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Hybrid localization with temporal post-processing

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Abstract – Precise localization of mobile devices is a promising and challenging task. Due to multipath propagation state of the art localization techniques provide only coarse position estimates. An approach to improve position estimates consists in hybrid localization, i.e., in combining several coarse position estimates to obtain one better position estimate. The position estimates to be combined can stem from different spatial measurements taken at the same time instant or from the same spatial measurement performed at different time instants.

Index Terms – localization, SPIDER, post-processing

1. Introduction

Localization is gaining importance in various fields of application as mobile communication networks, sensor networks or Ambient Intelligence (AI). The satellite navigation systems Global Positioning System (GPS) and Galileo provide accurate position estimates, but in several scenarios of interest they are not applicable, for instance in indoor or dense urban environment due to lack of line-of-sight satellite connections or since they cannot be integrated in small sensors due to cost, size and weight restrictions or due to long acquisition time. Therefore, hybrid localization combining several less reliable sources of information on the position is a promising approach.

In order to locate a device, measurements of characteristic quantities which depend on the position of the device must be available. The function that describes the relationship between the device position and the characteristic quantity is termed characteristic function. Ideally, the characteristic function should provide a sharp one-to-one relationship between the device position and the characteristic quantity so that even in the presence of measurement errors reliable position estimates can be obtained.

In reality this is often not the case. State-of-the-art characteristic quantities are e.g. the propagation time, signal strength, angle of arrival or fingerprint like signatures. In general, the corresponding characteristic functions do only weakly depend on the position. Thus, a combination of multiple characteristic quantities and related measurements from different sources is required to reliably estimate the unknown device position.

The accuracy of the localization method depends on three influencing factors. First, the accuracy of the localization method depends on the accuracy with which the measurements of the characteristic quantities can be obtained. Second, the accuracy of the localization method depends on the characteristic function.

On the one hand, a sharp dependency of the corresponding value of the characteristic function on the device position is desirable. On the other hand, the stronger the characteristic function depends on the position of the device, the more exact the characteristic function has to be known to determine the unknown position of the device. Third, the accuracy of the localization method depends on the way the measurement values of different characteristic quantities are combined.

In this paper, several localization methods based on a combination of measurements of different characteristic quantities are presented. First, the concept of SPIDER (smart position identification rationale) is introduced which jointly determines location estimates of several devices using distance estimates, e.g. obtained by propagation time measurements, on the one hand between the devices and installed stations of known position and on the other hand also between different devices. Second, the concept of temporal post-processing is introduced. It is shown that both concepts lead to considerable accuracy improvements.

2. SPIDER

2.1 Idea and motivation

As motivated above, a reliable estimation of the position of a device may be obtained by including as many characteristic quantities and their measurements as possible in the localization process. Typical characteristic quantities used for locating a device are its (measured) distances from pre-installed stations of known position. Therefore, for accurate localization many of such distances are helpful. The number of pre-installed stations in the surroundings of a device is typically limited; cf. for instance the localization of a mobile terminal (MT) in a cellular mobile radio system where only a few base stations (BS) act as pre-installed stations. Consequently, for applications of this kind the localization accuracy is limited. In order to overcome the aforementioned problem one can resort to the fact that typically several devices of unknown positions are active in the same area. Assuming this, the mentioned distance based localization scheme may be extended and enhanced: not only estimates of the distances of the devices to be located to surrounding pre-installed stations, but, as additional information, also estimates of all or some distances inbetween the devices can be utilized. This approach, which in the following is referred to as SPIDER (smart position identification rationale), implies the joint estimation of the positions of all devices. In the following, devices are referred to as mobile terminals (MTs) and pre-installed stations are termed base stations (BSs).

