Throughput analysis of Multi-user OFDMA-Systems using imperfect CQI feedback and diversity techniques

Alexander Kühne, *Student Member, IEEE,* and Anja Klein , *Member, IEEE*

***Abstract*—In this paper, the throughput of an adaptive multi-user SISO-OFDMA/FDD system with channel quality information (CQI) signalled digitized over a feedback channel to the transmitter is investigated, where the instantaneous signal-to-noise ratio (SNR) of the different subcarriers is used as CQI to exploit multi-user diversity using adaptive subcarrier allocation. The CQI available at the transmitter is assumed to be imperfect due to estimation errors and quantization at the receiver side, time delay and feedback errors. In this paper, a closed form expression of the average throughput of an adaptive multi-user OFDMA system using imperfect CQI and uncoded M-QAM modulation is derived. Furthermore, a closed form expression of the average throughput of an OFDMA system exploiting frequency diversity, which does not require CQI at the transmitter, is presented. Both throughput performances are compared in order to identify the optimal transmission strategy depending on the grade of CQI imperfectness.**

***Index Terms*—OFDMA, adaptive modulation, multi-user diversity, imperfect channel knowledge, frequency diversity**

I. Introduction

In the recent research, the multicarrier scheme OFDM [1] is regarded as groundwork for future mobile radio systems, supporting very high data rates. In a multicarrier scheme like OFDM, the overall channel is divided into several subchannels in time and frequency dimension, so called subcarriers, which can be allocated to different connections. If the knowledge about the channel quality of the subcarriers is available at the transmitter side in a multi-user system, the subchannels can be allocated adaptively to the different users in order to exploit multi-user diversity [2, 3]. Having perfect channel knowledge at the transmitter, adaptive subchannel allocation schemes achieve very good performances [4, 5].

However, in a realistic scenario perfect instantaneous channel knowledge is not available at the transmitter which leads to a performance degradation using adaptive techniques compared to the performance when having perfect channel knowledge. In this case, the use of diversity techniques can be beneficial. Theoretically, the exploitation of diversity, which does not require instantaneous channel knowledge at the transmitter, leads to an averaging of the channel qualities of the different subchannels resulting in a performance enhancement. Applying frequency hopping [6] or applying a DFT-precoding of the data together with interleaved subcarrier allocation [7] are examples for techniques exploiting frequency diversity in an OFDM system. However, the achievable performance using diversity is worse compared to the performance using adaptive schemes with perfect channel knowledge. The assumption of perfect channel knowledge is rather unrealistic, while the abandonment of any kind of channel knowledge at the transmitter is again a too conservative approach. Hence, adaptive schemes using imperfect channel knowledge have to be considered. In a system with imperfect channel knowledge available at the transmitter the question is which of the two transmission strategies, adaption or diversity, provides the better performance depending on the considered scenario and the grade of channel knowledge imperfectness. Hence, a comparison between adaptivity with imperfect channel knowledge and diversity has to be made.

For single user transmission, adaptive OFDM having perfect channel state information (CSI) at the transmitter has been studied in previous works, for example in [8] - [10]. In [8], a power and bit loading algorithm for Discrete Multitone Modulation (DMT) systems is presented where the power and the data rate per subchannel is adapted to maximize the sum rate for a given target system performance margin. In [9] and [10], loading algorithms are proposed which maximize the system performance margin at a given target data rate.

For the case of single user transmission with imperfect or partial CSI at the transmitter, OFDM transmission schemes have also been studied, see [11]-[20] and references therein. In [11], adaptive OFDM with imperfect CSI for uncoded variable bit rates are studied, where the imperfect CSI arises from noisy channel estimates and the time delay of getting the CSI to the transmitter. The authors propose the use of multiple estimates to improve the performance. In [12], the impact of imperfect CSI is investigated for an adaptive OFDM system using the bit and power loading algorithm of [9]. In [13], a subchannel loading algorithm is proposed combating the negative effects resulting from channel errors in coherent detection at the receiver. In [14], the impact of imperfect one bit per subcarrier CSI feedback is studied. In [15], channel prediction is used to combat the impact of outdated CSI and in [16] a statistical adaptive modulation scheme based on long-term statistics is proposed. In [17], the minimum feedback rate required to determine the set of active subchannels using an on-off power allocation in a multicarrier transmission scheme is studied. Optimizing the activation threshold results in an achievable

Manuscript received November 1, 2007; revised April 15, 2008. The work in this paper was supported by the Deutsche Forschungsgemeinschaft (DFG) in the framework of the TakeOFDM project.

The authors are with Technische Universität Darmstadt, Communication Engineering Lab, Germany (e-mail: a.kuehne@nt.tu-darmstadt.de; a.klein@nt.tu-darmstadt.de).

data rate, which is shown to be asymptotically equivalent to the channel capacity. In [18], an optimal power loading algorithm for OFDM based on average and outage capacity criteria is presented assuming imperfect CSI at the transmitter. In [19], a limited feedback OFDM power loading algorithm is proposed using a codebook of power loading vectors. In [20], a loading algorithm is presented which aims at minimizing transmit power under rate and error probability constraints using quantized CSI. For single user OFDM systems with multiple antennas, the use of imperfect or partial CSI has been also investigated, e.g. in [21] and [22].

All above mentioned references considered the single user case. For the case of multi-user transmission, adaptive schemes exploiting multi-user diversity based on imperfect or partial CSI have also been studied, for example, in [23]-[27]. In [23], selective multi-user diversity is introduced, where only channel gains are fed back which are above a given threshold. In [24], the impact of partial CSI is studied in an OFDMA system, where each user only feeds back the CSI of the M best subcarriers. In [25], multi-user diversity with outdated channel information is studied. In [26], combinations of frequency and space based diversity techniques for a multi-user scenario with limited feedback are discussed. In [27], a multi-user scenario with either outdated or noisy estimated CSI is analysed.

In [28], a first comparison between adaptivity and diversity is drawn for a single user Multiple Input Multiple Output (MIMO) system, where space-time coding is compared to adaptive bit-and power loading. For perfect channel knowledge, the adaptive scheme provides the better performance as expected. In [29], a theoretical consideration between two special MIMO techniques, spatial multiplexing and spatial diversity is presented for the single user case. In [30, 31], adaptive subcarrier allocation with imperfect channel knowledge is compared to diversity techniques in a multi-user OFDMA system, where the ergodic capacity is taken as performance criterion, which only takes into account the effect of an erroneous user selection on the performance of the system, but not the effect of an erroneous channel adaption resulting from an inaccurate modulation scheme selection. A theoretical comparison between adaptive schemes and diversity techniques in a multi-user scenario under the consideration of the grade of channel knowledge imperfectness is missing as known to the authors.

In this paper, we study the impact of imperfect channel knowledge on the performance of an adaptive multi-user OFDMA downlink system, where we use the instantaneous signal-to-noise ratio (SNR) of the different subcarriers of the different users as channel quality information (CQI) to perform adaptive subcarrier allocation and adaptive M-QAM or M-PSK modulation, respectively. In the following, this transmission scheme is referred to as adaptive transmission mode. As performance criterion the average throughput is applied. For the CQI, the following assumptions are made:

- The CQI measured at the mobile station (MS) is only an estimate with a certain estimation error.
- The CQI is digitized before it is fed back over a feedback channel to the base station (BS).
- When detecting the feedback bits at the BS errors may occur.
- The CQI available at the BS is outdated due to time delays.

We provide a closed form expression of the average throughput under the assumption of imperfect CQI. Furthermore, we compare this throughput to the throughput achievable by exploiting frequency diversity without any channel knowledge at the transmitter referred to as non-adaptive transmission mode in order to identify the optimal transmission mode depending on the grade of CQI imperfectness.

The remainder of this paper is organised as follows. Section II describes the system model. In Section III, the adaptive and non-adaptive transmission modes are presented. In Section IV, the different sources of errors of the CQI are introduced together with parameters describing the CQI imperfectness. In Section V, the average throughput is derived analytically for the case of imperfect CQI using the adaptive transmission mode. Section VI provides the analytical derivation of the average throughput in the non-adaptive transmission mode using frequency diversity. Section VII presents how to choose between the two transmission modes based on the available channel knowledge. In Section VIII, the achievable throughputs for the adaptive and non-adaptive transmission are illustrated and compared for a realistic OFDMA scenario. Finally, conclusions are drawn in Section IX.

