A convex quadratic SDMA grouping algorithm based on spatial correlation

Tarcisio F. Maciel and Anja Klein

Darmstadt University of Technology, Communications Engineering Lab
Merckstrasse 25, 64283 Darmstadt, Germany
{t.maciel, a.klein}@nt.tu-darmstadt.de

***Abstract*— Space Division Multiple Access (SDMA) is a promising solution to improve the spectral efficiency of future mobile radio systems. However, finding the group of MSs that maximizes system capacity using SDMA is a complex combinatorial problem, which can only be assuredly solved through an Exhaustive Search (ES). Because an ES is usually too complex, there are several sub-optimal SDMA grouping algorithms to solve this problem. Such algorithms, however, usually depend on the precoding matrices of candidate SDMA groups and are also considerably complex. In this work, an SDMA grouping algorithm is proposed for the downlink of multi-user multiple input multiple output systems. It is based on the spatial correlation and gains of the MSs' channels in the SDMA group, thus not depending on precoding and having low complexity. The proposed algorithm is formulated as a convex quadratic optimization problem and is efficiently solved by convex optimization methods. It is analyzed considering zero-forcing precoding and it is shown to almost achieve the performance of an ES for the SDMA group that maximizes the system capacity.**

I. Introduction

Multiple Input Multiple Output (MIMO) techniques are a promising solution for high throughput provision in future mobile radio systems [1]. In the downlink of Multi-User MIMO (MU-MIMO) systems, if Channel State Information (CSI) is available at the transmitter, a group of Mobile Stations (MSs) can be multiplexed in space using Space Division Multiple Access (SDMA) in order to improve spectral efficiency. In the following, such a group of MSs is termed an SDMA group.

The MSs in an SDMA group share the same resource in frequency and time while being separated in space, e.g., using a MIMO precoder such as the transmit Zero-Forcing (ZF) precoder [2,3]. Through SDMA, the system can serve more MSs without needing extra radio resources and, therefore, its spectral efficiency can be increased. Indeed, if MSs' spatial channels are close to orthogonal, SDMA gains are obtained by placing MSs in the same SDMA group. Oppositely, placing MSs with spatially correlated channels in the same SDMA group can even lead to spectral efficiency losses. MSs with correlated channels must belong to different SDMA groups, which are multiplexed on different resources in frequency or time. Therefore, the SDMA grouping algorithm must determine whether MSs are spatially compatible, i.e., whether they can efficiently share the same radio resource through SDMA.

Tarcisio F. Maciel is scholarship supported by the German Academic Exchange Service (DAAD).

The problem of finding the SDMA group that maximizes the system capacity is a complex combinatorial problem. It is similar to the well-known knapsack problem and is a Non-deterministic Polynomial time Complete (NPC) problem [4]–[6]. Its optimum solution is assuredly found through an Exhaustive Search (ES). However, an ES has exponential complexity and can reach prohibitive computational cost even for a moderate number of MSs. Indeed, having the SDMA group capacity as grouping metric requires to compute precoding matrices in order to compare different candidate groups, thus increasing the complexity of each step in the ES. Therefore, sub-optimal SDMA algorithms able to find an efficient SDMA group with reduced complexity are attractive.

Two relevant aspects can then be identified in order to design an efficient SDMA grouping algorithm with acceptable complexity:

1. A metric with low complexity should be considered in order to determine whether MSs pass in the same SDMA group and to compare the performance of different groups. Herein, such a metric is termed grouping metric.
2. An algorithm is required to find an SDMA group that maximizes (or minimizes) the grouping metric without needing to compare all the possible SDMA groups, thus, avoiding the ES. Herein, such an algorithm is termed SDMA algorithm.

The above discussion has been concerned with finding the SDMA group that maximizes the system capacity. However, the two sub-problems above are much more general and apply for many different grouping metrics, i.e., different optimization objectives. The best SDMA group is the one optimizing the group metric.

Several sub-optimal SDMA grouping algorithms that fit into this framework are proposed in the area. For example, in [5] the Signal-to-Interference plus Noise Ratio (SINR) margin, defined as the minimum difference between the expected and the target SINR of the MSs in an SDMA group, is used as grouping metric. Therein, several heuristic SDMA algorithms are proposed, such as the First Fit (FF) and Best Fit (BF) algorithms. Considering for example FF, the best SDMA group is built by sequentially adding MSs to an SDMA group as long as the SINR margin is kept non-negative. In [7], group capacity and the average Signal-to-Noise Ratio

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).

(SNR) are used as grouping metrics while a tree structure is employed by the SDMA algorithm in order to avoid an ES. In [8], the best SDMA group is built by selecting G channels among the available ones in order of singular values. The sum of the singular values is the grouping metric and the SDMA algorithm avoids the the ES by simply selecting the channels with highest singular values, thus disregarding spatial compatibility among the channels.