2.2 Considered Scenario

We consider a geographic observation area $\mathbb{X} \subseteq \mathbb{R}^D$ in the D-dimensional space, $D \in \{2,3\}$, comprising K_B BSs $\mathrm{B}^{(k_\mathrm{B})}$, $k_\mathrm{B} = 1 \ldots K_\mathrm{B}$, and K MTs $\mathrm{M}^{(k)}$, $k = 1 \ldots K$. The key parameters of such an area are its geographic extension, the number K_B of known fixed positions of the BSs $\mathrm{B}^{(k_\mathrm{B})}, k_\mathrm{B} = 1 \ldots K_\mathrm{B}$, and the number K of MTs $\mathrm{M}^{(k)}$, $k = 1 \ldots K$. The K MTs are at positions $\mathbf{x}_\mathrm{M}^{(k)} \in \mathbb{X}$, $k = 1 \ldots K$, to be determined by localization. The K MT positions $\mathbf{x}_\mathrm{M}^{(k)}$ are stacked in the position vector

$$\mathbf{x}_\mathrm{M} = \left(\mathbf{x}_\mathrm{M}^{(1)\,\mathrm{T}} \ldots \mathbf{x}_\mathrm{M}^{(K)\,\mathrm{T}} \right)^\mathrm{T}. \tag{1}$$

In the ensemble of K_B BSs and K MTs a number of

$$K_\rho = K_\mathrm{B} K + K(K-1)/2 \tag{2}$$

mutual distances $\rho^{(k_\rho)}$, $k_\rho = 1 \ldots K_\rho$, exist, where each

$$\rho^{(k_\rho)} = \left\| \mathbf{x}_\mathrm{B}^{(k_\mathrm{B})} - \mathbf{x}_\mathrm{M}^{(k)} \right\|_2 \in \mathbb{R}_0^+, \tag{3}$$

$$\forall k_\rho = (k_\mathrm{B} - 1)K + k \in \{1 \ldots K K_\mathrm{B}\},$$

denotes the distance between the BS $\mathrm{B}^{(k_\mathrm{B})}$ and the MT $\mathrm{M}^{(k)}$, and each

$$\rho^{(k_\rho)} = \left\| \mathbf{x}_\mathrm{M}^{(k)} - \mathbf{x}_\mathrm{M}^{(k')} \right\|_2 \in \mathbb{R}_0^+, \tag{4}$$

$$\forall k_\rho = K_\mathrm{B} K + (k-1)K - \frac{1}{2}k(k+1) + k' \in \{KK_\mathrm{B}+1 \ldots K_\rho\}$$

denotes the distance between two MTs $\mathrm{M}^{(k)}$ and $\mathrm{M}^{(k')}, k' > k$. The distances $\rho^{(k_\rho)}$ of (3) and (4) can be stacked in the distance vector

$$\boldsymbol{\rho} = \left(\rho^{(1)} \ldots \rho^{(K_\rho)}\right)^\mathrm{T} \in \mathbb{R}_0^{+K_\rho}. \tag{5}$$

As a consequence of (3) to (5), for fixed BS positions $\mathbf{x}_\mathrm{B}^{(k_\mathrm{B})}$, the distance vector $\boldsymbol{\rho}$ of (5) is a function $\boldsymbol{\rho}(\mathbf{x}_\mathrm{M})$ of the position vector $\mathbf{x}_\mathrm{M}$ of (1).

In practical system operation only estimates $\hat{\rho}^{(k_\rho)}$ of the distances $\rho^{(k_\rho)}$ of (3) and (4) are available, which quite generally can be expressed with the measurement error $n^{(k_\rho)}$ as

$$\hat{\rho}^{(k_\rho)} = \rho^{(k_\rho)} + n^{(k_\rho)}. \tag{6}$$

Stacking $n^{(k_\rho)}$, $k_\rho = 1 \ldots K_\rho$, to the error vector $\mathbf{n} \in \mathbb{R}^{K_\rho}$, we obtain the estimate