II. System model

As system model a one cell SISO downlink scenario in an FDD system with one BS and U MSs with user index $u = 1, \cdots, U$ is considered. Furthermore, it is assumed that each user has the same requirements in terms of data rate. OFDMA is employed to subdivide the downlink bandwidth into N orthogonal subcarriers, where the channel response of each subcarrier is assumed to be flat. Note, that a subcarrier can also be interpreted as a representative of a block of subcarriers, so called chunks or clusters [32]. The transfer factor $H_u(n,k)$ of the n-th subcarrier with $n = 1, \cdots, N$ of user u at time slot $k \in \mathbf{N}$ is modeled by a complex Gaussian distributed random process with variance one. In the following, only fast fading is considered, i.e. the effects of path loss and shadowing [33] are assumed to be ideally compensated by power control. From this it follows that all users experience the same average SNR $\bar{\gamma}$ and that the instantaneous SNR $\gamma_u(n,k)$ of user u on the subcarrier with index n in time slot k is given by

$$\gamma_u(n,k) = \bar{\gamma} \cdot |H_u(n,k)|^2 . \quad (1)$$

III. Two transmission modes

The considered adaptive OFDMA system has the ability to switch between an adaptive transmission mode and a non-adaptive transmission mode which exploits frequency diversity, like it is proposed in [34]. Depending on the grade of imperfectness of the available channel knowledge, the mode that provides the highest throughput is activated, which is explained in details in Section VII. In the following, the two transmission modes are presented.

A. Adaptive transmission mode

When using the adaptive transmission mode, an adaptive subcarrier allocation and modulation using the CQI at the BS is performed. As CQI, the digitized instantaneous available SNR of different users when allocated to different subcarriers is applied. By doing so, one can benefit from multi-user diversity [2, 3]. In this paper, a Max-SNR Scheduler is employed that favours the users with the best SNR conditions when allocating the different subcarriers to the different users. One subcarrier is allocated to only one user exclusively. In case of several equally strong users, the subcarrier is allocated randomly among these users. Since each user experiences the same average SNR $\bar{\gamma}$, the probability of getting access to a subcarrier is equal for all users and given by $P_a = \frac{1}{U}$, i.e. the scheduler is long-term fair [23]. After assigning all subcarriers to the different users, a modulation scheme is selected for each subcarrier based on the CQI, where it is assumed that the transmit power is equal for each subcarrier. In this work, the following modulation schemes are considered: BPSK, QPSK, 8-PSK, 16-QAM, 32-QAM, 64-QAM and 128-QAM.

B. Non-adaptive transmission mode

In contrast to the adaptive transmission mode, the non-adaptive transmission mode does not require any instantaneous knowledge about the channel quality of different users. Now, all N subcarriers are allocated to one user u exclusively at each time slot. A transmission scheme is used to exploit frequency diversity, e.g. DFT precoded OFDM [7]. As with the adaptive transmission mode, each user gets the same amount of channel accesses. Assuming, that the average SNR $\bar{\gamma}$ is known to the transmitter, one fixed modulation scheme is selected for all subcarriers.

IV. Modelling imperfect CQI

In this section, the four different sources of error for imperfect CQI are presented together with the model and the parameters describing the imperfectness. In the following, we assume that these parameters are the same for each user.

A. CQI with an estimation error

In a realistic scenario, the measured channel transfer function is only an estimate of the actual channel transfer function. Assuming minimum mean square error (MMSE) estimation, and skipping the user, subcarrier and time-slot indices, the MMSE estimate is denoted by $\hat{H}$ and the estimation error by $E = H - \hat{H}$. The random variables $\hat{H}$ and E are uncorrelated, where E is complex Gaussian distributed with variance σ_E^2. The estimated channel transfer function $\hat{H}$ is also complex Gaussian distributed with variance $1 - \sigma_E^2$, where $\sigma_E^2 \in [0, 1]$ depends on the conditions of the channel and the applied estimation scheme. In [35], the authors consider a block Rayleigh fading channel, where orthonormal training signals are used. In this case, the estimation error variance is given by

$$\sigma_E^2 = \frac{1}{1 + T_\tau P_\tau} \tag{2}$$

where T_τ is the number of training symbols and P_τ the SNR during the training phase. It is assumed that σ_E^2 is known both to the transmitter and receiver.

B. Digitized CQI

In order to decrease the amount of feedback, the CQI of each subcarrier n in each time slot k is digitized at each MS u. In this case, the scheduler at the BS can not distinguish between the channel qualities of different users as precisely as with analog CQI, since there is only a limited numbers of CQI levels. The digitalization is done in two steps. First, each measured SNR value is quantized in $W = 2^{N_Q}$ quantization levels with $W + 1$ quantization bounds s_l with $l = 0, \cdots, W$, where $s_0 = 0, s_W = \infty$ and N_Q denotes the number of quantization bits per subcarrier. Second, the quantized CQI feedback is digitized according to a certain bit coding scheme which is defined by a $W \times W$ matrix $\mathcal{B}$. The (i, j)-th element $b_{i,j}$ of matrix $\mathcal{B}$ with $i, j = 1, \cdots, W$ contains the number of bits which differ comparing the bit coding of the i-th quantization level $[\gamma_{i-1}, \gamma_i]$ to the bit coding of the j-th quantization level $[\gamma_{j-1}, \gamma_j]$.

C. Digitised CQI with feedback errors

In a realistic scenario, the transmission of the digital CQI over the feedback channel can not be assumed to be error-free. Depending on the condition of the feedback channel and the used modulation and coding scheme, errors may occur when detecting the feedback bits with a certain bit error rate p_b. If an error occurs when detecting the feedback bits, an SNR value, which was measured to be in the i-th quantization level $[\gamma_{i-1}, \gamma_i]$ is now assumed to be in the j-th quantization level $[\gamma_{j-1}, \gamma_j]$. To determine the probability of this event, the $W \times W$ matrix $\mathbf{D}$ is introduced. The (i, j)-th element $d_{i,j}$ of $\mathbf{D}$ with $i, j = 1, \cdots, W$ denotes the probability that an SNR value which was measured at the MS to be in the j-th quantization level $[\gamma_{j-1}, \gamma_j]$ is assumed to be in the i-th quantization level $[\gamma_{i-1}, \gamma_i]$ at the BS. Matrix $\mathbf{D}$ is calculated using the bit coding matrix $\mathcal{B}$ according to

$$d_{i,j} = (1 - p_b)^{N_Q - b_{i,j}} \cdot p_b^{b_{i,j}}, \tag{3}$$

where $(1-p_b)^{N_Q - b_{i,j}}$ determines the probability that $N_Q - b_{i,j}$ bits are received correctly and $p_b^{b_{i,j}}$ determines the probability that $b_{i,j}$ bits are received incorrectly.

D. Outdated CQI

Due to the fact that there is always a time delay between the instant of SNR measuring and the actual instant of transmission of the data to the scheduled users, the available CQI at the transmitter is outdated. Outdated CQI is modelled by correlation, i.e. the outdated channel transfer function and the actual channel transfer function are modelled as two complex Gaussian distributed random variables with a correlation coefficient ρ. Assuming Jakes scattering model, the correlation coefficient $\rho \in [0, 1]$ is given by

$$\rho = J_0 (2\pi f_D T), \tag{4}$$

with $J_0(x)$ denoting the 0-th order Bessel function, where f_D denotes the maximum Doppler frequency and T the delay time between the outdated and actual channel transfer function realisation. The vehicular speed corresponding to this Doppler frequency is given by $v = c \cdot (f_D/f_c)$ with f_c denoting the carrier frequency and c the speed of light.

V. Average throughput using the adaptive transmission mode

In this section, we analytically derive expressions for the average throughput for the different types of imperfect CQI introduced in Section IV using the adaptive transmission mode.

