Interleaved Orthogonal Frequency Division Multiple Access with Variable Data Rates

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***Abstract*— Multicarrier transmission systems with a specific subcarrier allocation such as equidistantly interleaved OFDMA or Interleaved Frequency Division Multiple Access (IFDMA) are promising candidates for future mobile radio systems, because they provide many advantages such as high flexibility, good performance and low computational complexity. In this paper, a new method for accommodation of different data rates per user is introduced. It keeps the orthogonality of different users' signals and at the same time maintains an equidistant subcarrier allocation. It is shown that the performance of equidistantly interleaved OFDMA and of IFDMA for coded transmission over a frequency selective mobile radio channel improves with increasing data rate.**

I. Introduction

Presently, research for beyond 3rd generation mobile radio systems is in progress worldwide. A future mobile radio system has to meet challenging requirements. On the one hand, it should enable different types of services with data rates from a few kbit/s up to several Mbit/s. Moreover, it should provide high flexibility and granularity as well as good performance. On the other hand, low cost and hence, low complexity implementation is requested, especially for mobile terminals.

A promising candidate for future mobile radio systems is the well known OFDM based multiple access scheme OFDMA [1]. For OFDMA, different data rates can be provided by assignment of different numbers of subcarriers to each user. OFDMA provides high flexibility and granularity in terms of different data rates. In general, for OFDMA blockwise subcarrier allocation or, alternatively, arbitrarily interleaved subcarrier allocation is possible for different numbers of subcarriers per user. Compared to blockwise subcarrier allocation, OFDMA with subcarrier interleaving provides higher frequency diversity and thus, better performance for transmission over a frequency selective mobile radio channel. Moreover, for an equidistant arrangement of the interleaved subcarriers the computational complexity can be significantly reduced [2]. In the following, we consider an OFDMA system with interleaved subcarriers that are equidistantly distributed over the total available bandwidth. In the sequel, this system is designated as Interleaved OFDMA (I-OFDMA). We regard I-OFDMA with linear precoding designated as precoded I-OFDMA and I-OFDMA without precodig designated as non-precoded I-OFDMA. A special form of precoded I-OFDMA is obtained by choosing the Discrete Fourier Transform (DFT) as the precoding operation. This is known as Interleaved Frequency Division Multiple Access (IFDMA) [3-8]. In addition to the advantages of I-OFDMA, IFDMA provides low envelope fluctuations of the transmitted signal.

As for I-OFDMA in general and for IFDMA in particular, the subcarriers for each user are equidistantly distributed over the total bandwidth, the accommodation of multiple users with different data rates in one system while maintaining orthogonality of different users' signals is not straightforward. A first approach for solving this problem for a variation of IFDMA designated as CDMA using Frequency Domain Signature Sequences (FDOSS) [9] can be found in [10]. Therein, it is assumed that the total number of subcarriers and the number of subcarriers that can be assigned to each user are powers of 2.

In this paper, a new method is introduced that extends the method described in [10] to the more general case of I-OFDMA signals. Since IFDMA and CDMA using FDOSS can be interpreted as precoded I-OFDMA, the new method is also applicable for IFDMA and CDMA using FDOSS. Furthermore, for the method introduced in this paper the numbers of subcarriers in total and for each user are not necessarily assumed to be powers of 2. Additionally, some extensions of the new method such as its combination with CDMA and special characteristics of the method in a system under operation are discussed.

The paper is organized as follows: In Section II, a system model for I-OFDMA and for IFDMA is given. In Section III, the new method for accommodation of different data rates per user is introduced. In Section IV, special cases and possible extensions of the new method are discussed. Finally, in Section V, simulation results for coded performance of I-OFDMA and IFDMA with different data rates per user are presented and discussed.

II. System Model

In this section, a system model for precoded I-OFDMA, i.e., OFDMA with interleaved subcarriers equidistantly distributed over the total available bandwidth, is described. As IFDMA can also be understood as unitary precoded OFDMA with interleaved subcarriers [5], [7], the model also holds for IFDMA.