$$\hat{\boldsymbol{\rho}} = \boldsymbol{\rho} + \mathbf{n} \tag{7}$$

of the distance vector $\boldsymbol{\rho}$ of (5). The error vector $\mathbf{n}$ is a random quantity and can therefore be statistically characterized by its probability density function $\mathrm{p}_\mathbf{n}(\mathbf{n})$ which is assumed to be independent of $\boldsymbol{\rho}$ in the following. With $\mathrm{p}_\mathbf{n}(\mathbf{n})$ and $\boldsymbol{\rho}$ of (5) and under consideration of (7) we can express the probability density function of the estimate $\hat{\boldsymbol{\rho}}$ conditioned on $\mathbf{x}_\mathrm{M}$ as

$$\mathrm{p}_{\hat{\boldsymbol{\rho}}}\left(\hat{\boldsymbol{\rho}} \,\middle|\, \mathbf{x}_\mathrm{M}\right) = \mathrm{p}_\mathbf{n}\left(\hat{\boldsymbol{\rho}} - \boldsymbol{\rho}\left(\mathbf{x}_\mathrm{M}\right)\right). \tag{8}$$

With the estimate $\hat{\boldsymbol{\rho}}$ of $\boldsymbol{\rho}$ determined by measurements and the conditioned probability density function $\mathrm{p}_{\hat{\boldsymbol{\rho}}}\left(\hat{\boldsymbol{\rho}} \,\middle|\, \mathbf{x}_\mathrm{M}\right)$ of $\hat{\boldsymbol{\rho}}$, see (8), the Maximum Likelihood (ML) estimate of $\mathbf{x}_\mathrm{M}$ is given by

$$\hat{\mathbf{x}}_\mathrm{M} = \underset{\forall \mathbf{x}_\mathrm{M} \in \mathbb{X}^K}{\arg\max} \left\{ \mathrm{p}_{\hat{\boldsymbol{\rho}}}\left(\hat{\boldsymbol{\rho}} \,\middle|\, \mathbf{x}_\mathrm{M}\right) \right\}. \tag{9}$$

If no further information on the statistics of the error vector $\mathbf{n}$ of (7) is available, an obvious assumption would be that $\mathbf{n}$ is Gaussian [1]. Then, $\mathrm{p}_\mathbf{n}(\mathbf{n})$ is completely characterized by the expectation

$$\boldsymbol{\mu} = \mathrm{E}\{\mathbf{n}\} \tag{10}$$

and the covariance matrix

$$\mathbf{R} = \mathrm{E}\left\{(\mathbf{n}-\boldsymbol{\mu})(\mathbf{n}-\boldsymbol{\mu})^\mathrm{T}\right\} \tag{11}$$

of $\mathbf{n}$ and takes the form

$$\mathrm{p}_\mathbf{n}(\mathbf{n}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{n}-\boldsymbol{\mu})^\mathrm{T}\mathbf{R}^{-1}(\mathbf{n}-\boldsymbol{\mu})\right)}{(2\pi)^{K_\mathrm{B}/2}\sqrt{\det(\mathbf{R})}}. \tag{12}$$

With $\mathrm{p}_\mathbf{n}(\mathbf{n})$ of (12) the estimate of (9) can be expressed as

$$\hat{\mathbf{x}}_\mathrm{M} = \underset{\forall \mathbf{x}_\mathrm{M} \in \mathbb{X}^K}{\arg\min} \left(\left(\hat{\boldsymbol{\rho}} - \boldsymbol{\rho}(\mathbf{x}_\mathrm{M}) - \boldsymbol{\mu}\right)^\mathrm{T} \mathbf{R}^{-1} \left(\hat{\boldsymbol{\rho}} - \boldsymbol{\rho}(\mathbf{x}_\mathrm{M}) - \boldsymbol{\mu}\right) \right). \tag{13}$$