In [5,7,8], the considered grouping metrics depend either on precoding matrices, such as group capacity, SNR, and SINR, or on a complex operation, such as the sum of the singular values which requires the Singular Value Decomposition (SVD) of MSs' channels.

In [9], a grouping metric based on the spatial correlation is computed for every pair of MSs and the best SDMA group is selected as the group containing G MSs, with $G_{min} \leq G \leq G_{max}$, and having the minimum sum of the grouping metric for all pairs of MSs in the group. In this case, however, selection is done on a user basis, channel gains are not explicitly considered in the grouping metric and, in spite of having a limited number of candidate groups, the SDMA algorithm searches exhaustively for the best SDMA group. In [9], for a selection on a channel basis, the SVD of MSs' channels is also required.

In [10], a grouping metric based on the spatial correlation is considered and the search for the best SDMA group is performed by heuristic SDMA algorithms similar to the FF and BF SDMA algorithms in [5].

The spatial correlation among channels is easily computed and does not depend on precoding matrices [4]. Therefore, designing a grouping metric based on the spatial correlation, as in [9,10], can save computational costs while being considerably efficient. Additionally, channel gains should also be taken into account, as in [10], in order to enhance the overall performance of the SDMA grouping algorithm.

In this work, a new SDMA grouping algorithm is proposed for the downlink of MU-MIMO systems. The algorithm has the following attractive properties:

- It uses a new grouping metric, which is a function of the spatial correlation among the MSs' channels in the SDMA group, as well as of the MSs' channel gains. It is efficient, low complex, and depends neither on precoding matrices, as do [5,7], nor on a complex operation, as the SVD in [8,9]. Differently from [8], spatial compatibility is suitably taken into account. In contrast to [10], it allows to control the importance given to spatial correlation and to channel gain.
- The SDMA algorithm is formulated as a convex quadratic optimization problem, which can be efficiently solved by convex optimization methods [11]. It requires neither exhaustive searches, as that in [9], nor heuristic searches, as those in [5,7,10]. It works on a channel basis without incurring in excessive extra complexity, differently from [9].

The proposed SDMA grouping algorithm is analyzed considering ZF precoding and its performance is compared with an ES having the SDMA group capacity as grouping metric.

In section II, the adopted system model is described. In section III, the proposed SDMA algorithm is presented. Its performance is analyzed in section IV. Finally, in section V conclusions are drawn.

II. System model

This work focuses on the downlink of a MU-MIMO system. Without loss of generality, a single Base Station (BS) sector is considered in the problem modeling. The sector is equipped with an Antenna Array (AA) having n_T elements. A total number of K active MSs are located in the sector and each MS k has an AA with n_{Rk} elements.

In the sector a single frequency channel is considered, which is shared in space by MSs in an SDMA group. The channel response is assumed to be flat and perfect CSI is considered at the transmitter. This scenario can be seen as a single subcarrier, or a chunk of adjacent sub-carriers [12] for which a single sub-carrier is a good representative, in a system using Orthogonal Frequency Division Multiplexing (OFDM), Time Division Duplexing (TDD), and having perfect channel estimation at the BS.

Interference arriving from other sectors is assumed to be Gaussian distributed and is directly incorporated as part of the Gaussian noise in the system.

Each link between the BS sector and an MS k has an associated channel matrix $\mathbf{H}_k \in \mathbb{C}^{n_{Rk} \times n_T}$, which is known at the BS. Let $n_R = \sum_{k=1}^{K} n_{Rk}$ denote the total number of receive antennas in the sector and $(\cdot)^T$ denote vector or matrix transposition. Then, the channel matrix $\mathbf{H}$ of all MSs in the sector can be written by stacking the channel matrices $\mathbf{H}_k$ as

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \dots & \mathbf{H}_K^T \end{bmatrix}^T \in \mathbb{C}^{n_R \times n_T}. \tag{1}$$

Each row $\mathbf{h}_i \in \mathbb{C}^{1 \times n_T}$, $i = 1, \dots, n_R$ of $\mathbf{H}$ is a vector channel that can be selected for transmission using SDMA.

Considering this scenario, the SDMA grouping problem corresponds to building a group $\mathcal{G}$ containing a total number of $G \leq n_T$ vector channels optimally selected among all the n_R ones existing in the sector, i.e., to optimally select G rows of $\mathbf{H}$.

In this sector, the BS transmits the data symbols s_g, $g = 1, \dots, G$ to the MSs in the group $\mathcal{G}$. The data symbols s_g are assumed to be uncorrelated with average power $\sigma_s^2 = 1$ and are organized in the input data vector $\mathbf{s} \in \mathbb{C}^{G \times 1}$. This vector is modulated using the modulation matrix $\mathbf{M} \in \mathbb{C}^{n_T \times G}$, transmitted through the SDMA group channel $\mathbf{G} \in \mathbb{C}^{G \times n_T}$, and distorted by noise, which is represented by $\mathbf{n} \in \mathbb{C}^{G \times 1}$ and is considered to be spatially white with spectral density power σ_n^2. The received signal is demodulated using the demodulation matrix $\mathbf{D} \in \mathbb{C}^{G \times G}$ producing at the receivers the estimated output data vector of the transmitted data symbols

$$\hat{\mathbf{s}} = \mathbf{D}(\mathbf{GMs} + \mathbf{n}) \in \mathbb{C}^{G \times 1}, \tag{2}$$

as illustrated in Fig. 1.

