# A resource allocation strategy for SDMA/OFDMA systems

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*Abstract*— **Optimum Resource Allocation (RA) in Space Division Multiple Access (SDMA)/Orthogonal Frequency Division Multiple Access (OFDMA) systems is a prohibitively complex task for which efficient sub-optimal strategies are preferable. In this work, a new Space-Frequency/Time Resource Allocation (S-FT RA) is proposed, which divides RA in two tasks: the SDMA grouping, for which a new SDMA grouping algorithm is proposed, and the joint frequency/time RA, which is solved using Munkres' algorithm. It is shown that the proposed strategy is flexible and that it achieves a considerable fraction of the maximum achievable average system capacity.**

## I. INTRODUCTION

Space Division Multiple Access (SDMA) and Orthogonal Frequency Division Multiple Access (OFDMA) are promising technologies for the provision of flexible high-rate services in future mobile radio networks [**?**,1]. However, due to the large number of degrees of freedom in space, frequency, and time, SDMA/OFDMA systems need a sophisticated Resource Allocation (RA) strategy to efficiently assign resources to User Terminals (UTs).

If Channel State Information (CSI) about UTs' channels is available at the Base Station (BS), either through direct estimation or feedback, SDMA can be used to serve a group of UTs simultaneously on the same radio resource. In the following, such a group is termed an SDMA group. The efficiency of the RA strategy can be influenced, first, by selecting which UTs to place in the same SDMA group, and second, by deciding whether and when to grant a resource to a group.

In general, performing optimum RA in space, frequency, and time is a very complex problem. For such optimum RA, the tasks of building SDMA groups and assigning resources to groups cannot be decoupled, thus requiring to compare the efficiency of assigning every possible SDMA group on every possible resource. This problem can be recognized as Non-deterministic Polynomial time Hard (NP-H) and has exponentially increasing complexity. To simplify this problem, it is proposed here to sub-optimally divide it into two tasks:
*Task 1:* Arranging UTs in SDMA groups, which is done by an SDMA grouping algorithm that places spatially compatible UTs in the same SDMA group. Spatial compatibility, i.e., how efficiently UTs can be separated in space, is measured by a grouping metric computed using the available CSI.
*Task 2:* Allocating resources to the SDMA groups, which is accomplished by a joint frequency/time RA algorithm.

Finding the SDMA group that optimizes an adopted grouping metric on a given resource is known to be an NP-H problem [2,3] while joint frequency/time RA of groups to resources can also become NP-H depending on the problem constraints [4]. Therefore, tasks 1 and 2 also ask for sub-optimal solutions.

Considerable work on SDMA grouping has been done to sub-optimally solve task 1 [2,3], [5]–[11]. In [3], greedy algorithms are proposed to iteratively build SDMA groups based on the Signal-to-Interference plus Noise Ratios (SINRs) of UTs. In [7]–[9], greedy SDMA algorithms are proposed considering successive precoding of the signals of UTs being added to the group. In [6], several SDMA groups are built and compared afterwards in terms, e.g., of the group capacity, in order to determine the best group.

Developing an SDMA algorithm requires designing an efficient low-complexity grouping metric and a selection algorithm that uses that metric to build candidate SDMA groups. The algorithms in [3], [6]–[9], among others, achieve high system capacity, but their grouping metrics depend on precoding to elect the best SDMA group, which increases the complexity of the algorithms. In [2,5,10,11], SDMA grouping algorithms based on the spatial correlation are proposed and they are shown to be very efficient and to have low complexity.

In this work, a new sub-optimal SDMA grouping algorithm, named Greedy Regularized Correlation-Based Algorithm (GRCBA), is proposed to solve task 1. It is a greedy variant of Regularized Correlation-Based Algorithm (RCBA) [11] and is based on the correlation and gains of the spatial channels of the UTs. It has almost the same performance as RCBA, but lower complexity.

Joint frequency/time RA algorithms have been often studied in the context of resource assignment, bit- and power-loading, cf. e.g. [12]–[14]. Such algorithms usually aim at efficiently allocating resources to SDMA groups while adjusting the number of bits sent to each UT on the radio resources and the power allocated on each resource. Indeed, they are very efficient and enhance system throughput, but are also considerably complex.