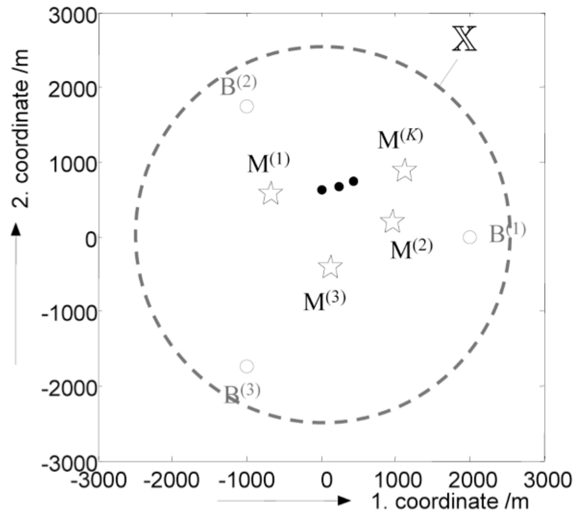

Fig. 1: Scenario with three base stations $\mathrm{B}^{(k_\mathrm{B})}$, $k_\mathrm{B} = 1 \ldots K_\mathrm{B}$, (○) and K mobile terminals $\mathrm{M}^{(k)}$, $k=1 \ldots K$, (☆).

Since the function $\boldsymbol{\rho}(\mathbf{x}_\mathrm{M})$ is non-linear, the minimization required to determine the estimates $\hat{\mathbf{x}}_\mathrm{M}$, see (9) and (13) cannot be performed in closed form. However, a closed form approximate solution can be found using linear Taylor approximation originally proposed by Torrieri [2] and extended in [3].

2.3 Performance Analysis

To evaluate the localization accuracy of SPIDER, the observation domain $\mathbb{X}$ shown in Fig. 1 is considered. Moreover, the estimates $\hat{\rho}^{(k_\rho)}$ were obtained according to (6), where the values $n^{(k_\rho)}$ were randomly chosen based on (12) with zero mean and

$$\mathbf{R} = (70\mathrm{m})^2 \cdot \mathbf{I}. \tag{14}$$

We assume that K MTs are active and that they are located at positions $\mathbf{x}_\mathrm{M}^{(k)}$, $k=1 \ldots K$, which are uniformly distributed within $\mathbb{X}$. The localization accuracy for a MT $\mathrm{M}^{(k)}$, $k=1 \ldots K$, is quantified by the localization error

$$\delta^{(k)} = \left\| \hat{\mathbf{x}}_\mathrm{M}^{(k)} - \mathbf{x}_\mathrm{M}^{(k)} \right\|_2, \tag{15}$$

i.e., the distance between estimated position $\hat{\mathbf{x}}_\mathrm{M}^{(k)}$ and real position $\mathbf{x}_\mathrm{M}^{(k)}$ of the MT $\mathrm{M}^{(k)}$. As both $\mathbf{n}$ of (7) and $\mathbf{x}_\mathrm{M}^{(k)}$, $k=1 \ldots K$, of (1) are random quantities, also the localization error $\delta^{(k)}$ of (15) is a random quantity. Therefore, the localization accuracy of SPIDER is evaluated in a statistical sense by means of the complementary cumulative distribution function (CCDF) $\Pr\{\delta^{(k)} > \Delta\}$ of the localization error $\delta^{(k)}$ of a MT $\mathrm{M}^{(k)}$, $k=1 \ldots K$. Fig. 2 shows such CCDFs for K equal to 4, 8, 16, 32 and 64 for both SPIDER and the conventional scheme. If the conventional scheme is applied, K has no influence on $\Pr\{\delta^{(k)} > \Delta\}$, because localization of an MT $\mathrm{M}^{(k)}$ is based on the distances between this MT $\mathrm{M}^{(k)}$ and the BSs $\mathrm{B}^{(k_\mathrm{B})}$, $k_\mathrm{B} = 1 \ldots K_\mathrm{B}$, only and is, therefore, independent of other MTs $\mathrm{M}^{(k')}$, $k \neq k'$. As a consequence, the probability that the localization error $\delta^{(k)}$ is larger than 50m is higher than 70%. In the case of SPIDER, with increasing K the localization accuracy is massively improved. Consequently, the probability that the localization error $\delta^{(k)}$ exceeds 50m is significantly reduced compared to the conventional scheme ranging from a probability of 55% for K equal to four down to less than 1% for K equal to 64. These figures impressively demonstrate the superiority of

SPIDER and underline the high attractiveness of including as many measurements of characteristic quantities as possible to significantly enhance localization accuracy.