A. Derivation of average throughput using imperfect CQI

The average throughput for perfect channel knowledge is defined as the expectation value of the number of bits which are correctly received during a data transmission packet of length L using a modulation scheme with index m, i.e.,

$$\bar{\eta} = c_m \cdot \int_0^\infty (1 - SER_m(\gamma))^L \cdot p_\gamma(\gamma)\, d\gamma, \tag{5}$$

where c_m is the number of bits per symbol corresponding to the modulation scheme and $p_\gamma(\gamma)$ denotes the probability density function (PDF) of the actual SNR. The expression $PER = (1 - SER_m(\gamma))^L$ denotes the packet error rate (PER). The symbol error rate (SER) of M-QAM modulation can be approximated by

$$SER_m(\gamma) = \alpha_m \cdot \exp(-\beta_m \gamma) \tag{6}$$

with $\alpha_m = 0.2 \cdot c_m$, $\beta_m = \frac{1.6}{2^{c_m} - 1}$ using M-QAM modulation and $\beta_m = \frac{7}{2^{1.9 c_m} + 1}$ using M-PSK modulation, respectively [36]. For the special case $m = 1$ (BPSK), $\beta_1 = 1$.

For adaptive modulation, different modulation schemes are applied for different channel conditions. The bounds γ_{m-1} and γ_m , with $m = 1, ..., card(\mathcal{M})$, determine the interval, in which a particular modulation scheme is applied. In case of digitized CQI, these modulation bounds are identical to the quantization bounds introduced in Section IV. The average throughput is given by

$$\bar{\eta} = \sum_{m=1}^{card(\mathcal{M})} c_m \cdot \int_{\gamma_{m-1}}^{\gamma_m} (1 - SER_m(\gamma))^L \cdot p_\gamma(\gamma)\, d\gamma, \tag{7}$$

where $\mathcal{M}$ denotes a certain selection of modulation schemes, with $card(\mathcal{M})$ the cardinality of $\mathcal{M}$.

In the case of imperfect CQI, the selection of the scheduled user and the applied modulation scheme is based on possibly erroneous channel knowledge, i.e. the scheduler possibly selects a user with a weak channel and in addition selects a modulation scheme for this user which is only suitable for high SNR channels. On the other hand, a robust modulation scheme could be selected for a supposed weak user due to imperfect CQI, resulting in a waste of channel capacity. Both effects, the possibly erroneous user and modulation scheme selection, result in a throughput degradation and have to be taken into account when determining the average throughput using imperfect CQI, leading to

$$\begin{aligned}\bar{\eta} &= \sum_{m=1}^{card(\mathcal{M})} c_m \cdot \int_{\gamma_{m-1}}^{\gamma_m} p_{\hat{\gamma}}(\hat{\gamma}) \\ &\cdot \left[\int_0^\infty (1 - SER_m(\gamma))^L p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})\, d\gamma\right] d\hat{\gamma},\end{aligned} \tag{8}$$

where $p_{\hat{\gamma}}(\hat{\gamma})$ denotes the PDF of the assumed SNR values of the scheduled users at the BS and $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ the conditional PDF of the actual SNR γ and the assumed SNR $\hat{\gamma}$. The PDF $p_{\hat{\gamma}}(\hat{\gamma})$ of the assumed SNR values depends on the subcarrier scheduling and the type of available CQI, which will be explained later on. The conditional PDF $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ only depends on the ρ, σ_E^2 and the average SNR $\bar{\gamma}$ and is given by, (see Appendix A),

$$\begin{aligned}p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) &= \frac{1}{\bar{\gamma}\sigma_r^2} \cdot I_0\left(\frac{2\rho\sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}\sigma_r^2}\right) \\ &\cdot \exp\left(-\frac{(\rho^2 \cdot \hat{\gamma} + \gamma)}{\bar{\gamma} - \sigma_r^2}\right),\end{aligned} \tag{9}$$

with $\sigma_r^2 = 1 - \rho^2(1 - \sigma_E^2)$, where $I_0(x)$ denotes the 0th-order modified Bessel function of the first kind.

B. Average throughput using analog CQI

In the following, analog CQI denotes outdated CQI with estimation errors. In the case of analog CQI feedback, the throughput degradation is caused by the fact that the CQI is only an outdated estimation of the actual channel. The Max-SNR scheduler at the BS selects the best user out of U users, where the SNR of each user is Rayleigh distributed. Hence, the PDF $p_{\hat{\gamma}}(\hat{\gamma})$ of the assumed SNR of the selected user is given by

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{U}{E\{\hat{\gamma}\}} \sum_{k=0}^{U-1} \binom{U-1}{k} \cdot (-1)^k \exp\left(-\frac{\hat{\gamma}(k+1)}{E\{\hat{\gamma}\}}\right) \tag{10}$$

using order statistics [38], where $E\{\hat{\gamma}\} = \bar{\gamma}(1 - \sigma_E^2)$. In Fig. 1, the PDF of the assumed SNR of the selected user is illustrated for a system with $U = 5$ users and an average SNR $\bar{\gamma} = 10$ dB. The solid line represents the PDF evaluated from a simulation with 10000 snapshots and the dashed line represents the theoretical PDF of 10. As one can see, the theoretical curve is consistent with the simulative one. Inserting (10), (9) and (6) in (8) and using the identities [39, Eq. 6.643.4], [39, Eq. 8.406.3], [39, Eq. 8.970.1] and [39, Eq. 1.111], the average throughput $\bar{\eta}_{A,an}$ for analog CQI feedback is given by

$$\begin{aligned}\bar{\eta}_{A,an} &= U \cdot \sum_{m=1}^{card(\mathcal{M})} c_m \sum_{k=0}^{U-1} \binom{U-1}{k} \cdot (-1)^k \\ &\sum_{l=0}^{L} \binom{L}{l} \frac{(-1)^l \alpha_m^l}{\beta_m l \bar{\gamma}(1 + k\sigma_r^2) + (k+1)} \\ &\cdot \left[\exp\left(-\frac{\gamma_{m-1}[\beta_m l \bar{\gamma}(1 + k\sigma_r^2) + (k+1)]}{E\{\hat{\gamma}\}(1 + \beta_m l \bar{\gamma}\sigma_r^2)}\right)\right. \\ &\left. - \exp\left(-\frac{\gamma_m[\beta_m l \bar{\gamma}(1 + k\sigma_r^2) + (k+1)]}{E\{\hat{\gamma}\}(1 + \beta_m l \bar{\gamma}\sigma_r^2)}\right)\right]\end{aligned} \tag{11}$$

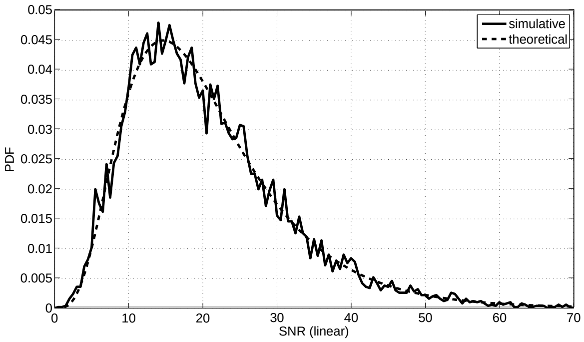

Fig. 1. PDF of the assumed SNR of scheduled user in linear scale for $U = 5$ users, $\sigma_E^2 = 0.09$ and $\bar{\gamma} = 10$ dB for analog CQI

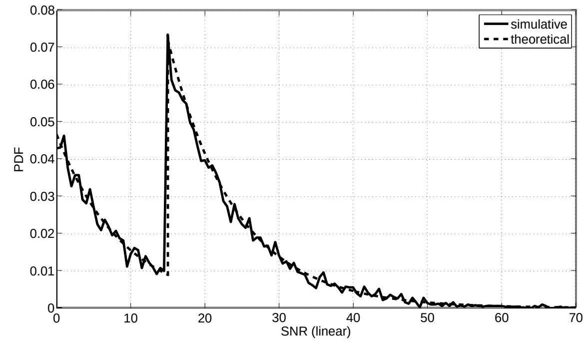

Fig. 2. PDF of the assumed SNR of scheduled user in linear scale for $U = 5$ users, $\sigma_E^2 = 0.09$, $\bar{\gamma} = 10$ dB, $N_Q = 1$ and $\gamma_1 = 15$ for digitized CQI

with $E\{\hat{\gamma}\} = \bar{\gamma}(1 - \sigma_E^2)$ and $\sigma_r^2 = 1 - \rho^2(1 - \sigma_E^2)$.

