IST Mobile \& Wireless Communications Summit, Jun 2006, Mykonos, Greece

Performance of Single and Multi-Antenna Amplify-and-Forward Relays in a Manhattan Street Grid Scenario

Markus Herdin
Wireless Solution Laboratory
DoCoMo Communications Laboratories
Europe GmbH
80687 Munich, Germany
herdin@docomolab-euro.com

Timo Unger
Institute of Telecommunications
Communications Engineering Group
Technische Universität Darmstadt
64283 Darmstadt, Germany
T.Unger@nt.tu-darmstadt.de

Abstract— **Amplify-and-Forward (AF) is a simple but effective relaying concept for multihop networks that combines transparency regarding modulation format and coding scheme with ease of implementation. In this paper, we analyze the mutual information achieved with SISO or MIMO AF relays in a Manhattan street grid scenario. We show that with relaying, a significant coverage extension is possible. Furthermore, we consider the performance gain through simultaneous transmission from all relay stations. No explicit cooperation between different relays is assumed but each relay optimizes retransmission, separately. Therefore, the focus lies in particular on the rank improvement of the effective channel from a MIMO base station to a MIMO mobile station. Below rooftop and above rooftop relays are compared. It is shown that for SISO relays always a significant gain is obtained, whereas for MIMO relays the rank improvement by simultaneous transmission from several relays only becomes significant for above rooftop relays.**

I. Introduction

Multihop networks have recently gained a lot of interest in the mobile radio research community. The basic idea is to introduce intermediate relay stations (RSs) that forward data packets to a destination that is otherwise out of reach. Using RSs can bring a number of advantages. Apart from increasing the range [1], cooperative transmission from several RSs and the base station (BS), simultaneously, allows for increased diversity [2], [3]. This can be further seized by applying distributed space-time coding [4]. RSs can also be used as virtual antenna arrays to increase capacity [5] or for rank-improvement resulting in increased capacity between a multi-antenna BS and mobile station (MS) [6].

An often considered relaying scheme is amplify-and-forward (AF), which means that a RS receives and stores the data packets in a first time slot (TS) without actually decoding it and retransmits a processed version of it during a second TS. Recently, this concept was extended to the case of multiple-input multiple-output (MIMO) RSs. Muñoz et at. [7] proposed a MIMO AF scheme that allow to maximize capacity between a multi-antenna BS and MS when instantaneous channel knowledge is available at the RS. In [8], this approach was extended to the case of having correlated MIMO channels and only knowledge about the average RS to MS channel. There, it appeared that a spatial filter at the RSs can bring large SNR gains when the MIMO channel shows significant correlations.

In this paper, the performance of SISO and MIMO AF RSs in a Manhattan street grid scenario is considered. Realistic models for the spatial structure and also the pathloss are applied, which allows to investigate the gain that is actually achieved through relaying. Furthermore, the case of below rooftop and above rooftop RSs is compared. The basis for the comparison are SISO or MIMO AF RSs, where for MIMO RSs, a spatial filter at the RS is used.

II. Scenario

We consider the downlink in a single cell Manhattan street grid scenario with one base station (BS), several AF RSs and a single MS (see Fig. 1). The Manhattan street grid consists of 80m long blocks of houses and streets with 20m width which results in a distance of 100m between two adjacent crossings. For sake of simplicity, the BS and also the RSs are assumed to be located at street crossings, hence having LOS into the four streets connected to the crossing. The BS is located above rooftop level at a height of 20m. The locations of the RSs were chosen such that a balanced SNR and capacity distribution is achieved within the whole cell. All RSs are assumed to be either mounted on lamp masts at 4*m* height, i.e. below rooftop level, or above rooftop level at the same height as the BS. For below rooftop RSs, it turned out that placing four RSs at LOS and four RSs at NLOS positions to the BS gives a relatively even capacity distribution in the whole cell. The considered RS distribution also has the advantage that a large number of streets are covered with LOS to at least one RS. Only the streets next to the BS (marked with green arrows in the figure) have no LOS to either the BS or a RS. For the case of RSs above rooftop, the same RS positions