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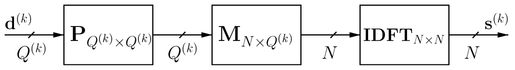

Fig. 1: Precoded I-OFDMA signal generation

We consider a linearly precoded OFDMA system with N subcarriers. Let $Q^{(k)}$ designate the number of subcarriers assigned to the user with user index k. Let

$$\mathbf{d}^{(k)} = (d_0^{(k)}, \ldots, d_{Q^{(k)}-1}^{(k)})^{\mathrm{T}} \tag{1}$$

designate the vector of transmitted data symbols at symbol rate $1/T_s$ for the k-th user. We assume that $\mathbf{P}_{Q^{(k)}\times Q^{(k)}}$ designates a linear $(Q^{(k)} \times Q^{(k)})$ precoding matrix and that $\mathbf{IDFT}_{N\times N}$ designates the matrix representation of the N-point Inverse Discrete Fourier Transform (IDFT). Let us further assume that $\mathbf{M}_{N\times Q^{(k)}}^{(k)}$ designates the user dependent mapping matrix with elements $M_{n,q}^{(k)}$ for $n = 0, \ldots, N-1$ and $q = 0, \ldots, Q^{(k)} - 1$. Matrix $\mathbf{M}_{N\times Q^{(k)}}^{(k)}$ places the subcarriers assigned to the k-th user equidistantly distributed over the total available bandwidth. Let us assume that the N available subcarriers are numbered in ascending order starting with index 0 for the subcarrier at the lowest frequency. The positions of the subcarriers are dependent on the number of subcarriers $Q^{(k)}$ assigned to the k-th user and a user specific start index $I^{(k)}$ that designates the index of the subcarrier with the lowest frequency assigned to the k-th user. Thus, the elements $M_{n,q}^{(k)}$ of the mapping matrix are given by

$$M_{n,q}^{(k)} = \begin{cases} 1 \;, & n = I^{(k)} + q \cdot \frac{N}{Q^{(k)}} \\ 0 \;, & \text{else} \end{cases} \;. \tag{2}$$

We define an OFDMA signal vector at chip rate $1/T_c = N/T_s$ according to

$$\mathbf{s}^{(k)} = (s_0^{(k)}, \ldots, s_{N-1}^{(k)})^{\mathrm{T}}. \tag{3}$$

Thus, a linearly precoded I-OFDMA signal is given by

$$\mathbf{s}^{(k)} = \mathbf{IDFT}_{N\times N} \cdot \mathbf{M}_{N\times Q^{(k)}}^{(k)} \cdot \mathbf{P}_{Q^{(k)}\times Q^{(k)}} \cdot \mathbf{d}^{(k)}. \tag{4}$$

The corresponding block diagram is given in Fig. 1. If the precoding matrix is chosen according to

$$\mathbf{P}_{Q^{(k)}\times Q^{(k)}} = \mathbf{DFT}_{Q^{(k)}\times Q^{(k)}}, \tag{5}$$

where $\mathbf{DFT}_{Q^{(k)}\times Q^{(k)}}$ designates the matrix representation of a $Q^{(k)}$-point Discrete Fourier Transform (DFT), Eq. (4) describes an IFDMA signal. The choice of $\mathbf{P}_{Q^{(k)}\times Q^{(k)}}$ as a $(Q^{(k)} \times Q^{(k)})$ identity matrix results in non-precoded I-OFDMA.

III. Different Data Rates per User

In this section, a method is introduced that enables the accommodation of different data rates per user for I-OFDMA systems. The method keeps the specific interleaved subcarrier scheme of I-OFDMA and at the same time maintains the orthogonality of different users' signals.