C. Average throughput using digitized CQI

In the following, digitized CQI denotes outdated digitized CQI with estimation errors. In the case of digitized CQI, three effects result in a throughput degradation compared to perfect analog CQI. First, the SNR values available at the MS are estimates, i.e. the SNR values is possibly quantized in the wrong quantization level. Secondly, due the quantization, the scheduler at the BS can not distinguish between users within the same quantization level, i.e. the scheduler has to choose randomly between those users. Finally, the signalled CQI is already outdated at the actual time instant of data transmission to the scheduled users. The PDF of the assumed SNR of the scheduled user in case of digitized CQI is given by

$$\begin{aligned} p_{\hat{\gamma}}(\hat{\gamma}) &= \sum_{m=1}^{card(\mathcal{M})} \frac{a_m}{E\{\hat{\gamma}\}} \exp\left(-\frac{\hat{\gamma}}{E\{\hat{\gamma}\}}\right) \\ &\cdot \quad [\sigma(\hat{\gamma} - \gamma_{m-1}) - \sigma(\hat{\gamma} - \gamma_m)] \end{aligned} \tag{12}$$

with

$$a_m = \frac{\left(1 - \exp\left(-\frac{\gamma_m}{E\{\hat{\gamma}\}}\right)\right)^U - \left(1 - \exp\left(-\frac{\gamma_{m-1}}{E\{\hat{\gamma}\}}\right)\right)^U}{\exp\left(-\frac{\gamma_{m-1}}{E\{\hat{\gamma}\}}\right) - \exp\left(-\frac{\gamma_m}{E\{\hat{\gamma}\}}\right)}, \tag{13}$$

see Appendix B, where $\sigma(x)$ denotes the step function. In Fig. 2, the PDF of the assumed SNR of the scheduled user is depicted for a system with digitized CQI, where $N_Q = 1$ quantization bit is used and the quantization bound is $\gamma_1 = 15$. As one can see, both the simulative and theoretical curve correspond to each other, where at an SNR of $\hat{\gamma} = 15$ the step in the PDF function is clearly visible. Inserting (12), (9) and (6) in (8) leads to the average throughput $\bar{\eta}_{A,dig}$ for digitized outdated CQI feedback with estimation errors given by (14) as shown on the top of the next page.

D. Average throughput using digitized CQI with feedback errors

In the following, digitized CQI with feedback errors denotes outdated digitized CQI with estimation and feedback errors. In addition to the three effects described before, there occur a fourth effect using imperfect digitized CQI with feedback errors. Due to the feedback bit errors, the SNR values are possibly assumed to be in the wrong quantization level at the BS resulting in a throughput degradation. In order to determine the PDF of the assumed SNR $\hat{\gamma}$ of the scheduled user, the $W \times 1$ vector $\mathbf{z}$ is introduced containing the elements

$$z_i = \exp\left(-\frac{\gamma_{i-1}}{E\{\hat{\gamma}\}}\right) - \exp\left(-\frac{\gamma_i}{E\{\hat{\gamma}\}}\right), \tag{15}$$

with $i = 1, \cdots, W-1$, which determine the probability that a measured SNR value at the MS is in the i-th quantization level $[\gamma_{i-1}, \gamma_i]$. The $W \times 1$ vector $\mathbf{p}$ is calculated according to $\mathbf{p} = \mathbf{D} \cdot \mathbf{z}$, where the j-th element p_j of vector $\mathbf{p}$ with $j = 1, \cdots, W-1$ denotes the probability that the signalled SNR value is assumed to be in the j-th quantization level $[\gamma_{j-1}, \gamma_j]$ at the BS. The PDF of the assumed SNR $\hat{\gamma}$ of the scheduled user in case of digitized CQI with feedback errors is then given by

$$\begin{aligned} p_{\hat{\gamma}}(\hat{\gamma}) &= \sum_{m=1}^{card(\mathcal{M})} \tilde{a}_m \sum_{k=1}^{card(\mathcal{M})} \frac{d_{m,k}}{E\{\hat{\gamma}\}} \\ &\cdot \exp\left(-\frac{\hat{\gamma}}{E\{\hat{\gamma}\}}\right) [\sigma(\hat{\gamma} - \gamma_{k-1}) - \sigma(\hat{\gamma} - \gamma_k)] \end{aligned} \tag{16}$$

with

$$\tilde{a}_m = \frac{\left(\sum_{j=1}^{m} p_j\right)^U - \left(\sum_{j=1}^{m-1} p_j\right)^U}{p_m} \tag{17}$$

see Appendix B. In Fig. 3, the PDF of the assumed SNR of the scheduled user is presented for a system with digitized CQI with feedback errors, where $N_Q = 1$ quantization bit is used, the quantization bound is $\gamma_1 = 15$ and the average BER of the feedback transmission $p_b = 10^{-1}$. Again, the simulative and theoretical curves correspond to each other. Comparing the PDF of Fig. 2 with the PDF of Fig. 3, one can see that the probability for an assumed SNR value to be in the first quantization level increases. This originates from the fact that, due to the feedback errors, weak users from the first quantization level are wrongly assumed to be in the second quantization level and thus selected for transmission. Inserting (16), (9) and (6) in (8) leads to the average throughput $\bar{\eta}_{A,dig,FB}$ for digitized CQI feedback with feedback errors

$$\bar{\eta}_{A,dig} = \sum_{m=1}^{card(\mathcal{M})} a_m \cdot c_m \sum_{l=0}^{L} \binom{L}{l} \frac{(-1)^l \alpha_m^l}{1+\beta_m l \bar{\gamma}} \cdot \left[\exp\left(-\frac{\gamma_{m-1}(1+\beta_m l\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m l \bar{\gamma}\sigma_r^2)}\right) - \exp\left(-\frac{\gamma_m(1+\beta_m l\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m l\bar{\gamma}\sigma_r^2)}\right)\right] \quad (14)$$

$$\bar{\eta}_{A,dig,FB} = \sum_{m=1}^{card(\mathcal{M})} \tilde{a}_m \cdot c_m \sum_{k=1}^{card(\mathcal{M})} d_{m,k} \cdot \sum_{l=0}^{L} \binom{L}{l} \frac{(-1)^l \alpha_m^l}{1+\beta_m l\bar{\gamma}} \cdot \left[\exp\left(-\frac{\gamma_{k-1}(1+\beta_m l\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m l\bar{\gamma}\sigma_r^2)}\right) - \exp\left(-\frac{\gamma_k(1+\beta_m l\bar{\gamma})}{E\{\hat{\gamma}\}(1+\beta_m l\bar{\gamma}\sigma_r^2)}\right)\right] \quad (18)$$

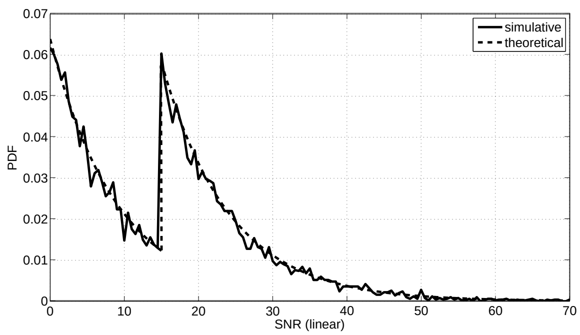

Fig. 3. PDF of the assumed SNR of scheduled user in linear scale for $U = 5$ users, $\sigma_E^2 = 0.09$, $\bar{\gamma} = 10$ dB, $N_Q = 1$, $\gamma_1 = 15$ and $p_b = 10^{-1}$ for digitized CQI with feedback errors

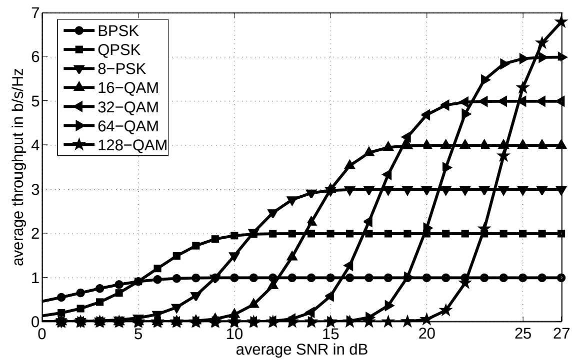

Fig. 4. Average throughput using non-adaptive transmission mode vs. average SNR $\bar{\gamma}$ for $N = 128$ and $L = 10$

given by (18) as depicted on the top of the next page.

