

ADAPTIVE SUBCARRIER ALLOCATION WITH IMPERFECT CHANNEL KNOWLEDGE VERSUS DIVERSITY TECHNIQUES IN A MULTI-USER OFDM-SYSTEM

Alexander Kühne, Anja Klein

Darmstadt University of Technology, Communications Engineering Lab,
Merckstr. 25, 64283 Darmstadt, Germany, a.kuehne@nt.tu-darmstadt.de

ABSTRACT

In this paper, the performance of an adaptive multi-user OFDM system with imperfect channel knowledge at the transmitter is investigated, where the ergodic capacity is taken as performance criterion. This performance is then compared to the performance achievable with the use of diversity, where no channel knowledge at the transmitter is required. For that reason, different models of imperfect channel knowledge are introduced: noisy, outdated and quantised channel knowledge, where we provide an analytical derivation of the ergodic capacity for the combinations of the different models. By means of the ergodic capacity it is shown to which grade of channel knowledge imperfectness the use of adaptive subcarrier allocation with imperfect channel knowledge is still better than the use of diversity.

I. INTRODUCTION

OFDMA is considered to be a suitable candidate for future radio network systems where high data rates are required. It is applicable in multi-user systems and can be combined with Multiple Input Multiple Output (MIMO) techniques [1] [2]. With a multicarrier scheme like OFDMA, the overall channel can be divided in several resources in the frequency domain which can be combined with a division in the time and space dimension. The aim of a multi-user system is to find an optimal allocation of the resources to the different users. Here, the quality of the resources of the different users and the interference between the different users have to be known at the transmitter. Having perfect channel knowledge at the transmitter, adaptive subchannel allocation schemes achieve very good performances [3] [4] [5]. However, in a realistic scenario perfect channel knowledge is not available at the transmitter and thereby, the performance of adaptive schemes that requires channel knowledge at the transmitter decreases. The use of diversity is an alternative way to achieve performance enhancements without channel knowledge at the transmitter. Applying frequency hopping [6] or applying a DFT-precoding of the data [7] together with interleaved carrier allocation are examples for techniques to exploit frequency diversity in a OFDMA system. In a frequency selective channel, diversity techniques lead to an averaging over strongly and weakly attenuated frequencies which results in a performance averaging. Due to the fact that in a realistic scenario only imperfect channel knowledge is achievable with acceptable effort, a comparison between adaptivity with imperfect channel knowledge and diversity has to be made. In [8] a first comparison between adaptivity and diversity is

presented where in a MIMO system space-time coding is compared to adaptive bit- and power loading for single connections. With perfect channel knowledge, the adaptive scheme showed the better performance as expected. In [9], an application of special orthogonal space-time-block codes with partial channel knowledge is analysed. In [10], combinations of spatial and frequency based diversity techniques for a multiuser scenario with limited feedback are discussed.

In this paper, we analytically derive the performance in terms of ergodic capacity for a multiuser SISO-OFDM system with adaptive subcarrier allocation under the assumption of imperfect channel knowledge. As channel knowledge we employ the instantaneous SNR of the different subcarriers, which are assumed to be measured at the BS in case of a TDD system or fed back over a feedback channel in case of a FDD system. For the imperfect channel knowledge we introduce different models: noisy, outdated and quantised channel knowledge and furthermore their combinations. The capacity achievable with adaptive subcarrier allocation with imperfect channel knowledge at the transmitter is compared to the capacity achievable by exploiting frequency diversity without any channel knowledge at the transmitter. The aim is to find a bound where adaptivity with imperfect channel knowledge is still better than diversity. The paper is organised as follows. Section II describes the system model. The considered adaptive subcarrier allocation is given in Section III, where Section IV presents the different models for imperfect channel knowledge. In Section V the ergodic capacity is analytically derived for the case of adaptive subcarrier allocation with imperfect channel knowledge. Section VI provides the analytical derivation of the ergodic capacity using frequency diversity. In Section VII the capacity using adaptive subcarrier allocation with imperfect channel knowledge is presented and compared to the capacity achievable with diversity techniques. Finally, conclusions are drawn in Section VIII.

