE is only dependent on E_s/N_0. Again, the data rate has only slight impact on the MSE for the Correlation Based Approach. Moreover, the MSE is nearly independent of E_s/N_0 because the estimate given in (11) is not affected by AWGN. Further on, it is shown that the number of users hardly influences the performance, as it has been derived in Section III-A2.

In Fig. 1(c), the MSE for the Subspace Based Approach is investigated with the same parameters as the MSE for the Correlation Based Approach in Fig. 1(b). The MSE shows the particular behavior of subspace methods, i.e., high MSE for high noise powers and an MSE that tends to zero for low noise powers, as signal and noise subspace are ideally separable in this case. Thus, for $E_s/N_0 > 15$ dB, the Subspace Based Approach outperforms the Correlation Based Approach (cf. Fig. 1(b)). For $E_s/N_0 > 15$ dB, the MSE for the Subspace Based Approach is the lower the higher the value of Q. This is due to the assumption, that the number M of channel coefficients is known. Therefore, the higher Q, the more coefficients only consisting of noise are discarded. In case of multi-user transmission, the MSE is degraded if the signals of more than one user are transmitted in the uplink. It holds true, that the higher the number of users in the system the poorer the performance. Nevertheless, for $E_s/N_0 \geq 25$ dB, the Subspace Based Approach with $U = 4$ users shows the same performance as the Correlation Based Approach (cf. Fig. 1(b)). In contrast to that, in downlink, the MSE of the Subspace Based Approach improves with increasing number of users as the energy of the total received signal is increased and, therefore, the identifiability of signal and noise subspace is improved especially for low E_s/N_0.

V. Conclusion

Two SOS based blind CE algorithms, the Correlation and the Subspace Based Approach, have been applied to IFDMA. Furthermore, the two approaches have been investigated regarding multi-user transmission for an uplink and a downlink scenario. It has been shown, that the performance of the Correlation Based Approach is nearly independent of the signal-to-noise ratio and the bit rate and is not influenced by multi-user transmission neither in uplink nor in downlink. Moreover, the algorithm converges very fast compared to the Subspace Based Approach whose convergences is greatly influenced by the bit rate. Additionally, a high number of users in an uplink scenario degrades the performance of the Subspace Based Approach whereas in the downlink, the signal of additional users can be exploited to improve the CE performance. Although the Subspace Based Approach shows slow convergence and poor performance at low signal-to-noise ratios, it exhibits best results for high signal-to-noise ratios.

References

[1] R. van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, 1st ed. Artech House, 2000.

[2] A. Filippi, E. Costa, and E. Schulz, "Low Complexity Interleaved Sub-carrier Allocation in OFDM Multiple Access Systems," in *Proc. IEEE Vehicular Technology Conference*, Los Angeles, California, USA, September 2004, pp. 1890–1893.

[3] D. Galda, H. Rohling, E. Costa, H. Haas, and E. Schulz, "A Low Complexity Transmitter Structure for OFDM-FDMA Uplink Systems," in *Proc. IEEE Vehicular Technology Conference*, Birmingham, United Kingdom, May 2002, pp. 1737–1741.

[4] U. Sorger, I. De Broeck, and M. Schnell, "IFDMA - A New Spread-Spectrum Multiple-Access Scheme," in *Proc. ICC'98*, Atlanta, Georgia, USA, June 1998, pp. 1013–1017.

[5] T. Frank, A. Klein, E. Costa, and E. Schulz, "IFDMA - A Promising Multiple Access Scheme for Future Mobile Radio Systems," in *Proc. PIMRC 2005*, Berlin, Germany, Sep. 2005.

[6] T. Frank, A. Klein, E. Costa, and A. Kuehne, "Low complexity and power efficient space-time-frequency coding for OFDMA," in *Proc. of 15th Mobile & Wireless Communications Summit*, Mykonos, Greece, June 2006.

[7] T. Frank, A. Klein, and T. Haustein, "A Survey on the envelope fluctuations of DFT precoded OFDMA signals," in *Proc. of ICC 2008*, Beijing, China, May 2008.

[8] A. Sohl, T. Frank, and A. Klein, "Channel estimation for DFT precoded OFDMA with blockwise and interleaved subcarrier allocation," in *Proc. International OFDM Workshop 2006*, Hamburg, Germany, August 2006.

[9] A. Sohl and A. Klein, "Comparison of localized, interleaved and block-interleaved FDMA in terms of pilot multiplexing and channel estimation," in *Proc. of the 15th European Signal Processing Conference (EUSIPCO 2007)*, Poznan, Poland, September 2007.

[10] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second order statistics: a time domain approach," *IEEE Trans. on Information Theory*, vol. 40, no. 2, pp. 340–349, March 1994.

[11] M. K. Tsatsanis and G. B. Giannakis, "Transmitter induced cyclostationarity for blind channel equalization," *IEEE Trans. on Signal Processing*, vol. 45, no. 7, pp. 1785–1794, July 1997.

[12] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. on Signal Processing*, vol. 43, no. 2, pp. 516–525, February 1995.

[13] B. Muquet, M. de Courville, and P. Duhamel, "Subspace-Based blind and semi-blind channel estimation for OFDM systems," *IEEE Trans. on Signal Processing*, vol. 50, no. 7, pp. 1699–1712, July 2002.

[14] F. Gao and A. Nallanathan, "Blind channel estimation for OFDM systems via generalized precoding," *IEEE Trans. on Vehicular Technology*, vol. 56, no. 3, pp. 1155–1164, May 2007.