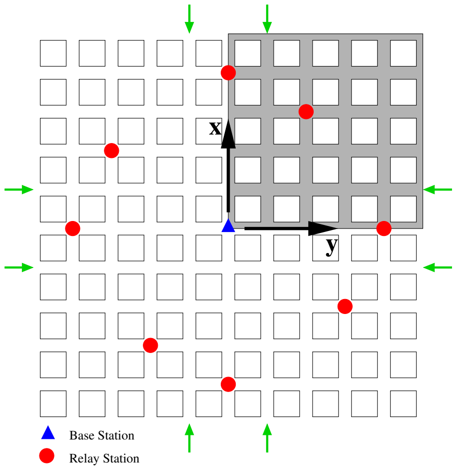

Fig. 1. Manhattan street grid with centrally located base station and several relay stations

are used. In that case, however, each RS has LOS to the BS. To achieve a sensible coverage extension through relaying, it is essential to have a good connection from the BS to the RSs. This can be achieved by placing the RSs such that they have LOS to the BS and/or by making use of directional antennas at the RSs [9]. The latter one, which we consider here, means that two antennas are necessary at each RS. One directional antenna that is dedicated for communication with the BS and one omni-directional antenna for communication with the MSs. An antenna gain of 10dB is assumed here.

We consider the case of a multi-antenna BS and a multi-antenna MS. At the RS, the case of having either a single antenna or an antenna array per link is considered. The antenna arrays at BS, MS and eventually each RS are 4-element uniform circular arrays with an antenna spacing of half the wavelength.

A maximum transmit power of 5W is assumed for the BS and each RS. For the analysis, a total bandwidth of $100MHz$ is assumed, however, we focus on the spatial domain, only. This means we do not consider frequency selective fading. No interference from neighboring cells is considered, only additive white Gaussian thermal noise with a noise power density of $-174dBm/Hz$ is assumed. In the simulation, only the upper right corner (gray region in Fig. 1) is considered due to the symmetry of the scenario.

III. System Model

The RSs operate in an AF fashion. This means that communication takes place in two phases. During TS 1, the BS transmits a signal to the RS, where the signal is received and stored, without decoding it. During TS 2, an amplified and (in case of a MIMO RS) spatially filtered signal is retransmitted to the MS. A system model for the general case of having several RS and multiple antennas at BS, RS and MS is developed in the following. The receive signal at the mth RS and the MS during TS 1 is given by

$$\mathbf{r}_1^{(m)} = \mathbf{H}_1^{(m)}\mathbf{x} + \mathbf{n}_0^{(m)} \quad (1)$$

$$\mathbf{y}_1 = \mathbf{H}_0\mathbf{x} + \mathbf{n}_1. \quad (2)$$

Here, $\mathbf{H}_1^{(m)}$ is the BS-to-RS channel matrix for the mth RS, $\mathbf{H}_0$ the BS-to-MS channel matrix, $\mathbf{x}$ the transmitted symbol vector at the BS and $\mathbf{n}_0^{(m)}$ and $\mathbf{n}_1$ the received additive white Gaussian noise vector at RS and MS, respectively.

During TS 2, the RS retransmit an amplified and spatially filtered version of the receive signal. Therefore, the received signal vector at the MS during TS 2 becomes

$$\mathbf{y}_2 = \sum_{m=1}^{M} \mathbf{H}_2^{(m)}\mathbf{G}^{(m)}\mathbf{r}_1^{(m)} + \mathbf{n}_2, \quad (3)$$

where $\mathbf{G}^{(m)}$ denotes the spatial gain and filter matrix applied to the receive signal at the mth RS before retransmission, $\mathbf{H}_2^{(m)}$ the corresponding RS-to-MS channel and $\mathbf{n}_2$ the received additive white Gaussian noise vector at the MS. A more compact description is achieved by stacking the receive and noise vectors at the RS into single column vectors