II. SYSTEM MODEL

In this paper, we consider a one cell OFDMA downlink scenario with one base station (BS) and U mobile stations (MS) with user index $u = 1, \dots, U$, where it is assumed that each user has the same requirements in terms of data rate. The number of subcarriers is N with subcarrier index $n = 1, \dots, N$. Furthermore, we assume perfect time and frequency synchronization. It is assumed that real and imaginary part of the channel transfer factor $H_u(n, k)$ of each user u at each subcarrier with index n at each time-slot index $k \in \mathbb{N}$ can be modelled by an independent identically Gaussian distributed process with zero

mean and variance one half. Assuming perfect power control leading to an average SNR $\bar{\gamma}$ for all users, the instantaneous SNR $\gamma_u(n, k)$ of user u of subcarrier with index n of time-slot k is given by

$$\gamma_u(n, k) = \hat{\gamma} \cdot |H_u(n, k)|^2. \quad (1)$$

III. ADAPTIVE SUBCARRIER ALLOCATION

In order to be able to benefit from multi-user diversity [11] [12], the scheduler at the BS needs to have channel knowledge, in our consideration an estimate $\hat{\gamma}_u(n, k)$ of the instantaneous SNR $\gamma_u(n, k)$ of the n -th subcarrier of user u at time-slot k , which is measured at the BS (FDD) or fed back over an feed-back channel from the MS to the BS (TDD). Throughout this paper we apply a Max-SNR Scheduler that favours the users with the best SNR conditions. From this it follows that one subcarrier is allocated to only one user exclusively. Hence, in time-slot k the subcarrier n is allocated to the user u with the highest instantaneous SNR $\gamma_u(n, k)$:

$$\gamma_{u_{max}}(n, k) = \arg \max_u \{\gamma_1(n, k), \dots, \gamma_u(n, k), \dots, \gamma_U(n, k)\}. \quad (2)$$

Since each user experience the same average SNR $\bar{\gamma}$, the probability P_a that a subcarrier is assigned to a user is equal for all users and given by $P_a = \frac{1}{U}$.

IV. MODELS FOR IMPERFECT CHANNEL KNOWLEDGE

In this section, two models for imperfect channel knowledge are presented. First, noisy and outdated channel knowledge and second quantised channel knowledge are considered.

A. Noisy and outdated channel knowledge

In a realistic scenario, the measurement of the channel is not perfect, i.e. the measured channel is only a erroneous estimate of the actual channel. It is assumed that this estimation error can be modelled by an additive complex Gaussian error term e_u with variance $\sigma_{e_u}^2$. The second source of error is the time delay, i.e., there is always a time delay between the instant of SNR measuring and the actual instant of transmission of the data to the scheduled users. From this it follows that the channel knowledge available at the transmitter is outdated. This can be modelled by correlation, i.e. the measured channel and the actual channel are two realisations of one complex Gaussian process with correlation coefficient ρ_u . Assuming a Jakes' scattering model, $\rho_u = J_0\left(2\pi v_u \frac{f_0 T}{c}\right)$, where v_u denotes the velocity of user u , f_0 the carrier frequency and T the delay time. For simplicity, we assume that $\rho_u = \rho$ and $\sigma_{e_u}^2 = \sigma_e^2$ for each user u . Since the following considerations are valid for each subcarrier n of user u at each timeslot k , the indices can be skipped. Now, the measured channel can be modelled by

$$\hat{H} = \tilde{H} + e \quad (3)$$

where $\tilde{H}$ denotes the channel correlated to the actual channel H with correlation coefficient ρ and e the complex Gaussian

distributed error term with variance σ_e^2 . In order to describe the relationship between the signalled channel $\hat{H}$ and the actual channel H , the conditional probability density function (PDF) $p_{H|\hat{H}}(H|\hat{H})$ has to be determined. It can be shown that the conditional PDF is given by

$$p_{H|\hat{H}}(H|\hat{H}) = \frac{1}{\sqrt{2\pi} \sqrt{\frac{1-\rho^2+\sigma_e^2}{1+\sigma_e^2}}} \times \exp\left(-\frac{\left(H - \frac{\rho}{1+\sigma_e^2} \hat{H}\right)^2}{2 \left(\frac{1-\rho^2+\sigma_e^2}{1+\sigma_e^2}\right)}\right). \quad (4)$$