The model in (2) encompasses the signals of all the MSs in the SDMA group $\mathcal{G}$ under consideration. Since the demodulation process is distributed among the MSs, $\mathbf{D}$ can be written

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).

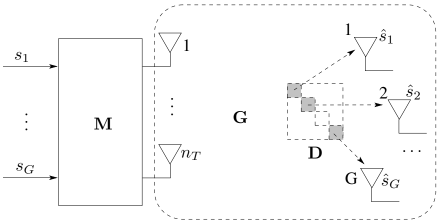


Fig. 1. Transmission chain.

in a diagonal form (or block diagonal form, if MSs have multiple antennas), decoupling signals received by different MSs. Both matrices $\mathbf{M}$ and $\mathbf{D}$ in (2) are defined according to the precoding technique used in the system.

Let $(\cdot)^H$ denote the conjugate transpose of a matrix and $\mathbf{I}$ an identity matrix of suitable dimension. Then, using (2) and (6), the group capacity C of the SDMA group $\mathcal{G}$ is given by

$$C = \log_2\left(\det\left(\mathbf{I} + \sigma_n^{-2}\mathbf{DGM}(\mathbf{DGM})^H\right)\right), \qquad (3)$$

which will be used in order to calculate the system spectral efficiency.

The spatial correlation between two vector channels $\mathbf{h}_i$ and $\mathbf{h}_j$ is measured by the normalized scalar product ρ_{ij} [4,5,13]. Denoting the absolute value of a complex scalar and the l_2-norm of a vector by $|\cdot|$ and $\|\cdot\|_2$, respectively, ρ_{ij} is given by

$$\rho_{ij} = \left|\mathbf{h}_i\mathbf{h}_j^H\right| / \left(\|\mathbf{h}_i\|_2 \|\mathbf{h}_j\|_2\right) \qquad (4)$$

Let $\operatorname{diag}\{\cdot\}$ denote a diagonal matrix whose diagonal elements are given as arguments. Then, using (1) and (4), it is possible to write a real non-negative matrix $\mathbf{R} \in \mathbb{R}_+^{n_R \times n_R}$ containing the correlation coefficient ρ_{ij} for every pair of channels $\mathbf{h}_i$ and $\mathbf{h}_j$ as

$$\mathbf{R} = \left|\mathbf{NHH}^H\mathbf{N}\right|, \quad \text{with} \qquad (5a)$$

$$\mathbf{N} = \operatorname{diag}\left\{\|\mathbf{h}_1\|^{-1}, \|\mathbf{h}_2\|^{-1}, \ldots, \|\mathbf{h}_{n_R}\|^{-1}\right\}, \qquad (5b)$$

where $|\cdot|$ is applied to $\mathbf{R}$ element-wise. In the next section, $\mathbf{R}$ is used as input for the proposed SDMA grouping algorithm.

ZF precoding, which a simple and linear precoding technique, is considered in this work. Other precoding techniques could however be considered [2]–[4]. Let $\|\cdot\|_F$ denote the Frobenius norm of a matrix. Then, the modulation and demodulation matrices in (2) become, respectively,

$$\mathbf{M} = \sqrt{P}\mathbf{G}^H\left(\mathbf{GG}^H\right)^{-1} \Big/ \left\|\mathbf{G}^H\left(\mathbf{GG}^H\right)^{-1}\right\|_F \quad \text{and} \qquad (6a)$$

$$\mathbf{D} = \mathbf{I}, \qquad (6b)$$

where P is the available transmission power.

III. SDMA GROUPING ALGORITHM

A. General problem

In this section, the general SDMA grouping problem is briefly discussed. In general, finding the SDMA group that optimizes a given group metric is a selection problem. It corresponds to select G, with $1 \leq G \leq n_T$, channels from a total number of n_R vector channels as to optimize the grouping metric. This problem is combinatorial and hard to solve.

Since the number of active MSs and, consequently, the total number of receiving antennas in the sector is usually much larger then the number of transmit antennas ($n_R \gg n_T$), searching for the best SDMA group by evaluating and comparing the grouping metric for every possible SDMA group can become prohibitively complex even for small values of n_R. This happens, e.g., if the maximization of the throughput of the system is pursued, which requires to compute the group capacity according to (3) for $\sum_{g=1}^{n_T}\binom{n_R}{g}$ groups. When n_R increases, the computational cost of this approach increases exponentially and it becomes rapidly unfeasible.