For I-OFDMA systems the data rate per user is dependent on the number of subcarriers assigned to each user. The subcarriers are equidistantly distributed over the total available bandwidth. Therefore, the subcarrier assignment for I-OFDMA systems for different data rates is not completely flexible but has to meet the following specific conditions: Firstly, the possible numbers of subcarriers $Q^{(k)}$ that can be assigned to each user have to be integer divisors of N in order to assure that in a fully loaded system no subcarriers are wasted. Secondly, the signals of different users have to be orthogonal to each other, i.e., each subcarrier has to be assigned exclusively to one user in order to avoid multiple access interference (MAI). However, avoiding MAI for different data rates while keeping the specific interleaved subcarrier scheme of I-OFDMA is not straightforward. In the following, a solution for this problem is introduced.

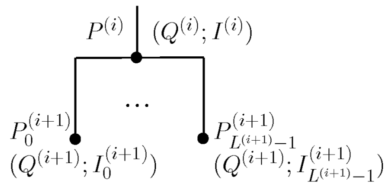

Fig. 2: Tree structure

As already mentioned in Section II, the positions of all subcarriers of an I-OFDMA signal, respectively, are completely defined by the knowledge of $Q^{(k)}$ and the knowledge of the start index $I^{(k)}$. In order to cope with different possible numbers of $Q^{(k)}$ and different start indices $I^{(k)}$, we introduce a tree structure. The tree structure can be recursively determined from the structure depicted in Fig. 2 and explained below.

Let $P^{(i)}$ designate the node on a hierarchical position given by index i and let further $L^{(i+1)}$ designate the number of nodes originating from node $P^{(i)}$. Then, $P_l^{(i+1)}$, $l = 0, \ldots, L^{(i+1)} - 1$ designates the $L^{(i+1)}$ nodes originating from node $P^{(i)}$.

Node $P^{(i)}$ is related to a pair of values $Q^{(i)}$ and $I^{(i)}$ and the nodes $P_l^{(i+1)}$, $l = 0, \ldots, L^{(i+1)} - 1$, are related to pairs of values $Q_l^{(i+1)}$ and $I_l^{(i+1)}$. In order to keep the equidistant subcarrier distribution for the signals of all users and to provide a high granularity of the tree structure, we define the values of $Q_l^{(i+1)}$ to be equal for all l, $l = 0, \ldots, L^{(i+1)} - 1$. Thus, in the following, the index l is omitted.

The values $Q^{(i)}$ and $Q^{(i+1)}$, respectively, designate specific numbers of subcarriers. The values $I^{(i)}$ and $I_l^{(i+1)}$, respectively, designate the corresponding start indices. Hence, each node represents a specific data rate and a corresponding start index. In the following, the rules for recursive determination of possible tree structures are described:

1. Let $i = 0$ designate the root of the tree. Thus, $Q^{(i=0)}$ is set to $Q^{(i=0)} = N$, where N designates the total number of subcarriers available. Furthermore, $I^{(i=0)}$ is set to $I^{(i=0)} = 0$.

2. For all i, the number of subcarriers $Q^{(i+1)}$ has to be an integer divisior of the number of subcarriers $Q^{(i)}$ related to node $P^{(i)}$. This is equivalent to the condition of

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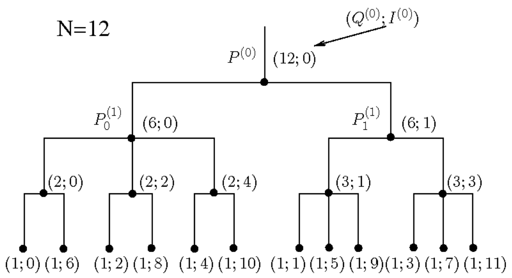

Fig. 3: Tree structure, example for $N = 12$

not wasting any subcarriers in a fully loaded system mentioned above. All nodes $P_l^{(i+1)},\ l = 0, \ldots, L^{(i+1)} - 1$ originating from node $P^{(i)}$ represent the same number of subcarriers. Nodes with index $i + 1$ that originate from different nodes with index i may represent different numbers of subcarriers.