For solving task 2, a new sub-optimal joint frequency/time RA algorithm is proposed here, namely the Maximum Weighted Capacity Algorithm (MWCA). MWCA aims at maximizing the total weighted capacity of the system, takes into account both capacity and Quality of Service (QoS) aspects, and formulates joint frequency/time RA as a standard assignment problem, which is efficiently solved by employing

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Munkres' algorithm [15]. Power is equally divided among subcarriers [2] and is allocated to users in an SDMA group according to transmit Zero-Forcing (ZF) precoding [1,16]. This makes MWCA considerably simple.

In order to sub-optimally solve the RA problem in space, frequency and time, GRCBA and MWCA are combined into the proposed Space-Frequency/Time Resource Allocation (S-FT RA) strategy. The remainder of this work is organized as follows. Section II describes the adopted system model. Section III introduces the proposed GRCBA and MWCA, whose performance is analyzed in section IV. Finally, section V draws some conclusions.

## II. SYSTEM MODEL

This section describes the system model used in this work. The downlink of a single BS is considered in the modeling. Interference from other BSs is assumed as Gaussian and is incorporated directly as part of the Gaussian noise in the system.

The BS has an $n_T$-element Antenna Array (AA) and there are $K$ single-antenna UTs associated with the BS, thus totalizing $n_R = K$ receive antennas.

OFDMA is used and the BS bandwidth is divided into $S$ orthogonal subcarriers whose channel response is assumed to be flat. A block of $Q$ adjacent subcarriers, also called cluster or chunk [17], is defined as radio resource. A total number of $N = \lfloor S/Q \rfloor$ resources is assumed, where $\lfloor\cdot\rfloor$ is the nearest integer lower than or equal to the argument. $Q$ is chosen such that channel responses do not vary much within a chunk, thus reducing the required signaling/estimation effort without much degradation of the system performance. Considering chunks of $Q$ adjacent subcarriers reduces the number of resources on which SDMA groups have to be built, thus simplifying task 1, as well as it reduces the dimension of the joint frequency/time RA problem and also simplifies task 2.

On a subcarrier $s$, each link between the BS and a UT $k$ has an associated channel matrix $\mathbf{H}_{ks} \in \mathbb{C}^{1\times n_T}$. Denoting transposition by $(\cdot)^T$, the channel matrix $\mathbf{H}_s \in \mathbb{C}^{n_R\times n_T}$ on subcarrier $s$ of all UTs can be written by stacking the channel matrices $\mathbf{H}_{ks}$ as

$$\mathbf{H}_s = \begin{bmatrix}\mathbf{H}_{1s}^T & \mathbf{H}_{2s}^T & \ldots & \mathbf{H}_{Ks}^T\end{bmatrix}^T. \tag{1}$$

Each resource is represented by its middle subcarrier, whose channel response is assumed to be perfectly known at the BS. The channel response $\mathbf{H}_n$ of all UTs on resource $n$ is written as in (1) using the referred middle subcarrier.

On resource $n$, building an SDMA group $\mathcal{G}$ corresponds to adequately selecting a total of $G \leq n_T$ vector channels $\mathbf{h}_{in} \in \mathbb{C}^{1\times n_T}$, $i = 1,\ldots,n_R$, of $\mathbf{H}_n$, i.e., to optimally select $G$ out of the $n_R$ rows of $\mathbf{H}_n$ according to the adopted grouping metric and problem constraints. $G$ is the cardinality of $\mathcal{G}$, i.e., $G = \operatorname{card}\{\mathcal{G}\}$ is the number of vector channels in $\mathcal{G}$.