It can be seen that $H|\hat{H}$ is a complex Gaussian distributed random variable with mean $\mu = \frac{\rho}{1+\sigma_e^2} \hat{H}$ and variance $\sigma^2 = \frac{1-\rho^2+\sigma_e^2}{1+\sigma_e^2}$. Since not the complex channel transfer factors are measured, but the received SNR $\hat{\gamma}$ it is essential to determine the conditional PDF $p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma})$. It can be shown that $\gamma|\hat{\gamma}$ is a noncentral chi-square random variable with 2 degrees of freedom. With [13, p. 43] the conditional PDF is determined by

$$p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) = \frac{1+\sigma_e^2}{\bar{\gamma}(1-\rho^2+\sigma_e^2)} \cdot I_0\left(\frac{2\rho\sqrt{\gamma\cdot\bar{\gamma}}}{\bar{\gamma}(1-\rho^2+\sigma_e^2)}\right) \times \exp\left(-\frac{\frac{\rho^2}{1+\sigma_e^2} \hat{\gamma} + \gamma(1+\sigma_e^2)}{\bar{\gamma}(1-\rho^2+\sigma_e^2)}\right), \quad (5)$$

where $I_0(x)$ denotes the 0th-order modified Bessel function of the first kind.

B. Quantised channel knowledge

In order to decrease the feedback in case of a FDD system, the SNR values of each subcarrier n of each user u in each time-slot k are quantised in quantisation intervals with $W = 2^{N_Q} + 1$ quantisation bounds s_q with $q = 0, \dots, W$ where $s_0 = 0$, $s_W = \infty$ and N_Q the number of quantisation bits per subcarrier. The feedback is then digitised before transmission. Using the indices once again, the channel knowledge available at the BS of subcarrier n of user u at time-slot k is given by

$$\hat{\gamma}_u(n, k) = Q\{\gamma_u(n, k)\} \quad (6)$$

where $Q\{x\}$ denotes the quantisation operation. If we combine quantised channel knowledge with noisy and outdated channel knowledge referred to as outdated noisy quantised channel knowledge, the conditional PDF is also determined by (5).

V. ERGODIC CAPACITY USING ADAPTIVE SUBCARRIER ALLOCATION

In this section, we provide an analytical derivation of the ergodic capacity per scheduled subcarrier for the case of noisy and outdated channel knowledge and outdated noisy quantised channel knowledge.

A. Ergodic capacity

The ergodic capacity of a scheduled subcarrier is determined by

$$\bar{C} = \int_0^\infty \log_2(1 + \gamma) \cdot p_\gamma(\gamma) d\gamma \quad (7)$$

where $p_\gamma(\gamma)$ denotes the PDF of the actual SNR γ of the scheduled subcarrier, which is calculated by

$$p_\gamma(\gamma) = \int_0^\infty p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) \cdot p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}. \quad (8)$$

B. Noisy and outdated channel knowledge

In case of noisy and outdated channel knowledge using a Max-SNR Scheduler, the PDF $p_{\hat{\gamma}}(\hat{\gamma})$ of the selected SNR $\hat{\gamma}$ of a subcarrier is the same for all subcarriers N for all users U and obtained using order statistics [14] by

$$\begin{aligned} p_{\hat{\gamma}}(\hat{\gamma}) &= \frac{U}{E\{\hat{\gamma}\}} \left(1 - \exp\left(-\frac{\hat{\gamma}}{E\{\hat{\gamma}\}}\right)\right)^{U-1} \\ &= \frac{U}{E\{\hat{\gamma}\}} \sum_{i=0}^{U-1} \binom{U-1}{i} (-1)^i \exp\left(-\frac{\hat{\gamma}(i+1)}{E\{\hat{\gamma}\}}\right) \end{aligned} \quad (9)$$