$$\mathbf{r}_1 = \left[\begin{array}{ccc} \mathbf{r}_1^{(1)^T} & \ldots & \mathbf{r}_1^{(M)^T} \end{array}\right]^T, \quad (4)$$

$$\mathbf{n}_1 = \left[\begin{array}{ccc} \mathbf{n}_1^{(1)^T} & \ldots & \mathbf{n}_1^{(M)^T} \end{array}\right]^T \quad (5)$$

and the BS-to-RS channel matrices into a block channel matrix

$$\mathbf{H}_1 = \left[\begin{array}{ccc} \mathbf{H}_1^{(1)^T} & \ldots & \mathbf{H}_1^{(M)^T} \end{array}\right]^T. \quad (6)$$

With the block diagonal gain and filter matrix

$$\mathbf{G} = \operatorname{diag}\left\{\mathbf{G}^{(1)}, \ldots, \mathbf{G}^{(M)}\right\} \quad (7)$$

the received signal vector at the MS during TS 2 now becomes

$$\mathbf{y}_2 = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{H}_2\mathbf{G}\mathbf{n}_1 + \mathbf{n}_2. \quad (8)$$

To fulfill the power constraint at the RSs, each spatial gain and filter matrix has to fulfill

$$\frac{P_{t,bs}}{n_{t,bs}} \operatorname{tr}\left\{\mathbf{G}^{(m)}\mathbf{H}_1^{(m)}\mathbf{H}_1^{(m)^H}\mathbf{G}^{(m)^H}\right\} + \quad (9)$$

$$\sigma_n^2 \operatorname{tr}\left\{\mathbf{G}^{(m)}\mathbf{G}^{(m)^H}\right\} = P_{t,rs}, \quad (10)$$

where $P_{t,bs}$ and $P_{t,rs}$ is the total transmit power of the BS and RS, respectively, and $n_{t,bs}$ the number of transmit antennas at the BS. Note that *each* RS transmits with $P_{t,rs}$. The corresponding pathloss is already included in the channel matrices $\mathbf{H}_0$, $\mathbf{H}_1^{(m)}$ and $\mathbf{H}_2^{(m)}$ and the noise power density and therefore σ_n^2 is assumed to be the same at all RSs and the MS. The actual SNR at the RS and MS is therefore implicitly given.

In case of single-antenna RSs, the matrix $\mathbf{G}^{(m)}$ simplifies to a gain coefficient. For multi-antenna RSs, however, $\mathbf{G}^{(m)}$ can be an arbitrary gain and filter matrix that only has to fulfill the power constraint. The most simple approach is to choose $\mathbf{G}^{(m)}$ as scaled identity matrix, hence, to amplify each antenna signal by the same gain coefficient before retransmission. In correlated MIMO channels, however, this has the disadvantage that implicit beamforming occurs which can lead to a large SNR loss at the MS and therefore to a significantly reduced capacity [8]. One possibility to overcome this problem is to use a spatial filter as it was proposed by Muñoz et al. [7]. This requires instantaneous channel knowledge about both the BS-to-RS and the RS-to-MS channel at the RSs. Since this might be too demanding, in particular, if the MS moves relatively fast, we assume here that only average channel knowledge about the RS-to-MS channel is available at the RSs. The BS-to-RS channel is still assumed to be known instantaneously at the RSs. This leads to the filter approach [8]

$$\mathbf{G}^{(m)} = \mathbf{V}_{2,LT}^{(m)} \mathbf{\Lambda}_f^{(m)} \mathbf{U}_1^{(m)H}, \tag{11}$$