Whenever $\mathcal{G}$ is scheduled for transmission on resource $n$, all the subcarriers of this resource are allocated to all UTs of the group. On each subcarrier $s$ of the resource, the BS transmits data symbols $d_{gs}$, $g = 1,\ldots,G$, to all UTs in the group. The data symbols $d_{gs}$ are assumed to be uncorrelated with average power $\sigma_d^2 = 1$ and are arranged in the input data vector $\mathbf{d}_s \in \mathbb{C}^{G\times 1}$, which is precoded using the modulation matrix $\mathbf{M} \in \mathbb{C}^{n_T\times G}$, transmitted through the SDMA group channel $\mathbf{G}_s \in \mathbb{C}^{G\times n_T}$, and distorted by noise, which is represented by $\mathbf{z}_s \in \mathbb{C}^{G\times 1}$. $\mathbf{z}_s$ is considered to be spatially white with average power $\sigma_z^2$. The received signal is demodulated using the demodulation matrix $\mathbf{D} \in \mathbb{C}^{G\times G}$ producing at the receivers the output data vector

$$\hat{\mathbf{d}}_s = \mathbf{D}(\mathbf{G}_s\mathbf{M}\mathbf{d}_s + \mathbf{z}_s) \in \mathbb{C}^{G\times 1}. \tag{2}$$

Both $\mathbf{M}$ and $\mathbf{D}$ in (2) depend on the adopted precoding technique and are applied to all the subcarriers of resource $n$. Since the demodulation process is distributed among the UTs, $\mathbf{D}$ is diagonal and decouples signals received by different UTs.

Assuming Gaussian signaling, the group capacity $C(\mathcal{G})$ of the SDMA group $\mathcal{G}$ on resource $n$ is estimated as

$$C(\mathcal{G}) = Q\sum_{i=1}^{G}\log_2\left[1+\frac{\mathbf{m}_i^H\mathbf{g}_{in}^H\mathbf{g}_{in}\mathbf{m}_i}{\sigma_z^2+\sum\limits_{j=1,\,j\neq i}^{G}\mathbf{m}_j^H\mathbf{g}_{in}^H\mathbf{g}_{in}\mathbf{m}_j}\right], \tag{3}$$

where $\mathbf{g}_{in}$ is the $i^{th}$ row of the group channel matrix $\mathbf{G}_n$ given by the middle subcarrier of resource $n$, and $\mathbf{m}_i$ is the $i^{th}$ column of $\mathbf{M}$.

The spatial correlation between two vector channels $\mathbf{h}_i$ and $\mathbf{h}_j$ is measured by the normalized scalar product [2,3,10,11]. Let $|\cdot|$ denote the absolute value of a complex scalar, $\|\cdot\|_2$ denote the 2-norm, and $\operatorname{diag}\{\cdot\}$ denote a diagonal matrix whose diagonal elements are given in the vector argument. Using (1), a matrix $\mathbf{R}_n \in \mathbb{R}_+^{n_R\times n_R}$ containing the spatial correlation of every pair of channels $\mathbf{h}_{in}$ and $\mathbf{h}_{jn}$ from $\mathbf{H}_n$ can be written as

$$\mathbf{R}_n = \left|\operatorname{diag}\{\mathbf{n}_n\}\,\mathbf{H}_n\mathbf{H}_n^H\operatorname{diag}\{\mathbf{n}_n\}\right|, \text{ with} \tag{4a}$$

$$\mathbf{n}_n = \begin{bmatrix}\|\mathbf{h}_{1n}\|_2^{-1} & \|\mathbf{h}_{2n}\|_2^{-1} & \ldots & \|\mathbf{h}_{n_R n}\|_2^{-1}\end{bmatrix}^T, \tag{4b}$$

where $|\cdot|$ is applied to $\mathbf{R}_n$ element-wise. $\mathbf{R}_n$ is used as input for the SDMA grouping algorithms in the next sections.

## III. SPACE-FREQUENCY/TIME RESOURCE ALLOCATION

This section introduces the S-FT RA strategy proposed in this work. Section III-A describes the proposed SDMA grouping algorithm. Section III-B introduces the joint frequency/time RA algorithm. Section III-C describes how the algorithms in sections III-A and III-B integrate into the S-FT RA strategy.

### *A. SDMA grouping algorithm*

In this section, the proposed SDMA grouping algorithm, namely GRCBA, is introduced. Its grouping metric is based on the spatial correlation and gains of the spatial channels of the UTs. Consequently, it does not depend on the precoding matrices and has low complexity.