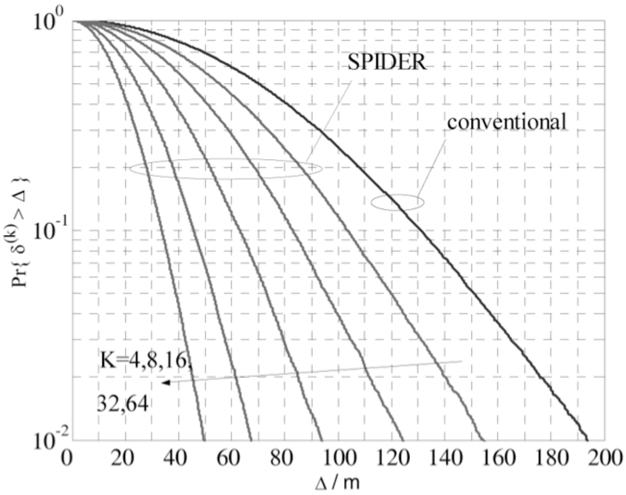

Fig. 2: CCDF of the localization estimation error $\delta^{(k)}$ for $K_B = 3$ and $K = 4,8,16,32,64$.

3. Temporal post-processing of position estimates

Temporal post-processing aims at improving position estimates by observing the movement of an MT over a certain time period. Consequently, a single MT will be considered in the following. To render the post-processing techniques computationally feasible one has to resort to discrete coordinates. In the following capital symbols will be used for discrete coordinates. We assume that the observation area inside which the MT is located during the measurement period is known, e.g., due to the knowledge of the BS to which the MT is assigned. The observation area is divided into $N \times N$ square pixels of size $l \times l$. The time instants at which the initial position estimates are taken are denoted by $p = 1 \ldots P$, i.e., in total P initial position estimates are taken. The position of the MT at time instant p is denoted by $\mathbf{X}_p$. During the measurement period the MT moves along the path $\mathbf{P} = \mathbf{X}_1 \ldots \mathbf{X}_P$.

The potential of temporal post-processing of position estimates stems from the fact that MTs can not move in a totally random way. Basically one needs to know the mobility model which can be described by the a-priori probabilities $\Pr\{\mathbf{P}\}$ of all combinatorial possible paths. However, from a complexity point of view temporal post-processing based on such a very general mobility model would not be feasible. Fortunately, at least for pedestrian users it is reasonable to assume that the movement of the MTs can be described by a Markoff model [4,5], i.e., that the future movement only depends on the current position. With the a-priori probabilities $\Pr\{\mathbf{X}_1\}$ of the positions at time instant 1 and the transition probabilities $\Pr\{\mathbf{X}_p | \mathbf{X}_{p-1}\}$ the path probabilities can be expressed as

$$\Pr\{\mathbf{P}\} = \Pr\{\mathbf{X}_1\} \cdot \prod_{p=2}^{P} \Pr\{\mathbf{X}_p | \mathbf{X}_{p-1}\} . \tag{16}$$

In a practical implementation it would be necessary to estimate the transition probabilities $\Pr\{\mathbf{X}_p | \mathbf{X}_{p-1}\}$ based on long term observation of the scenario and the MTs' movements, e.g., with the Baum-Welch algorithm [4,5]. In the present paper we assume that the transition probabilities $\Pr\{\mathbf{X}_p | \mathbf{X}_{p-1}\}$ are known. For numerical investigations throughout this paper it is assumed that