where $\mathbf{V}_{2,LT}^{(m)}$ is the long term transmit eigenmode matrix of the mth RS-to-MS channel given by the eigenvalue decomposition (EVD) of $E\{\mathbf{H}_2^{(m)H}\mathbf{H}_2^{(m)}\} = \mathbf{V}_{2,LT}^{(m)} \mathbf{\Lambda}_{2,LT}^{(m)} \mathbf{V}_{2,LT}^{(m)\,H}$, $\mathbf{\Lambda}_f^{(m)}$ a diagonal gain matrix and $\mathbf{U}_1^{(m)}$ the instantaneous receive eigenmode matrix of the BS-to-RS channel as given by the EVD of $\mathbf{H}_1^{(m)}\mathbf{H}_1^{(m)H} = \mathbf{U}_1^{(m)} \mathbf{\Lambda}_1^{(m)} \mathbf{U}_1^{(m)H}$. The diagonal gain matrix $\mathbf{\Lambda}_f^{(m)}$ is chosen according to [8] which is equivalent to [7] with the short-term eigenvalues of the RS-to-MS channel replaced by the long-term eigenvalues. This filter approach shows a significant gain compared to having no filter at the RS, even though only the average RS-to-MS channel is known.

Note that we assume here that the BS does not transmit during the second TS. For further investigations, the received signals at the MS within both TSs is combined into one vector:

$$\mathbf{y} = \underbrace{\begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{I}_{n_{r,ms}} & \mathbf{0}_{n_{r,ms}\times M n_{r,rs}} & \mathbf{0}_{n_{r,ms}} \\ \mathbf{0}_{n_{r,ms}} & \mathbf{H}_2\mathbf{G} & \mathbf{I}_{n_{r,ms}} \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}}_{\mathbf{n}} \tag{12}$$

Here, $n_{r,rs}$ and $n_{r,ms}$ are the number of receive antennas at RS and MS, respectively.

The mutual information for unknown channel at transmit side (BS) can now be calculated by [10]

$$C = \frac{1}{2}\log_2\left(\det\left(\mathbf{I}_{2n_{r,ms}} + \frac{P_{t,bs}}{n_{t,bs}\,\sigma_n^2}\mathbf{A}\mathbf{A}^H\left(\mathbf{B}\mathbf{B}^H\right)^{-1}\right)\right). \tag{13}$$

The factor 1/2 comes from the fact that transmission takes place in two TSs, hence, double resources are needed. For the evaluations, we consider in the following the mean capacity that is achieved at different positions in the Manhattan street grid, where the mean is taken over a large number of channel realizations.

IV. Channel Model

The channel models used in the analysis are based on the interim IST-WINNER channel models [11]. Three propagation links are considered:

(i) above rooftop level BS or RS to below rooftop RS or MS,
(ii) above rooftop level BS to above rooftop level RS and
(iii) below rooftop level RS to MS.

The pathloss at distance d for Scenario (i) for LOS and NLOS is modeled by

$$a^{(LOS)}(d) = 43.3 + 23.4 \cdot \log_{10}(d) \tag{14}$$

$$a^{(NLOS)}(d) = 53.5 + 28.3 \cdot \log_{10}(d). \tag{15}$$

with d the distance in meter. For Scenario (ii), there exists only the LOS case, where the pathloss is equally modeled as in Scenario (i). In case of Scenario (iii), the pathloss is modeled by

$$a^{(LOS)}(d) = 41.0 + 22.7 \cdot \log_{10}(d) \tag{16}$$

$$a^{(NLOS)}(d_1, d_2) = 47.3 + 20.1 \log_{10}\left(d_1\left(\frac{d_2}{11.2}\right)^{1.79}\right). \tag{17}$$

Note, that the NLOS case of Scenario (iii) means that propagation goes around a corner. Here, d_1 measures the distance from the source to the corner and d_2 the from the corner to the destination. The pathloss values given are valid for a center frequency of $5GHz$. The spatial structure of the channel is modeled according to [11] with the only difference that different realizations of the channel are created by using random phases for each multipath component [12]. This means that propagation is modeled using scattering clusters with a scenario dependent number of clusters, inter- and intra-cluster spread and cluster power. The frequency characteristic of the channel is not modeled but frequency flat fading is assumed.