In the above problem, the complexity of the SDMA algorithm is directly affected by the precoding technique used. By using ZF precoding [2]–[4], which is a simple linear technique, a lower computational cost can be achieved by the algorithm compared to more sophisticated precoding techniques, such as Block Diagonalization [14]. However, performance losses are expected. By assuming a fixed SDMA group size G, the problem can also be simplified. In this case, only $\binom{n_R}{G}$ groups must be compared. However, performance can be degraded if G does not match the optimum SDMA group size $G^\star$ and, moreover, there is no fixed rule to determine $G^\star$ a priori.

B. Regularized Correlation-Based Algorithm (RCBA)

In this section, a sub-optimal SDMA grouping algorithm named Regularized Correlation-Based Algorithm (RCBA) is proposed. It is formulated as a convex optimization problem based on the spatial correlation among MSs' channels, thus avoiding an ES and not needing to compute precoding matrices. ZF precoding and a given group size G are assumed. The choice of the SDMA group size G is discussed in the next section.

Under ZF, SDMA groups containing correlated channels result into poor performance in terms of group capacity [15]. With ZF, however, building an SDMA group whose channels are as uncorrelated as possible represents an effective sub-optimal approach. The SDMA group of size G with minimum total correlation, i.e., whose sum of the spatial correlation values between every pair of vector channels in the SDMA group is minimal, represents a good solution.

Using (5), the SDMA group with minimum total correlation can be found by solving the following integer optimization problem:

$$\mathbf{x}^\star = \arg\min_{\mathbf{x}}\left\{\frac{1}{2}\mathbf{x}^T\mathbf{R}\mathbf{x}\right\}, \qquad (7a)$$

$$\text{s.t.:} \quad \mathbf{1}^T\mathbf{x} = G, \qquad (7b)$$

$$x_i \in \{0,1\},\ i = 1,\ldots,n_R, \qquad (7c)$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \ldots & x_{n_R}\end{bmatrix}^T$ is a binary selection vector and $\mathbf{1}$ is a $n_R \times 1$ vector of ones. If $\mathbf{x}^\star$ is the optimum solution of (7), the optimum SDMA group $\mathcal{G}^\star$ corresponds to select the rows $\mathbf{h}_i$ of $\mathbf{H}$ for which $x_i^\star = 1$, $i = 1,\ldots,n_R$.

The problem in (7) is still combinatorial and can be solved by evaluating the cost function at the $\binom{n_R}{G}$ values of $\mathbf{x}$ and selecting the one with minimum cost. However, when n_R

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).

increases, complexity still grows exponentially.

Avoiding this exponential complexity increase is desired here. This is accomplished by relaxing the constraint (7c) to allow continuous values for x_i, which is replaced by $\tilde{x}_i \in [0,1]$. The factor $1/2$, which does not affect the optimization, was also removed from (7a). Then, (7) is rewritten as

$$\tilde{\mathbf{x}}^\star = \arg\min_{\tilde{\mathbf{x}}} \left\{\tilde{\mathbf{x}}^T \mathbf{R} \tilde{\mathbf{x}}\right\}, \tag{8a}$$

$$\text{s.t.:}\ \mathbf{1}^T \tilde{\mathbf{x}} = G, \tag{8b}$$

$$\tilde{x}_i \in [0,1],\ i = 1, \ldots, n_R, \tag{8c}$$

which is a convex quadratic problem with linear constraints. Since $\mathbf{R}$ is positive definite in the feasible set of (8), the problem can be efficiently solved, i.e., with non-exponential complexity, using convex optimization [11].

The optimum solution $\tilde{\mathbf{x}}^\star$ of (8) is non-integer and its components $\tilde{x}_i^\star$ can be interpreted as the probabilities of the corresponding channels being in the optimum SDMA group $\mathcal{G}^\star$. In order to convert $\tilde{\mathbf{x}}^\star$ into an integer solution, their G largest components are simply set to 1 and the other $n_R - G$ components to 0. Nevertheless, more sophisticated rounding strategies could be applied [6]. In spite of not being necessarily the optimum solution for (7), $\tilde{\mathbf{x}}^\star$ often coincides with $\mathbf{x}^\star$.