Thus, it is possible to determine closed form expressions of the average throughput as a function of all types of imperfect CQI introduced in Section IV. By setting $p_b = 0$, $\sigma_E^2 = 0$ and $\rho = 1$, the different sources of errors caused by channel estimation, feedback transmission and time delay can be switched off.

VI. Average throughput using non-adaptive transmission mode

Using a transmission technique that exploits frequency diversity leads to an averaging over the N different subcarrier SNR conditions. By assuming that the channel responses of adjacent subcarriers are independent from each user, the PDF of the resulting SNR is a chi-square distribution with $2N$ degrees of freedom [37] and given by

$$p_\gamma(\gamma) = \left(\frac{N}{\bar{\gamma}}\right)^N \cdot \frac{\gamma^{N-1}}{(N-1)!} \cdot \exp\left(-\frac{\gamma N}{\bar{\gamma}}\right). \quad (19)$$

Inserting (19) in (5) and using the identities [39, Eq. 1.111] and [39, 3.351.3], the average throughput using a diversity technique with a fixed modulation scheme with index m is given by

$$\bar{\eta}_D = c_m \sum_{l=0}^{L} \binom{L}{l} (-1)^l \alpha_m^l \left[\frac{N}{N+\beta_m \bar{\gamma} l}\right]^N. \quad (20)$$

In Fig. 4, the average throughput according to (20) is depicted as a function of the average SNR $\bar{\gamma}$ for a system with $N = 128$ subcarrier and a packet length of $L = 10$ for the different modulation schemes. For this set of parameters, the mapping rule between the average SNR and the modulation level is given by Table I. The different threshold values can be directly read off from the intersection points in Fig. 4.

TABLE I
MAPPING RULE

Modulation	BPSK	QPSK	8-PSK	16-QAM
SNR $\bar{\gamma}$ in dB	$[-\infty, 5]$	$[5, 11]$	$[11, 15]$	$[15, 18.7]$

Modulation	32-QAM	64-QAM	128-QAM
SNR $\bar{\gamma}$ in dB	$[18.7, 22.3]$	$[22.3, 25.7]$	$[25.7, \infty]$

VII. Adaptive transmission mode with imperfect CQI vs. non-adaptive transmission mode

With the derived formulas of the average throughput for the adaptive and non-adaptive transmission mode, the optimal transmission mode regarding the throughput can be identified. As seen in Section IV, the parameters defining the quality of the channel knowledge are the correlation coefficient ρ between the actual and the outdated channel transfer function, which corresponds to the normalized Doppler frequencies $f_D T$, the estimation error variance σ_E^2, which corresponds to a number T_τ of training symbols, and the average BER p_b of the feedback channel. The parameters defining the scenario, which are assumed to be known to both the transmitter and receiver, are the number U of active users, the number N_Q of quantization bits per subcarrier, the number N of subcarriers, the packet length L and the average SNR $\bar{\gamma}$. The parameters which can be adaptively changed by the system are the quantization levels $[\gamma_{m-1}, \gamma_m]$ with $m = 1, ..., card(\mathcal{M})$, the bit coding scheme $\mathcal{B}$ and the applied modulation schemes $\mathcal{M}$. Now, for a given set of channel knowledge parameters (ρ,σ_E^2,p_b) and scenario parameters (U,N_Q,N,L,$\bar{\gamma}$) the optimal throughput $\bar{\eta}_{A,opt}$ using the adaptive transmission mode can be found by optimising the throughput $\bar{\eta}_A$ according to (18) with regard to the set of adaptive parameters ($[\gamma_{m-1}, \gamma_m]$,$\mathcal{B}$,$\mathcal{M}$), i.e.,

$$\bar{\eta}_{A,opt} = \max_{[\gamma_{m-1},\gamma_m],\mathcal{B},\mathcal{M}} \left(\bar{\eta}_A\right). \tag{21}$$

This can be done by numerical optimisation. Note that the computational complexity of this optimisation is not considered in this work. However, this computation can be done offline for several possible sets of parameters and the results can be stored in a look-up table. Next, the modulation scheme which maximises the average throughput using the non-adaptive transmission mode (20) has to be found. The optimal average throughput $\bar{\eta}_{D,opt}$ using the non-adaptive transmission mode results in

$$\bar{\eta}_{D,opt} = \max_{m} \left(\bar{\eta}_D\right). \tag{22}$$

Finally, the two optimised throughput values of the adaptive and non-adaptive transmission modes are compared and the maximum is determined, leading to

$$\bar{\eta}_{opt} = \max\left(\bar{\eta}_{A,opt}, \bar{\eta}_{D,opt}\right). \tag{23}$$

VIII. Numerical results

In the following, the achievable throughputs using the adaptive and non-adaptive transmission modes are illustrated for an OFDMA/FDD system with $N = 128$ subcarriers. Due to the high amount of feedback in such an FDD system, we restrict ourselves to a maximal number of $N_Q = 2$ feedback bits per subcarrier to be signalled as CQI to the BS. Furthermore, we assume a packet length of $L = 10$ and $T_\tau = 1$ training symbols with $P_\tau = \bar{\gamma}$, leading to $\sigma_E^2 = (1 + \bar{\gamma})^{-1}$.

In a first example, we assume a system with $U = 25$ users, where the average SNR is $\bar{\gamma} = 10$ dB, using $N_Q = 1$ bits feedback per subcarrier. As discussed in Section VII, the modulation scheme which maximises the average throughput using the non-adaptive mode has to be determined. From Fig. 4 and Table I, it can be seen that the QPSK modulation scheme provides the highest throughput for $\bar{\gamma} = 10$ dB, resulting in $\bar{\eta}_{D,opt} = 1.96$ b/s/Hz. Next, the average throughput using the adaptive mode is maximised for the given set of scenario and channel knowledge parameters with regard to the set of adaptive parameters ($[\gamma_{m-1}, \gamma_m]$,$\mathcal{B}$,$\mathcal{M}$).