V. Results

The first aspect that is considered here is the coverage extension by relaying. Figure 2 (left) shows the mutual information at different MS positions in the cell when only the BS transmits, but within both TSs (top), and when the BS transmits in the first and the RSs in the second TS (bottom). In case of BS only transmission, the mutual information decreases relatively fast with increasing distance from the BS, simply because the receive power decreases. For relaying, i.e. when using the BS only in the first TS and then transmitting from all RSs, together, in the second TS, it appears that a relatively even distribution of the mutual information

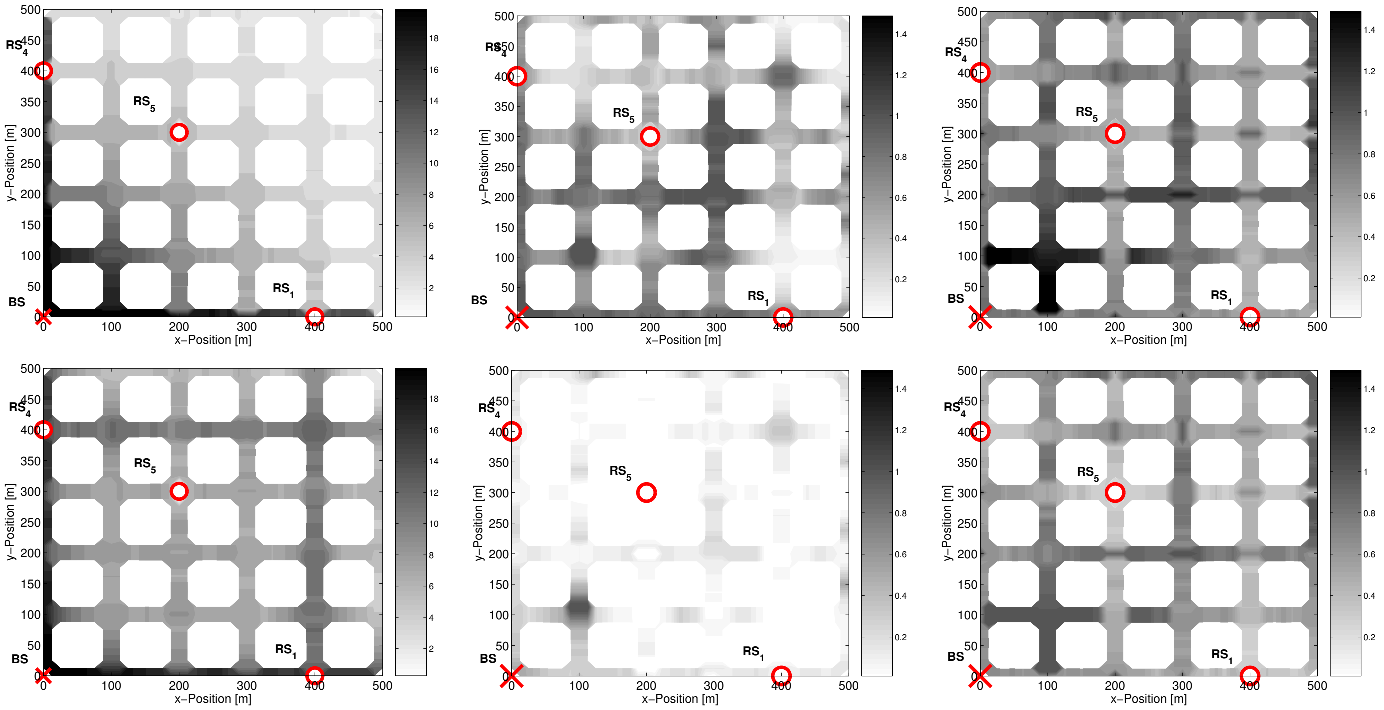

Fig. 2. **Left**: relays *below* rooftop, average mutual information for transmission from only the BS in both TSs (top) or from the BS in the first TS and all RSs in the second TS (bottom); **Middle**: relays *below* rooftop, gain in average mutual information (γ_{mean}) for simultaneous transmission from all RSs compared to transmission from only the best RS in case of SISO RSs (top) and MIMO RSs (bottom); **Right**: relays *above* rooftop: gain in average mutual information (γ_{mean}) for simultaneous transmission from all RSs compared to transmission from only the best RS in case of SISO RSs (top) and MIMO RSs (bottom)