The performance of (8) can be improved in terms of capacity by formulating a regularized version for it. Regularization is a common scalarization method used to solve multi-criterion problems [11]. Because group capacity depends not only on the spatial correlation among the channels, but also on the channel gains, the idea is to introduce a second term in (8) to favor SDMA groups whose channels have high gains. The optimization problem of the Regularized Correlation-Based Algorithm (RCBA) is formulated as

$$\tilde{\mathbf{x}}^\star = \arg\min_{\tilde{\mathbf{x}}} \left\{\frac{(1-\alpha)}{\|\mathbf{R}\|_F}\tilde{\mathbf{x}}^T \mathbf{R} \tilde{\mathbf{x}} + \alpha \frac{\|\mathbf{N}\tilde{\mathbf{x}}\|_1}{\|\mathbf{N}\|_F}\right\}, \tag{9a}$$

$$\text{s.t.:}\ \mathbf{1}^T \tilde{\mathbf{x}} = G, \tag{9b}$$

$$\tilde{x}_i \in [0,1],\ i = 1, \ldots, n_R, \tag{9c}$$

where $\|\cdot\|_1$ is the l_1-norm of a vector and $0 \leq \alpha \leq 1$ is a weighting factor. For $\alpha = 0$, (9) is equivalent to (8), and by increasing α towards 1, channels with higher gain become more and more preferential.

Note that (9) can be modified in order to take MSs' priorities into account. In RCBA, the constraint

$$\tilde{x}_c = 1, \quad c \in [1, n_R] \tag{10}$$

can be added to (9) in order to force a high priority MS channel, indexed here by c, to be present in the SDMA group.

Computing $\mathbf{R}$ and evaluating the cost function in (9a) roughly require $\mathcal{O}\left(n_T n_R^2\right)$ and $\mathcal{O}\left(n_R^2\right)$ operations, respectively. RCBA complexity is therefore roughly of $\mathcal{O}\left(n_R^2\right)$, assuming $n_R \gg n_T$, which is much lower than that of the ES in section III-A, which is roughly of $\mathcal{O}\left(2^{n_R}\right)$.

C. Sequential Removal Algorithm (SRA)

In the previous section, RCBA is proposed assuming a fixed SDMA group size G. In this section, a Sequential Removal Algorithm (SRA) is proposed to adjust the size of the SDMA group obtained by solving (9) and to compensate for possible mismatches between G and the optimum group size $G^\star$. Because a larger group does not necessarily imply better performance, at each step of the SRA, G is reduced by one by removing the channel having the highest total correlation with respect to the other channels in the SDMA group. This is a simple strategy, but more sophisticated algorithms could also be applied [4].

1. Apply RCBA with an initial fixed group size $G \leq n_T$ and generate $\mathcal{G}$.
2. Compute the group capacity C of $\mathcal{G}$ using (3).
3. Define $C^\star = C$ and $\mathcal{G}^\star = \mathcal{G}$.
4. While the size of $\mathcal{G} > 1$
   a. Generate the correlation matrix $\mathbf{R}_\mathcal{G}$ for $\mathcal{G}$ using the data in $\mathbf{R}$, given by (5).
   b. Define $\mathcal{G} = \mathcal{G} \setminus \{\arg\max_g \mathbf{R}_\mathcal{G}\mathbf{1}\}$, with $g \in \{1, \ldots, G\}$.
   c. Compute the group capacity C of $\mathcal{G}$ using (3).
   d. If $C > C^\star$, define $C^\star = C$ and $\mathcal{G}^\star = \mathcal{G}$.

Since C is not necessarily monotonic in G, all the SDMA group sizes are considered in step 4. Note that SRA can be modified to ensure that a given high priority MS be never removed from the SDMA group in the step 4b. In the following, SRA is used with RCBA to improve its performance compared with RCBA using a fixed group size.

IV. Simulation Results

Simulations considering one sector with an Uniform Linear Array (ULA) having $n_T = 4$ or 8 elements are performed to assess the performance of the RCBA. A total number of $K = 16$ active single-antenna MSs are randomly placed in the sector. MSs' priority management is not considered.

MSs' channel matrices $\mathbf{H}_k$ are obtained using

$$\mathbf{H}_k = \sqrt{K_R/(1+K_R)}\overline{\mathbf{H}} + \sqrt{1/(1+K_R)}\mathbf{H}_w \tag{11}$$

where K_R is the Rice factor, and $\overline{\mathbf{H}}$ and $\mathbf{H}_w$ are the Line-Of-Sight (LOS) and Non-LOS (NLOS) channel components, respectively [15,16]. The NLOS component of (1) has zero-mean circularly symmetric complex Gaussian components. NLOS and LOS scenarios are considered, in which $K_R = -\infty$ and $K_R = 10$ dB, respectively. In LOS scenario, MSs' channels become strongly correlated if the angular separation among the MSs is small. Slow fading and path loss are assumed to be ideally compensated by power control and only the fast fading is considered. Group capacity is calculated using (3). The most relevant simulation parameters are summarized in Table I.

First, it is investigated how close the total spatial correlation of the solution obtained by RCBA is to that of the optimum solution of (7) found by checking all possible SDMA groups of fixed size G. In Fig. 2, the Cumulative Distribution Function (CDF) of the quotient between the total correlation of the SDMA groups obtained by RCBA with $\alpha = 0$ and by (7) is shown. Whenever this quotient is equal to 1, RCBA matches the optimum solution of (7). This happens in over 60% of the

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).