- the transition probabilities to all neighboring pixels take the same value q ,
- the transition probability to the same pixel, i.e., for staying at the current pixel, takes the value

$$\Pr\{\mathbf{X}_p | \mathbf{X}_{p-1} = \mathbf{X}_p\} = 1 - \sum_{\mathbf{X}_p \neq \mathbf{X}_{p-1}} \Pr\{\mathbf{X}_p | \mathbf{X}_{p-1}\} \tag{17}$$

and,

- all remaining transition probabilities take the value zero.

Please note that this very simple mobility model is just used for first simulations and that our techniques can also be used in conjunction with more sophisticated Markoff models.

First, some measurements $\overline{\mathbf{X}}_p$, $p = 1 \ldots P$, are obtained, e.g., by the localization method SPIDER described in the previous section. The measurements $\overline{\mathbf{X}}_p$, $p = 1 \ldots P$, are modeled as independent random variables for which the conditioned distribution $\Pr\{\overline{\mathbf{X}}_p | \mathbf{X}_p\}$ are assumed to be known.

For the numerical investigations throughout this paper we assume a sampled two dimensional Gaussian distribution. With the distance $\delta(\overline{\mathbf{X}}_p, \mathbf{X}_p)$ between the pixels $\overline{\mathbf{X}}_p$ and $\mathbf{X}_p$ and the standard deviation σ this sampled two dimensional Gaussian distribution reads

$$\Pr\{\overline{\mathbf{X}}_p | \mathbf{X}_p\} = \frac{\exp\left(-\frac{1}{2\sigma^2}\delta^2\left(\overline{\mathbf{X}}_p, \mathbf{X}_p\right)\right)}{\sum_{X'_p=1}^{N^2} \exp\left(-\frac{1}{2\sigma^2}\delta^2\left(\overline{\mathbf{X}}'_p, \mathbf{X}_p\right)\right)} . \tag{18}$$

With the Bayes' theorem the a-posteriori probabilities can be calculated as

$$\Pr\{\mathbf{X}_p | \overline{\mathbf{X}}_p\} = \frac{\Pr\{\overline{\mathbf{X}}_p | \mathbf{X}_p\} \cdot \Pr\{\mathbf{X}_p\}}{\sum_{\mathbf{X}'_p=1}^{N^2} \Pr\{\overline{\mathbf{X}}_p | \mathbf{X}'_p\} \cdot \Pr\{\mathbf{X}'_p\}} . \tag{19}$$

First initial position estimates could be obtained by taking the most probable positions

$$\hat{\mathbf{X}}_p = \arg\max_{\mathbf{X}_p} \left\{\Pr\{\mathbf{X}_p | \overline{\mathbf{X}}_p\}\right\} \tag{20}$$

based on the individual measurements as the position estimates. The initial position estimates differ from the measurements mainly by the fact that the a-priori probabilities of the positions are taken into account.

The goal of conventional path estimation is to determine the most probable path

$$\hat{\mathbf{P}} = \hat{\mathbf{X}}_1 \ldots \hat{\mathbf{X}}_P = \arg\max_{\mathbf{P}} \left\{\Pr\{\mathbf{P} | \overline{\mathbf{X}}_1 \ldots \overline{\mathbf{X}}_P\}\right\} \tag{21}$$

on which the MT moved based on the measurements. The positions $\hat{\mathbf{X}}_p$ on the most probable path $\hat{\mathbf{P}}$ are position estimates. If the movement of the MT can be described by a Markoff model the most probable path $\hat{\mathbf{P}}$ can be determined in a computational efficient way by using the Viterbi algorithm [6]. However, these are not the most probable positions which could be found based on the measurements $\overline{\mathbf{X}}_p$, $p = 1 \ldots P$.