where $E\{\hat{\gamma}\}$ denotes the expectation value of the measured SNR $\hat{\gamma}$ of a subcarrier and is given by $E\{\hat{\gamma}\} = \bar{\gamma}(1 + \sigma_e^2)$. Inserting (9) and (5) in (8) and using the identities [15, Eq. 6.643.4] [15, Eq. 8.406.3] and [15, Eq. 8.970.1] the PDF of γ is given by

$$\begin{aligned} p_\gamma(\gamma) &= U \sum_{i=0}^{U-1} \binom{U-1}{i} (-1)^i \\ &\times \frac{1 + \sigma_e^2}{\bar{\gamma}(\rho^2 + (i+1)(\sigma_e^2 + 1 - \rho^2))} \\ &\times \exp\left(\frac{1 + \sigma_e^2}{\bar{\gamma}(\rho^2 + (i+1)(\sigma_e^2 + 1 - \rho^2))}\right). \end{aligned} \quad (10)$$

Inserting (10) in (7) and using the identity [15, Eq. 4.337.2], the ergodic capacity per scheduled subcarrier is determined by

$$\begin{aligned} \bar{C} &= \frac{U}{\ln(2)} \sum_{i=0}^{U-1} \binom{U-1}{i} (-1)^i \frac{1}{i+1} \\ &\times \exp\left(\frac{(i+1)(1 + \sigma_e^2)}{\bar{\gamma}(\rho^2 + (i+1)(\sigma_e^2 + 1 - \rho^2))}\right) \\ &\times E_1\left(\frac{(i+1)(1 + \sigma_e^2)}{\bar{\gamma}(\rho^2 + (i+1)(\sigma_e^2 + 1 - \rho^2))}\right). \end{aligned} \quad (11)$$

as a function of the number U of users, the correlation coefficient ρ , the error variance σ_e^2 and the average SNR $\bar{\gamma}$, where E_1 is standing for the first order exponential integral function ($E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt$). By setting $\sigma_e^2 = 0$ and $\rho = 1$ in (12) one obtains the ergodic capacity for perfect channel knowledge

$$\begin{aligned} \bar{C}_{per} &= \frac{U}{\ln(2)} \sum_{i=0}^{U-1} \binom{U-1}{i} (-1)^i \\ &\times \frac{1}{i+1} \exp\left(\frac{i+1}{\bar{\gamma}}\right) E_1\left(\frac{i+1}{\bar{\gamma}}\right). \end{aligned} \quad (12)$$

In the worst case of no channel knowledge ($\rho = 0$ or $\sigma_e^2 \rightarrow \infty$), i.e. the measured channel is either completely uncorrelated to

the actual channel or completely false, the ergodic capacity is given by

$$\bar{C}_{wc} = \frac{1}{\ln(2)} \exp\left(\frac{1}{\bar{\gamma}}\right) E_1\left(\frac{1}{\bar{\gamma}}\right). \quad (13)$$

This capacity corresponds to the case where the subcarrier are randomly allocated to the user. Depending on the grade of channel knowledge imperfectness (ρ, σ_e^2), the ergodic capacity lies between these two capacities $\bar{C}_{per}$ and $\bar{C}_{wc}$ which corresponds to the results obtained in [16].

C. Outdated noisy quantised channel knowledge

In a realistic scenario, the signalled SNR values are outdated. Furthermore, the SNR values available at the MS are SNR estimates. Hence, the quantisation decision is based on erroneous channel estimates. Thus, there are three effects leading to performance degradation. First, the SNR values are already outdated at the time instant of transmission. Second, the SNR value is quantised in the wrong quantisation interval due to the erroneous SNR estimate. And finally, due to the quantisation, the BS can not distinguish between users with the same quantisation value, i.e. the scheduler has to choose randomly. Therefore, the PDF of the selected SNR $\hat{\gamma}$ is given by

$$p_{\hat{\gamma}}(\hat{\gamma}) = \frac{1}{E\{\hat{\gamma}\}} \exp\left(-\frac{\hat{\gamma}}{E\{\hat{\gamma}\}}\right). \quad (14)$$