In Fig. 5 the average throughput of the adaptive transmission mode is depicted as a function of the average BER p_b of the feedback channel for different normalized Doppler frequencies f_DT and an estimation error variance $\sigma_E^2 = 0.09$. Assuming that the channel knowledge parameters ρ and p_b are not known to the BS, the transmitter assumes feedback error and delay-free CQI, i.e. $\rho = 1$ and $p_b = 0$. Using (21), the optimal set of adaptive parameters is determined, which are $\gamma_{1,opt} = 19.3$, $\mathcal{M}_{opt}$ ={QPSK,8-PSK} and $\mathcal{B}_{opt} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, where for $N_Q = 1$ quantization bit only one possible bit coding matrix $\mathcal{B}$ exists. The average throughput is calculated according to (18) with the fixed set of optimal adaptive parameters ($\gamma_{1,opt}$, $\mathcal{M}_{opt}$, $\mathcal{B}_{opt}$) for each set of channel knowledge parameter (f_DT, p_b). When being aware of the channel knowledge, the optimal adaptive parameters are determined for each set of channel knowledge parameters, i.e. for each set of (f_DT, p_b) there exists an optimal set of adaptive parameters. The average throughput is again calculated according to (18) using the optimal set of adaptive parameters for each set of channel knowledge parameter (f_DT, p_b). The dashed lines represent

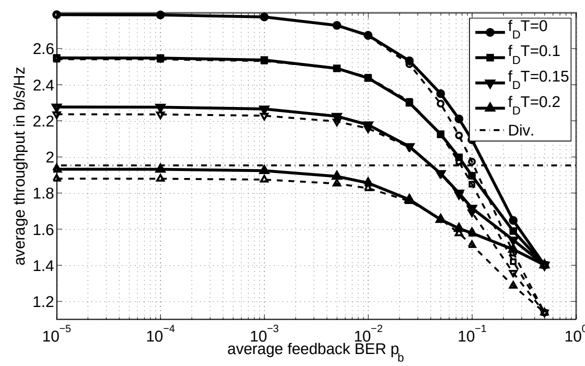

Fig. 5. Comparsion average throughput adaptive mode vs. non-adaptive mode for $\bar{\gamma} = 10$ dB, $N = 128$, $L = 10$, $\sigma_E^2 = 0.09$ and $N_Q = 1$, (dashed line: feedback error and delay-free CQI assumed; solid line: CQI imperfectness known)

the achievable throughput assuming feedback error and delay-free CQI. The solid lines represent the achievable throughput when being aware of the channel knowledge parameters. The constant dot-dashed curve represents the throughput achievable with the non-adaptive transmission mode. From Fig. 5 it can be seen that being aware of the fact that the CQI feedback is not perfect, one can achieve a better throughput by adapting the parameters ($[\gamma_{m-1}, \gamma_m]$,$\mathcal{B}$,$\mathcal{M}$) to this CQI imperfectness. This effect becomes apparent especially in the BER region $p_b > 10^{-1}$. Furthermore, it can be seen that as long as the BER $p_b < 10^{-3}$, the effect of the feedback errors can be neglected. Comparing the throughput of the adaptive mode with the throughput achievable with the non-adaptive mode, it can be seen, that for a BER $p_b < 10^{-3}$ and $f_DT < 0.2$, the adaptive mode provides a higher throughput than the non-adaptive mode. For cases with $f_DT > 0.2$, the non-adaptive mode is the better choice. Assuming a time delay of $T = 1$ ms and a carrier frequency of $f_c = 5$ GHz, $f_DT = 0.2$ corresponds to a vehicular speed of $v = 43.2$ km/h.

In Fig. 6, the adaptation of the quantization bound γ_1 is illustrated, where γ_1 is depicted as a function of the average BER p_b of the feedback channel for different normalized Doppler frequencies f_DT. It can be seen that for a particular f_DT, the quantization bound γ_1 decreases with increasing p_b. This behaviour results from the fact that in the majority of cases users having an SNR value out of the second quantization level are selected for transmission. For an increasing BER p_b, the probability increases, that the SNR value of a selected user is located in the first quantization level [0,γ_1],

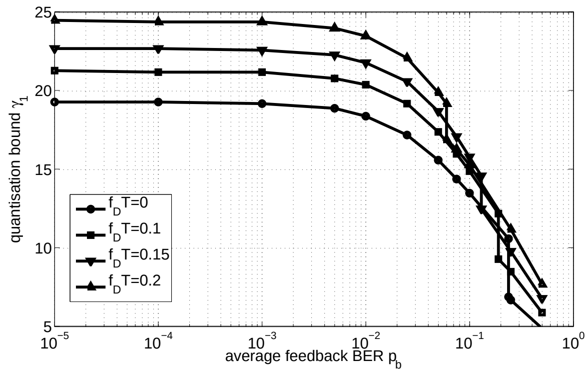

Fig. 6. Quantisation bound γ_1 adaptive mode for $\bar{\gamma} = 10$ dB, $L = 10$, $\sigma_E^2 = 0.09$ and $N_Q = 1$

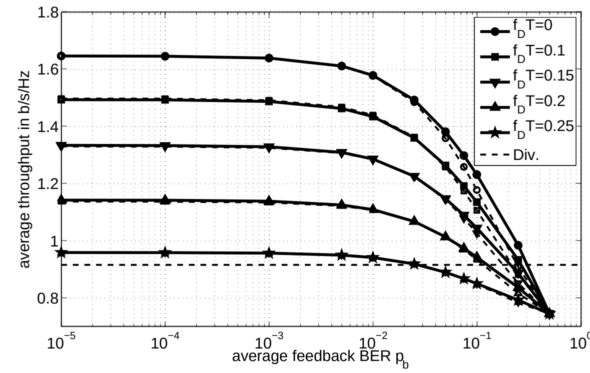

Fig. 7. Comparsion average throughput adaptive mode vs. non-adaptive mode for $\bar{\gamma} = 5$ dB, $N = 128$, $L = 10$, $\sigma_E^2 = 0.24$ and $N_Q = 1$, (dashed line: feedback error and delay-free CQI assumed; solid line: CQI imperfectness known)

although it is assumed to be in the second quantization level [γ_1,∞], causing throughput degradation due to a wrong user and modulation scheme selection. By decreasing the bound γ_1, the probability for this event is decreased.
Furthermore, for a particular f_DT, a discontinuity can be observed at a certain value of p_b corresponding to the change of the modulation schemes. In this example, $\mathcal{M}$={QPSK,8-PSK} changes to $\mathcal{M}$={BPSK,QPSK}, i.e. at a certain value of p_b, the modulation scheme of the first and second quantization level becomes more robust to cope with the increased feedback BER.
For a particular p_b, the bound γ_1 increases with increasing f_DT. This behaviour can be explained by the fact that with an increasing f_DT, the probability increases, that a selected user assumed to be in the second quantization level [γ_1,∞] is actually in the first quantization level [0,γ_1], causing a throughput degradation due to a wrong user and modulation scheme selection. Increasing the first quantization level [0,γ_1] decreases the probability of this event.

In Fig. 7, all parameters remain the same compared to Fig. 5 except for the average SNR which is now set to $\bar{\gamma} = 5$ dB. In this case, the optimal adaptive parameters assuming feedback error and delay-free CQI are $\gamma_{1,opt} = 5.5$ and $\mathcal{M}_{opt}$ ={BPSK,QPSK}. Analog to Fig. 5, the average throughput decreases with increasing feedback BER p_b, where the throughput values are lower compared to the case $\bar{\gamma} = 10$ dB due to the lower level of the used modulation schemes. Since $\mathcal{M}$ ={BPSK,QPSK} is the optimal solution for all possible values of p_b in Fig. 7, there is only a small different between the solid and dashed curves, i.e. due to the fact that we already use robust modulation schemes, the throughput does not decrease that much being unaware of the CQI imperfectness.

In the following, $N_Q = 2$ bit quantization is considered. In contrast to the one bit feedback, there exist three possible bitcoding matrices $\mathcal{B}$ using a 2 bit feedback:

$$\mathcal{B}_1 = \begin{pmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{pmatrix}, \mathcal{B}_2 = \begin{pmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{pmatrix}$$

$$\text{and } \mathcal{B}_3 = \begin{pmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix},$$

where bitcoding matrix $\mathcal{B}_3$ corresponds to a Gray Mapping. In Fig. 8, the average throughput of a system with $\bar{\gamma} = 10$ dB, $N_Q = 2$ bit feedback and $U = 25$ users is depicted as a function of the feedback BER p_b for the three different bitcoding schemes. Again, the solid curves represent the throughput being aware of the CQI imperfectness and the dashed curves represent the case assuming feedback error and delay-free CQI. For this case, the optimal adaptive parameters are given by $\gamma_{1,opt} = 12$, $\gamma_{2,opt} = 19$, $\gamma_{3,opt} = 33$ and $\mathcal{M}_{opt}$ ={BPSK,QPSK,8-PSK,16-QAM}. From Fig. 8, it can