3. For all i, the number $L^{(i+1)}$ of nodes that originate from node $P^{(i)}$ is given by the number of subcarriers $Q^{(i)}$ related to node $P^{(i)}$ divided by the number of subcarriers $Q^{(i+1)}$:

$$L^{(i+1)} = \frac{Q^{(i)}}{Q^{(i+1)}}. \tag{6}$$

As $Q^{(i+1)}$ has to be an integer division of $Q^{(i)}$, also $L^{(i+1)}$ is an integer value.

4. For all i, the start indices $I_l^{(i+1)}$ related to the nodes $P_l^{(i+1)}$ are given by

$$I_l^{(i+1)} = I^{(i)} + l \cdot \frac{N}{Q^{(i)}}; \quad l = 0, \ldots, L^{(i+1)} - 1, \tag{7}$$

where $I^{(i)}$ designates the start index related to node $P^{(i)}$.

5. For recursive determination of the tree, a new subtree according to Fig. 2 may originate from each node $P_l^{(i+1)}$, $l = 0, \ldots, L^{(i+1)} - 1$ until $Q^{(i+1)} = 1$ is reached.

We regard an example as it is depicted in Fig. 3. It is assumed that $N = 12$ subcarriers are available. As index $i = 0$ designates the root of the tree, the number of subcarriers that is related to node $P^{(0)}$ is given by $Q^{(0)} = N = 12$. The start index related to the node $P^{(0)}$ is $I^{(0)} = 0$, cf. rule 1. The number of subcarriers $Q^{(1)}$ related to the node $P_l^{(1)}$, $l = 0, 1$, is chosen to be $Q^{(1)} = 6$ which is an integer divisor of 12, cf. rule 2. There are $L^{(1)} = Q^{(0)}/Q^{(1)} = 2$ nodes $P_l^{(1)}$, $l = 0, 1$, originating from node $P^{(0)}$, cf. rule 3. The indices related to the 2 nodes $P^{(1)}$ are given by $I_0^{(1)} = 0$ and $I_1^{(1)} = 1$ according to Eq. (7), cf. rule 4. All other nodes of the tree can be calculated in a similar way.

In the following, it is described how the introduced tree structure can be used for accommodation of different data rates per user.

Accommodation of a specific number of subcarriers with a specific position is possible by assignment of an adequate node of the tree to the k-th user, i.e.,

$$Q^{(k)} = Q^{(i)} \tag{8}$$

and

$$I^{(k)} = I_l^{(i)}. \tag{9}$$

In this context, an adequate node means a node which represents the smallest possible number of subcarriers $Q^{(k)}$ greater than or equal to the number of subcarriers that is requested by the k-th user. We designate a node that is assigned to a specific user as occupied. By assignment of a node to a specific user, not only this node, but also recursively all nodes originating from it are occupied. As a major rule for assignment of nodes to the different users, the user can be only assigned to nodes that are not yet occupied. As already mentioned, the two values $Q^{(k)}$ and $I^{(k)}$ are sufficient for determination of the position of the subcarriers of an I-OFDMA signal, cf. Eq. (4). Thus, the problem of accommodation of different data rates per user while keeping the specific subcarrier distribution for I-OFDMA signals and at the same time maintaining the orthogonality for different users' signals is solved by the assignment of the nodes of the tree to the different users.

IV. Special Cases and Extensions

In this section, special tree structures and their properties are discussed.

For practical implementation, it is reasonable to choose the total number of subcarriers N as an arbitrary integer value with many preferably small divisors. This choice enables a tree structure with many different possible values for $Q^{(k)}$, i.e., high granularity in terms of different data rates. However, since the different possible values for $Q^{(k)}$ have to be divisors of N, the tree provides discrete values for the different possible data rates. In order to meet data rate requests inbetween two discrete data rates provided by the tree, link adaptation methods such as puncturing, repetition, or different modulation and coding schemes can be used.