The PDF of the actual selected SNR γ is calculated according to

$$\begin{aligned} p_\gamma(\gamma) &= \sum_{i=1}^W p_\gamma^{(i)}(\gamma) \\ &= \sum_{i=1}^W \int_{s_{i-1}}^{s_i} a_i \cdot p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) \cdot p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}, \end{aligned} \quad (15)$$

where a_i is a factor to ensure that

$$\begin{aligned} \int_0^\infty p_\gamma^{(i)}(\gamma) d\gamma &= \\ \left(1 - \exp\left(-\frac{s_i}{E\{\hat{\gamma}\}}\right)\right)^U - \left(1 - \exp\left(-\frac{s_{i-1}}{E\{\hat{\gamma}\}}\right)\right)^U. \end{aligned} \quad (16)$$

The expression on the right side of (16) corresponds to the probability that a measured SNR value $\hat{\gamma}$ lies in the quantisation interval $[s_{i-1}, s_i]$. Inserting (14) and (5) in (15) and using (16) leads to

$$a_i = \frac{\left(1 - \exp\left(-\frac{s_i}{E\{\hat{\gamma}\}}\right)\right)^U - \left(1 - \exp\left(-\frac{s_{i-1}}{E\{\hat{\gamma}\}}\right)\right)^U}{\exp\left(-\frac{s_{i-1}}{E\{\hat{\gamma}\}}\right) - \exp\left(-\frac{s_i}{E\{\hat{\gamma}\}}\right)}. \quad (17)$$

In order to determine the ergodic capacity for outdated noisy quantised channel knowledge, we insert (15) in (7) which re-

sults in

$$\begin{aligned} \bar{C} &= \sum_{i=1}^W a_i \int_0^\infty \log_2(1 + \gamma) \cdot \frac{1}{\bar{\gamma}(1 + \sigma_e^2)} \\ &\times \int_{s_{i-1}}^{s_i} \frac{1 + \sigma_e^2}{\bar{\gamma}(1 - \rho^2 + \sigma_e^2)} \exp\left(-\frac{\hat{\gamma}}{\bar{\gamma}(1 + \sigma_e^2)}\right) \\ &\times \exp\left(-\frac{\left(\frac{\rho^2}{1 + \sigma_e^2} \hat{\gamma} + \gamma(1 + \sigma_e^2)\right)}{\bar{\gamma}(1 - \rho^2 + \sigma_e^2)}\right) \\ &\times I_0\left(\frac{2\rho\sqrt{\gamma \cdot \hat{\gamma}}}{\bar{\gamma}(1 - \rho^2 + \sigma_e^2)}\right) d\hat{\gamma} d\gamma. \end{aligned} \quad (18)$$

For the case of perfect quantised channel knowledge ($\sigma_e^2 = 0$ and $\rho = 1$) the double integral in (18) can be solved resulting in

$$\begin{aligned} \bar{C} &= \frac{1}{\ln(2)} \sum_{i=1}^W a_i \left[\exp\left(\frac{1}{\bar{\gamma}}\right) \left(E_1\left(\frac{s_{i-1} + 1}{\bar{\gamma}}\right) \right. \right. \\ &- \left. E_1\left(\frac{s_i + 1}{\bar{\gamma}}\right) \right) \exp\left(-\frac{s_{i-1}}{\bar{\gamma}}\right) \\ &\times \left. \ln(1 + s_{i-1}) - \exp\left(-\frac{s_i}{\bar{\gamma}}\right) \ln(1 + s_i) \right]. \end{aligned} \quad (19)$$