is achieved. For MS positions next to one of the RS, the mutual information is not limited by the received power at the MS but by the amplified noise received from the RS, i.e. the SNR at the RS. This is the reason why the mutual information along streets with LOS to a RS is nearly constant. Due to the favorable distribution of RSs such that within nearly every street there is LOS possible to at least one RS, a relatively constant average mutual information is achieved at different positions within the cell. Only at the cell border, the mutual information decreases. This highlights one important conclusion: The placement of the RSs determines essentially the performance that can be achieved. This means that as long as not a large number of RSs is deployed, it is necessary to carefully select positions for them in order to make them effective.

In the following, we investigate the gain in average mutual information that is achieved when transmitting from all RSs simultaneously, which allows for rank improvements and SNR gains due to a higher receive power compared to the case of using only a single RS where that RS is selected which gives the highest mutual information. For that purpose, we define the gain in average mutual information as:

$$\gamma_{mean} = \frac{C^{(2)}_{mean} - C^{(2)}_{mean,i}}{C^{(2)}_{mean,i}}, \qquad (18)$$

Here, $C^{(2)}_{mean}$ is the average mutual information that is achieved by simultaneously transmitting from all RSs during the second TS and $C^{(2)}_{mean,i}$ the average mutual information that is achieved by transmitting only from that RS which gives the highest mutual information at the MS. Note, that we focus on the relay slot, only, which means that we do not consider direct transmission from the BS to the MS during the first TS, although this would be advantageous at several MS positions near to the BS. This means that the first row in (12) is removed (but not the factor 1/2).

Figure 2 (middle, top) shows γ_{mean} for the case of SISO RSs but MIMO BS and MS. A gain in mutual information of up to 100% percent is obtained for significantly large areas of the cell. The main reason is the rank improvement, since each single RS supplies the MS only with a rank one channel. Additionally, there is a gain due to the increased receive power (each RS has an individual power constraint), however, this gain is only significant at MS positions where the power and the supplied SNR from two or more RSs is in the same range.

In the MIMO RS case (Fig. 2 (middle, bottom)), the gain through rank improvement disappears. Already a single RS allows spatial multiplexing, and it appears that the additional rank improvement by transmitting from several RSs is negligible. Whereas for SISO RS, even weak signals coming from other RSs then the strongest RS have a significant influence on the channel rank and therefore the average mutual information, this is not the case for MIMO RSs. The mutual information is only increased significantly where the receive power and the SNR that is supplied from different RSs is similar. This is true for position (400,400) and in particular for (100,100). At the latter position, the receive power from three RSs appears to be in the same range. Since the noise at the RSs is not dominating but the noise at the MS is the limiting factor, the SNR at the MS is increased. However, position (100,100) would actually be much better supplied by a direct link from the BS. To summarize, it appears that nearly no gain through rank improvement is obtained for MIMO RSs below rooftop.

In case of having RSs above rooftop, the situation changes totally. Figure 2 (right) depicts the case of having the RS above rooftop with LOS to the BS and SISO (top) or MIMO (bottom) RSs, respectively. Now, not only for SISO RSs, there is a significant gain through simultaneous transmission from several RSs. Also for MIMO RSs, the average mutual information is largely increased. The reason is simply the better connection from the RSs to the MS, in particular for NLOS. Typically, more than one RSs have a similar pathloss to the mobile station, which leads to rank improvements and power gains, also in the MIMO RS case.

The clear result from the considered scenarios is that above rooftop RSs allow for large performance improvements compared to below rooftop RS. There are, however, also drawbacks of above rooftop relays. As soon as several cells have to be supplied, the advantage of having good connection to several RS is likely to turn into the disadvantage of increasing the inter-cell interference. Also in case of reusing the second TS in the same cell the small range that is achieved by below rooftop RSs is actually an advantage. Furthermore, below rooftop relays can easily be mounted on lamp posts without the need of additional constructions, whereas above rooftop relays will likely need an own mast, like a base station, to be effective. This means that one important advantage of using RSs would be lost, namely the small effort and the low cost for mounting a RS.