In a communication system under operation the tree structure described in Section III can be adapted dynamically. Again, we regard the tree structure depicted in Fig. 3 as an example. This tree provides 3 nodes related to a number of 2 subcarriers. If this number is not sufficient it is possible, e.g., to replace the right subtree, i.e., all nodes originating from node $P_1^{(1)}$, by another subtree which is equal to the left subtree, i.e., all nodes originating from node $P_0^{(1)}$. In this case, the tree would provide 6 nodes related to a number of 2 subcarriers instead. In a communication system under operation where, e.g., many users are requesting 2 subcarriers, the structure given in Fig. 3 could be dynamically changed this way if the subcarriers corresponding to the right subtree are not assigned to other users.

Another way of dynamic adaptation of the tree in a system under operation can be described by the following example: We assume a system under operation and a user requesting a number of subcarriers that is less or equal to the number of subcarriers that are not occupied. The subcarriers that are not occupied shall be assumed to be related to different nodes of the tree. Thus, the request of the user cannot be served offhand. In this case, possibly a reassignment of the nodes to the different users brings to a favourable solution.

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It is also possible to further extend the tree structure that is described in Section III. E.g., in case of a communication system using I-OFDMA combined with CDMA, the tree structure for I-OFDMA systems and the tree structure known from OVSF can be combined. Combination of I-OFDMA and CDMA can be obtained if vector $\mathbf{d}^{(k)}$ in Eq. (4) represents a block of data symbols spread by a user specific CDMA-spreading sequence. An example for a system combining IFDMA and CDMA is given in [6].

In some cases it might be possible to choose the different possible values for $Q^{(k)}$ according to

$$Q^{(k)} = A^n; \quad n = 0, 1, 2, \ldots, \tag{10}$$

where A is an integer value with $A > 1$. In this case, the tree structure introduced in Section III becomes symmetric and, therefore, the rules for determination of the tree can be simplified. In this case, rule 3 can be expressed as

$$L^{(i)} = \frac{Q^{(i)}}{Q^{(i+1)}} = A. \tag{11}$$

For a further simplification, in some cases it might be possible to choose $A = 2$. In this case, the resulting tree structure provides a high number of different possible data rates, i.e., high granularity and rule 3 results in

$$L^{(i)} = \frac{Q^{(i)}}{Q^{(i+1)}} = 2. \tag{12}$$

According to Eq. (7), the start index that is related to $P_0^{(i+1)}$ is given by $I_0^{(i+1)} = I^{(i)}$ and the start index that is related to node $P_1^{(i+1)}$ is given by $I_1^{(i+1)} = I^{(i)} + N/Q^{(i)}$, cf. rule 4. For $A = 2$ for I-OFDMA systems instead of the DFT and the IDFT the Fast Fourier Transform (FFT) and the Inverse Fast Fourier Transform (IFFT) algorithms can be used without dummy bits.

The described tree structure according to Eq. (10) with $A = 2$ is the same as for Orthogonal Variable Spreading Factor (OVSF) codes [11]. Similar to OVSF codes, in this case, also for I-OFDMA systems it is possible to ensure that a user requesting a specific number of subcarriers can be served as long as the number of subcarriers that are not assigned to other users is larger than or equal to the requested number of subcarriers. Possibly that means that the current assignment of nodes to users has to be reorganized.

V. Performance Results

In Fig. 4 and 5 the simulation results for transmission of non-precoded I-OFDMA and IFDMA over a mobile radio channel using the described method for accommodation of different data rates while keeping orthogonality of different users' signals are depicted. E_s designates the average energy of the signal per QPSK symbol and N_0 designates the energy of the noise. For both schemes a $1/2$-rate forward error correction code is used. The simulation parameters are given in Table I. The performance of both schemes depends on the data rate. The different