If spatial channels of the UTs in the same SDMA group are close to orthogonal, spectral efficiency gains are obtained. However, if they are spatially correlated, SDMA can even

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lead to spectral efficiency losses. Considering ZF precoding [16] and a fixed group size $G$, building an SDMA group $m$ on resource $n$, indicated by $\mathcal{G}_{mn}$, whose $G$ channels are as uncorrelated as possible, is a good candidate solution for the SDMA grouping problem.

In the GRCBA, shown in Table I, an initial spatial channel $\mathbf{h}_{cn}$, indexed by $c$, is selected and a one-channel SDMA group $\mathcal{G}_{mn} = \{c\}$ is built. Then, at each iteration, the spatial channel most compatible with respect to all the channels already admitted to $\mathcal{G}_{mn}$ is added to the group. This procedure is repeated until the desired group size $G$ is reached.

TABLE I

GREEDY REGULARIZED CORRELATION-BASED ALGORITHM (GRCBA).

```
1. Set the best SDMA group $\mathcal{G}_{mn} = \{c\}$.
2. For $g = 1$ to $G - 1$
    a. Set $\mathcal{G}_{mn} = \mathcal{G}_{mn} \cup \arg\min_{c} \left\{ \frac{(1-\alpha)}{\|\mathbf{R}_n\|_F} \sum_j [\mathbf{R}_n]_{jc} + \frac{\alpha}{\|\mathbf{n}_n\|_1} [\mathbf{n}_n]_c \right\}$,
       with $c \in \{1, \ldots, n_R\} \setminus \mathcal{G}_{mn}$, and $j \in \mathcal{G}_{mn}$.
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GRCBA is a greedy variant of RCBA introduced in [11]. GRCBA and RCBA have almost the same grouping metric. However, GRCBA builds the SDMA group based on a simple greedy algorithm while RCBA builds the SDMA group by solving the optimization problem

$$\mathbf{x}_n^\star = \arg\min_{\mathbf{x}_n} \left\{ (1-\alpha)\mathbf{x}_n^T \frac{\mathbf{R}_n}{\|\mathbf{R}_n\|_F} \mathbf{x}_n + \alpha \frac{\mathbf{n}_n^T}{\|\mathbf{n}_n\|_1} \mathbf{x}_n \right\}, \quad (5a)$$

$$\text{subject to: } \|\mathbf{x}_n\|_1 = G, \quad (5b)$$

$$x_{cn} = 1, \ c \in \{1, \ldots, n_R\}, \quad (5c)$$

$$x_{jn} \in [0, 1], \ j = 1, \ldots, n_R, \quad (5d)$$

where $\|\cdot\|_1$ is the 1-norm of a vector, and $0 \leq \alpha \leq 1$ is a parameter controlling the preference for highly uncorrelated UTs or UTs with high channel gain. Constraint (5c) allows to force a given UT, indicated by $c$, to be in the SDMA group. Problem (5) is the convex relaxation of the associated integer optimization problem and can be solved with non-exponential complexity using convex optimization methods [11,18]. Anyway, solving RCBA might require a considerable number of iterations of a convex optimization algorithm, and the Greedy Regularized Correlation-Based Algorithm (GRCBA) is proposed here to further simplify the SDMA grouping problem.

### B. Frequency/Time Resource Allocation

This section describes the proposed MWCA algorithm. Its objective is to find a scheduling of SDMA groups on the resources that maximizes a weighted sum of revenues.

Consider total numbers of $M$ SDMA groups and $N$ resources, with SDMA groups built using either RCBA or GRCBA. For each SDMA group $m$, a non-negative group priority $w_{mn}$, a non-negative allocation revenue $p_{mn}$ obtained by allocating the group $m$ to resource $n$, and an assignment variable $u_{mn} \in \{0, 1\}$ indicating whether group $m$ is allocated on resource $n$, are defined. Then, the total weighted revenue of the system corresponds to $\sum_{n=1}^{N} \sum_{m=1}^{M} w_{mn} p_{mn} u_{mn}$.