TABLE I

SIMULATION PARAMETERS.

Parameter	Value
Scenario	NLOS ($K_R = -\infty$ dB), LOS ($K_R = 10$ dB)
Communication link	Downlink
# of active MSs	16
User distribution	Uniform
Propagation	Fast fading only
Channel response	Flat
# of sub-carriers	1
Sector antenna	ULA with $n_T = 4$ or $n_T = 8$ (half wavelength element separation)
MS antenna	Single antenna
Average SINR	10 dB
SDMA algorithms	RCBA with fixed group size, RCBA with SRA, ES
Precoding	ZF according to (6)
Signaling	Gaussian

cases for both the NLOS and LOS scenarios. In over 90% of the cases, the described quotient is no larger than 1.05 showing that the solution of RCBA approximates very well the optimum solution of the problem (7).

Now, the system capacity achieved by using RCBA is compared to that one achievable through ES. Comparisons are performed considering the capacity that the system achieves in 90% of the cases, named here C_{90} capacity, and which corresponds to the well-known 10% outage capacity of the system. Fig. 3 shows, for different values of α, the ratio between the C_{90} capacity obtained by applying the RCBA normalized with respect to that obtained by means of the ES, i.e., $C_{90,RCBA}/C_{90,ES}$. In the ES, the group with maximum capacity is found by comparing all possible SDMA groups. Capacity values are determined using (3).

In Fig. 3(a), the performance of RCBA with fixed group size $G = n_T$ can be observed. For $G = n_T = 8$, the normalized capacity figures are below 20% of the optimum obtained by means of the ES. The $C_{90,ES}$ corresponds to 19.6 bps/Hz and 21.3 bps/Hz in the NLOS and LOS scenarios, respectively. On the other hand, for the case in which $G = n_T = 4$ and considering $0 \leq \alpha \leq 0.3$, capacity figures of at least 70% of the optimum capacity are achieved. In this case, $C_{90,ES}$ corresponds to 11.9 bps/Hz and 12.7 bps/Hz in the NLOS and LOS scenarios, respectively. The better performance when $n_T = 4$ transmit antennas are considered is due to the fact that a value of $G = 4$ is a much better suited SDMA group size in this case than $G = 8$ in the case with $n_T = 8$ transmit

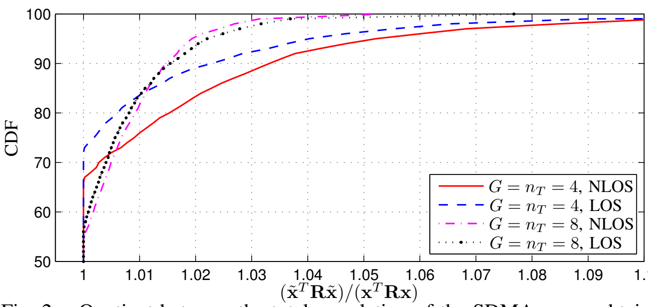
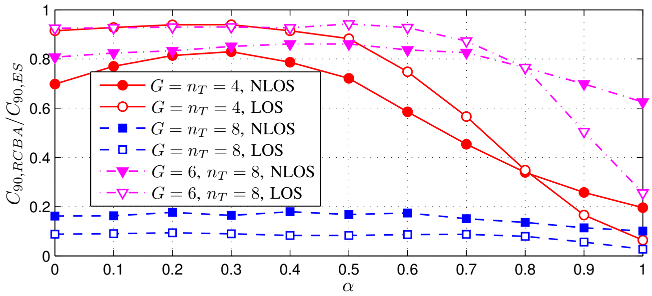
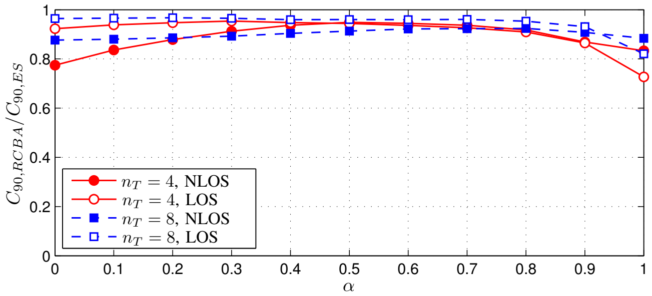


Fig. 2. Quotient between the total correlation of the SDMA group obtained by RCBA and the total correlation of the optimum SDMA group for (7): $(\tilde{\mathbf{x}}^{\star T}\mathbf{R}\tilde{\mathbf{x}}^{\star})/(\mathbf{x}^{\star T}\mathbf{R}\mathbf{x}^{\star})$. NLOS: $K_R = -\infty$ dB. LOS: $K_R = 10$ dB.



(a) RCBA with fixed group size.



(b) RCBA with initial $G = n_T$ and SRA.