VI. ERGODIC CAPACITY USING DIVERSITY

In contrast to adaptive subcarrier allocation, the use of diversity techniques does not require any channel knowledge at the transmitter. If we assume that at each time-slot k all N subcarriers are allocated to a different user u , a transmission scheme can be used to exploit frequency diversity, e.g. Interleaved Orthogonal Frequency Division Multiple Access (IFDMA) [7] or Multicarrier Code Division Multiple Access (MC-CDMA) [18]. Compared to adaptive subcarrier allocation, each user gets the same amount of channel accesses. In theory, exploitation of diversity leads to an averaging over the N different subcarrier SNR conditions. Since we assume that adjacent subcarriers are independent from each other, the PDF of the resulting average SNR γ over N subcarriers is a chi-square distribution with $2N$ degrees of freedom [13, p. 41] and given by

$$p_\gamma(\gamma) = \left(\frac{N}{\bar{\gamma}}\right)^N \frac{\gamma^{N-1}}{(N-1)!} \exp\left(-\frac{\gamma N}{\bar{\gamma}}\right). \quad (20)$$

Inserting (20) in (7) and using the identity [15, Eq. 3.381.3] the ergodic capacity per scheduled subcarrier is determined by

$$\bar{C}_D = \frac{1}{\ln(2)} \exp\left(\frac{N}{\bar{\gamma}}\right) \sum_{m=1}^N \frac{\Gamma\left(m - N, \frac{N}{\bar{\gamma}}\right)}{\left(\frac{N}{\bar{\gamma}}\right)^{m-N}}, \quad (21)$$

with $\Gamma(\alpha, x)$ the incomplete gamma function, which corresponds to the result given in [17].

VII. COMPARISON

In this section we illustrate the achievable capacity of adaptive subcarrier allocation with imperfect channel knowledge. We

assume an OFDM system with $N = 8$ subcarriers and $U = 10$ users, where each user has an average SNR of $\bar{\gamma} = 10$ dB. Furthermore, we compare the achievable capacity with imperfect channel knowledge to the capacity using diversity techniques without channel knowledge at the transmitter. In Fig. 1 we depict the ergodic capacity per scheduled subcarrier as a function of the error variance σ_e^2 in dB and the correlation coefficient ρ . The capacity is represented in terms of gray values. As one can see, for a small error variance and a high correlation, good capacity performances are achievable. By decreasing the correlation coefficient and increasing the error variance, the performance decreases. The lowest capacity value corresponds to the capacity one achieves by randomly allocating the subcarriers to the different users. It can be seen that for an error variance $\sigma_e^2 < -5$ dB, the capacity only depends on the correlation coefficient ρ . The white line represents the capacity one achieves using diversity techniques without using any channel knowledge at the transmitter side, in this example the achievable capacity using frequency diversity is $C_D = 3.38$ bits/s/Hz. Hence, in the region above this diversity line, the achievable capacity with imperfect channel knowledge is still better than the capacity using diversity. For $U = 10$ users, adaptivity with imperfect channel knowledge outperforms diversity in the region $\rho > 0.5$ and $\sigma_e^2 < -5$ dB. For a carrier frequency of $f_0 = 2$ GHz and an assumed time delay of $T = 2$ ms, $\rho = 0.5$ corresponds to a MS velocity of $v_{MS} = 33$ km/h. Fig. 2 shows the