By arranging $w_{mn}$, $p_{mn}$, and $u_{mn}$ into a weighted revenue matrix $\mathbf{W} \in \mathbb{R}_+^{M \times N}$ and an assignment matrix $\mathbf{U} \in \mathbb{B}^{M \times N}$ as follows

$$\mathbf{W} = [\mathbf{W}]_{mn} = [w_{mn} p_{mn}], \quad (6a)$$

$$\mathbf{U} = [\mathbf{U}]_{mn} = [u_{mn}], \quad (6b)$$

the problem of maximizing the total weighted revenue of the system can be written as

$$\mathbf{U}^\star = \arg\max_{\mathbf{U}} \left\{ \mathbf{1}_M^T (\mathbf{W} \odot \mathbf{U}) \mathbf{1}_N \right\} \quad (7a)$$

$$\text{subject to: } \mathbf{1}_M^T \mathbf{U} = \mathbf{1}_N^T, \quad (7b)$$

where $\odot$ is the Hadamard product, $\mathbf{1}_L$ is an $L \times 1$ vector of ones, and (7b) ensures assigning only one group to each resource. Problem (7) is a standard assignment problem [19] and is efficiently solved herein by applying Munkres' algorithm [15]. Note that by suitably defining $w_{mn}$ and $p_{mn}$ different optimization objectives can be pursued.

Herein, the group priority $w_{mn}$ is defined as sum of the non-negative priorities $\nu_{kn}$ of the UTs within the group, i.e.,

$$w_{mn} = \sum \nu_{kn}, \text{ with } k \in \mathcal{G}_{mn}, \quad (8)$$

so that the higher the UTs' priorities are, the higher the group priority becomes. Moreover, UT priority management is kept reasonably decoupled from the SDMA grouping algorithm. Herein, $\nu_{kn}$ is defined as

$$\nu_{kn} = R_k / \overline{R}_k, \quad (9)$$

where $R_k$ and $\overline{R}_k$ are the contracted and measured average throughputs of UT $k$, respectively. This makes group priorities frequency-independent and priviledges groups containing UTs whose QoS levels are below the contracted ones.

In order to exploit Multi-User Diversity (MUD) gains the allocation revenue $p_{mn}$ of the SDMA group $\mathcal{G}_{mn}$ should be high if its current achievable rate on this resource is high. Therefore, $p_{mn}$ is defined herein as

$$p_{mn} = C(\mathcal{G}_{mn}), \quad (10)$$

with $C(\mathcal{G}_{mn})$ given by (3). Therefore, groups whose UTs are in good channel conditions have increased chances of being allocated.

As a result, (7) incorporates a trade-off between the current achievable rates of SDMA groups and their QoS levels providing some degree of proportional fairness.

### C. Resource Allocation Strategy

This section describes how GRCBA and MWCA are combined in the new S-FT RA strategy proposed here.

Spatial correlation is frequency-dependent and the best SDMA group on resource $n$ might not be optimum for resource $n' \neq n$. Therefore, in S-FT RA first a candidate SDMA group is created for every UT $k$ on each resource $n$, thus totalizing $M = KN$ candidate groups. Forcing UT $k$ to be in a group is done by setting $c = k$ in (5c), for RCBA, or in step 1 of Table I, for GRCBA. After that, the product $w_{mn} p_{mn}$ is computed for each SDMA group $m$ on the resources it appeared to. Otherwise it is set to 0 to avoid

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assigning a group to an unsuitable resource. Then, MWCA solves (7) with Munkres' algorithm.

S-FT RA is applied for each allocation period, which corresponds in this work to one Time-Slot (TS). One TS contains multiple OFDMA symbols. Priorities $w_{mn}$ are updated on a TS basis. Allocation revenues $p_{mn}$ can be updated at a lower rate, since channel responses only change considerably after a few TSs. This is a reasonable assumption for short frame lengths, low to moderate UTs' speeds, and channel estimation at a low rate. Moreover, modifying (7) in the proposed S-FT RA in order to apply it on a frame basis is straightforward.