Fig. 3. C_{90} capacity of the RCBA normalized with respect to C_{90} of the ES: $C_{90,RCBA}/C_{90,ES}$. NLOS: $K_R = -\infty$ dB. LOS: $K_R = 10$ dB.

antennas. For $G = n_T = 8$, some incompatible MSs are often included in the SDMA group, which causes the group capacity to be low.

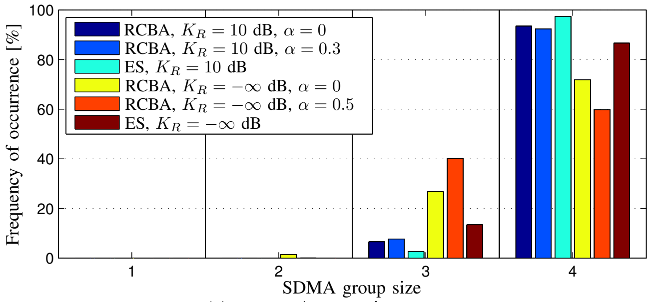
Because the optimum group size $G^{\star}$ cannot be predicted, the selection of G has a considerable impact on the performance of RCBA. This impact can be seen in Fig. 3(a) by comparing the curves for $G = n_T = 8$, indicated by squares, with the curves for $G = 6$ and $n_T = 8$, indicated by triangles. It can be seen that the performance obtained by RCBA with the group size $G = 6$ is much better than with $G = 8$.

Applying the RCBA for every group size $1 \leq G \leq n_T$ and selecting the best SDMA group among the obtained ones is an alternative to overcome the group size problem. However, it is more complex than considering the proposed SRA. In Fig. 3(b), RCBA with SRA is considered, which adapts the SDMA group size. In this case, SRA tests $G = n_T$ groups and selects the best among them, thus resulting in a considerable improvement of the capacity figures. In fact, with SRA, the capacity figure obtained by RCBA can reach up to 95% of the capacity of the ES. Moreover, over 70% of the capacity of the ES is achieved for all values of α.

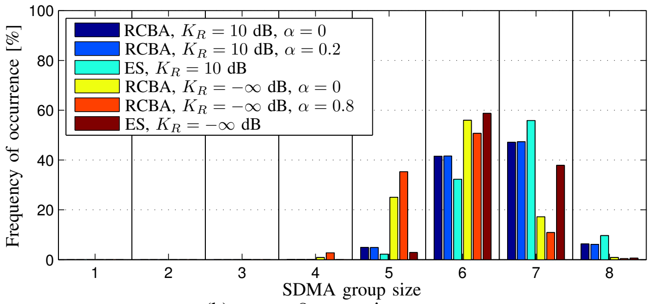
It can also be noted that setting $G = n_T$ and using SRA to adjust the group size, as in Fig. 3(b), provides better capacity figures than using the fixed group sizes assumed in Fig. 3(a). Nevertheless, it incurs in extra complexity due to SRA.

It can be seen in Fig. 3(b) that, by varying α, the performance of the RCBA with SRA can be improved by 22% in the NLOS scenario with $n_T = 4$ transmit antennas when compared with $\alpha = 0$. RCBA with $\alpha = 0$ corresponds to the non-regularized case in (8). In the NLOS scenario, spatial correlation and channel gain have comparable roles and the best capacity figures are obtained for $\alpha = 0.5$. For the scenarios considering $n_T = 8$ transmit antennas, the gain

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).



(a) $n_T = 4$ transmit antennas.



(b) $n_T = 8$ transmit antennas.

Fig. 4. SDMA group size distribution.

in the NLOS scenario approximates 10% for $\alpha = 0.8$ when compared to RCBA with $\alpha = 0$. In the LOS scenarios, spatial correlation plays a more decisive role and the RCBA with $\alpha = 0$ already achieves over 90% of $C_{90,ES}$. Nevertheless, small gains are obtained for $\alpha = 0.3$ when $n_T = 4$, and $\alpha = 0.8$ when $n_T = 8$. In practice, a control loop could be implemented by the system in order to learn from the environment and adaptively select an adequate value for α.

The obtained group sizes for the above configurations are included in Fig. 4. It shows the distribution of the SDMA group size obtained by RCBA with SRA, as well as the optimum group size found by means of the ES. In Fig. 4(a), it can be observed that $G = 4$ is the most frequent group size when using the ES and $n_T = 4$ transmit antennas. On the other hand, in Fig. 4(a) in which $n_T = 8$, the most frequent values for G are 6 and 7 in the case of the ES. This explains the better performance of RCBA with fixed group size $G = n_T$ in Fig. 3(a) when $n_T = 4$ compared to the case when $n_T = 8$ in the same figure, as well as the improvement observed for $G = 6$ and $n_T = 8$. Anyway, it can be seen in Fig. 4 that the optimum SDMA group size found using RCBA with SRA often corresponds to the optimum one obtained through an ES in all the considered scenarios.