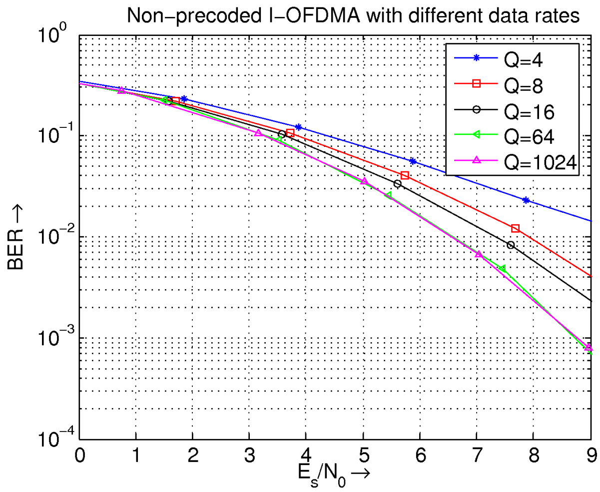

Fig. 4: Coded Performance of non-precoded I-OFDMA

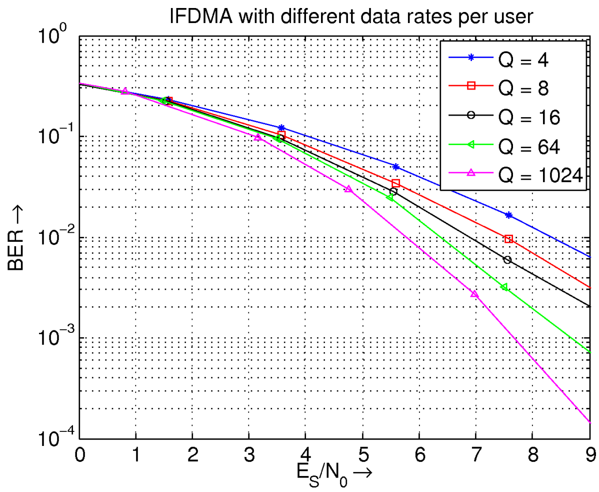

Fig. 5: Coded Performance of IFDMA

TABLE I: Simulation parameters

Carrier frequency	5 GHz
Bandwidth	20 MHz
Number of subcarriers	1024
Modulation	QPSK
Code	Convolutional Code
Code rate	1/2
Constraint length	6
Decoder	MaxLogMAP
Equalizer	MMSE Frequency Domain Equalizer
Interleaving	Random
Interleaving depth	0.5 ms
Guard interval	5 μs
Channel	Urban Macro
Velocity	70 km/h

TABLE II: Data rates per user

Q	1024	64	16	8	4
R_b/(Mbit/s)	20	1.25	0.312	0.156	0.078

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net bit rates R_b corresponding to the different numbers Q of subcarriers per user are given in II. The higher the data rate, the higher the number of subcarriers per user. Thus, a signal of a user with high data rate provides higher frequency diversity compared to signal of a user with low data rate and hence, better performance.

The given performance results for IFDMA and non-precoded OFDMA transmission cannot be directly compared to each other because for simulation the effect of a realistic receiver filter has not been taken into consideration. A realistic receiver filter would significantly suppress the sidelobes of the received signal outside the bandwidth of 20 MHz. Since for non-precoded I-OFDMA and IFDMA the amount of energy in the sidelobes is different, a direct comparison without regarding the effect of a realistic receiver filter is not fair.

VI. Conclusions

In this paper it is shown that even for multiple access systems with constraints on the subcarrier allocation as it is the case for I-OFDMA and IFDMA, high flexibility in terms of different data rates can be achieved. As the method for accommodation of different data rates described in this paper is similar to OVSF codes, some of the principles of OVSF may be adopted also for multicarrier transmission systems. The granularity of the possible data rates can be further improved using link adaptation methods such as puncturing, different code rates or different modulation techniques. Moreover, performance results for coded transmission over a mobile radio channel have been presented and it is shown that due to different frequency diversity for different data rates the performance of IFDMA and non-precoded OFDMA improves with increasing data rate.

VII. Acknowledgements

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