In order to reduce the total number of candidate SDMA groups $M = KN$ and consequently reduce computational costs, one can impose that a given SDMA group is not allocated on more than one resource during one allocation period. This is done by assuming that each of the $M$ groups appears only once, i.e., that each group is unique, thus resulting in a number $M' \leq KN$ of candidate groups. Note that in the next allocation period, i.e., in the next TS, a given candidate SDMA group can appear again and be allocated to new resources. In the next section, it will be shown that considering $M'$ unique groups results in negligible performance losses compared to the case with $M$ non-unique groups.

## IV. ANALYSIS AND SIMULATION RESULTS

The performance of the S-FT RA is studied through simulations in this section. A BS with a Uniform Linear Array (ULA) of $n_T = 4$ elements separated by half wavelength is assumed. The BS serves $K = 16$ single-antenna UTs. There are $S = 96$ subcarriers grouped in blocks of $Q = 12$ adjacent subcarriers, thus resulting in $N = 8$ resources. Equal transmit power and the same average noise power are assumed for all subcarriers. A fixed SDMA group size $G = 4$ is assumed. All UTs have the same rate requirement and are assumed to always have data to transmit. Precoding matrices are determined applying transmit ZF [16]. Channel matrices are obtained using the WINNER Phase I Channel Model (WIM) [20]. Slow fading and path loss are assumed to be ideally compensated by power control and only the fast fading is considered.

In order to compare the performance of the proposed S-FT RA strategy, which employs MWCA, Maximum Capacity Assignment (MCA) and Round Robin Assignment (RRA) are also considered for joint frequency/time RA. The MCA allocates a resource to the SDMA group with highest capacity, while the RRA is a fair resource strategy. Thus, they allow to get some useful insight on how capacity and fairness are handled by S-FT RA. In all the cases, candidate SDMA groups are built using either RCBA or GRCBA and are combined with the referred joint frequency/time RA algorithms. Moreover, the maximum achievable capacity, i.e., the Sato bound [21] is provided. The most relevant parameters adopted in the simulations are given in Table II.

First, the average system capacity achieved by S-FT RA is investigated. Fig. 1 shows the average system capacity in bps/Hz achieved by the different joint frequency/time RA algorithms combined with RCBA and GRCBA as a function of the average Signal-to-Noise Ratio (SNR). In this case, MCA provides the highest capacity figures as expected.

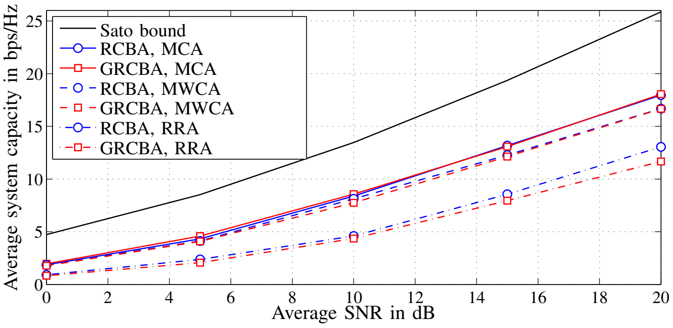
TABLE II

SIMULATION PARAMETERS.

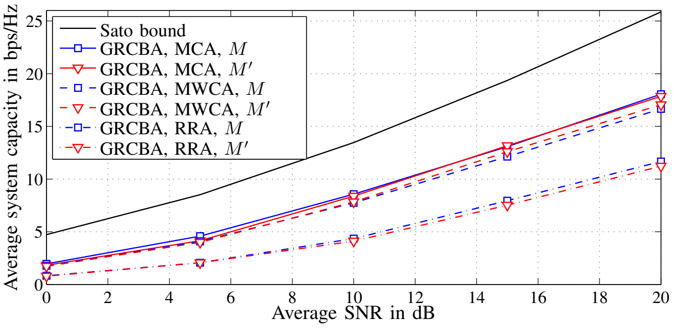
| Parameter | Value |
|---|---|
| **System bandwidth** | 937.5 kHz |
| **Center frequency** | 5 GHz |
| **# of subcarriers** | 96 |
| **# of subcarriers per resource** | 12 |
| **TS duration** | 0.25 ms |
| **Update rate of $w_{mn}$ and $p_{mn}$** | 1 TS and 4 TSs |
| **WIM scenario** | C2 |
| **# of single-antenna UTs** | 16 |
| **UTs' speed** | 10 km/h |
| **Transmit ULA** | 4 omni elements separated by half wavelength |
| **SDMA grouping algorithm** | RCBA, GRCBA |
| **SDMA group size** | 4 |
| **$\alpha$ parameter (see (5), Table I, and [11])** | 0.5 |
| **RA** | MCA, MWCA, RRA |