In fact, information about the group size distribution can be learnt by the system from the environment by applying the SRA algorithm during an initial phase. After that phase, the SRA could be simplified by discarding too large or too small group sizes or, alternatively, by using even a fixed group size.

V. Conclusions

In this work, the SDMA grouping problem has been studied. An SDMA grouping algorithm, namely the RCBA, is proposed in order to find a sub-optimal but efficient solution with reduced complexity compared to an ES. It uses a new grouping metric, which is based on the spatial correlation and gains of the MSs' channels in the SDMA group. Thus, it optimizes a trade-off between the minimization of the total spatial correlation and the selection of channels with high gains. It formulates the SDMA algorithm as a convex quadratic problem, which can be efficiently solved using convex optimization. In order to adjust the SDMA group size, which impacts the overall performance of SDMA grouping algorithms, a sub-optimal but effective algorithm, namely the SRA, is also proposed. The performance of RCBA is investigated in two extreme scenarios: a NLOS scenario with uncorrelated MSs' spatial channels and a LOS scenario with strongly correlated MSs' spatial channels. Considering ZF precoding, RCBA with a fixed group size and RCBA with SRA have been shown to achieve up to 95% of the $C_{90,ES}$. SRA has been shown to find an adequate SDMA group size in a considerable number of cases in all the investigated scenarios.

References

[1] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[2] M. Meurer, P. W. Baier, and W. Qiu, "Receiver orientation versus transmitter orientation in linear MIMO transmission systems," *EURASIP J. on Applied Signal Proc.*, vol. 9, pp. 1191–1198, 2004.

[3] M. Joham, "Optimization of linear and nonlinear transmit signal processing," Ph.D. dissertation, Munich University of Technology, Munich, Germany, Jun. 2004.

[4] D. B. Calvo, "Fairness analysis of wireless beamforming schedulers," Ph.D. dissertation, Technical University of Catalonia, Spain, Nov. 2004.

[5] F. Shad, T. D. Todd, V. Kezys, and J. Litva, "Dynamic Slot Allocation (DSA) in indoor SDMA/TDMA using a smart antenna basestation," *IEEE/ACM Trans. Networking*, vol. 9, no. 1, pp. 69–81, Feb. 2001.

[6] G. Nemhauser and L. Wosley, *Integer and combinatorial optimization*. Wiley & Sons, 1999.

[7] M. Fuchs, G. D. Galdo, and M. Haardt, "A novel tree-based scheduling algorithm for the downlink of multi-user MIMO systems with ZF beamforming," in *Proc. of the IEEE Internat. Conf. on Acoustics, Speech, and Signal Proc. (ICASSP)*, vol. 3, Mar. 2005, pp. 1121–1124.

[8] A. Tölli and M. Juntti, "Scheduling for multiuser MIMO downlink with linear processing," in *Proc. of the IEEE Personal, Indoor and Mob. Radio Commun. (PIMRC)*, 2005.

[9] Q. Spencer and A. L. Swindlehurst, "Channel allocation in multi-user MIMO wireless communications systems," in *Proc. of the IEEE Internat. Conf. on Commun. (ICC)*, vol. 5, Jun. 2004, pp. 3035–3039.

[10] T. F. Maciel and A. Klein, "A low-complexity SDMA grouping strategy for the downlink of Multi-User MIMO systems," in *Proc. of the IEEE Personal, Indoor and Mob. Radio Commun. (PIMRC)*, Sept. 2006.

[11] S. Boyd and L. Vandenberghe, *Convex optimization*, 1st ed. Cambridge Univ. Press, 2004.

[12] M. Doettling, M. Sternad, G. Klang, J. von Hafen, and M. Olsson, "Integration of spatial processing in the WINNER B3G air interface design," in *Proc. of the IEEE Vehic. Tech. Conf. (VTC)*, vol. 1, May. 2006, pp. 246–250.

[13] C. Farsakh and J. A. Nossek, "A real time downlink channel allocation scheme for an SDMA mobile radio system," in *Proc. of the IEEE Personal, Indoor and Mob. Radio Commun. (PIMRC)*, vol. 3, Oct. 1996, pp. 1216–1220.

[14] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[15] A. Paulraj, R. Nabar, and D. Gore, *Introduction to space-time wireless communications*, 1st ed. Cambridge Univ. Press, 2003.

[16] F. R. Farrokhi, A. Lozano, G. J. Foschini, and R. A. Valenzuela, "Spectral efficiency of wireless systems with multiple transmit and receive antennas," in *Proc. of the IEEE Personal, Indoor and Mob. Radio Commun. (PIMRC)*, vol. 1, Sept. 2000, pp. 18–21.

T. F. Maciel and A. Klein, "A convex quadratic SDMA grouping algorithm based on spatial correlation," IEEE International Conference on Communications (ICC 2007), 24-28 June 2007, Glasgow, Scotland, (accepted for publication).