VI. Conclusions

In this paper, the performance that is achieved with SISO and MIMO Amplify-and-Forward (AF) relay stations was analyzed. A Manhattan street grid scenario is considered, where relays can be placed either at e.g. lamp posts below the rooftop level or above rooftop level, similarly mounted as base stations. It was shown that with below rooftop relays a significant coverage extension can be achieved and that the disadvantage of needing double resources can easily be overcome. One essential parameter to overcome this disadvantage was the assumption of having an antenna gain at the relay station. However, it turned out that the locations of the relays have to be chosen carefully for a good performance.

Furthermore, it was shown that the gain through rank improvement of the effective MIMO channel from the base station to the mobile station was relatively high for SISO relays but very low if MIMO relays below rooftop level were used. In case of SISO relays, even weak signals from other relays than the strongest one contribute a lot to the average mutual information, both for relays below and above rooftop. In contrast to this, such a rank improvement effect was only observed for MIMO relays above rooftop, where the signals arriving at the mobile station from different relays have similar power. For below rooftop MIMO relays, the pathloss from different relays differs too much as that weak relay signals contribute significantly to the rank and therefore the average mutual information. In other words, each single relay allows already for spatial multiplexing and a reasonably high rank of the channel. This makes cooperative approaches for MIMO relays questionable.

References

[1] Daniel Christian Schultz, Ralf Pabst, and Tim Irnich, "Multi-hop based Radio Network Deployment for efficient Broadband Radio Coverage," in *Proc. International Symposium on Wireless Personal Multimedia Communications, WPMC*, Yokosuka, Japan, October 2003, vol. 2, pp. 377–381.

[2] J. Nicholas Laneman, David N.Č. Tse, and Gregory W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Transactions on Information Theory, accepted for publication*.

[3] Andrew Sendonaris, Elza Erkip, and Behnaam Aazhang, "User Cooperation Diversity, Part I and II," *IEEE Transactions on Communications*, vol. 15, pp. 1927–1948, November 2003.

[4] Ingmar Hammerstroem, Marc Kuhn, and Boris Rankov, "Space-Time Processing for Cooperative Relay Networks," in *Proc. IEEE Vehicular Technology Conference*, October 2004.

[5] M. Dohler, J. Dominguez, and H. Aghvami, "Link capacity analysis for virtual antenna arrays," in *Proc. IEEE Vehicular Technology Conference*, September 2002, vol. 1, pp. 440–443.

[6] Armin Wittneben and Boris Rankov, "Impact of Cooperative Relays on the Capacity of Rank-Deficient MIMO Channels," in *Proceedings of the 12th IST Summit on Mobile and Wireless Communications*, Aveiro, Portugal, June 2003, pp. 421–425.

[7] Olga Munoz, Josep Vidal, and Adrián Agustín, "Non-Regenerative MIMO Relaying with Channel State Information," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, March 2005.

[8] Markus Herdin, "MIMO Amplify-and-Forward Relaying in correlated MIMO channels," *accepted for presentation at ICICS 2005, Thailand, Bangkok*.

[9] Bernhard Walke, Ralf Pabst, and Daniel Christian Schultz, "A Mobile Broadband System based on Fixed Wireless Routers," in *Proc. of ICCT 2003 International Conference on Communication Technology*, April 2003.

[10] Patrick Herhold, Ernesto Zimmermann, and Gerhard Fettweis, "On the performance of cooperative amplify-and-forward relay networks," in *Proc. ITG Conference on Source and Channel Coding (SCS)*, Erlangen, Germany, 2004.

[11] *IST-2003-507581 WINNER D5.3 ver 2.4 - Interim Channel Models*, www.ist-winner.org.

[12] A.F. Molisch, M. Steinbauer, M. Toeltsch, E. Bonek, and R.S. Thomä, "Capacity of MIMO Systems Based on Measured Wireless Channels," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 3, pp. 561–569, April 2002.