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Position estimation aims at determining the most probable positions

$$\overline{\mathbf{X}}_p = \underset{\mathbf{X}_p}{\arg\max}\left\{\Pr\left\{\mathbf{X}_p \middle| \overline{\mathbf{X}}_1 \ldots \overline{\mathbf{X}}_P\right\}\right\} \quad (22)$$

based on the measurements $\overline{\mathbf{X}}_p$, $p = 1 \ldots P$. The most probable positions can be determined in a computational efficient way utilizing the BCJR algorithm [7] if the movement of the MT can be described by a Markoff model.

The performances of

- initial position estimation,
- determining the most probable path with the Viterbi algorithm and taking the positions on the most probable path as position estimates, and,
- optimum position estimation with the BCJR algorithm

are compared based on simulations. For all simulations a scenario of $N \times N = 10 \times 10$ pixels of size $l \times l = 20\text{m} \times 20\text{m}$ is considered. The path length is $P = 10$. For the simulations random paths were generated with the Markoff mobility model and measurements were generated using the sampled two dimensional Gaussian distribution. The simulation results are presented as complementary cumulative distribution function (CCDF) of the position estimation error $\delta\left(\overline{\mathbf{X}}_p, \mathbf{X}_p\right)$. In order to fulfill the requirements of E911 [8] the CCDFs must not cross the forbidden region shown in the figure.

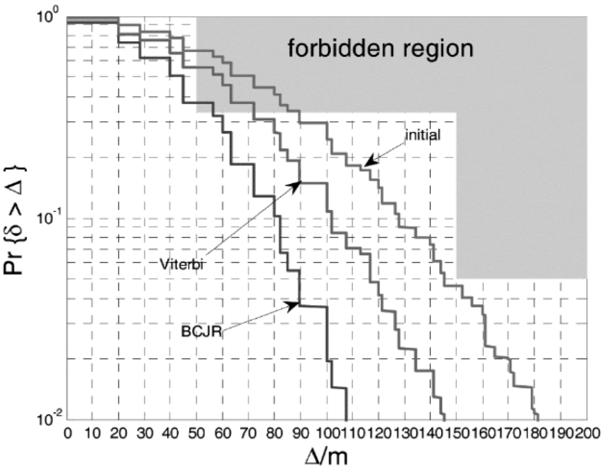

Fig. 3: CCDF of the position estimation error $\delta\left(\overline{\mathbf{X}}_p, \mathbf{X}_p\right)$ for $N = 10$, $l = 20\text{m}$, $P = 10$, $\sigma = 78\text{m}$ and $q = 0.05$.

Fig. 3 shows the simulation results for a scenario with a moderate mobility described by $q = 0.05$. This shows that temporal post-processing can offer significant performance gains over the initial position estimation. Furthermore, it can be clearly seen that the novel position estimation based on the BCJR algorithm offers significantly better performance than the conventional technique based on the Viterbi algorithm.

4. Conclusion

In the present paper hybrid localization techniques which combine several estimates in space or time in order to obtain improved position estimates were introduced.

Conventional distance based localization schemes rely solely on estimates of the distances between devices to be located to surrounding pre-installed stations of known positions. Intuitively, localization accuracy could be improved by additionally resorting to estimates of the distances between the devices to be located. This spatial post-processing rationale which is followed in the localization scheme SPIDER presented in this paper significantly outperforms the conventional scheme.

The temporal post-processing techniques considered in the present paper are based on a Markoff model describing the movement of the mobile terminal. The measurements one obtains are random variables for which distributions are influenced by the true positions of the mobile terminals, which correspond to the states of the Markoff model. Thus, the theory of hidden Markoff models is applicable. The novel temporal post-processing with the BCJR algorithm offers significant performance improvements over the initial position estimation and also the state of the art temporal post-processing based on the Viterbi algorithm.

In a final step both techniques can be combined in order to do some post-processing in both space and time in order to further improve position estimates.

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