In Fig. 1(a), it can be seen that RCBA and GRCBA achieve similar results in all of the considered configurations, thus showing that the proposed GRCBA effectively approximates the performance of the RCBA. It can also be seen that the S-FT RA strategy which combines GRCBA and MWCA achieves almost the same average capacity achieved by the RA strategy employing RCBA and MCA. Because the RRA algorithm is not capacity-oriented, it achieves the lowest capacity figures.

In Fig. 1(b), the performance of S-FT RA with non-unique groups in the curves marked with $M$ is compared to the case with unique groups in the curves marked with $M'$. It can be noted that similar results are achieved in both cases. Therefore, considering only unique groups and avoiding the same group



(a) Non-unique groups ($M = KN$).



(b) Unique groups ($M' \leq KN$).

Fig. 1. Average capacity of the system in bps/Hz for the different RA schemes.

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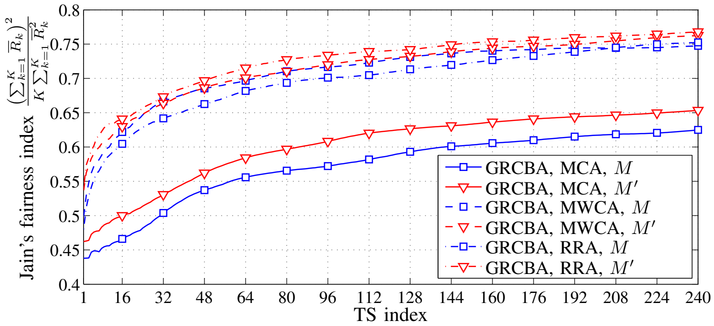

Fig. 2. Jain's fairness index for the UTs' average throughput after a given number of TSs. SNR = 10 dB.

to be assigned to multiple resources does not degrade capacity significantly. On the other hand, this significantly reduces the number of groups considered, thus reducing the complexity of S-FT RA.

In spite of employing equal power allocation across frequencies and linear ZF spatial precoding, it can be seen in Fig. 1 that the proposed S-FT RA scheme obtains at high SNR about 65% of the maximum achievable capacity drawn by the Sato bound.

In order to provide a fairness measure for the different strategies, the Jain's Index of Fairness (JIF) $\frac{(\sum_{k=1}^{K}\overline{R}_k)^2}{K\sum_{k=1}^{K}\overline{R}_k^2}$ [2] is computed with the average UTs throughputs $\overline{R}_k$ calculated for a varying number of TSs. Fig. 2 shows the average JIF for an increasing number of TSs and an SNR of 10 dB. It can be seen that S-FT RA is almost as fair as the RA strategy with RRA. This is due to the fact that MWCA pursues proportional throughput fairness. It can also be noted that S-FT RA obtains a JIF value above 0.7 just after a few TSs.

In Fig. 2, when comparing the cases with non-unique SDMA groups, indicated by $M$, and with unique groups, indicated by $M'$, only slight variations of JIF can be observed. Because in the case of unique groups the same group is not allocated to multiple resources, fairness is slightly increased at the expenses of a very small capacity loss compared to the case with non-unique groups.

From the obtained results, it can be seen that the proposed S-FT RA strategy is relatively efficient, providing high capacity with high degree of fairness among UTs. Most of the complexity of the proposed S-FT RA strategy resides on the group capacity estimation required to build $\mathbf{W}$, which needs to compute precoding matrices. Further simplifications of the studied S-FT RA strategy are left for future investigation.

## V. CONCLUSIONS

In this work, an S-FT RA strategy is proposed, which divides RA into an SDMA grouping task and and a joint frequency-time RA task, which are solved by the proposed GRCBA and MWCA, respectively. The proposed strategy has been shown to provide high average system capacity, reaching over 65% of the maximum achievable capacity at high SNRs. It is almost as fair as an RA strategy using simple RRA. Compared to an RA strategy using MCA, MWCA obtains almost the same capacity (over 90%) while providing a considerably higher degree of throughput fairness among UTs.

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