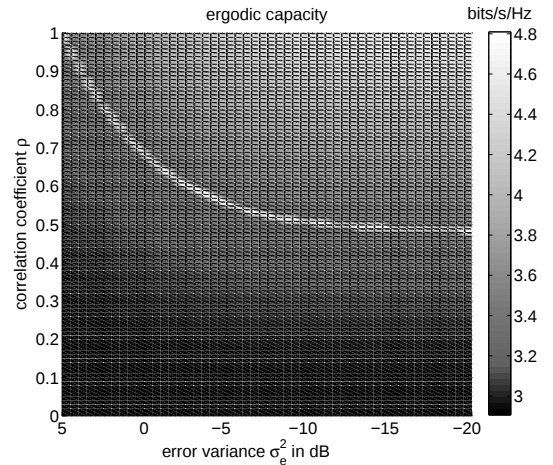
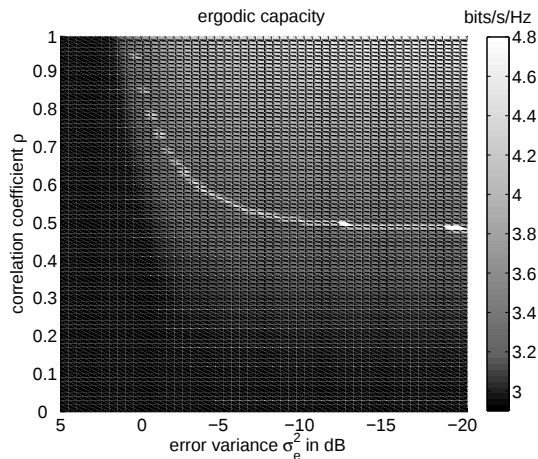
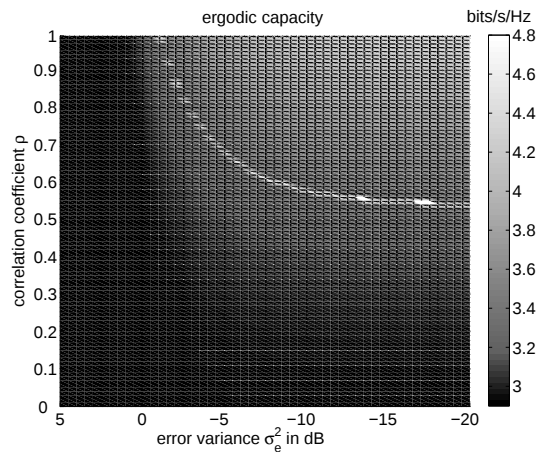
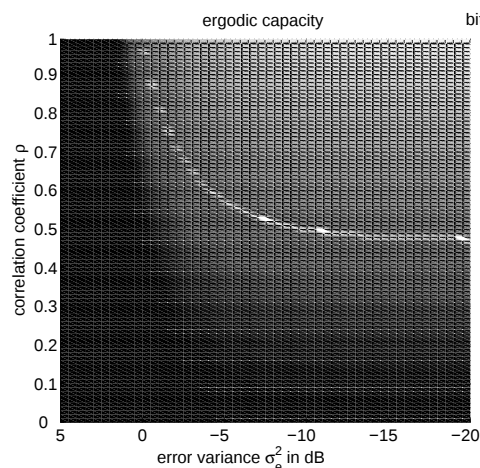


Figure 1: Ergodic capacity

capacity in case of outdated noisy quantised channel knowledge with $N_Q = 3$ quantisation bits, where the quantisation intervals s_i are set to maximise the ergodic capacity with perfect quantised channel knowledge given by (19). Compared to Fig. 1, the region where adaptivity outperforms diversity diminishes. If we decrease the number of quantisation bits to $N_Q = 2$, cf. Fig. 3, and $N_Q = 1$, cf. Fig. 4, the region further decreases and also the achievable capacity values. However in the region $\rho > 0.5$ and $\sigma_e^2 < -5$ dB already $N_Q = 3$ quantisation bits are sufficient to achieve a good capacity performance.


Figure 2: Ergodic capacity in case of $N_Q = 3$ quantisation bits

Figure 4: Ergodic capacity in case of $N_Q = 1$ quantisation bits

Figure 3: Ergodic capacity in case of $N_Q = 2$ quantisation bits

VIII. CONCLUSIONS

We have derived analytical expressions for the ergodic capacity in case of noisy and outdated channel knowledge and outdated noisy quantised channel knowledge. By comparing the achievable capacity using adaptive subcarrier allocation with imperfect channel knowledge with the capacity achievable exploiting frequency diversity, we showed that in the region $\rho > 0.5$ and $\sigma_e^2 < -5$ dB, adaptivity with imperfect channel knowledge is still better than diversity, even with quantised feedback. However, this consideration does not take into the account the effort in terms of capacity we have to spend in order to feed back the channel information in case of a FDD system, which will be the task for future work together with the consideration of a MIMO scenario.

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