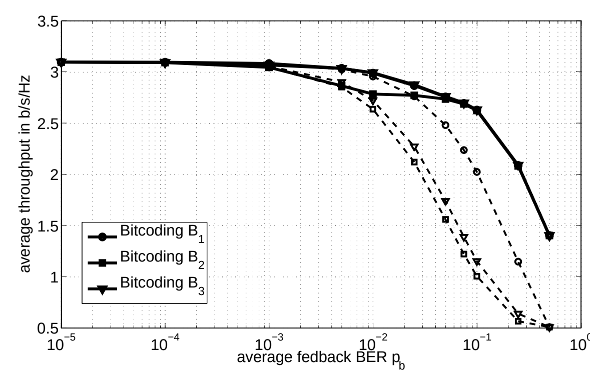

Fig. 8. Average throughput adaptive mode for $\bar{\gamma} = 10$ dB, $N = 128$, $L = 10$, $\sigma_E^2 = 0.09$, $f_DT = 0$ and $N_Q = 2$

be seen that assuming feedback error and delay-free CQI, bitcoding matrix $\mathcal{B}_1$ provides the best throughput. Since in the majority of cases users with SNR values from the third and fourth quantization level are selected for transmission, these levels have to be protected against the lower quantization levels. Using $\mathcal{B}_1$, the bitcoding of the highest and lowest quantization level differ in two bits, whereas for $\mathcal{B}_2$ and $\mathcal{B}_3$, the difference is only one bit. Hence, it is more likely using bitcoding matrices $\mathcal{B}_2$ and $\mathcal{B}_3$ that an SNR value from the lowest quantization level is wrongly assumed to be an SNR value from the highest quantization level. Being aware of the CQI imperfectness, the use of bitcoding matices $\mathcal{B}_1$ and $\mathcal{B}_3$ lead to the same performance while outperforming bitcoding matrix $\mathcal{B}_2$, since using bitcoding matrix $\mathcal{B}_2$, the bitcoding of the third and fourth quantization level only differ in one bit compared to the bitcoding of the first and second level. Furthermore, it can be seen that analog to the one bit feedback, the effect of the feedback bit errors can be neglected for $p_b < 10^{-3}$.

In Fig. 9 the optimal throughput according to (23) is depicted as a function of the normalized Doppler frequency $f_D T$ for different number U of users (solid lines: $U = 50$, dot-dashed lines: $U = 25$, dashed lines: $U = 10$), where we assume that the channel knowledge parameters are known. The number of quantization bits are $N_Q = 1$ and $N_Q = 2$, the average SNR is $\bar{\gamma} = 5$ dB and the feedback BER is set to $p_b = 10^{-3}$. Using $N_Q = 2$ bits feedback, bitcoding scheme $\mathcal{B}_1$ is employed. From the first solid line, representing

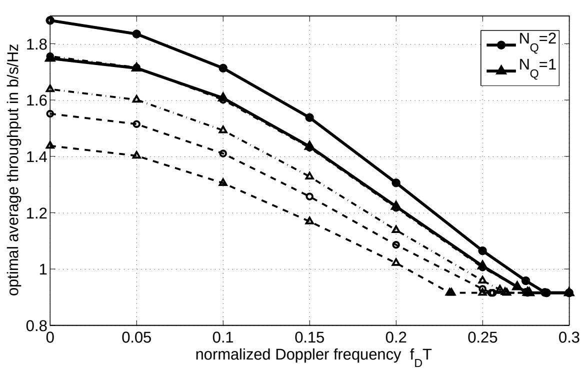

Fig. 9. Optimal throughput vs. $f_D T$ for $p_b = 10^{-3}$, $L = 10$, $N = 128$, $\bar{\gamma} = 5$ dB and $\sigma_E^2 = 0.24$; solid lines: $U = 50$; dot-dashed lines: $U = 25$; dashed lines: $U = 10$

the optimal throughput in a system with $U = 50$ users and $N_Q = 2$ feedback bits, it can be seen that for a normalized Doppler frequencies $f_D T < 0.29$ the adaptive transmission mode is the optimal transmission mode, i.e. for $f_D T > 0.29$, which corresponds to a vehicular speed of $v = 62.6$ km/h, the BS switches from the adaptive mode with imperfect CQI to the non-adaptive mode that provides a constant throughput independent from $f_D T$. Having less users in the system results in throughput degradation due to a lower multi-user diversity. Hence, the normalized frequency $f_D T$, up to which the adaptive mode outperforms the non-adaptive mode also decreases. Furthermore, using $N_Q = 1$ quantization bits instead of $N_Q = 2$ also results in a throughput degradation due to a less precise user selection and a smaller range of modulation levels. In the case of $U = 10$ users and $N_Q = 1$, the adaptive transmission mode outperforms the non-adaptive mode up to $f_D T = 0.23$ ($v = 49.7$ km/h). Note, that this throughput degradation however results in a reduction of the feedback by factor two.

In Fig. 10 we use the same parameters as in Fig. 9 except for the average SNR which is now set to $\bar{\gamma} = 10$ dB. Analog to Fig. 9 it can be seen that having more users in the system or using more quantization levels result in a throughput enhancement. Comparing the slopes of the throughput curves having an average SNR of $\bar{\gamma} = 10$ dB and $\bar{\gamma} = 5$ dB, it appears, that having a higher SNR, the slope is steeper, i.e. the system is more sensitive to outdated CQI. This results in a lower $f_D T$ up to which the adaptive mode outperforms the non-adaptive mode for $\bar{\gamma} = 10$ dB. In this case, the maximal normalized Doppler frequency $f_D T$ for a system with $U = 50$ users and $N_Q = 2$ bits feedback is $f_D T = 0.22$ ($v = 47.5$ km/h) and $f_D T = 0.16$ ($v = 34.6$ km/h) for a system with $U = 10$ users and $N_Q = 1$ bits feedback.

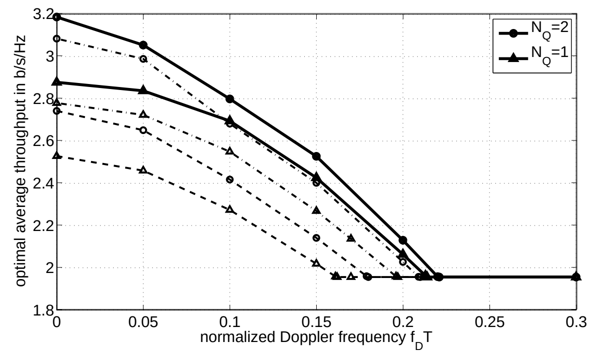

Fig. 10. Optimal throughput vs. $f_D T$ for $p_b = 10^{-3}$, $L = 10$, $N = 128$, $\bar{\gamma} = 10$ dB and $\sigma_E^2 = 0.09$; solid lines: $U = 50$; dot-dashed lines: $U = 25$; dashed lines: $U = 10$

IX. CONCLUSIONS

In this paper, we have derived a closed form expression for the average throughput of an adaptive multi-user SISO-OFDMA FDD system using imperfect CQI performing adaptive subcarrier allocation and uncoded adaptive modulation. The CQI is assumed to be outdated and digitized with estimation and feedback errors. We compare the achievable throughput using this adaptive transmission mode with imperfect CQI with the throughput achievable by a non-adaptive transmission mode exploiting frequency diversity in order to identify the optimal transmission mode regarding the throughput depending on the grade of CQI imperfectness. For both $N_Q = 1$ and $N_Q = 2$ bits feedback, the effect of feedback bit errors can be neglected for $p_b < 10^{-3}$. Being aware of the CQI imperfectness and the parameters describing the imperfectness, one can achieve higher throughput using the adaptive mode by adapting to this CQI imperfectness compared to the case where feedback error and delay-free CQI is assumed. Furthermore, it appears that having more users in the system and using more quantization levels result in a throughput enhancement. This again results in a higher normalized Doppler frequency $f_D T$ up to which the adaptive transmission mode outperforms the non-adaptive transmission mode. However, these considerations do not take into account the effort we have to spend in order to feed back the channel information to the BS, which will be the task for future work.

APPENDIX A
DERIVATION OF CONDITIONAL PDF OF (9)

With the assumptions made in Section IV, the relation between the actual channel H and an outdated version $\hat{H}$ of the channel with estimation errors can be modeled by

$$H = \rho \cdot (\hat{H} + E) + \sqrt{1-\rho^2} \cdot X, \tag{24}$$

where X is a complex Gaussian distributed random variable independent of $\hat{H}$ and E with variance one. It can be shown that the conditional probability density function (PDF) of the actual channel H and the estimated and outdated channel $\hat{H}$ is given by

$$p_{H|\hat{H}}(H|\hat{H}) = \frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2(1-\sigma_E^2)}} \cdot \exp\left(-\frac{\left(H-\rho\hat{H}\right)^2}{2(1-\rho^2(1-\sigma_E^2))}\right). \tag{25}$$

Using (1), (25) and [37, p. 43], the conditional PDF $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$ for the actual SNR γ and the estimated and outdated SNR $\hat{\gamma}$ is given by (9).

APPENDIX B
DERIVATION OF PDF OF THE ASSUMED SNR OF THE SCHEDULED USER USING DIGITIZED CQI WITH AND WITHOUT FEEDBACK ERRORS

In case of digitized CQI with feedback errors, the PDF $p_{\hat{\gamma}}(\hat{\gamma})$ of the assumed SNR of the scheduled user is a sum of $card(\mathcal{M})$ PDF functions $p_{\hat{\gamma}}^{(m)}(\hat{\gamma})$, each determining the distribution of the SNR values, which are assumed to be in the m-th quantization level. The PDF $p_{\hat{\gamma}}^{(m)}(\hat{\gamma})$ is not limited to the quantization bounds [γ_{m-1},γ_m] of the m-th quantization level, but it is defined for all possible SNR values, since, due to feedback errors, a selected user with an SNR value from another quantization level can wrongly be assumed to be a user with an SNR value in the m-th quantization. The probability that the SNR value of a selected user which is assumed to be in the i-th quantization level is actually from the j-th quantization level is determined by the elements $d_{i,j}$ of matrix $\mathbf{D}$, introduced in Section IV. Hence, the PDF $p_{\hat{\gamma}}^{(m)}(\hat{\gamma})$ is a sum of $card(\mathcal{M})$ Rayleigh distributed PDF functions $p_{\hat{\gamma}}^{(m,k)}(\hat{\gamma})$, with $k = 1, ..., card(\mathcal{M})$, each limited to the $card(\mathcal{M})$ quantization levels and weighted by the probability $d_{m,k}$ and is given by

$$p_{\hat{\gamma}}^{(m)}(\hat{\gamma}) = \tilde{a}_m \sum_{k=1}^{card(\mathcal{M})} \frac{d_{m,k}}{E\{\bar{\gamma}\}} \exp\left(-\frac{\hat{\gamma}}{E\{\bar{\gamma}\}}\right) \cdot \left[\sigma(\hat{\gamma}-\gamma_{k-1}) - \sigma(\hat{\gamma}-\gamma_k)\right], \tag{26}$$

where $\tilde{a}_m$ is a scaling factor. In order to determine $\tilde{a}_m$, the probability $P_m = P(\gamma_{m-1} < \hat{\gamma} < \gamma_m)$ of an SNR value of the scheduled user to be in the m-th quantization level has to be derived. Using a Max-SNR scheduler P_m can be determined by

$$P_m = \sum_{k=1}^{U} \binom{U}{k} P_1^{U-k} \cdot P_2^k, \tag{27}$$

where P_1 denotes the probability that the assumed SNR of a user is in a quantization level below the m-th level and P_2 denotes the probability that the assumed SNR of a user is in the m-th quantization level, i.e. at least one user is located in the m-th quantization level and no other user is located in a quantization level above the m-th level. In order to determine the probabilities P_1 and P_2, the probability p_m is introduced. The probability p_m of an SNR value assumed to be in the m-th quantization level is the sum of the probabilities of the $card(\mathcal{M})$ events that an SNR value originally from the j-th quantization level, with $j = 1, ..., card(\mathcal{M})$, is assumed to be in the m-th quantization level. This can be expressed by

$$p_m = \sum_{j=1}^{card(\mathcal{M})} d_{m,j} \cdot \left[\exp\left(-\frac{\gamma_{j-1}}{E\{\bar{\gamma}\}}\right) - \exp\left(-\frac{\gamma_j}{E\{\bar{\gamma}\}}\right)\right], \tag{28}$$

which is equivalent to the calculation $\mathbf{p} = \mathbf{D} \cdot \mathbf{z}$ introduced in Section V. Hence, P_1 is given by

$$P_1 = \sum_{j=1}^{m-1} p_j \tag{29}$$

and

$$P_2 = p_m. \tag{30}$$

Eq. (27) can be transformed to can be rewritten as

$$P_m = \sum_{k=0}^{U} \binom{U}{k} P_1^{U-k} \cdot P_2^k - P_1^U \tag{31}$$

leading to

$$P_m = (P_1 + P_2)^U - P_1^U \tag{32}$$

using [39, 1.111]. Inserting (29) and (30) in (32) leads to

$$P_m = \left(\sum_{j=1}^{m} p_j\right)^U - \left(\sum_{j=1}^{m-1} p_j\right)^U. \tag{33}$$

The scaling factor $\tilde{a}_m$ is determined by

$$\int_0^\infty p_{\hat{\gamma}}^{(m)}(\hat{\gamma}) d\hat{\gamma} = P_m \tag{34}$$

resulting in (17).

For the case of digitized CQI without feedback errors ($p_b = 0$), matrix $\mathbf{D}$ is equal to an identity matrix ($\mathbf{D} = \mathbf{I}$). Hence, the PDF (26) of the assumed SNR of a selected user can be rewritten as

$$p_{\hat{\gamma}}^{(m)}(\hat{\gamma}) = a_m \cdot \frac{1}{E\{\bar{\gamma}\}} \exp\left(-\frac{\hat{\gamma}}{E\{\bar{\gamma}\}}\right) \cdot \left[\sigma(\hat{\gamma}-\gamma_{m-1}) - \sigma(\hat{\gamma}-\gamma_m)\right]. \tag{35}$$

Furthermore, the probabilities P_1 and P_2 reduces to

$$P_1 = 1 - \exp\left(-\frac{\gamma_{m-1}}{E\{\bar{\gamma}\}}\right) \tag{36}$$

and

$$P_2 = \exp\left(-\frac{\gamma_{m-1}}{E\{\bar{\gamma}\}}\right) - \exp\left(-\frac{\gamma_m}{E\{\bar{\gamma}\}}\right), \tag{37}$$

leading to the scaling factor a_m given by (13).

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Alexander Kühne (S’07) received the Dipl.-Ing. degree in electrical engineering from Technische Universität Darmstadt, Germany, in 2006. He is currently pursuing the Ph.D. degree in electrical engineering at the Communications Engineering Lab of Technische Universität Darmstadt. His current research interests include adaptive multi-user transmission schemes with imperfect channel knowledge.

Anja Klein (S’93-M’96) received the diploma and Dr.-Ing. (Ph.D.) degrees in electrical engineering from University of Kaiserslautern, Germany in 1991 and 1996, respectively. From 1991 to 1996, she was a member of the staff of Research Group for RF Communications at University of Kaiserslautern. In 1996, she joined Siemens AG, Mobile Network Division, Munich and Berlin. She was active in the standardization of third generation mobile radio in ETSI and in 3GPP. She was vice president, heading a development department and a system engineering department. In May 2004, she joined Technische Universität Darmstadt, Germany, as full professor, heading the Communications Engineering Lab. Her main research interests are in mobile radio, including multiple access and transmission schemes like single and multi carrier schemes and multi antenna systems on the one hand, and network aspects like resource management, network planning and dimensioning, cross-layer design and relaying and multi-hop on the other hand.

Dr. Klein has published over 115 refereed papers and has contributed to four books. She is inventor and co-inventor of more than 50 patents in the field of mobile radio. In 1999, she was inventor of the year of Siemens AG. Dr. Klein is a member of IEEE and of Verband Deutscher Elektrotechniker-Informationstechnische Gesellschaft (VDE-ITG).